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Research Article

Modeling a Risk-Based Dynamic Bus Schedule Problem under No-Notice Evacuation Incorporated with Dynamics of Disaster, Supply, and Demand Conditions

Menghui Li,[1] **Jinliang Xu**,[1] **Leyu Wei**,[1] **Xingli Jia**,[1] **and Chao Sun**[2]

[1] *College of Highway Engineering, Chang'an University, Xi'an 710064, China*
[2] *School of Automotive and Traffic Engineering, Jiangsu University, Zhenjiang 212013, China*

Correspondence should be addressed to Jinliang Xu; xujinliang@chd.edu.cn

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Apart from private traffic, the evacuation of transit-dependent population is also an essential component of emergency preparedness, especially under no-notice evacuation scenarios with limit evacuation horizon. In literature, most bus-based evacuation models for no-notice evacuation are established under implicit assumptions of uniform evacuation horizon among different pick-up locations or fixed bus fleet in the evacuation area. These constraints will distance their models from real-world situations, where evacuation horizon is various due to spatial distribution of pick-up locations and fleet size of bus available for allocation will increase over time in no-notice evacuation. This research presents a risk-based bus schedule model which is differentiated from the vehicle routing problem (VRP) and bus evacuation problem (BEP) in literature, including the objective and the time-dependent parameters. A quantified definition of evacuation risk for pick-up location with concerns of disaster dynamics and time-varying supply-demand conditions is proposed in this paper as a criterion for bus allocation, also acting as a reflection of social equity to some extent. A notion of time-evolving disadvantageous evacuation units (DEU) is introduced to represent the pick-up locations selected for bus allocation with limited resource. The binary integer linear programming (BILP) named risk-based bus schedule model incorporated with DEU notion can provide a reference for resource allocation in stage of both evacuation planning and operation for transit-dependent population. The proposed model structure can effectively capture the changes of evacuation risk among pick-up locations over time to realize real-time bus schedule. Numerical experiments are conducted using the transportation network of the city of Xi'an, China, to test the performance of the model. The applicability and comparison of different bus evacuation models are also discussed in this paper. This research provides insights into dealing with disaster dynamics and time-varying supply conditions in realistic bus-based no-notice evacuation operations.

1. Introduction

Apart from private traffic, the evacuation of transit-dependent population is also an essential component of emergency preparedness, especially under no-notice evacuation scenarios with dynamic disaster impacts. The researches on bus-based evacuation have drawn worldwide attention after Hurricane Katrina [1–10]. Compared with short notice evacuation, the planning of bus-based evacuation under no-notice scenario is more difficult because of limited evacuation horizon and insufficient preparedness. As noted by the TRB special report 294 [11], although the probability of occurrence of no-notice disasters is low, the spatiotemporal randomness and catastrophic impact justify the rationality of research on such disasters. Modeling for bus-based evacuation planning under no-notice evacuation with limited resources has drawn more attention in recent years [1, 7, 12–20]. The literature on bus-based evacuation mainly solves problems of pickup location selection (Kulshrestha et al., 2014) [4], route optimization (Tunc et al., 2011) [7], and resource allocation (Kulshrestha et al., 2014) [1]. The bus-based evacuation problem in this paper is more like a dynamic resource allocation (DRA) problem, which has become an active area in recent years.

He et al. (2015) modeled the problem of DRA of movable resources for large-scale evacuation as a mixed integer linear program and developed a solution algorithm based on the Benders decomposition technique to solve the proposed model.

Existing bus-based evacuation models solve the bus evacuation problem (BEP) by minimizing evacuation duration [7], Goerigk and Grün 2013 [14]. While under no-notice evacuation, the primary concern is not to reduce the network clearance time, but instead to maximize the safe evacuation numbers or minimize the total amount of casualties or exposure to the threat area [13, 21]. Meanwhile, since the available number of buses is limited, the social equity should also be considered in the bus schedule model. Recently, Aalami and Kattan [1] proposed a DRA model in bus-based evacuation with consideration of "proportional fairness" and developed an algorithm based on Lagrange dual approach to find proportional fair allocation of resources. However, both the bus fleet size and bus schedule are unchangeable and a complex objective function is utilized in their model; therefore only the demand aspect is considered in resource allocation. Further, the bus schedule scheme should be adjustable to real-time information. It is noteworthy that, in bus-based no-notice evacuation, the supply conditions and disaster impacts may change dynamically during evacuation procession based on evacuation response and disaster analysis [18, 20]. However, most existing bus-based evacuation models are established under implicit assumptions of uniform evacuation horizon among different pick-up locations and fixed bus fleet in the evacuation network. These constraints will distance their models from real-world situations, where evacuation horizon is various due to spatial distribution of pick-up locations and fleet size of bus available for allocation will increase over time in no-notice evacuation. Therefore, the spread of disaster and bus supply condition combined with evacuation demand should be incorporated in the bus-based evacuation modeling to better adapt to realistic situation.

To bridge the aforementioned gaps in incorporation of dynamic disaster characteristics and supply-demand conditions in bus-based evacuation, the bus-based evacuation problem under no-notice evacuation is addressed as a risk-based no-notice bus schedule problem (RNBSP) proposed in this paper. The quantified evacuation risk of pick-up locations derived from the interactions of aforementioned factors acts as criterion of bus allocation with consideration of social equity. To the best of the authors' knowledge, there is no research on bus-based evacuation modeling under no-notice evacuation incorporated with dynamics of disaster impacts and supply-demand conditions.

The rest of the paper is organized as follows. The preliminaries for modeling the problem are provided in the next section. Section 3 describes the evacuation risk assessment of bus-based evacuation in this paper. The method of determination of initial disadvantageous evacuation units (DEU) is proposed in Section 4; Section 5 introduces the modeling and solution method of RNBSP. Numerical results and discussions are presented in Section 6. The study is summarized in Section 7.

2. Preliminaries

2.1. Evacuation Scenarios and Scope of Research. Consider the scenario when a disaster such as a hazmat explosion strikes an urban area: citizens in the affected area (predefined by the disaster characteristics and wind directions) are required to evacuate the area within a very short time horizon, e.g., less than an hour. Under this situation, evacuees with access to automobile will evacuate by personal vehicles, while, for transit-dependent citizens, public transportation systems coordinated by governmental agencies might be their only available means of evacuation. Although there might be other mode choices available for the transit-dependent evacuees (e.g., walking and waterway in 9/11 and carpools in Hurricane Katrina), this study focuses on bus-based evacuation under no-notice evacuation scenario where evacuees are guided to designated pick-up locations (e.g., bus termini, landmarks like park, and schools) and depend on public transit service. It is understandable that deadlines of different locations within the area are different through the endangered area because of spatial distribution and disaster propagation characteristics, interpreted as spatial difference of evacuation risk associated with characteristics of disaster among pick-up locations within the endangered area.

Under this circumstance, buses within and around evacuation area will dismiss their scheduled path and follow different paths to the temporary scheduled pick-up locations as required. It is noteworthy that not only the buses within the evacuation area is available for allocation to associated pick-up locations, the buses operated outside the evacuation area would also be assigned to endangered area with a time delay. That is, at the starting point of no-notice evacuation, the available bus fleet only contains the buses currently in the evacuation area, but the size of fleet will increase over time as more buses out of the evacuation area arriving in the evacuation area. Therefore, a dynamic supply condition is considered based on available resources for allocation over time.

2.2. Analysis of Compliance Rate and Decision-Making Process of Evacuees under Emergencies with Different Type. Evacuation demand for each pick-up location should be estimated first before resource allocation. A possible direction for demand estimation for each predefined pick-up location from demand side is discussed in this subsection. Although the evacuation demand at each pick-up location is assumed as a model input in this study, it is noteworthy that the uncertainty of the evacuees' compliances in the evacuation plan is a main concern in real operation. Currently, the main flow of studies on compliance of evacuees is focused on evacuation of private vehicles, i.e., compliance behavior of car owners [22–25]. Unless it is a mandatory evacuation performed by the law enforcement units, it is very likely to have a certain proportion of the population that would not comply with the order or the recommendations. Compliance rate is an aggregate behavioral characteristic that indicates the percentage of evacuees that follow a recommendation or order by EMAs. It is a vital influential factor for the effectiveness of control strategies during evacuation operation.

According to Abdelgawad and Abdulhai [8], noncompliance percentage acts as a sensitivity parameter in their traffic flow optimization model for mass evacuation.

Compliance of evacuees in evacuation includes compliance with evacuation warnings, departure time, destination, and evacuation route, etc. Evacuees' compliance with evacuation warning would impact the determination of total amount of traffic in the network. Risk perception is one of the key factors in understanding the evacuation decision-making process. However, the heterogeneity among population will turn out different level of perceived risk with given warning massage and circumstance. Furthermore, current practices in conducting evacuations are very general; for example, same set of instructions are given to evacuees in the endangered area containing available evacuation time and evacuation route (for car-owners). However, evacuees need much more detailed instructions, which are thus more difficult to disseminate [26]. Current advances in communications offer several feasible options in this arena, where providing customized instructions is a feasible way to improve the level of compliance in the case of evacuations. Normally, under evacuations with relative longer notice, evacuees will select the departure time and pick-up location reasonably by service rate, *i.e.*, the number of evacuees transported out from a pick-up location per unit time, queue size *i.e.*, the number of evacuees waiting at the pick-up location currently, and arrival rate, *i.e.* the number of evacuees arriving at the pick-up location per unit time; it is regarded as a tradeoff between perceived cost and risk exposure. Under this scenario, the behavior of evacuees might be not in accordance with the recommendation of emergency management agencies (EMAs) because of heterogeneity among population. Evacuees will compare the recommendation with the information that they received from other sources and make their own decision after deep consideration. The accuracy of demand estimation in terms of evacuees' compliance of recommendations can directly influence the performance of evacuation plan. Therefore, the evacuees' compliance is probably one of main factors impacting the evacuation risk. However, under scenario of no-notice evacuation, evacuees intend to get out of the endangered area as soon as possible out of the fear of risk; thus the departure time can be perceived as the time of disaster occurrence. Meanwhile, the decision-making process of evacuees may depend on simple principle with approximate judgment on a few factors and alternatives, due to the time pressure associated with high level of emergency caused by the disaster [25]. For evacuees with access to automobiles, the decision-making process is mainly about route choice; here, evacuees' compliance to route recommendation will impact the traffic assignment of evacuation network and further impact the evacuation operation, while, for transit-dependent evacuees, the decision is mainly about selection of pick-up locations. Transit-dependent population is vulnerable under no-notice evacuation, especially for situations with low information penetration, where disaster-related information provided by media associated with personal vehicles is not accessible to these groups of people. Compliance of transit-dependent population will impact the initial demand distribution among pick-up locations, which acts as one of the determinants of bus resource allocation. Therefore, given the limitations of mobility of transit-dependent population, EMAs should assign evacuees with most convenient pick-up locations to obtain higher compliance rate. The initial distribution of transit-dependent population can be obtained from historical statistics and daily trip data. It is assumed that evacuees have no idea of the current situation of each pick-up location (e.g., queue size, service rate, and arrival rate) near their current location under no-notice evacuation. Whether a pick-up location selected by an evacuee or not is only relied on the intuitionistic perceived cost comparing to other pick-up locations, which can be simply interpreted as seeking nearest pick-up locations of no information received. Since the decision-making process of evacuees may depend on simple principle with approximate judgment on a few factors and alternative, we circumvent the estimation of compliance rate of evacuees by directly assigning them corresponding pick-up locations based on the minimum distance principle. That is, it is assumed that the evacuees would choose the nearest pick-up location as no other valuable information is available.

The decision-making mechanism highlights the possibility of using simple principles for decision-making during no-notice evacuation. However, under short-notice evacuation scenario, transit-dependent evacuees' compliance would be a vital factor that impacts demand level of each pick-up location. Therefore, compliance remains a practical issue to be addressed in the future. In evacuation operation, the buses usually have fixed routes compared to private vehicles. In future, the compliance of bus drivers on evacuation route should be studied, as drivers might change the assigned route based on their judgement on traffic conditions.

2.3. Parameters and Problem Setting. The evacuation area can be abstracted to topological structure of a directed network $\boldsymbol{G}$ $\boldsymbol{(N, A)}$, where $\boldsymbol{N}$ and $\boldsymbol{A}$, respectively, denote the set of nodes and arcs. Each node $i \in N$, representing a location, is assumed to cover a predefined geographical area in its vicinity. $\boldsymbol{N}$ is composed of two subsets of nodes; $\boldsymbol{P}$, a set of pre-defined pick-up locations $\{P_1, \ldots, P_n\}$ serving associated geographical area in its vicinity; and S, a set of shelters $\{S_1, \ldots, S_m\}$. The network is considered at a set $\boldsymbol{T}$ of discrete time intervals $\{t_0 + t\varepsilon\}$, where $t = 0, 1, 2, \ldots$, T, ε is a constant increment of time, and let t_0 represent the starting point of evacuation. The end of evacuation is defined by $t_0 + T\varepsilon$. It is reasonable to set $T\varepsilon$ equivalent to the longest evacuation time horizon of the pick-up locations in the endangered network. Thus, there are T periods in total evacuation horizon. Here, to capture the dynamics of supply condition and guarantee relative accuracy of travel time, a time increment of 1 minute is taken in this paper.

Since the evacuation demand increases explosively under no-notice evacuation, the arrival rate of evacuees is higher than service rate because of limit resources available for allocation, and thus the total demand is given as a constant instead of a monotone increasing function over time without violation of reality. We denote the total number of evacuees with $D = \sum_{i \in P} d_i$, where d_i is demands of pick-up location i. The value of d_i is acquired by assigning evacuees to

certain pick-up locations based on the minimum distance. The approximate distribution of population is obtained by historical statistics of the area. Since each pick-up location is expected to receive a relatively large number of evacuees, it is assumed that each vehicle will always be fully loaded at its scheduled pick-up location, and thus it will always travel directly between its pick-up facility and the shelter (instead of traveling to other locations for additional pick-ups). We also assume the number of evacuees at every pick-up location $i \in \boldsymbol{P}$ is known, in terms of integer multiples of bus capacities denoted by c. The capacity of each shelter is uncapacitated as the aim of evacuation under this circumstance is to transport the evacuees out of endangered area timely; thus the destination is not necessary a place that can provide accommodation to evacuees. $\boldsymbol{B}(t)$ is a given parameter denoting the total number of buses available for assignment at time t, determined by the initial number of buses in endangered area and arrival rate of buses outside the area. Although the evacuation time horizon for no-notice evacuation is limited, multiple trips between shelters and pick-up locations should be concerned instead of a one-time transportation; τ_{ij} represents the estimated round trip time between pick-up location i and shelter j, in terms of integer multiples of time increment. In the case of τ_{ij} various over time (multimodal evacuation with influence of private vehicles), the notation τ_{ij}^t is used to refer to τ_{ij} at time t. δ_{ij}^{bt} is introduced to denote whether bus b operates between P_i and S_j at time t. If the answer is yes, δ_{ij}^{bt} equals one; otherwise, δ_{ij}^{bt} equals zero. $c\sum_{j\in S}\sum_{b\in B}(\delta_{ij}^{bt}/\tau_{ij}^t)$ can denote the number of evacuees who can leave P_i in time unit t, *i.e.*, the service rate in time unit t.

3. Evacuation Risk Assessment for Transit-Dependent Population

3.1. Related Work. In this paper, evacuation risk is utilized to rank the priority of pick-up locations for bus allocation with limited resources. The principle for bus assignment in this paper is to allocate limit resources where most in need is based on current situation (disaster dynamics, supply condition, level of demand). The definition of evacuation risk is still obscure in literature, especially for bus-based evacuation. Generally, the evacuation risk is notional according to the conceptual danger as research needs. The evacuation risk in this paper is defined in terms of whether the transit-dependent population of a pick-up location can be safely evacuated before the impact of disaster on it under current supply condition. It is an integration of disaster dynamics, supply, and demand conditions. Murray-Tuite and Wolshon [27] provide a comprehensive overview of evacuation transportation modeling. The majority of researches are focused on traffic management strategies from supply side, while the disaster characteristics are treated as a notional concept, except for Hsu and Peeta [28], they incorporate disaster characteristics into risk measurement for evacuation of private vehicles in a dynamic manner. The risk of a location is mainly determined by the potential disaster impact on it associated with geographical features in the natural hazards management literature. Method of "Hazard Scores" has been proposed to rank level of risk for planning purposes [29]. Church and Cova [30] quantify evacuation risk in terms of network and demographic characteristics to produce maps of evacuation risk or vulnerability. However, the measurement method of risk with concerns of various factors combination is questionable. Social vulnerability has also been introduced into risk calculation to represent the level of needs for evacuation [31]. Most evacuation risk proposed in literature is addressed in a static planning context mainly based on disaster analysis and has key gaps to adapt its application on dynamic evacuation operations from transportation side. Further, as the measures are developed as research needs, most existing measures for risk assessment are specific to particular disaster types and lack general applicability. As mentioned in Hsu and Peeta [28], it is a challenge to measure risk robustly to the affected region using a unified model considering different disaster types and dynamics. The evacuation risk measurement for bus-based evacuation in this paper comprehensively considering dynamics of disaster, demand, and supply condition is still missing in literature. The risk-based bus schedule principle is of significance as it put social equity in priority instead of convenience and efficiency, *i.e.*, serving the population in need first rather than population with shortest round-trip cost.

3.2. Evacuation Risk Formulation. The main challenge of no-notice bus-based evacuation is formed by the evolving characteristics of three factors: disaster dynamics, demand level, and supply condition. The disaster dynamics have tempospatial influence on evacuation risk among the pick-up locations within the evacuation area as the spread of disaster impact. The demand level at different pick-up locations will determine the time duration for evacuation with given supply condition, and vice versa. With aim of developing a formulation of evacuation risk measurement that is consistent with the bus-based evacuation scenario, a formulation of evacuation risk that can capture the dynamics of disaster characteristics, demand, and supply condition is proposed in this subsection. Therefore, to obtain the evacuation risk of transit-dependent population in a comprehensive manner, the evacuation risk should be formulated by proxy parameters of the aforementioned three factors.

The proposed measure for evacuation risk in the RNBSP context accounts for disaster characteristics using the notion of a residual evacuation time. Residual evacuation time of a location refers to the estimated length of time available for evacuation before the arrival of disaster impact on that location. In this paper, $H_i(\Gamma)$ means the available evacuation time left for location i at time Γ. Residual evacuation time is assumed to be known based on analysis of disaster characteristics, which is obtained by meteorological department or other related agencies with assistance of relevant prediction models of disaster impact and spread, such as meteorological, hydrologic, or plume dispersion models, etc. It characterizes disaster-related risk, as the potential for significantly increased casualties if evacuation of transit-dependent population is not finished by the end of its residual

evacuation time, *i.e.* $H_i(\Gamma) = 0$. It is understandable that the risk increases as the end of residual evacuation time approaching. At the starting point of evacuation, t_0, the residual evacuation time of P_i can be expressed as follows:

$$H_i(t_0) = \frac{L_i}{v_0} + t_p \quad \forall i \in \boldsymbol{P} \tag{1}$$

If the propagation speed is constant, the residual evacuation time of P_i at time unit Γ, $H_i(\Gamma)$ is formulated as follows:

$$H_i(\Gamma) = \frac{L_i - v_0\Gamma}{v_0} + t_p \quad \forall i \in \boldsymbol{P},\ \Gamma \in \boldsymbol{T} \tag{2}$$

or

$$H_i(\Gamma) = H_i(t_0) - \Gamma + t_p \quad \forall i \in \boldsymbol{P},\ \Gamma \in \boldsymbol{T} \tag{3}$$

If the propagation speed is various over time, the residual evacuation time of P_i at time unit Γ, $H_i(\Gamma)$ can be written as follows:

$$H_i(\Gamma) = \frac{L_i - \sum_{t=1}^{t=\Gamma} v(t)\,\Delta t}{v(\Gamma)} + t_p \quad \forall i \in \boldsymbol{P},\ t, \Gamma \in \boldsymbol{T} \tag{4}$$

where L_i is distance between P_i and location of disaster occurrence. v_0, $v(t)$, and $v(\Gamma)$ are the propagation speed of disaster impact at different time unit. t_p denotes tolerable exposure time, which is predefined based on disaster type.

In the RNBSP context, utilization of residual evacuation time as proxy of disaster impact not only reflects the time available to evacuate of a disaster relative to the location of interest, but also manifests the spatial differentiation of disaster-induced risk across various pick-up locations in the endangered area. Residual evacuation time is negative related to evacuation risk of a location.

The evacuation risk formulation considers the effects of demand and supply conditions by a notion of evacuation completion time. Evacuation completion time for a location is defined as the time required to evacuate all the evacuees left in that location at current time. The estimated evacuation completion time required to evacuate citizens left in P_i at time Γ, $E_i(\Gamma)$, can be written as follows:

$$E_i(\Gamma) = \frac{d_i(\Gamma)}{c\sum_{j\in\boldsymbol{S}}\sum_{b\in\boldsymbol{B}}\left(\delta_{ij}^{b(\Gamma-1)}/\tau_{ij}^{\Gamma-1}\right)} \quad \forall i \in \boldsymbol{P},\ \Gamma \in \boldsymbol{T} \tag{5}$$

where $d_i(\Gamma)$ is the number of evacuees left at P_i at time Γ.

$$d_i(\Gamma) = d_i - c\sum_{t=1}^{t=\Gamma-1}\sum_{j\in\boldsymbol{S}}\sum_{b\in\boldsymbol{B}}\frac{\delta_{ij}^{bt}}{\tau_{ij}^{t}} \quad \forall i \in \boldsymbol{P},\ t \in \boldsymbol{T} \tag{6}$$

Both evacuation completion time and residual evacuation time are time-dependent itself measured by same unit, thereby enabling the incorporation of the evolving traffic dynamics resulting from the demand–supply–performance interactions. As mentioned before, the evacuation risk in this paper is defined in terms of whether the transit-dependent population of a pick-up location can be safely evacuated before the impact of disaster on it with current supply condition. Therefore, evacuation risk of P_i at time Γ without adding new arrived buses, $risk_i(\Gamma)$, can be expressed as follows:

$$risk_i(\Gamma) = -\left(H_i(\Gamma) - E_i(\Gamma)\right) \quad \forall i \in \boldsymbol{P},\ \Gamma \in \boldsymbol{T} \tag{7}$$

It is noticeable that $E_i(\Gamma)$ is a nonlinear component in risk formulation, which will dramatically increase the computation complexity of the problem. Thus, instead of representing risk in terms of time, we transform (7) into (8) without changing the generality.

$$r_i(\Gamma) = d_i(\Gamma) - H_i(\Gamma)\cdot c\sum_{j\in\boldsymbol{S}}\sum_{b\in\boldsymbol{B}}\frac{\delta_{ij}^{b(\Gamma-1)}}{\tau_{ij}^{\Gamma-1}} \quad \forall i \in \boldsymbol{P}, \Gamma \in \boldsymbol{T} \tag{8}$$

where $d_i(\Gamma)$ denotes the demands left in P_i at time Γ, and $H_i(\Gamma)\cdot c\sum_{j\in\boldsymbol{S}}\sum_{b\in\boldsymbol{B}}(\delta_{ij}^{b(\Gamma-1)}/\tau_{ij}^{\Gamma-1})$ specifies the number of evacuees that can escape from the disaster before $H_i = 0$ based on current supply condition. The proxy of evacuation risk in (8) is in terms of the number of evacuees instead of evacuation time. If the value of $r_i(\Gamma)$ is non-positive, then all the evacuees in P_i are able to leave the area in time under current supply condition. Otherwise, the value of $r_i(\Gamma)$ is equal to the number of casualties. After assignment of available bus fleet at Γ, the evacuation risk $r_i(\Gamma)$ will change into $R_i(\Gamma)$:

$$R_i(\Gamma) = d_i(\Gamma) - H_i(\Gamma)\cdot\sum_{j\in\boldsymbol{S}}\sum_{b\in\boldsymbol{B}}\frac{\delta_{ij}^{b\Gamma}}{\tau_{ij}^{\Gamma}} \quad \forall i \in \boldsymbol{P},\ \Gamma \in \boldsymbol{T} \tag{9}$$

The linear structure of the risk formulation can capture the effect of bus assignment on evacuation risk.

4. Disadvantageous Evacuation Units (DEU) Determination

In this paper, a notion of disadvantageous evacuation units (DEU) is introduced to represent the pick-up locations selected for bus assignment based on ranking of evacuation risk; in other words, $\mathbf{DEU}^t$ consists of a set of $\{P_i\}$ with highest evacuation risk at time unit t. Bus-based evacuation in this paper is more like a dynamic resource allocation (DRA) problem; it is a supply side operation instead of action taken by demand side (evacuation of private vehicles), although a pick-up location represents associated geographical area in its vicinity, it is not a graph partition problem, thus contiguity constraint is not a necessity for DEU determination [32]. The confusion of evacuees in automobile evacuation scenarios mentioned in Hsu and Peeta [28] is also perceived not happening in bus-based evacuation scenarios because (a) the distance between two pick-up locations is relatively far for walking distance and the available information is limited under no-notice evacuation; (b) evacuation of these citizens completely depends on transit system instead of themselves. Thus, the areas covered by pick-up locations are allowed to be discrete as long as they can better capture the evacuation risk

of different pick-up locations. Unlike automobile evacuation operation [28], the bus-based evacuation is totally conducted by supply side (EMA); thus it can perfectly adapt the bus schedule scheme to minimize evacuation risk of the total endangered area for whole evacuation horizon.

4.1. Initial DEU Determination. In this section, DEU formulation at the starting point of evacuation is developed, acting as the initial resource allocation under RNBSP context. The dynamic expanding DEU over time is equivalent to the dynamic bus schedule scheme in evacuation operation. $\boldsymbol{B}(t_0)$ is set of available buses that can be assigned to pick-up locations to assist evacuation operation at initial time stage, B_0 is the total number of bus available for assignment and $B_0 \geq n+1$, and n is number of pick-up locations. Superscript $t = t_0$ is omitted for convenience.

Decision Variables. δ_i is binary variable that equals one if pick-up location i is selected into DEU at t_0; otherwise, it equals zero. $\forall i \in \boldsymbol{P}$.

δ_{ij}^b is binary variable that equals 1 if bus b is assigned to operate between P_i and S_j at t_0; otherwise, it equals zero. $\forall i \in \boldsymbol{P}, j \in \boldsymbol{S}, b \in \boldsymbol{B}(t_0)$.

The determination of DEU can be formulated in the following manner:

$$\min \quad R \quad \forall i \in \boldsymbol{P} \tag{10}$$

where $R_i = d_i - H_i \cdot \sum_{j\in S}\sum_{b\in B}(\delta_{ij}^b/\tau_{ij})$.

$$s.t. \quad R \geq R_i \quad \forall i \in \boldsymbol{P} \tag{11}$$

$$\sum_{i\in P}\sum_{j\in S}\delta_{ij}^b \leq 1 \quad \forall b \in \boldsymbol{B} \tag{12}$$

$$\sum_{b\in \boldsymbol{B}}\sum_{i\in \boldsymbol{P}}\sum_{j\in \boldsymbol{S}}\delta_{ij}^b \leq B_0 \tag{13}$$

$$\sum_{b\in B}\sum_{j\in S}\delta_{ij}^b \geq 1 \quad \forall i \in \boldsymbol{P} \tag{14}$$

$$c\sum_{b\in B}\sum_{j\in S}\delta_{ij}^b \leq d_i \quad \forall j \in \boldsymbol{S} \tag{15}$$

$$M\delta_i \geq \sum_{j\in S}\sum_{b\in B}\delta_{ij}^b - 1 \quad \forall i \in \boldsymbol{P} \tag{16}$$

$$\delta_i \leq \sum_{j\in S}\sum_{b\in B}\delta_{ij}^b - 1 \quad \forall i \in \boldsymbol{P} \tag{17}$$

$$\delta_i \in \{0,1\} \quad \forall i \in \boldsymbol{P} \tag{18}$$

$$\delta_{ij}^b \in \{0,1\} \quad \forall i \in \boldsymbol{P},\ j \in \boldsymbol{S},\ b \in \boldsymbol{B} \tag{19}$$

Equation (11) requires R to be greater than or equal to the maximum risk R_i of any P_i, which is then minimized by the objective function (10), *i.e.*, minimizing the highest risk. Because of this, we will refer to this objective function as the "min–max" objective. The above model can be utilized to identify pick-up locations with highest evacuation risk based on limited supply condition of the endangered area. The aim of the model is to select a set of $\{P_i\}$ with higher risk compared to others and decrease the evacuation risk by assigning buses to them. It can be treated as an iteration procedure that continuously decrease the current highest risk to second highest risk with current available buses B_0. Therefore, the objective function is developed by minimizing the maximization of evacuation risk of pick-up locations within the evacuation area after bus assignment. For better understanding the objective function, the set of pick-up locations can be settled in x-y rectangular coordinates, and let x coordinates be pick-up location number and y coordinates be corresponding evacuation risk. Linking the nodes by number, we can get a line chart. The objective is to shave the upper peak value of the chart and obtain a chart with lowest upper peak by assignment of bus. Equation (12) allows a bus to make at most one trip at a time. Equation (13) specifies that the total number of buses allocated to pick-up locations does not exceed B_0. Equation (14) is a social equity constraint, which specifies that at least one bus should be assigned to each pick-up location regardless of risk. Equation (15) ensures that supply should not surpass demand. Equations (16) and (17) together explain that P_i is selected into DEU if any bus is assigned to it. M is a big number bigger than B_0. Equations (18) and (19) are the logical binary restrictions on variables δ_i and δ_{ij}^b.

4.2. Numerical Experiments. A simple network is given in Figure 1 to test the validity of the model. Only one shelter is determined for simplicity. The travel time for each bold link in Figure 1 is assumed to be identical as 5 min. The evacuation demand, residual evacuation time, number of buses, and round-trip time of pick-up locations at initial stage are given in Table 1. The schedule scheme of different supply condition is also shown in Table 1, calculated by the model above; the bold and italic numbers denote pick-up locations for each added bus to the evacuation network.

The value of R_i is nonnegative for all P_i; *i.e.*, all evacuees can be safely evacuated, after assignment of 32 buses into the current network. The initial DEU determined for different initial supply condition (given number of B_0) is shown as the bold and italic numbers in Table 1. For example, for B_0=17, the initial DEU consists of $\{P_2,P_6,P_7,P_8\}$, and DEU=$\{P_6,P_7\}$ when B_0=10. The bold numbers interpret the pick-up locations for added buses under different supply condition. The DEU formulation in this section is a static resource allocation deployment to understand the principle and mechanism of DEU in bus-based evacuation. However, in real operation, the bus schedule scheme will change dynamically by the dynamics of disaster, supply, and demand conditions reflected on risk. Actually, the real number of buses required is less than the static model as a bus can be assigned to other pick-up locations after the evacuation of current served pick-up location is finished under dynamic situation. The principle for DEU-based bus schedule is to assign available buses to pick-up locations with highest evacuation risk in any time unit considering dynamics of disaster, supply, and demand conditions. The dynamic model formulation of RNBSP and the solution approach will be introduced in next section.

Table 1: Pick-up location attributes and bus schedule scheme for different supply conditions.

Pick-up location	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
d_i/person	500	700	650	600	500	850	900	650
H_i/min	150	100	100	150	100	50	50	100
t_{ij}/min	30	20	10	20	40	30	20	30
B_0=8	1	1	1	1	1	1	1	1
B_0=10	0	0	0	0	0	**1**	**1**	0
B_0=11	0	0	0	0	0	**1**	*0*	0
B_0=12	0	0	0	0	0	*0*	**1**	0
B_0=13	0	0	0	0	0	**1**	*0*	0
B_0=16	0	0	0	0	0	**1**	**1**	**1**
B_0=17	0	**1**	0	0	0	*0*	*0*	*0*
B_0=19	0	*0*	0	0	**1**	**1**	*0*	*0*
B_0=20	0	*0*	0	0	*0*	*0*	**1**	*0*
B_0=23	**1**	*0*	0	0	*0*	**1**	*0*	**1**
B_0=25	*0*	*0*	0	**1**	**1**	*0*	*0*	*0*
B_0=27	*0*	*0*	0	*0*	*0*	**1**	**1**	*0*
B_0=28	*0*	**1**	0	*0*	*0*	*0*	*0*	*0*
B_0=32	*0*	*0*	**1**	*0*	**1**	**1**	*0*	**1**
$R_i(B_0=32)$	0	-200	-550	-200	-100	-50	0	-150
$\sum_{j\in S}\sum_{b\in B}\delta_{ij}^b$	2	3	2	2	4	9	6	4

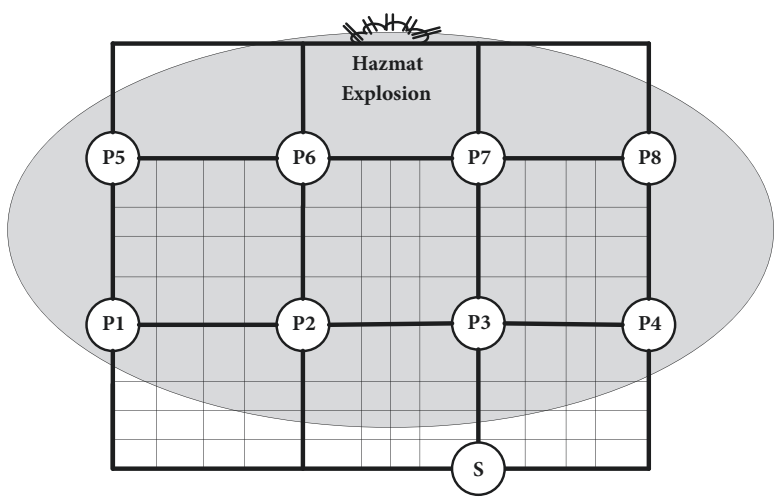


Figure 1: Simplified network for DEU determination in RNBSP context.

5. RNBSP Formulation and Solution Method

5.1. RNBSP Formulation. This section develops the RNBSP formulation of a dynamic bus schedule scheme for bus-based evacuation operation. The model aims to optimize the bus schedule scheme according to the dynamic changing evacuation risk among pick-up locations.

5.1.1. Objective Function. With consideration of time attribute in dynamic resource assignment, the evacuation risk $R_i(t)$ denotes the evacuation risk of pick-up location i at t after assignment of current available buses in the evacuation network. Since the evacuation operation lasts for the entire evacuation horizon, represented by $\boldsymbol{T}$, the RNBSP is equivalent to determination of dynamic expanding DEU during T time units by assignment of the buses available to pick-up locations with highest evacuation risk over t. After considering time attribute, the objective function is developed as follows:

$$\min \quad R(\Gamma) \quad \forall \Gamma \in \boldsymbol{T} \tag{20}$$

where $R(\Gamma) \geq \max\{R_i(\Gamma)\}$; the expression of $R_i(\Gamma)$ is given in (9). The objective function is to minimize the risk of P_i with highest evacuation risk for $\forall \Gamma \in T$ from the start of evacuation operation.

5.1.2. Dynamics of Supply Condition. In the model proposed in Section 4, the supply of bus B is treated as a given input

to identify the initial DEU and determine the number of buses required to evacuate the transit-citizen in time. In realistic operation, the bus will be assigned to the evacuation network continuously over time. At the initial stage, only the buses within the endangered area can serve the pick-up locations immediately, with process of the evacuation operation, buses relative far from the evacuation area can also arrive in the pick-up locations and provide service. The dynamic set of supply is represented by $\boldsymbol{B}(\boldsymbol{t})$. $\boldsymbol{B}(\boldsymbol{t})$ contains the set of new arrival buses $\boldsymbol{b}(\boldsymbol{t})$ and the buses already in the evacuation network $\boldsymbol{B}(\boldsymbol{t}-\mathbf{1})$. It is a time-dependent parameter based on the travel time determined by distance between the assigned pick-up location and buses outside the endangered area and evolving traffic conditions over time that obtained by exogenous models. $B(\Gamma) = B(t_0) + \sum_{t=1}^{t=\Gamma} b(\Gamma), \forall t, \Gamma \in \boldsymbol{T}$, the relationship can be expressed as follows:

$$\sum_{i\in\boldsymbol{P}}\sum_{j\in\boldsymbol{S}}\sum_{b\in\boldsymbol{B}}\delta_{ij}^{bt} \le B(t) \quad \forall t \in \boldsymbol{T} \tag{21}$$

This is the core equation of supply dynamics in proposed bus schedule model.

5.1.3. Summarized RNBSP Formulation. In summary, the RNBSP for an endangered area is performed by solving the following formulation.

Objective

$$\min \quad R(\Gamma) \quad \forall \Gamma \in \boldsymbol{T} \tag{22}$$

where $R_i(\Gamma)$ can refer to (9), and $d_i(\Gamma)$ refers to (6).

$$s.t. \quad \sum_{i\in\boldsymbol{P}}\sum_{j\in\boldsymbol{S}}\sum_{b\in\boldsymbol{B}}\delta_{ij}^{bt} \le B(t) \quad \forall t \in \boldsymbol{T} \tag{23}$$

$$R(\Gamma) \ge R_i(\Gamma) \quad \forall \Gamma \in \boldsymbol{T} \tag{24}$$

$$\sum_{i\in\boldsymbol{P}}\sum_{j\in\boldsymbol{S}}\delta_{ij}^{bt} \le 1 \quad \forall b \in \boldsymbol{B}(t),\ t \in \boldsymbol{T} \tag{25}$$

$$\sum_{b\in\boldsymbol{B}}\sum_{j\in\boldsymbol{S}}\delta_{ij}^{b1} \ge 1 \quad \forall i \in \boldsymbol{P} \tag{26}$$

$$c\sum_{j\in\boldsymbol{S}}\sum_{b\in\boldsymbol{B}}\sum_{t\in\boldsymbol{T}}\frac{\delta_{ij}^{bt}}{\tau_{ij}^{t}} \le d_i \quad \forall i \in \boldsymbol{P} \tag{27}$$

$$\delta_{ij}^{b(t+m)} - \left(\delta_{ij}^{bt} - \delta_{ij}^{b(t-1)}\right) \ge 0 \quad \forall i \in \boldsymbol{P},\ j \in \boldsymbol{S},\ b \in \boldsymbol{B},\ t \in \boldsymbol{T},\ m = 1, \ldots, \tau_{ij} \tag{28}$$

$$M\delta_i^t \ge \sum_{j\in\boldsymbol{S}}\sum_{b\in\boldsymbol{B}}\delta_{ij}^{bt} - 1 \quad \forall i \in \boldsymbol{P} \tag{29}$$

$$\delta_i^t \le \sum_{j\in\boldsymbol{S}}\sum_{b\in\boldsymbol{B}}\delta_{ij}^{bt} - 1 \quad \forall i \in \boldsymbol{P} \tag{30}$$

$$\sum_{j\in\boldsymbol{S}}\sum_{b\in\boldsymbol{B}}\delta_{ij}^{bt} = 0 \quad \forall i \in \boldsymbol{P},\ t \in \boldsymbol{T},\ t \ge H_i \tag{31}$$

$$\delta_i^{bt} \in \{0, 1\} \quad \forall i \in \boldsymbol{P},\ b \in \boldsymbol{B},\ t \in \boldsymbol{T} \tag{32}$$

$$\delta_i^{t} \in \{0, 1\} \quad \forall i \in \boldsymbol{P},\ t \in \boldsymbol{T} \tag{33}$$

Equation (23) is constraint of dynamic supply condition. Equation (24) requires $R(\Gamma)$ to be greater than or equal to the maximum $R_i(\Gamma)$. Equation (25) ensures that a bus can make at most one trip at a time. Equation (26) specifies that at least one bus should be assigned to each pick-up location when evacuation starts. Equation (27) implies that all evacuees might not depend on bus to escape from the endangered area because of high level of urgency under no-notice evacuation scenario. Equation (28) specifies that a bus can be assigned again only after it finishes the current tour. Equations (29) and (30) explain the relationship between δ_{ij}^{bt} and δ_i^t. Equation (31) specifies that no bus will be assigned to P_i after $H_i(t) = 0$; we can decrease the search range effectively by adding this constraint. The last two constraints are logical binary restrictions on variables δ_{ij}^{bt} and δ_i^t.

5.2. Solution Method. For $\forall t \in \boldsymbol{T}$, if $\max\{R_i(t)\} \le 0$, then all the evacuees are safely evacuated. Actually, for $\forall i \in P$, as long as $R_i(t) \le 0$ when $t = H_i$, the safe evacuation is guaranteed. The mechanism of the objective function is divided into two steps: (a) identifying P_i with highest evacuation risk before its deadline at current stage; (b) consecutively, assigning buses to it until it is not the riskiest one, then assigning the remainder to the pick-up location with currently highest risk (the former second-highest one), and iterating this procedure until no bus is available at current time stage. This operation can ensure that the highest casualties of different P_i are always the lowest. In scenario with ideal bus fleet size, the evacuation risk is almost identical among pick-up locations. Therefore, each pick-up location is treated equally with consideration of social equity.

The aim of the RNBSP in this paper is to provide recommendations of real-time bus schedule scheme for EMA. The problem changes into a BILP after the transformation of risk formulation in Section 3. Under no-notice evacuation scenario, bus schedule scheme is required to be real-time adjustable. The solution steps of the RNBSP formulation can be summarized as in Algorithm 1.

Based on the calculated evacuation risk over the endangered area, the solution procedure for DEU determination is implemented in a time-dependent operational framework as illustrated in Figure 2. The evacuation operation begins in the first stage (t = 1) with the initial conditions of the evacuation network. The EMAs determine DEU continuously by solving the corresponding RNBSP. The shadowed part in Figure 2 is the mechanism of DEU determination for each t.

6. Numerical Experiments and Discussion

6.1. Objectives of Numerical Experiments. Numerical experiments are conducted in this section to test the performance of the dynamic risk-based bus schedule scheme under no-notice evacuation scenarios. These experiments focus on illustrating the practical applications of the proposed approach for DEU determination and bus schedule scheme in the operational context, where evacuation risk over the study region needs to be captured in a dynamic manner. In the experiments,

Inputs: evacuation demands in terms of population at each pick-up location(d_i), Residual evacuation time (H_i), set of bus available for each time stage($\boldsymbol{B}(t)$), evacuation time horizon ($\boldsymbol{T}$), roundtrip time between P_i and S_j (τ_{ij}), set of pick-up locations ($\boldsymbol{P}$). Set of shelters ($\boldsymbol{S}$). Residual evacuation time of pick-up locations in terms of $t(H_i(t))$.

Step 0: Initialization

Set initial value of: $t = 1$, decision variable $\delta_{ij}^{bt} = 0$, $\delta_{ij}^{b} = 0$, $\forall i \in \boldsymbol{P}$. Current *DEU* ($\boldsymbol{N_d^t} = \emptyset$). Calculate current value of evacuation risk $r_i(t)$ based on Eq. (8). Update $R_i(t)$ with one bus assigned to each pick-up location.

Step 1: Range Determination

Step 1.1: If $t \geq H_k$, then $\delta_{kj}^{bt} = 0, \forall k \in P, j \in S, b \in B$, remove k from P.

Step 1.2: Sort pick-up locations based on sequence of $r_i(t)$, and index the sorted sequence with k. Let *i(k)* represent the mapping from k to pick-up location index i, and let $\boldsymbol{W}$ be the vector of $r_i(t)$ for the sorted pick-up locations. $\boldsymbol{W} = \{W_1, \ldots, W_k, \ldots\}$.

Step 2: DEU_t Process

Step 2.1: For elements in P, search for the pick-up location k with $r_k(t) = \max_{i \in P}\{r_i(t)\}$, $\forall i, k \in P$.

Step 2.2: Consecutively assign buses in $B(t)$ to pick-up location W_1 based on Eq. (23) ~ Eq. (25) until $W_1 \leq W_2$, go to *Step 2.3*; else, $W_1 > W_2$ is true after assignment of all elements in $B(t)$, go to *Step 3.1*.

Step 2.3: Add W_1 into $\boldsymbol{N_d^t}$, update $\boldsymbol{N_d^t}$. Update $B(t) = B(t) - \sum_{b \in B^t} \delta_{kj}^{bt}$, $\delta_k^t = 1$. $r_k(t) = R_k(t)$, update the position of element in $\boldsymbol{W}$ according to renewed $r_k(t)$, go back to *Step 2.2*.

Step 3: DEU_t Acquirement

Step 3.1: Add W_1 into $\boldsymbol{N_d^t}$, update $\boldsymbol{N_d^t}$,

Step 3.2: Update $t = t + 1$ if $t \leq T - 1$, update $r_k(t) = R_k(t)$, update parameters in updated t, back to *Step 1.1;* else, end.

Output: $\delta_{ij}^{b,t}$, $\boldsymbol{N_d}$.

Algorithm 1

different disaster impact scenarios are created to test the robustness of the proposed approach to the related variability.

6.2. Numerical Experiments Scenario Descriptions and Operations. The transportation network of Xi'an city, China, is used in numerical experiment (see in Figure 3(a)); the research area is located in the southern part of the Xi'an city. In total, 238 bus stops are located on the daily route of transit vehicles in the research area. As mentioned in Bish [7], the run-time of min-max objective grows quickly with problem size. Thus, the number of pick-up locations for selection should be controlled; *i.e.*, the evacuation demands in each pick-up location are large enough for multiple times of bus capacity. This precondition is in accordance with the single-visit single-trip assumption. Three scenarios are created to test the robustness of the proposed risk-based bus schedule model under disasters with different dissemination pattern and locations of occurrence, described as follows:

Scenario A: A vehicle carrying hazardous materials explodes and creates a hazmat disaster in the southeast part of the network with diffusive impact uniformly spreading throughout the network in a radial pattern.10 among 51 bus stops are selected as pick-up locations in Figure 3(b).

Scenario B: The same incident happened in the central part of the network. 9 among 42 bus stops are selected as pick-up locations in Figure 3(c).

Scenario C: The same incident happened in the middle part of the network. The spread of hazmat is affected by the wind direction (from left to right, 0.7km/h); 9 among 44 bus stops are selected as pick-up locations in Figure 3(d).

The allowable exposure time duration of hazmat is assumed to be 15 min, after which the risk of poisoning and casualties will increase significantly. The diffusion speed of hazmat is assumed to be 3km/h and the effect radius is 5 km in this numerical experiment. Residual evacuation horizon for different pick-up locations in the affected area can be calculated by distance between the pick-up locations and the location of disaster occurrence divided by the spread speed of the disaster impact, then plus the value with allowable exposure time duration. Evacuation demand of transit-dependent population can be assumed as model input. The density of traffic in the network is collected from an output file that is generated by Dynasmart-P when simulating the abovementioned disaster scenarios. The density output file contains the traffic density on all road segments in every time unit. The travel time required to traverse a path is calculated by speed of traveling on the path and the length of the path.

The time interval is set as 5 minutes for each t, and extra buses will be assigned to the evacuation network within $t' = 6$; *i.e.*, no more buses can be assigned to assist evacuation after 30min. The value of t' is flexible based on the user's need or the realistic network supply condition. τ_{ij} is generated by

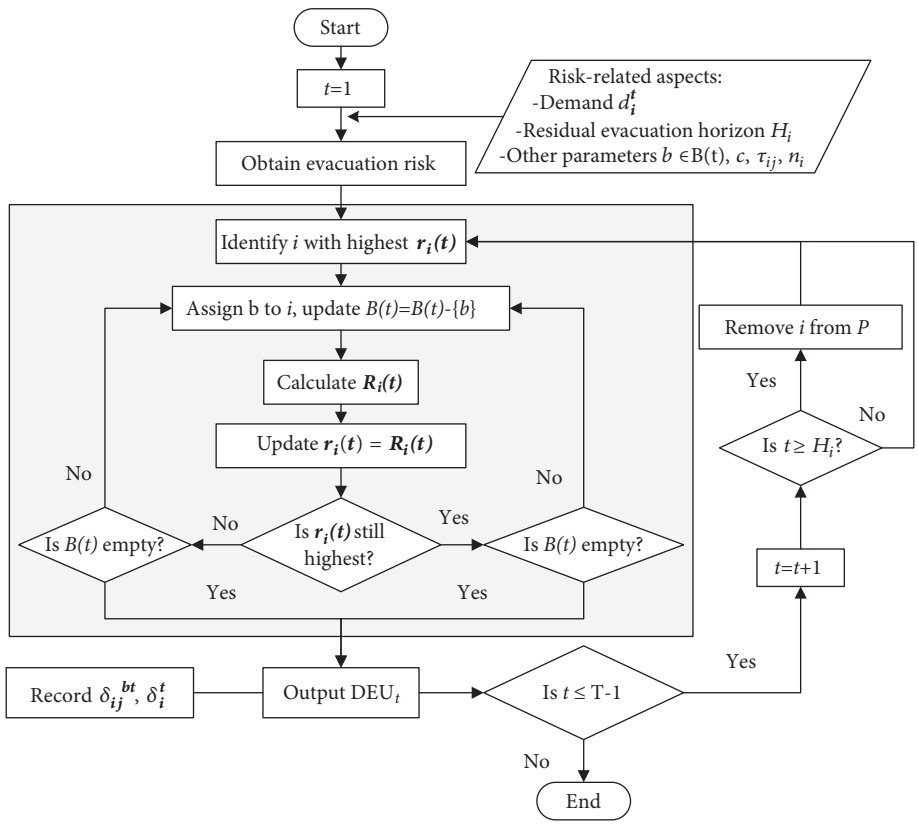


Figure 2: Time-dependent framework of bus schedule scheme for bus-based evacuation with DEU-based deployment.

the software DYNASMART-P with given input of personal vehicles in the network. The population of transit-dependent evacuees of each pick-up location is generated by six demand levels of 1000, 2000, 3000, 4000, 5000, and 6000. The initial number of buses available for assignment is 20, and for each t the bus assigned to evacuation network is 3, 5, 7, 9 for different level of resource. The capacity of bus is 50.

6.3. Criterion of Performance and Results. Since the RNBSP is focused on evacuation with high level of emergency, *i.e.*, the residual evacuation time is so short that all of the population might not be evacuated using the current available bus fleet, the number of evacuees that can be saved by bus fleet during the evacuation time horizon is a valid criterion of model performance, similar to the percentage of total evacuees saved by available bus fleet:

$$U = \frac{\sum_{i \in P} Q_i}{\sum_{i \in P} D_i} \times 100\% \tag{34}$$

It is noteworthy that the social equity is an important principle in evacuation resource allocation, especially in evacuation with high level of urgency. Social equity is reflected in terms of evacuation risk in this paper. We introduce a notion of social injustice index as another criterion for model performance, expressed as follows:

$$J = \sqrt{\frac{\sum_{i \in P} \left(Q_i/D_i - \left(\overline{Q_i}/\overline{D_i}\right)\right)^2}{n}} \tag{35}$$

where Q_i and D_i, respectively, represent the number of evacuees saved by bus fleet and total demands at P_i. The formulation of J can reflect the degree of unevenness in terms of variance in percentage of evacuees saved by bus fleet among various pick-up locations. It is easy to obtain the maximum value of J is 0.5, under an extreme situation of two pick-up locations with, respectively, 100% and 0% of safely evacuated citizens. Total percentage of evacuees saved for different level of supply condition is shown in Figure 4. Illustration of social injustice index is shown in Figure 5. From Figure 4, it is evident that U is positive with the increment of bus fleet and negative with increase of demand level. The social injustice index is stable and near to 0 even under situations where all evacuees are not able to escape the location in time according to Figure 5. Actually, the same pattern is also observed in Scenarios B and C.

6.4. Discussion. The applicability of the general used objective function for bus-based evacuation that minimizes travel cost of total evacuation network (MINEC), minimizes the duration of the evacuation (MINED), maximizes the total

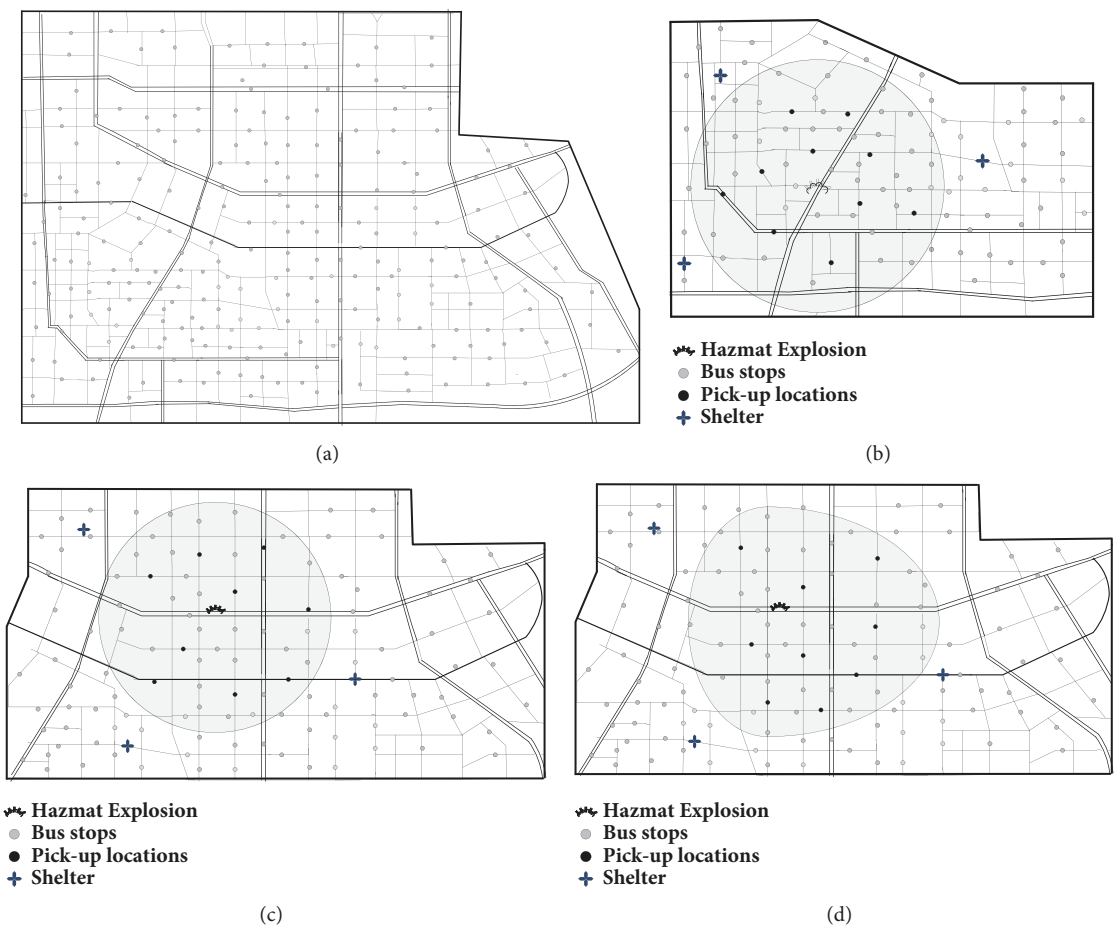


Figure 3: Illustration of network and evacuation scenarios: (a) network of southern part of Xi'an; (b) Scenario A; (c) Scenario B; (d) Scenario C.

number of rescued citizens (MAXEN), and maximizes evacuation rate (MAXER), *i.e.*, maximizes the number of evacuees who reach safety by any given deadline after the evacuation, is discussed in this part. The comparison of RNBSP, MAXEN, and MAXER is also discussed since similar evacuation scenarios are faced by EMA under these contexts.

6.4.1. Applicability Analysis of Different Objective Functions in Bus Evacuation Models. The objective function of the aforementioned bus evacuation models of MINEC, MINED, MAXEN, and MAXER is shown in following formulation, respectively:

$$\min \quad T_{total} = \sum_{t\in \boldsymbol{T}} \sum_{i\in \boldsymbol{P}} \sum_{j\in \boldsymbol{S}} \sum_{b\in \boldsymbol{B}} \delta_{ij}^{bt} \tag{36}$$

$$\min \quad T_{eva} = \underset{\Gamma}{\operatorname{argmin}} \left\{ c \sum_{t=1}^{t=\Gamma} \sum_{i\in \boldsymbol{P}} \sum_{j\in \boldsymbol{S}} \sum_{b\in \boldsymbol{B}} \frac{\delta_{ij}^{b\Gamma}}{\tau_{ij}} \geq \sum_{i\in P} d_i \right\} \tag{37}$$

$$\max \quad Q_{eva}(\Gamma) = c \sum_{t=1}^{t=\Gamma} \sum_{b\in \boldsymbol{B}} \sum_{i\in \boldsymbol{P}} \sum_{j\in \boldsymbol{S}} \frac{\delta_{ij}^{b\Gamma}}{\tau_{ij}} \tag{38}$$

$$\max \quad Q_{eva} = c \sum_{t\in \boldsymbol{T}} \sum_{i\in \boldsymbol{P}} \sum_{j\in \boldsymbol{S}} \sum_{b\in \boldsymbol{B}} \frac{\delta_{ij}^{bt}}{\tau_{ij}} \tag{39}$$

According to δ_{ij}^{bt} output by the model proposed in Section 5, value of objective function can be obtained accordingly with (36), (37), (38), and (39) under RNBSP context.

Though seldom considered in evacuation modeling, the disaster characteristics affect the level of evacuation risk spatially and temporally as the disaster impact spread through the affected area in terms of different value of H_i in this paper. With consideration of tempospatial difference of pick-up locations caused by disaster characteristics, the applicability of the aforementioned four objective functions for bus-based evacuation model is various due to different value of H_i. For deterministic H_i within the endangered area, if H_i is long enough compared with the network clearance time, *i.e.*, for $\forall i$, $H_i \gg T_{eva}$, it can be solved as a vehicle routing problem (VRP), represented by model of MINEC. The cost of evacuation operation is the main consideration for bus schedule as no casualties needs to worry about. If H_i is smaller or slightly larger than network clearance time, it is a bus evacuation problem (BEP) represented by model of MINED. Whether the transit-dependent citizens can be safely evacuated before the impacts of disaster can be determined by comparing the value of T_{evac} and H_i. This is also the criterion of whether extra bus resources are requirement. If the bus fleet cannot complete evacuation before the impacts

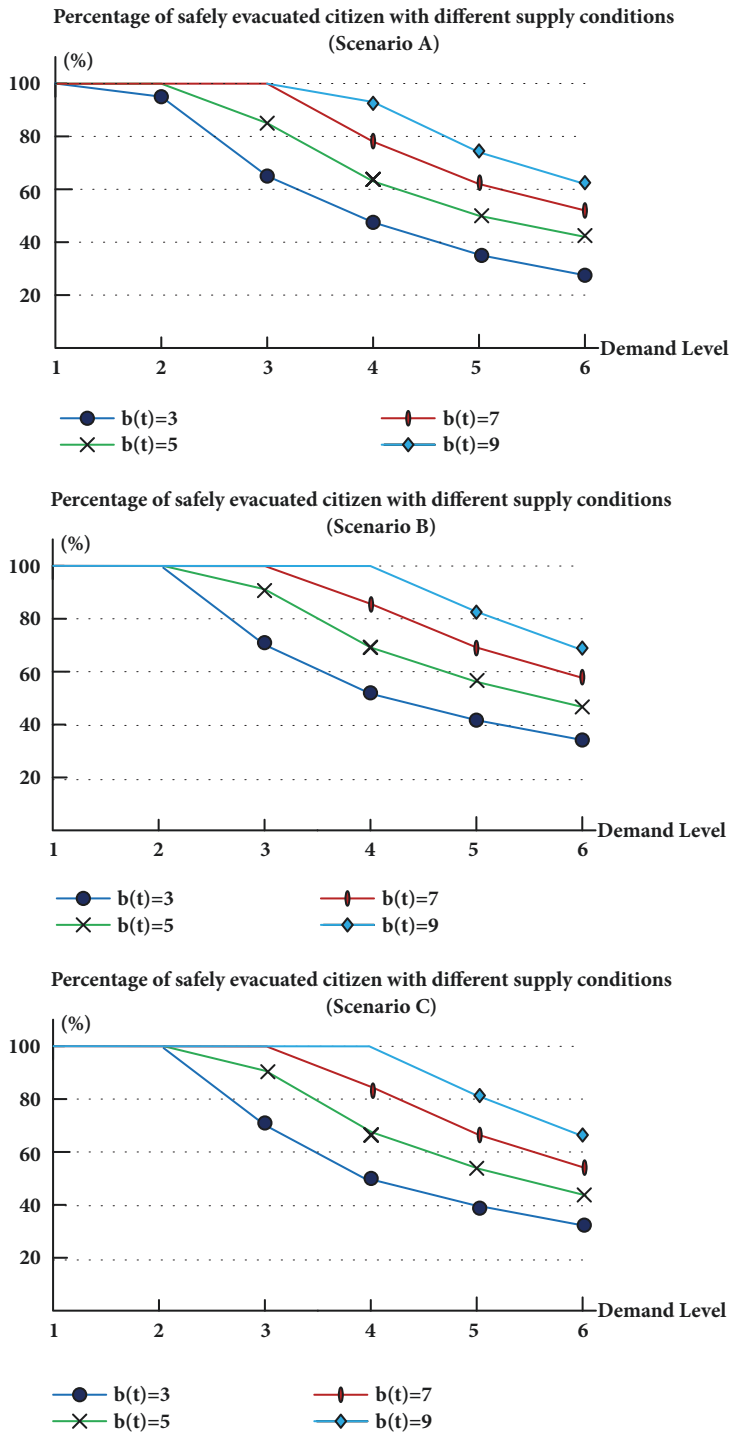


Figure 4: Operational performance under Scenarios A, B, and C in terms of percentage of citizens saved by buses.

of disaster, in other words, the deadline is so short that not all of the population can be evacuated using the current available bus fleet; generally, the objective is either to maximize the evacuation rate represented by MAXER or to maximize the total number of citizens saved by available bus fleet represented by MAXEN. Normally, MAXEN is a special situation of MAXER when $t = T$, where T is the deadline of entire evacuation region. After introducing dynamic supply condition $B(t)$ into the problem, the information provision of supply condition will distinguish these two objective functions by bus assignment. Both MAXER and MAXEN maximize the number of evacuees who leave pick-up locations in different

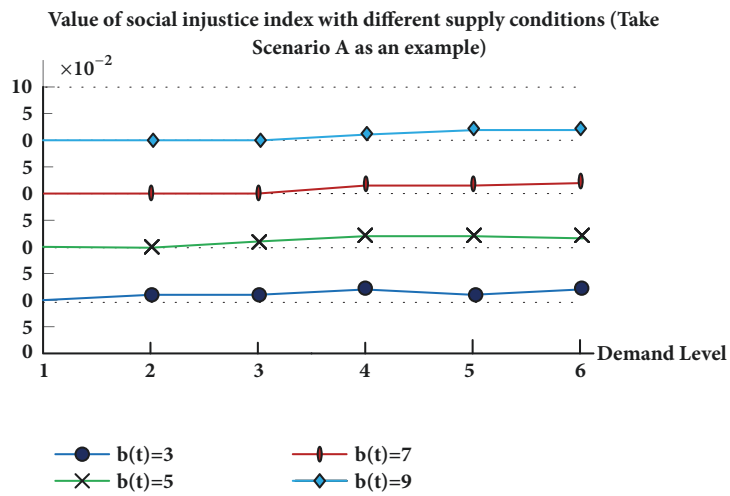


FIGURE 5: Operational performance under Scenario A in terms of social injustice index.

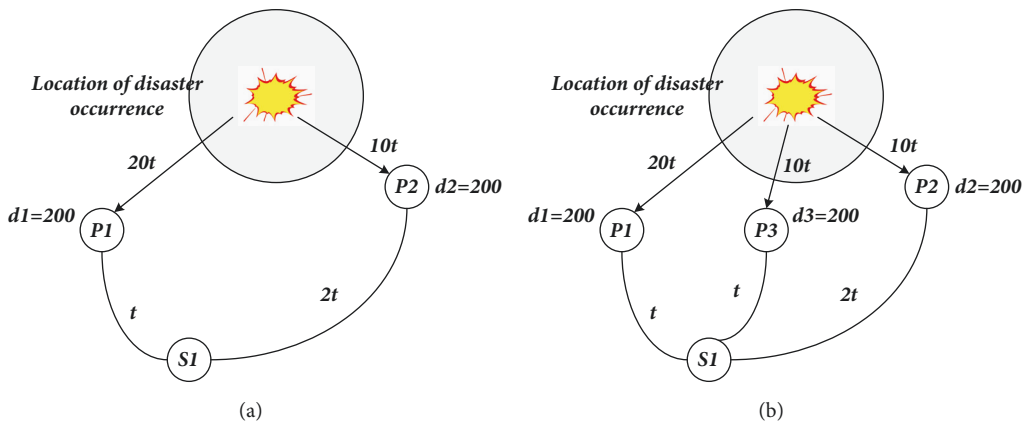


FIGURE 6: Illustration of evacuation scenarios: (a) one-shelter two-pick-up locations; (b) one-shelter three-pick-up locations.

time horizon without identifying the level of risk endured by evacuees at different pick-up locations.

6.4.2. RNBSP vs MAXER & MAXEN. Based on the aforementioned discussion, the evacuation scenario faced by EMA in RNBSP is similar to that in MAXER or MAXEN as the supply might not be sufficient to evacuate all the demands within the endangered area. Let Q_{eva} be total number of saved citizens. Let $S_j \overset{m}{\longleftrightarrow} P_i$ denote the bus operating from S_j to P_i for m times. The comparison of RNBSP, MAXER, and MAXEN is introduced by the following three examples.

Example 1. An evacuation scenario is given in Figure 6(a). Only one bus is available in the area with capacity of 50. In RNBSP context, the trip order of the bus is $S_1 \overset{3}{\longleftrightarrow} P_2 \longrightarrow S_1 \overset{4}{\longleftrightarrow} P_1 \longrightarrow S_1$, which is the same as that in MAXEN context. $Q_{eva} = 350$, U=87.5% and J=0.125 can obtained in both contexts. Under MAXER context, the trip order is $S_1 \overset{4}{\longleftrightarrow} P_1 \longrightarrow S_1 \longrightarrow P_2 \longrightarrow S_1$, and Q_{eva} is only 250, U=62.5% and J=0.375.

Example 2. Same evacuation scenario with *Example 1* is given in *Example 2*, only the capacity of bus is reduced to 20. The trip order in RNBSP context is still $S_1 \overset{3}{\longleftrightarrow} P_2 \longrightarrow S_1 \overset{4}{\longleftrightarrow} P_1 \longrightarrow S_1$, while, in MAXEN and MAXER context, the trip exists only between S_1 and P_1 for entire evacuation time horizon. *i.e.*, $S_1 \overset{10}{\longleftrightarrow} P_2 \longrightarrow S_1$. We can obtain Q_{eva}=140, U=35%, and J=0.05 in RNBSP context and Q_{eva}=200, U=50%, and J=0.5 in MAXEN and MAXER context. It is noteworthy that J achieves theoretical maximum under this scenario. Although the number of saved citizens in MAXER context is higher than that in RNBSP, the evacuees in P_2 are ignored by models of MAXEN and MAXER. The problem of social equity is exposed under this bus schedule scheme. The evacuees in different pick-up locations should have equal opportunity to be rescued in terms of human nature. Therefore, the limited resource should be sent to locations that are most in need.

Example 3. For evacuation scenarios in Figure 6(b) with bus capacity of 40, in RNBSP context, the trip order of the bus is $S_1 \overset{2}{\longleftrightarrow} P_2 \longrightarrow S_1 \longrightarrow P_3 \longrightarrow S_1 \overset{5}{\longleftrightarrow} P_1 \longrightarrow S_1$,

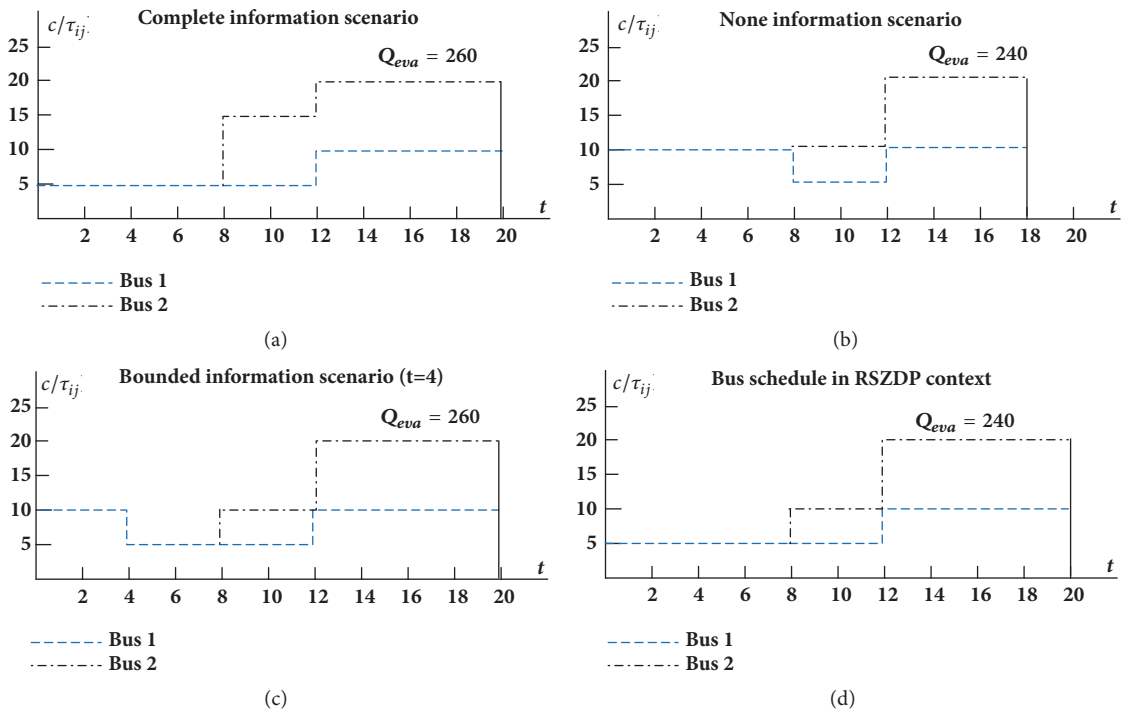
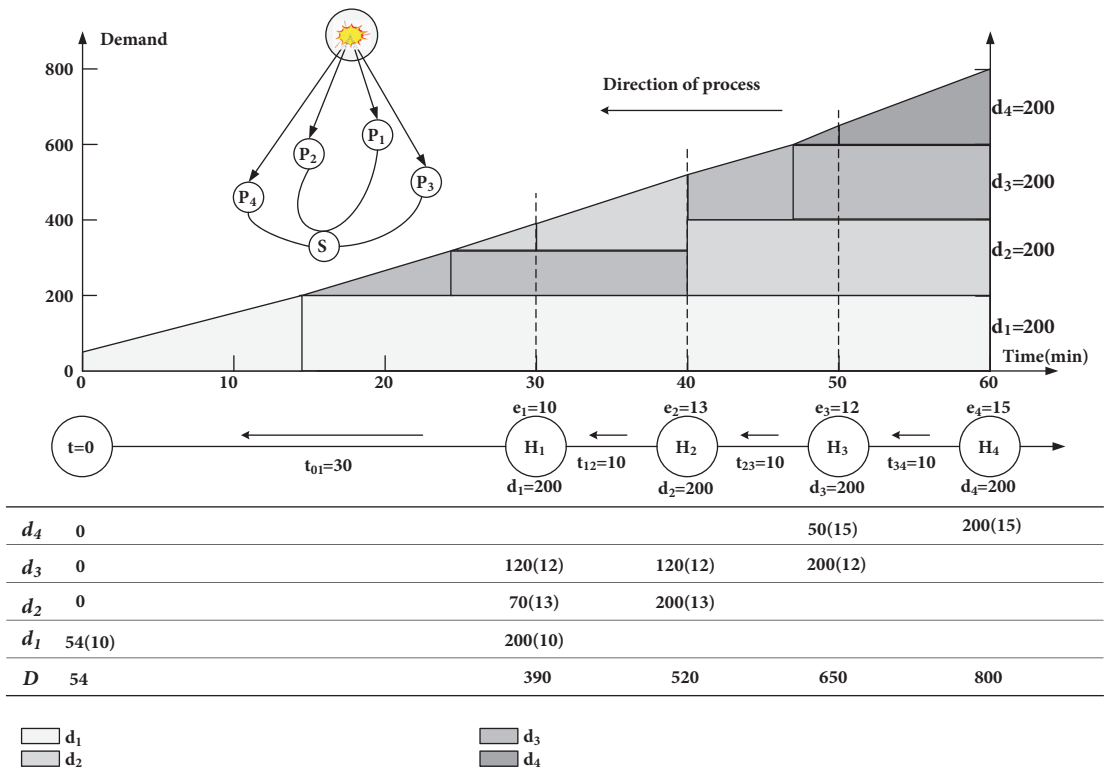


FIGURE 7: Trip orders of bus 1 and bus 2 under different scenarios with $t_2 = 8$.

the trip order in MAXEN context is $S_1 \overset{5}{\leftrightarrow} P_3 \longrightarrow S_1 \overset{5}{\leftrightarrow} P_1 \longrightarrow S_1$. It is interesting to find that the bus route in MAXER is various as assignment to P_1 or P_3 can provide the same amount evacuation rate. Here we find the route that can also provide maximum of total rescued population on premise of maximizing evacuation rate; it turns out that the trip order is the same as that in MAXEN. Q_{eva}=320, U=53.3% and J=0.305 in RNBSP context, and Q_{eva}=400, U=66.7% and J=0.471 in MAXEN and MAXER context. Actually, for the most adverse condition in MAXER context, the trip order is $S_1 \overset{5}{\leftrightarrow} P_1 \longrightarrow S_1$, we can obtain Q_{eva}=200, U=33.3% and J=0.471. P_2 is ignored in context of MAXEN and MAXER; further, P_3 is also ignored in context of MAXER for the most adverse condition.

Consider a scenario in which the whole population of a city must be evacuated to shelters outside of the city because of a disaster expanding from the downtown area. MAXER suggests to evacuate people who live in the suburb since they are closer to the shelters without considering the fact that people in downtown may be in a greater danger based on Aalami and Kattan [1], while, under MAXEN context, the buses available will also be allocated to pick-up locations near the city center with higher danger as long as the time left is enough to evacuate the citizens in the suburban area as discussed in *Example 1*. While if both pick-up locations are suffering from risk of insufficient evacuation time, as discussed in *Example 2*, the limited resource will be allocated to pick-up locations with highest evacuation rate in MAXEN context. The problem of social equity is exposed under bus schedule scheme both in MAXER and in MAXEN context. The evacuees in different pick-up locations should have equal opportunity to be rescued. Therefore, the limited resource should be sent to locations with higher evacuation risk. We can conclude from the three examples that, under situation of pick-up locations with different deadlines, the MAXER is not an appropriate objective function for bus-based evacuation under evacuation scenarios with short and various deadlines. Further comparison between MAXEN and RNBSP is given in next part.

6.4.3. Influence of Information Provision on Bus Schedule in MAXEN Context. In this part, the bus schedule scheme of MAXEN is discussed under different level of information provision. It is noteworthy that, in any case, $B(t)$ can only influence the bus schedule in and after t under MAXER or RNBSP context, which means the computation complexity of MAXER and RNBSP is not sensitive to the length of evacuation time. Three scenarios of information provision are listed as follows: (a) complete information scenario, *i.e.*, for $\forall t \in T$, $B(t)$ is known at the starting point $t = 1$; (b) bounded information scenario, *i.e.*, $B(t)$ is known a bounded time space before t; (c) none information scenario, *i.e.*, $B(t)$ is informed at t. Let t_b be the time point that bus b is available for assignment. Again, take the scenario in Figure 6(a) as an example; let $t_1 = 1$, $c_1 = 20$ and $t_2 = 8$, $c_2 = 20$. The trip order and Q_{eva} for the aforementioned three information scenarios in MAXER context combined with RNBSP context are, respectively, shown in Figure 7. In MAXEN context, under complete information scenario, U=65%, J=0.35, under bounded information scenario with 4 time unit advance, U=65%, J=0.35, under none information scenario, U=60%, J=0.4. In RNBSP context, U=60%, J=0.2. Comparing MAXER with RNBSP under different



d_4	0			50(15)	200(15)
d_3	0	120(12)	120(12)	200(12)	
d_2	0	70(13)	200(13)		
d_1	54(10)	200(10)			
D	54	390	520	650	800

Figure 8: Illustration of solution algorithm of MAXEN.

information scenarios in terms of U and J, we can conclude that the complete information scenario has a largest U, followed by bounded information scenario. Although ignoring the importance of social equity, the advantage of MAXEN under none information scenario is no better than RNBSP even in terms of U. We can conclude that information provision of supply condition has a critical influence on bus schedule in MAXEN context because it aims to maximize the number of safely evacuated citizens for entire time horizon instead of a certain time stage. The percentage of rescued population MAXERN is negatively related to degree of delay. However, the information has no influence on RNBSP as it is determined by current bus fleet size. Therefore, the robustness of proposed model is better compared with MAXEN under uncertain supply condition.

6.4.4. Solution Method of MAXEN. A solution method of MAXEN with consideration of different residual evacuation time among pick-up locations is developed in this part, which can fast identify the bus schedule and estimate total number of evacuees that can be rescued.

The MAXEN model can be solved by dynamic programing approach of back reasoning. First, the pick-up locations are sorted by its residual evacuation time at the starting point of evacuation. $H = \{H_1, \ldots, H_n\}$. The total evacuation time horizon of $T = H_n$ is divided into a series of stages with time intervals equal to the difference of two adjacent elements in H. Similar to finding the shortest path, here, the "shortest path" is equivalent to the "path" that possesses highest value of rescued population, the "cost" of link is expressed by Q_k^{k-1}, denoting the number of rescued evacuees during $H_k - H_{k-1}$. The approach searches backward from the last time period to the start of evacuation. If we seek the maximum value of Q_{eve}, which equals $Q_{eve}(H_n)$ as the total number of rescued evacuees at time H_n. By back reasoning method used in shortest path identification, the following equations are obtained:

$$\begin{aligned}
&Q_{eve}(H_n) = Q(H_{n-1}) + \max Q_n^{n-1} \\
&\max Q_n^{n-1} = \min\left\{d_n, (H_n - H_{n-1})\left(\frac{cB}{\tau_{nj}}\right)\right\} \\
&Q_{eve}(H_k) = Q(H_{k-1}) + \max Q_k^{k-1}, \quad \forall k,\ 1 \le k \le n \\
&\max Q_k^{k-1} = \min\left\{d_k\right. \\
&\quad \left. + d_k', \max\int_{t=H_{k-1}+1}^{t=H_k}\left(c\sum_{j\in S}\sum_{i\in P}\sum_{b\in B}\left(\frac{\delta_{ij}^{bt}}{\tau_{ij}}\right)\right)\right\}, \\
&\qquad \forall k,\ 1 \le k \le n
\end{aligned} \tag{40}$$

where d_k is evacuation demand of P_k, and d_k' denotes the demand left in pick-up locations with residual evacuation time larger than P_k. The illustration of this approach is shown in Figure 8 with a given example, where e_i denotes the

evacuation efficiency of P_i, interpreted as number of evacuees being rescued per unit time, equivalent to $\sum_{j \in S}(cB/\tau_{ij})$.

6.4.5. Summaries. This study quantifies evacuation risk in terms of number of casualties caused by impacts of disaster based on current bus schedule scheme. Modeled as a time-dependent variable, it is treated as a proxy of social equity and characterized by disaster dynamics, demand, and supply conditions. Since the objective of RNBSP is to shave the peak of evacuation risk among different pick-up locations in the network, the difference of casualties might be caused by disaster is lowest among pick-up locations. Not surprisingly, the run-time for model with either objective grows quickly with problem size, especially with the min-max objective. This is the reason that the problem size should be controlled in order to obtain feasible answer for realistic operation in dynamic manner. Out of consideration of social equity, bus schedule of RNBSP is recommended in evacuation operation. Actually, the disaster characteristics are not always predictable or just short-term predictable in realistic situation; thus the resource allocation based on current situation is more reliable than global allocation which covers entire evacuation time horizon. Based on the discussion above, MAXER is not suitable for bus-based evacuation with consideration of pick-up locations with different deadlines, and the efficiency of MAXEN is highly dependent on degree of advance and accuracy of predictable information of supply conditions.

The benefits of the risk-based dynamic bus schedule scheme are evident when considering social equity. The formulation of evacuation risk enables the DEU concept to be seamlessly applied to bus schedule scheme considering pick-up locations with different residual evacuation time; further, the estimation of evacuation risk is applicable for different types of disasters, providing a generalized framework for bus-based evacuation with dynamic supply condition and disaster impacts. It is interesting to find possibility that the performance of MAXER and MAXEN is no better than RNBSP even in aspect of number of rescued citizens. The results also highlight the importance of considering dynamics of disaster, demand, and supply conditions.

7. Conclusions

This paper presented a methodology of ability to model a risk-based dynamic bus schedule scheme for evacuation of transit-dependent citizens under no-notice scenarios. The proposed concept of evacuation risk can reflect the dynamic situation of a pick-up location within the endangered area considering disaster characteristics, demand patterns, and supply conditions. The concept of DEU introduced in this paper is a novel way for EMAs to design bus schedule scheme in a dynamic manner. Incorporation of disaster dynamics into evacuation modeling can better adapt the model to realistic situation.

The results of this research are of value to emergency management agencies. EMA can use the models proposed in this paper both in process of planning for a disaster and during disaster response. In the planning phase for deterministic disaster, the initial DEU determination model can be applied to identify the number of buses needed to evacuate all transit-dependent citizens in a static manner. In the operations phase of no-notice evacuation, the RNBSP model has the ability to identify optimal schedule scheme to minimize the evacuation risk of transit-dependent evacuees and maximize the vehicle utilization with limit resource in a dynamic manner. A future study direction is to take psychological experience into evacuation planning for transit-dependent populations. Under no-notice evacuation, normally, evacuees will care less about their physical experience during evacuation operation, but the psychological experience will have long-term influence on them. On premise of ensuring evacuation safety, seeking methods to improve psychological experience of transit-dependent evacuees during no-notice evacuation is an efficient way to take care of special needs and disadvantageous evacuees.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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