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Research Article

Optimal Coordination of Last Trains for Maximum Transfer Accessibility with Heterogeneous Walking Time

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Last train coordination aims to synchronize the arrival and departure times of the last feeder trains and the last connecting trains at transfer stations to improve the transfer accessibility of urban rail networks. This study focuses on the transfer accessibility between last trains with considering heterogeneous transfer walking time. Three mathematical models are developed on the last train timetable optimization. The first model fine-tunes the last train timetable under the given bound of the dwell time. The second one aims to allow the mutual transfers with the prolonged dwell time to maximize the transfer accessibility. A biobjective function is proposed to seek the trade-off between the maximal transfer accessibility and the minimal extension of dwell time. The third model considers the heterogeneity of transfer walking time that is represented as a random variable following a probability distribution. A discrete approximation method is proposed to reformulate the nonlinear model. The embedded Branch & Cut algorithm of CPLEX is applied to solve the models. A real case on the Shenzhen metro network is conducted to demonstrate the performance of the models. The three models all provide better last train timetable than the current timetable in practice. The sensitivity analysis manifests that the third model are always advantageous in the optimization of successful transfer passengers.

1. Introduction

Urban rail transit plays a very important role in the public transportation system to alleviate road congestion and reduce pollution, through the high requirements on safety and reliability and the service provision of mass capacity and environment-friendliness. In an urban rail network, it is very common for passengers to make an interchange from one line to another at transfer stations to complete their trips. For example, each passenger in the Shenzhen metro, one of the busiest systems in Southern China, needs to travel through more than 1.5 urban rail lines on average. It means that approximately half of passengers make a transfer during their trips. Improving transfer efficiency is therefore of significant importance to the overall service quality of an urban rail network.

During the daytime, transfer efficiency can be improved by reducing transfer waiting time with the coordination of train timetables among different lines [1]. However, urban rail systems seldom provide 24-hour train service because of regular maintenance work during early morning hours. The services on different lines close at slightly different times owing to various passenger demand patterns. There is thus a risk of transfer failure at transfer stations for passengers. *Transfer accessibility* is measured by whether passengers are able to make successful transfers between rail lines, which should be paid due attention in order to improve the network transfer efficiency before the closure of daily service.

The last trains provide the final opportunity for passengers to transfer to the connecting line. Transfer accessibility requires that the feeder trains arrive at the transfer station early enough to catch the last connecting train. The last train coordination aims to design a synchronized timetable for last trains in which the arrival of the last feeder trains and the departure of the last connecting trains coincide at the transfer stations to offer successful transfer for passengers [2]. The

last train coordination is becoming increasingly important with the expansion of urban rail networks and the growth of passenger demand at late nights for a megacity.

However, there are certain challenges in the last train coordination problem. The first one is the two-way transfer accessibility. For a pair of mutual transfers between two last trains, a designated last train should offer not only the connection service for transfer-in passengers from the other last train but also the feeder service for transfer-out passengers toward the train. Limited by the dwell time at the transfer station, it is difficult to keep the sufficient transfer time for both connections. In previous works, such as Kang et al. [3, 4] and Dou and Guo [5], only one transfer direction was considered. The heterogeneity in transfer walking time of passengers is the other key challenge. The transfer behaviors are usually affected by gender and age of individuals, as well as travel purposes, transfer flow, and exogenous facilities. The transfer walking time can vary widely among passengers over a range [6]. The transfer walking time of different passengers are however assumed to be a point value for the purpose of simplification in previous studies [2–4, 7, 8]. Such an assumption is unlikely to be valid at a real-life transfer station.

Noticing the challenges, this paper attempts to design a last train timetable that enables the two-way transfer accessibility with considering the heterogeneity in transfer walking time. Dwell time extension is first investigated. The dwell time of last trains is allowed to be prolonged at transfer stations so as to provide enough time for the mutual transfers. As the increased dwell time leads to the additional waiting time for onboard passengers, the optimal extension of dwell time is explored to find the trade-off between the improved transfer accessibility and the prolonged dwell time. On the other hand, the heterogeneity in transfer walking time is taken into account. Instead of a point value, the transfer walking time is represented by a random distribution at each transfer direction.

To this end, three last train timetable optimization models are developed to coordinate the last trains. One is the basic model, and the other two models undertake the above challenges. Specifically, the first model fine-tunes the running and dwell time of last trains under their bound constraints at the fixed transfer walking time. Focusing on two-way transfer accessibility, the second model sets a soft constraint on the upper bound of the dwell time. The constraint is converted into a cost function in the objective, which is to keep the extension of dwell time as minimal when maximizing the transfer accessibility. To consider the heterogeneity in the transfer walking time, the third model is extended as a nonlinear program based on the walking time distributions. Considering the nonlinearity of its probability distribution function, a discrete approximation method for walking time distribution is proposed to linearize the formulation for the sake of solution.

The contributions of this study are threefold. First, the dwell time extension is proposed to enable two-way transfer between last trains. With the consideration of both transfer passengers and onboard passengers, the maximal transfer accessibility is achieved at the minimal cost of the increased dwell time. Second, this is the first attempt to address the last train coordination under the heterogeneity of transfer walking time. The transfer accessibility is improved at the individual level through a nonlinear program and a linearization technique. Third, based on a real-life case of the Shenzhen metro network, the effectiveness of the three models is verified, and we demonstrate that the optimized timetable can be considered as operational benchmarks. Some managerial insights are obtained through the case study.

The remainder of this paper is organized as follows. Section 2 describes some related research work. Section 3 constructs three last train timetabling models. Section 4 conducts a case study of Shenzhen metro network to evaluate the proposed models. Finally, Section 5 gives the conclusions of this paper.

2. Literature Review

Transfer coordination has attracted a substantial amount of research attention for bus and rail networks. The problem seeks to maximize the transfer efficiency among different routes through the coordination of the arrival and departure times at transfer stations. Most of previous studies focused on the transfer coordination problem during peak hours. Domschke [9] first investigated the problem using a binary integer programming model. The model determines the departure times of routes at the starting station to minimize the transfer waiting time, and several heuristics algorithms with a branch and bound algorithm are proposed to solve the problem. Since then, Shafahi and Khani [10] built a mixed integer programming model to determine the departure time of the first bus on lines with given headway and proposed a genetic algorithm for large networks. Ceder et al. [11] optimized the departure times of successive buses in order to maximize the simultaneous arrivals at transfer nodes and developed a node-based heuristics algorithm. Ibarra-Rojas and Rios-Solis [12] proposed a multistart hill-climbing local search procedure to maximize the number of bus synchronizations. Wu et al. [13] investigated the resynchronization problem of bus timetable by a biobjective model to make a trade-off between the number of passengers benefited by smooth transfers and the difference from the existing timetable. A nondominated sorting genetic algorithm (NSGA-II) is applied for the biobjective model. The abovementioned researches all optimized the departure times of bus routes at terminal stations.

For rail networks, the train services are always scheduled with the arrival and departure times at each station. Chakroborty et al. [14] considered a problem with one transfer station and multiple routes at the station. A nonlinear model was developed to minimize the transfer waiting time of transferring passengers and the initial waiting time of arriving passengers, and the model was solved by a genetic algorithm. Wong et al. [1] studied the timetable coordination problem for the Mass Transit Railway system in Hong Kong. The study minimized the total transfer waiting time of transferring passengers by adjusting the departure times, running times, and dwell times of trains. Shi et al. [15] minimized the average waiting time of access passengers and

transfer passengers on a rail transit loop line by adjusting the headway and dwell times of trains. Instead of looking into the total/average transfer waiting time, Wu et al. [16] minimized the maximal passenger transfer waiting time among all transfer stations by a genetic algorithm combining with a binary variable substitute algorithm. With the time-dependent origin-destination demand, Niu et al. [17] proposed a train schedule synchronization approach to minimize the waiting time at stations and crowd disutility in trains for two high-speed rail lines. Shang et al. [18] built a timetable synchronization model to minimize passenger travel time for an urban subway network. Wang et al. [19] developed an event-driven model for the train scheduling problem of an urban rail network. Through this model, passenger travel time and energy consumption are both minimized. Guo et al. [20] focused on the first train timetable coordination problem in which the weighted transfer connection time is minimized. Ning et al. [21] focused on the robust timetable optimization for urban rail networks. The transfer buffer time is proposed, and the timetable robustness is optimized by adjustment of the running time supplement and transfer buffer time.

In recent years, a number of researches have attempted to address the last train coordination problem. The main objective of these researches is to maximize the successful transfer. Kang et al. [3] built a mean-variance model to determine the departure times, running times and dwell times of last trains in order to maximize the transfer redundant time and transfer binary variables and developed a hybrid algorithm of genetic simulated annealing algorithm. Kang et al. [4] proposed a mathematical formulation to maximize the transfer connection headway, which reflects the transfer accessibility and transfer waiting time. A genetic algorithm was designed to solve the model for Beijing subway network. Kang et al. [2] solved a last train timetabling problem by minimizing the standard deviation of transfer redundant time using a pattern search method. Dou and Guo [5] proposed a mixed integer nonlinear programming (MINLP) model for the transfer coordination of last trains. This study considered the minimization of schedule deviation. In these researches, the last train timetable is optimized regardless of passenger flows. Instead, the transfer demand is taken into account in the following researches. As an extension of Kang et al. [4], Kang et al. [7] proposed a mixed integer linear programming (MILP) model, which was solved by CPLEX upon applying a two-phase decomposition method. Yang et al. [8] further constructed a last train timetabling model considering the uncertainty of transfer demand based on the mean-variance theory. Yin et al. [22] formulated a bilevel programming model to decide the departure times and the dwell times of last trains, in which the upper level is to maximize the social service efficiency including social welfare (the sum of successful transfer passengers) and subsidy, and the lower level is to minimize the revenue loss for the operating companies, i.e., the difference between the operation cost and subsidy. Besides, several researches are conducted from the view of origin-destination (OD) demand, not just focusing on transfer connections. Zhou et al. [23] developed a model to optimize the departure times of the last trains for the minimization of inaccessible passenger volume. The travel time of alternative routes was discussed. Li. et al. [24] considered the last train problem with an objective to maximize the total number of passengers successfully reaching their destination. Chen et al. [25] developed a mixed integer programming model to consider the passenger rerouting in the network and multiple transfers in the routes. The difference between transfer accessibility and OD accessibility optimization is highlighted. The three studies all proposed customized genetic algorithms to solve their models.

Delay management [26] of the last trains is a new branch of the last train timetabling problem. Kang et al. [27] rescheduled the last train timetable with the minimal deviation from the original one, aiming to maximize the transfer accessibility and the transfer redundant time and to minimize the running times and the dwell times. Xu et al. [28] presented the last train delay management model with two objectives of maximizing the connecting passengers and minimizing the average waiting times of transfer passengers. Xu et al. [29] further investigated the timetable rescheduling problem during the daily end-of-service period. They proposed a biobjective optimization model to minimize the total waiting times for all transfer passengers and the deviation of the rescheduled timetable.

The abovementioned works have made valuable researches on the last train coordination. However, the existing researches have not fully studied the two-way transfer accessibility. A pair of transfer connections cannot be achieved simultaneously in the literature, due to the limited dwell time of last trains which is always shorter than the transfer walking time. Furthermore, to the best of our knowledge, there is no work considering the heterogeneity of passenger walking time during the last train transfer. The transfer walking time is often not fixed for passengers at a transfer direction. To address the drawbacks, this paper intends to propose a last train coordination approach that enables the two-way transfer accessibility with considering the heterogeneity of transfer walking time.

3. Last Train Timetable Modelling

In this section, three optimization models are developed for the last train coordination problem on an urban rail network. The last train timetable of different lines is synchronized by allocating the running and dwell time of last trains with the original departure time from the terminal station and the fixed timetable of previous trains. The objective of the models is to maximize the transfer accessibility for the last trains at all transfer stations. Specifically, Model (A) is a basic model to fine-tune the running and dwell time of last trains, while Model (B) is a more general model with the prolonged dwell time. Model (C) is a practical model considering the heterogeneity of transfer walking time.

(i) Model (A): basic model

(ii) Model (B): model with prolonged dwell time

(iii) Model (C): model with heterogeneous transfer walking time

Table 1: Notations and variables.

Model parameters	
L	Set of urban rail lines, indexed by l, l'. The upstream and downstream directions of a line are considered as two separated lines.
S_l	Set of two terminal stations and transfer stations on line l, $S_l = \{0, 1, \ldots, s, \ldots, n_l\}$, where $n_l + 1$ is the number of stations on line l
$i = sll'$	Transfer at station s from line l to l'
D_{0l}	Departure time of the last train from the starting station on line l
$\underline{R}_{sl}, \overline{R}_{sl}$	Lower and upper bound for the running time of the last train from station s-1 to station s on line l
$\underline{H}_{sl}, \overline{H}_{sl}$	Lower and upper bound for the dwell time of the last train at station s on line l
$\overline{T}_l$	Upper bound for the total travel time of the last train on line l
d_{sl}	Departure time of the penultimate train from station s on line l
$\underline{I}_l$	Minimal safety headway between successive departure of trains on line l
$\underline{I}_l^{DA}$	Minimal safety headway between the departure of the penultimate train and the arrival of the last train on line l
W_i	Transfer walking time of passengers at transfer direction i
$g_{W_i}(w)$	Probability distribution function of transfer walking time at transfer direction i
P_i	Transfer demand at transfer direction i
Decision variables	
R_{sl}	Running time of the last train from station s-1 to station s on line l
H_{sl}	Dwell time of the last train at station s on line l
A_{sl}	Arrival time of the last train at station s on line l
D_{sl}	Departure time of the last train from station s on line l
V_{sl}	Violation of upper bound for the dwell time at station s on line l
TR_i	Transfer redundant time for the last-train passengers in transfer direction i
x_i	Binary variable indicating transfer accessibility of transfer direction i with the fixed transfer walking time
z_i	Continuous variable indicating transfer accessibility of transfer direction i with the heterogeneous transfer walking time

3.1. Assumptions. For model formulation, the following assumptions are made.

Assumption 1. Only transfer stations on the network are considered in this study. Ordinary stations between two transfer stations are merged into a section [2]. The train running time between two transfer stations includes the running time and the dwelling time at nontransfer stations between them.

Assumption 2. The transfer demand from the last trains is constant and known. The passenger demand does not change with the adjustment of the last train timetable and can be obtained through the historical data of Automated Fare Collection (AFC) systems or demand survey.

Assumption 3. In Models (A) and (B), the transfer walking time is assumed to be a fixed value for all passengers in a transfer direction, whereas Model (C) assumes that transfer walking time varies and it follows a known probability distribution in order to incorporate the different walking time of various passengers.

3.2. Model Construction. We now construct the models to design the last train timetables. The necessary notations and variables are first defined in Table 1.

3.2.1. Model (A): Basic Model. In the basic Model (A), the transfer walking time is assumed to be a fixed value for all passengers in a transfer direction. In other words, all transfer passengers share the same transfer accessibility. To measure the transfer accessibility, two decision variables are defined:

x_i: binary variable. 1 if transfer direction i is accessible for the last train passengers; and 0 otherwise.

TR_i: transfer redundant time for the last train passengers in transfer direction i.

The transfer redundant time measures the remaining time for passengers to transfer to the last connecting train, which is the difference between the departure time of the last connecting train and the arrival time at the platform of passengers on the feeder train [3]. The transfer redundant time of last train passengers at transfer direction $i(l\text{-->}l')$ at station s is determined by the arrival time of the last feeder train, the departure time of the last connecting train, and transfer walking time of passengers between two trains, which is expressed by Constraint (1).

$$TR_i = D_{sl'} - A_{sl} - W_i, \quad \forall i \tag{1}$$

If $TR_i \geq 0$, the last train on line l has arrived early enough for passengers to catch the last train on line l', and the transfer direction x_i is accessible; otherwise, the transfer direction fails to realize the connection between the two last trains. Based on that, the transfer accessibility x_i satisfies the so-called "big M" constraint:

$$M(x_i - 1) \leq TR_i \leq Mx_i, \quad \forall i \tag{2}$$

where M is a large number. Constraint (2) ensures that x_i is equal to 1 if and only if $TR_i \geq 0$.

With the transfer demand from the last trains, the number of successful transfer passengers can be calculated by $\sum_i P_i x_i$. Model (A) is thus formulated as follows:

$$\max \sum_i P_i x_i \tag{3}$$

where

$$\begin{aligned} TR_i &= D_{sl'} - A_{sl} - W_i, \quad \forall i \\ M(x_i - 1) &\le TR_i \le M x_i, \quad \forall i \end{aligned} \tag{4}$$

s.t.

$$\underline{R}_{sl} \le R_{sl} \le \overline{R}_{sl}, \quad \forall l \in L, s \in S_l \backslash \{0\} \tag{5}$$

$$\underline{H}_{sl} \le H_{sl} \le \overline{H}_{sl}, \quad \forall l \in L, s \in S_l \backslash \{0, n_l\} \tag{6}$$

$$\sum_k^{n_l} R_{kl} + \sum_k^{n_l - 1} H_{kl} \le \overline{T}_l, \quad \forall l \in L \tag{7}$$

$$A_{sl} = D_{(s-1)l} + R_{sl}, \quad \forall l \in L, s \in S_l \backslash \{0\} \tag{8}$$

$$D_{sl} = A_{sl} + H_{sl}, \quad \forall l \in L, s \in S_l \backslash \{n_l\} \tag{9}$$

$$A_{sl} - d_{sl} \ge \underline{I}_l^{DA}, \quad \forall l \in L, s \in S_l \backslash \{0, n_l\} \tag{10}$$

$$D_{sl} - d_{sl} \ge \underline{I}_l, \quad \forall l \in L, s \in S_l \backslash \{0, n_l\} \tag{11}$$

The objective of the model is to maximize the number of successful transfer passengers in order to improve the transfer accessibility. Constraint (5) denotes the upper and lower bounds on the running time. The running time is limited due to train traction characteristics and the proper running time margin. Constraint (6) denotes the upper and lower bounds on the dwell time. The dwell time is lower bounded to ensure the adequate time for passengers to get on the last trains, and upper bounded to avoid the excessive passengers' waiting time. Constraint (7) defines a limited length of the total travel time. The constraint ensures the system maintenance requirement and working hours of manpower. Constraints (8)-(9) are built to determine the arrival and departure time at each station of the last trains. Constraints (10)-(11) define the minimal headway between the last trains and the penultimate trains, which assure the safety headway and the proper buffer time. As a whole, Model (A) is formulated as a mixed integer linear programming (MILP) problem.

The accessibility of a transfer direction is associated with its opposite transfer direction. A pair of mutual transfers $i(l\text{-->}l')$ and $i'(l'\text{-->}l)$ at transfer station s are illustrated in Figure 1. The two inequalities (12) need to be satisfied if the two transfer connections are both realized.

$$\begin{aligned} D_{sl'} - A_{sl} - W_i &\ge 0 \\ D_{sl} - A_{sl'} - W_{i'} &\ge 0 \\ &\Updownarrow \\ A_{sl'} + H_{sl'} &\ge A_{sl} + W_i \\ A_{sl} + H_{sl} &\ge A_{sl'} + W_{i'} \\ &\Updownarrow \end{aligned}$$

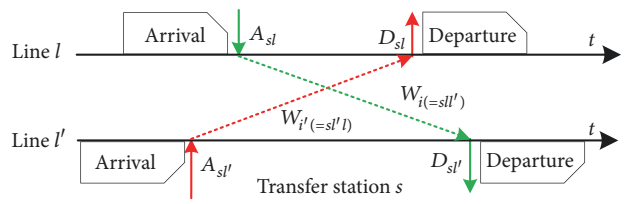


Figure 1: Transfer accessibility of two mutual directions.

$$\begin{aligned} H_{sl'} &\ge A_{sl} - A_{sl'} + W_i \\ H_{sl} &\ge A_{sl'} - A_{sl} + W_{i'} \end{aligned} \tag{12}$$

It shows that at least one of the two inequalities $H_{sl'} \ge W_i$ or $H_{sl} \ge W_{i'}$ is true if the mutual transfers are simultaneously successful [3]. However, if the dwell times are shorter than the transfer walking times, at most one inequality in (12) is valid. It can be concluded the dwell time of the last trains at transfer stations has a significant effect on the two-way transfer accessibility.

A strong constraint of Model (A) is that the dwell time at each transfer station is upper bounded by a fixed parameter. However, for the last trains, the dwell time is allowed to be prolonged to a certain extent, since there are no following trains behind the last trains. It is not easy to set a proper value for the upper bound. The dwell time at the transfer stations significantly affects the transfer accessibility, especially for the mutual transfers. An inadequate increase of the upper bound may not be sufficient to enable transfer connections between two last trains. On the other hand, the dwell time indicates the service requirement for passenger waiting time. Excessive dwell time will cause long waiting time at the station and thus result in the additional travel time for passengers. Nevertheless, Model (A) still provides the optimal last train timetable with a given upper bound on the dwell time. Experiments with different upper bounds are made to investigate the effects.

3.2.2. Model (B): The Model with Prolonged Dwell Time. To improve the two-way transfer accessibility, a soft constraint is proposed on the upper bound of the dwell time. The upper bound is preferred but not compulsorily enforced to be satisfied. Namely, the violation of these upper bound constraints is allowed but is associated with a penalty that is added to the objective function.

Let V_{sl} denote the violation for station s on line l, which can be calculated as $V_{sl} = \max(H_{sl} - \overline{H}_{sl}, 0)$. Model (B) is thus formulated as a biobjective model:

$$\max \sum_i P_i x_i \tag{13}$$

$$\min \sum_l \sum_s (V_{sl})^2 \tag{14}$$

s.t.

$$\underline{H}_{sl} \le H_{sl}, \quad \forall l \in L, s \in S_l \setminus \{0, n_l\} \tag{15}$$

$$V_{sl} \ge H_{sl} - \overline{H}_{sl}, \quad \forall l \in L, s \in S_l \setminus \{0, n_l\} \tag{16}$$

$$V_{sl} \ge 0, \quad \forall l \in L, s \in S_l \setminus \{0, n_l\} \tag{17}$$

(1)-(2), (5), and (7)-(11).

In Model (B), Constraint (15) denotes that the dwell time is lower bounded. The upper bound constraint of the dwell time is relaxed. Constraints (16) and (17) define the violation of the upper bound of the dwell time. The remaining constraints continue to hold for the feasible last train timetables. The two objectives in the model are to maximize the number of successful transfer passengers and to minimize the penalty of violations. The square function is applied to avoid the excessive dwell time at a transfer station.

A hierarchical approach is applied for the two objectives. The objectives are ranked in the order of their importance. Since the major goal of the last train coordination is to maximize the transfer accessibility for passengers, the model gives a priority to seeking the maximal accessibility over the minimal prolonged dwell time. The first-priority objective function is therefore the maximal number of successful transfer passengers on the network, and the secondary objective minimizes the squares of dwell time violations. To handle the ordered objectives, a small positive coefficient ω is introduced as the weight of the secondary objective. The coefficient should be so small that the optimal solution with respect to the first objective would not be affected.

Compared with Model (A), the upper bound constraint is converted into a penalty term for upper bound violations of the dwell time. Model (B) is ultimately a quadratic model with mixed integer linear constraints.

3.2.3. Model (C): The Model with Heterogeneous Transfer Walking Time. Model (C) is introduced to design the optimal last train timetable with heterogeneous transfer walking time. The heterogeneity in transfer walking time can trigger different transfer accessibility of passengers at a transfer direction. Model (C) takes a random distribution as input to describe the proportion of the heterogeneous transfer walking time. Let $g_{W_i}(w)$ denote the probability distribution function of transfer walking time at transfer direction i. Taken the log-normal distribution [30] as an example, the distribution function can be expressed as (18).

$$g_{W_i}(w) = \begin{cases} \left(\dfrac{1}{\sqrt{2\pi}\sigma_i}\right) \cdot \exp\left(\dfrac{-(\ln w - \mu_i)^2}{2\sigma_i^2}\right), & w > 0 \\ 0, & otherwise \end{cases} \tag{18}$$

where μ and σ^2 are the parameters in the probability distribution function. The two parameters can be deduced as $\mu = \ln[E(W)] - 1/2 \ln[1 + D(W)/E^2(W)]$ and $\sigma^2 = \ln[1 + D(W)/E^2(W)]$.

To measure the transfer accessibility under the heterogeneity of transfer walking time, a continuous decision variable is introduced:

z_i: the probability of accessible transfer at the transfer direction i, which indicates the proportion of passengers transferring successfully.

The transfer accessibility z_i can be calculated by the probability of transfer redundant time higher than 0. As expressed in Constraint (19), it is defined as the probability of transfer walking time W_i less than the time difference $D_{sl'} - A_{sl}$.

$$\begin{aligned} z_i &= \Pr(TR_i \ge 0) = \Pr(W_i \le D_{sl'} - A_{sl}) \\ &= \int_0^{D_{sl'} - A_{sl}} g_{W_i}(w)\, dw \end{aligned} \tag{19}$$

Model (C) is formulated as follows:

$$\max \sum_i P_i z_i \tag{20}$$

$$\min \sum_l \sum_s (V_{sl})^2 \tag{21}$$

where

$$z_i = \int_0^{D_{sl'} - A_{sl}} g_{W_i}(w)\, dw, \quad \forall i \tag{22}$$

s.t.

(5), (7)-(11), and (15)-(17).

Model (C) can be nonlinear and nonconvex due to the nonlinearity of Constraint (19). A discrete approximation method is proposed to linearize the constraint.

For each transfer direction i, the range of the transfer walking time is extracted according to 99.7% confidence interval as its probability distribution is unbounded. The time range is then divided uniformly into several tiny time intervals (e.g., one second). Let W_{ki} denote the kth time point for transfer direction i, i.e., $W_{ki} = \{W_{0i}, ..., W_{Ki}\}$, where K is the number of time intervals. The time interval is thus $\Delta w_i = (W_{Ki} - W_{0i})/K$.

To measure the time difference $D_{sl'} - A_{sl}$, an auxiliary binary decision variable is introduced:

y_{ki}: 1 if $D_{sl'} - A_{sl} \ge T_{ki}$, 0 otherwise, $k \in \{1, 2, ..., K\}$. It is equivalent to the linear constraint (23).

$$M(y_{ki} - 1) < D_{sl'} - A_{sl} - W_{ki} \le M y_{ki}, \quad \forall i, k \tag{23}$$

The transfer accessibility can thus be approximately calculated as the linear equation (24), as shown in Figure 2. The smaller the time interval is, the more accurate result can be attained.

$$z_i = \sum_{k=1}^{K} y_{ki} \cdot g_{W_i}(W_{ki}) \cdot \Delta w_i, \quad \forall i \tag{24}$$

4. Case Study

In this section, a real-life metro network in Shenzhen city is used as a case study to verify the three models.

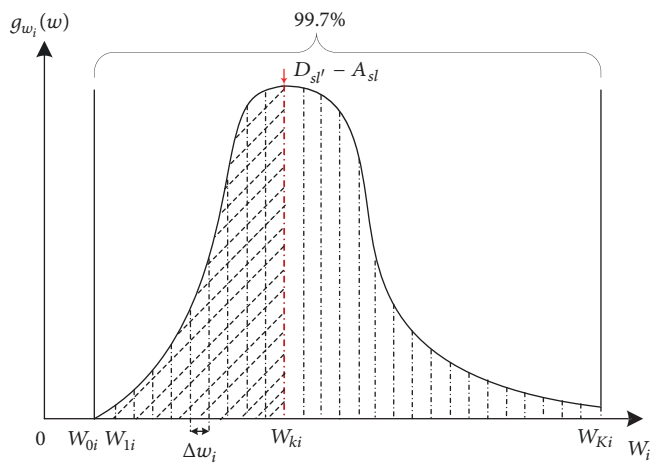


FIGURE 2: The distribution of transfer walking time.

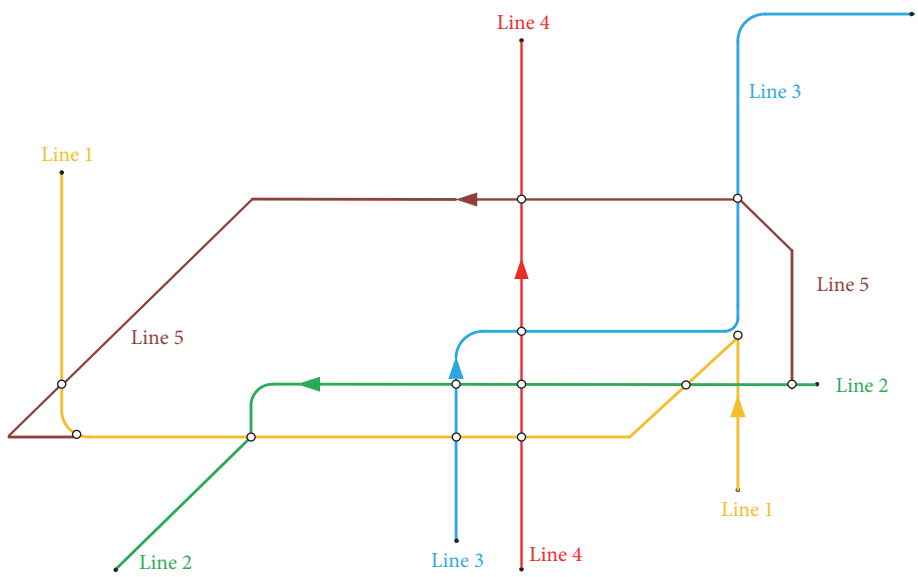


FIGURE 3: Network topology of Shenzhen metro.

4.1. Case Setup. Figure 3 illustrates the network structure of Shenzhen metro in 2016, where only transfer stations are considered. It consists of 5 bidirectional lines and 13 transfer stations, including 2 terminal transfer stations. There are 96 transfer directions in the network, of which 88 directions (44 pairs of mutual directions) are in the 11 nonterminal transfer stations and 8 directions are in the 2 terminal transfer stations.

The original last train timetable published by Shenzhen Metro Operator is adopted in this study. The departure times of the last trains on all lines are all 23:00. The dwell times are set to be 30 seconds at all transfer stations, whose lower and upper bound are set as 25 and 60 seconds, respectively. The running times between two adjacent transfer stations include all the running times and the dwelling times at nontransfer stations. The adjustment proportion of the running times is set as $[0.95, 1.20]$. The total travel time of the last train on each line is allowed to be extended by 5 minutes.

The transfer walking time at each transfer direction is assumed to be a point value for passengers in Models (A) and (B), which is 20% longer than the average walking time in order to cover most passengers. It ranges from 2 minutes to 4.5 minutes, varying with different transfer directions. Model (C) assumes that the transfer walking times follow the log-normal distributions. The number of discrete points, K, is set as 100 to approximate the continuous distribution function. The very small coefficient ω for the secondary objective is set as 0.001.

4.2. Optimized Results. Models (A), (B), and (C) are all solved by CPLEX 12.6 using the standard Branch-and-Cut (B&C) algorithm of the solver. The experiments are conducted on a personal computer with an Intel Core i5 2.40 GHz with 8 GB RAM. The optimal last train timetables can be obtained efficiently within 10 seconds. It confirms the effectiveness of the proposed linearization techniques.

Table 2: The transfer accessibility of the optimized last-train timetables.

	STC	SMT	STP	ASTP
Original timetable	41	0	1066	1075
Model (A)	45	0	1151 (+10.1%)	1135 (+5.6%)
Model (B)	48	3	1260 (+18.2%)	1234 (+14.8%)
Model (C)	48	3	1260 (+18.2%)	1270 (+18.1%)

Table 3: The dwell time after optimization.

	Sum of quadratic dwell-time violations	Average dwell time	Maximum dwell time
Model (A)	0	0.65 min	1.00 min
Model (B)	46.125	0.93 min	4.00 min
Model (C)	94.944	1.28 min	4.98 min

With the optimized last train timetables, the corresponding transfer accessibility is evaluated as the model performance, which is indicated by the number of successful transfer connections (STC), pairs of successful mutual transfers (SMT) and successful transfer passengers (STP), and the actual number of successful transfer passengers (ASTP). Taken the optimized last train timetables as input, STC, SMD, and STP are calculated under the point values of transfer walking times, while ASTP is computed based on the walking time distributions. The results are shown in Table 2.

All models indicate significant improvement in transfer accessibility over the original timetable. In Model (A), the number of STC is increased from 41 to 45. One from each pair of mutual transfer directions is guaranteed to be connected after optimization. However, no pair of mutual transfers is simultaneously connected because the dwell time is shorter than the transfer walking time. Model (B) makes 3 pairs of mutual connections since the upper bound of the dwell time is relaxed. The number of STC and STP is increased to 48 and 1260, respectively. Much higher transfer accessibility is realized. Although Model (C) provides the same number of STC as Model (B), the number of ASTP is further increased from 1234 to 1270. It indicates that Model (C) performs best in the transfer accessibility. Taken together, all the three models can provide better last train timetables than the original timetable.

The dwell time plays an important role in the last train coordination. The optimized results of the dwell time are shown in Table 3.

From Table 3, the dwell times are lengthened after optimization by the three models. The average dwell time of all last trains at all stations is increased from 30 to about 40, 60, and 75 seconds, respectively. And the maximum dwell time is extended to 4 minutes in Model (B), and even 4.98 minutes in Model (C). It still provides a feasible solution for last trains though it increases the waiting time for passengers. It is clear that the improved transfer accessibility is achieved at the cost of the increased dwell time. A balance of transfer accessibility and dwell time will be discussed in the next section.

4.3. Model Performance of Biobjective Optimization. In this section, the performance of the biobjective optimization is explored based on Model (B). The effectiveness of the secondary objective is first analyzed. The last train timetable is optimized only by the first-priority objective (i.e., the weight coefficient w for the secondary objective is set to 0). We compare the results of the dwell time with and without the secondary objective, which is shown in Table 4.

If the secondary objective is not involved, the result of transfer accessibility does not change, whereas the sum of quadratic violations is 121.768 for the default solution from CPLEX, which is more than twice as much as the optimal value 46.125. The maximum and average dwell time is also increased to 6.45 minutes and 1.23 minutes, respectively. It can be seen that there is a large room for the secondary optimization. The second objective is effective to reduce the dwell time.

The trade-off between the increased transfer accessibility and the prolonged dwell time is then discussed. Different weight coefficients are set for the violation of the dwell time. The weight is changed from 0.999 to the original coefficient 0.001. Note that the two objective functions are first normalized using Min-Max normalization method. The objective is expressed as

$$\max (1-\omega) \sum_{i} \frac{P_i x_i}{(1260-1151)} - \omega \sum_{l} \sum_{s} \frac{(V_{sl})^2}{(46.125-0)} \quad (25)$$

The results with different weights are illustrated in Figure 4.

As shown in Figure 4, when the weight coefficient ω is set to 0.999, the violation of the upper bound becomes the first-priority objective over the transfer accessibility. In this way, the upper bound constraint will be not violated. The model is naturally reduced to Model (A). With the decrease of the coefficient ω, the transfer accessibility is given a higher weight, and the much better transfer accessibility is acquired with spending a slight increase of dwell time violations. When the coefficient is decreased to 0.4 or 0.5, the dwell time is prolonged but the number of STP is also improved. And the dwell time and the number of STP are both continuously increased when the coefficient ω is less than 0.3. The optimal solutions are all the same as that of coefficient 0.001.

To jointly consider the transfer accessibility improvement and dwell time increase, passenger travel time is applied as an indicator from passengers' perspective. With the weight

Table 4: The results of the dwell time.

Model (B)	Sum of quadratic dwell-time violations	Average dwell time	Maximum dwell time
With the secondary objective	46.125	0.93 min	4.00 min
Without the secondary objective	121.768	1.23 min	6.45 min

Table 5: Passenger travel time of the optimized last-train timetables.

		Number of accessible OD pairs	Average travel time for accessible OD pairs	Average travel time for all OD pairs
Original timetable		5593	26.42 min	70.19 min
Model(B)	0.999	5708	26.57 min	69.64 min
	0.6	5719	26.75 min	69.82 min
	0.4	5746	26.76 min	69.52 min
	0.001	5811	27.08 min	69.31 min

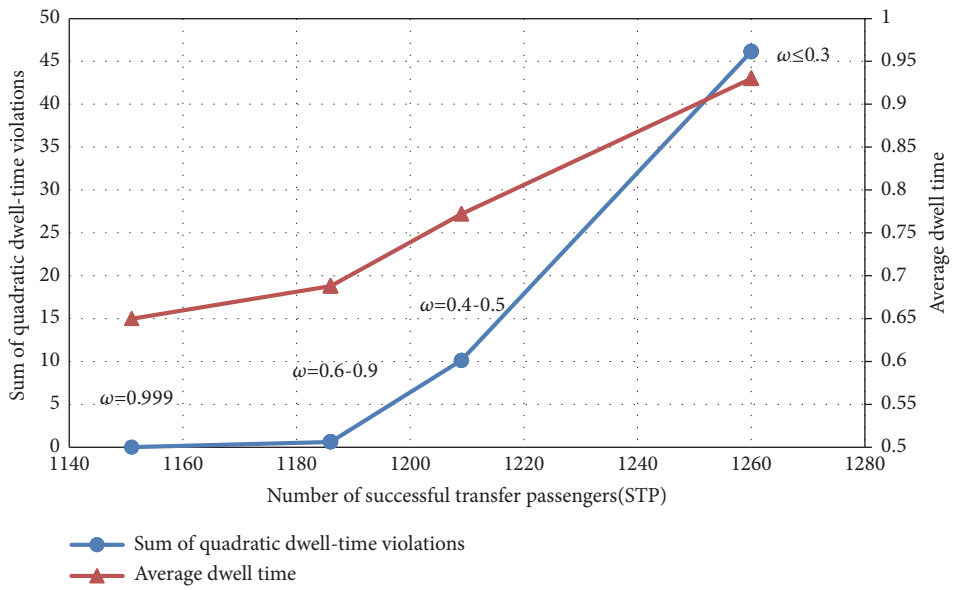


Figure 4: The optimal results with different weights.

coefficient of 0.001, 0.4, 0.6, and 0.999, the average travel time between OD pairs for last train service is shown in Table 5. If an OD pair is not accessible, the travel time is set as a penalty of 100 min.

As shown in Table 5, with the decrease of the weight coefficient, the number of accessible OD pairs is increased since the transfer accessibility is improved. There is also a slight increase in the travel time between the accessible OD pairs. However, the average travel time for all the OD pairs is decreased since more OD pairs are accessible.

4.4. Model Performance on Walking Time Distribution. To verify the performance of Model (C) on the walking time distribution, we also assume that the heterogeneous transfer walking times follow the uniform distributions. The means and variances of uniform distributions are the same as those of log-normal distributions. Based on the uniform distributions, the last train timetable is optimized by Model (C). The results of the optimized timetables are compared in Table 6.

With the two distributions, the last train timetables optimized by Model (C) both provide the same number of STP and the higher number of ASTP than Models (A) and (B). Model (C) performs best in the transfer accessibility. Moreover, the results of ASTP from Model (C) can keep stable with different probability distributions, whereas it would be affected by the probability distributions for Model (B).

4.5. Sensitivity Analysis. In this section, we conduct the sensitivity analysis to explore the effect of the dwell time and the transfer walking time on transfer accessibility.

4.5.1. Dwell Time. To explore the effect of the dwell time on the mutual transfers, the performance of Model (A) is firstly test with different upper bounds of the dwell time. The upper bounds are adjusted to 1, 2, 3, 4, and 5 minutes, respectively. Compared with the soft constraint of Model (B), the results of STC and SMT are shown in Figure 5.

As Figure 5 shows, the number of STC is improved with the increased upper bound of dwell time. When the

Table 6: The results of the optimized timetables.

	Uniform distributions		Log-normal distributions	
	STP	ASTP	STP	ASTP
Model (A)	1151	1133	1151	1135
Model (B)	1260	1227	1260	1234
Model (C)	1260	1269	1260	1270

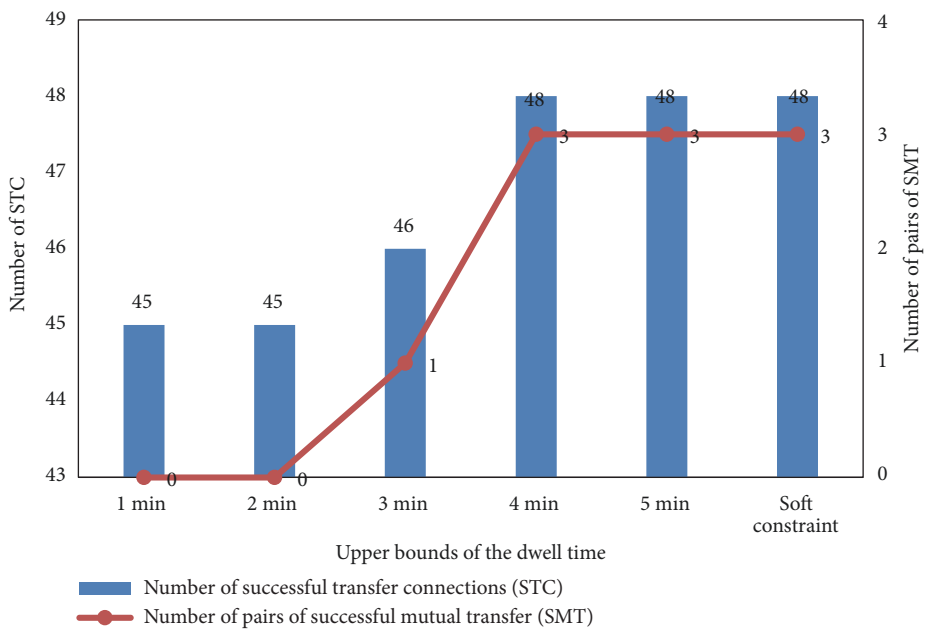


Figure 5: Effect of the dwell time on mutual transfers.

dwell time is less than 2 minutes, which is shorter than the transfer walking time, the mutual transfers at a station cannot be successful simultaneously. If the upper bound is set to 3 minutes, one pair of mutual transfers are successfully kept. When the upper bound is further increased to 4 minutes, the maximal pairs of successful mutual transfers are accomplished. It is concluded that the increased dwell time at transfer stations could result in the successful connections of mutual transfers and thus higher transfer accessibility. Through the sensitivity analysis of dwell time, Model (B) provides an effective approach directly to maximize the successful mutual transfers with the minimal dwell time. The model can achieve the maximal transfer accessibility by the optimal extension of the dwell time.

To further investigate the effect of the dwell time on transfer accessibility, an upper bound constraint of the dwell time is added to Model (C). With different upper bounds, the last train timetable is optimized by Models (A) and (C). And the corresponding results of STP and ASTP are illustrated in Figure 6.

As shown in Figure 6, the number of ASTP shows a trend of significant improvement with the increase of the dwell time both in Model (A) and (C). The better transfer accessibility can be acquired when the dwell time of the last trains is allowed to be prolonged, no matter whether the heterogeneity of transfer walking time is considered.

Specifically, the number of ASTP is always less than STP in Model (A) due to the heterogeneity of transfer walking time. Passengers may still miss the transfer because of the walking time longer than the average value. Nevertheless, the result of ASTP and STP are the same in Model (C) where the dwell time is limited to 1 or 2 minutes. All passengers at the kept transfer connection are guaranteed to transfer successfully. With the further increase of the dwell time, the number of ASTP is even higher than STP. This is the most evident when the upper bound is 4 minutes. It means that several passengers can succeed in making the transfer though the transfer connection fails to be kept at the assumption of the fixed transfer walking time. As a result, Model (C) is always superior to Model (A), particularly with a high upper bound of the dwell time. The last train timetable optimized by Model (C) is able to capture the heterogeneity in the transfer walking time. And a higher bound could provide a larger solution space, thus to be more consistent with the transfer walking time.

4.5.2. Transfer Walking Time. To quantify the benefit of Model (C) from considering the heterogeneity of transfer walking time, the performance of the three models (A), (B), and (C) is illustrated under different values of mean and variance adjusted from 80% to 120%. Model (A) is compared with Model (C) where the maximal dwell time is limited to

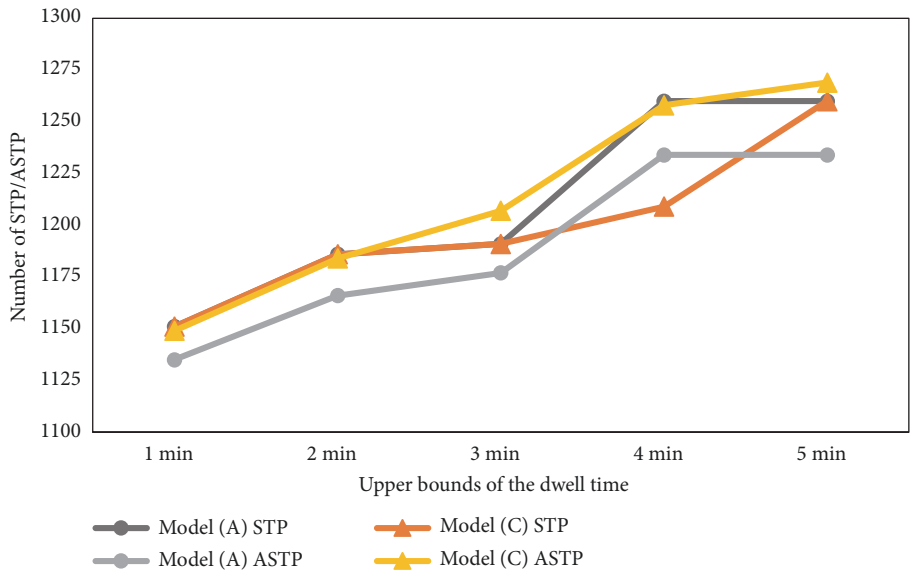


Figure 6: Effect of the dwell time on transfer accessibility.

1 minute. Model (B) is compared without the upper bounds. Figure 7 shows the sensitivity results of ASTP given the mean and variance value of transfer walking time.

As Figure 7 shows, Model (C) shows high consistency in ASTP under different mean and variance values. The number of ASTP remains nearly the same at different variance values. It is because Model (C) takes the heterogeneity in the transfer walking time as input explicitly. Besides, there is only a very slight decrease in ASTP when the mean value is high.

Models (A) and (B) are relatively more sensitive to the transfer walking time. The model performance shows a trend of fluctuation with the mean value. The number of ASTP would rise slowly if the mean value increases slightly, but it decreases drastically when the mean is improved to a certain value. The drastic decrease is caused by the failure of transfer connections because of the increased walking time, for example when the mean of transfer walking time is improved to 120% in Model (A) or to 100% in Model (B). On the other hand, the number of ASTP is reduced with the increased variance at the same mean values. Model (A) and (B) provide better results with a low variance value. It is because the two models do not consider the heterogeneity of the transfer walking time.

Taken together, Model (C) always shows an advantage in the transfer accessibility optimization over Models (A) and (B). The advantage is more evident with the increased variance value. It is therefore quite necessary to consider the heterogeneity if there is an obvious heterogeneity in the transfer walking time among different passengers.

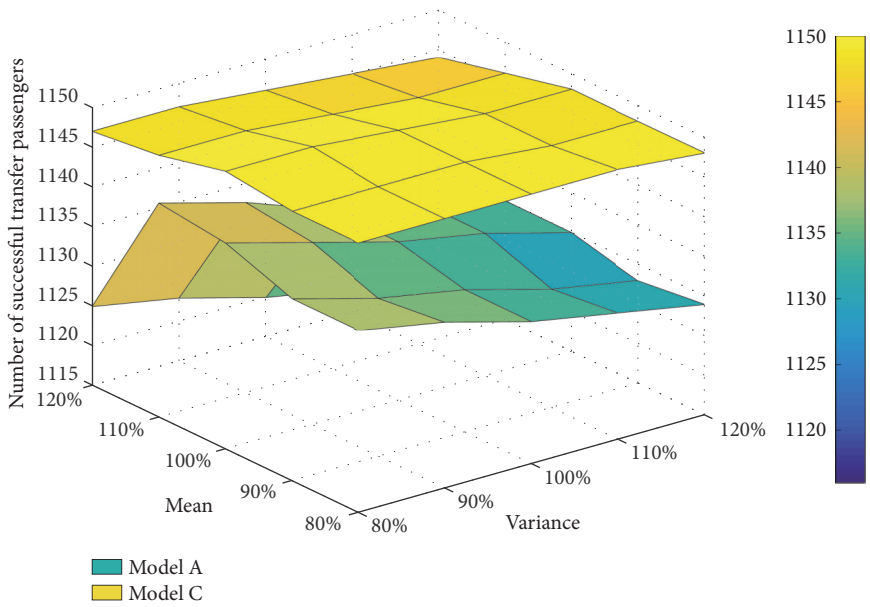
5. Conclusions

This paper proposes three optimization models on the timetable coordination among last trains of different lines on an urban rail network. Model (A) fine-tunes the running and dwell time of last trains with the given lower and upper bounds, which is formulated as a mixed integer linear programming. To enable the mutual transfers between two last trains, Model (B) allowing the extension of dwell time at transfer stations is proposed to maximize the transfer accessibility while keeping the penalty of the prolonged dwell time as minimal as possible. And Model (C) is the extension of taking the heterogenous transfer walking time into account, which is developed as a nonlinear program with the walking time distribution. To facilitate the solution method, Model (C) is reformulated as the mixed integer quadratic programming with linear constraints, which can be directly solved by the standard Branch & Cut algorithm of CPLEX.

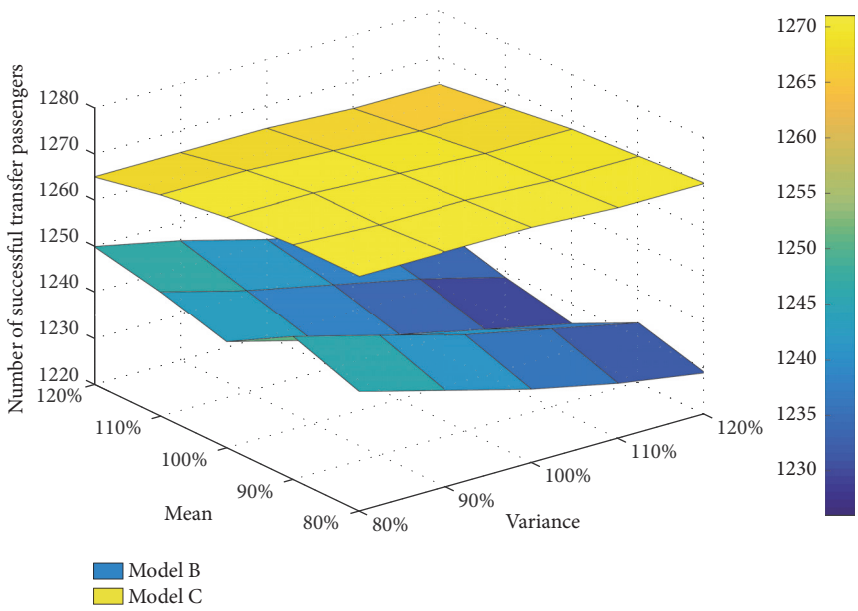
A real case on Shenzhen metro network is conducted to demonstrate the performance of the proposed models. The three models all provide better last train timetables than the practical one. All these timetables can be applied to improve network accessibility. Model (C) is always the most advantageous among these three ones. The kept transfer connections are improved from 41 to 48 and the number of successful transferring passengers from 1075 to 1270 yet at the cost of prolonging dwell time. The sensitivity analysis manifests that the longer dwell time at a transfer station is contributing to the better transfer accessibility. Furthermore, Model (C) always performs better than Model (A) without considering the heterogeneous transfer walking time, under the different dwell time constraints. The performance is more evident with higher upper bound of train dwell time. And Model (C) also demonstrates an overwhelming advantage with different distributions of the transfer walking time, especially with a high variance value.

Data Availability

The original last train timetable is published by Shenzhen Metro Operator (http://www.szmc.net/page/eng/index.html).



(a) The comparison results of Models (A) and (C) under the limited dwell time of 1 minute



(b) The comparison results of Models (B) and (C)

Figure 7: Sensitivity results of ASTP with different means and variances.

The other data used to support the findings of this study is stated within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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