Hindawi
Journal of Advanced Transportation
Volume 2020, Article ID 1291342, 16 pages
https://doi.org/10.1155/2020/1291342

Research Article

Research on Decision-Making Behavior of Discretionary Lane-Changing Based on Cumulative Prospect Theory

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Received 12 May 2019; Revised 13 November 2019; Accepted 13 December 2019; Published 9 January 2020

Academic Editor: Eleonora Papadimitriou

In this paper, the decision-making model of discretionary lane-changing is established using cumulative prospect theory (CPT). Through analyzing the vehicles' dynamic running states, safety spacing calculating approaches for discretionary lane-changing and lane-keeping have been put forward firstly. Then, based on CPT, a lane-changing decision model with accelerating space as its utility is proposed by estimating the difference between actual spacings and the safety spacings for discretionary lane-changing as well as lane-keeping. In order to calculate the utility of discretionary lane-changing, dynamic reference points and a parameter representing driver's risk preference are introduced into the model. With the real data collected from an urban expressway, the distribution of discretionary lane-changing duration is analyzed, and the model parameters are also calibrated. Furthermore, the applicability of the model is evaluated by comparing with the actual observation and random unity model. Finally, the sensitivity analysis of the model is carried out, that is, assessing the influence degree of each variable on the decision result. The study reveals that the CPT-based model can describe discretionary lane-changing behavior more accurately, which consider drivers' risk-aversion during decision-making.

1. Introduction

Lane-changing is a common traffic phenomenon. It is difficult for drivers to predict the lane-changing behavior of surrounding vehicles especially the discretionary lane-changing, which usually takes only a few seconds from intention generation to operation completion. Discretionary lane-changing usually interferes with traffic flow and may cause traffic jams or even traffic accidents. The purpose of this paper is to propose a method to describe the discretionary lane-changing behavior and predict whether the driver will change lanes.

As one of the most common traffic behaviors, lane-changing behavior has been paid extensive attention, and a series of achievements have been obtained in recent decades. The earliest study on lane-changing model dates back to 1986 when Gipps established a rule-based lane-changing model in the presence of obstacles from the perspective of gap acceptance threshold [1]. Later, many scholars developed some widely used models based on Gipps' model, such as microscopic traffic simulator (MITSIM), simulation of intelligent transport systems (SITRAS), and corridor simulation (CORSIM) [2–4]. Presently, cellular automata, game theory, and discrete selection model are principally used in lane-changing behavior study. However, the current studies either ignore the driver factors or assume the driver is perfectly rational. Lots of experiments have confirmed that the results generated by the models based on perfect rationality are quite different from the actual observations.

Simon pointed out that people are boundedly rational in the decision-making process because of limited perception and computing ability [5]. They are more likely to choose the satisfactory scheme rather than the optimal one calculated based on perfect rationality. Therefore, it is necessary to consider the driver's bounded rationality in the study of discretionary lane-changing decision-making.

In order to accurately describe the relationship between the driver's lane-changing intention and the surrounding traffic conditions as well as personal risk preference, the cumulative prospect theory (CPT) under the framework of bounded rationality has been introduced into the paper. Considering the unpredictability of lane-changing duration, this article explores that the minimum safety spacing varies with the dynamic lane-changing duration and describes how the accelerating space of objective vehicle varies with the adjacent vehicles' running states. On this basis, a decision-making model of discretionary lane-changing based on dynamic reference points is established. This study considers the limitation of driver's perception and the difference in attitudes towards losses and gains. It breaks through the limitation of perfect rationality assumption and can describe the driver's actual lane-changing process more accurately.

The rest of this paper is organized as follows. Section 2 provides a literature review on lane-changing behavior as well as the application of CPT in transportation. Section 3 analyzes the involved vehicles' dynamic running states during lane-changing process and the minimum safety spacing for lane-changing between the subjective vehicle and the leading and following vehicles in the destination lane. Section 4 analyzes the minimum safety spacing for lane-keeping between the subjective vehicle and the leading vehicle in the originating lane. Based on Sections 3 and 4, a lane-changing decision-making model with accelerating space as its utility is established in Section 5. Combining with the real collected data, Section 6 analyzes the distribution of discretionary lane-changing duration and calibrates drivers' risk preference coefficients. Further, driver's lane-changing behavior is predicted using actual samples, and the sensitivity of the lane-changing decision-making model is analyzed. Finally, Section 7 provides conclusions and suggestions for future studies.

2. Literature Review

2.1. Lane-Changing Behavior. The study of cellular automata in lane-changing behavior began in the 1990s. In 1997, based on the single-lane cellular automata model (Nasch model) proposed by Nagel and Schreckenberg [6], Chowdhury et al. established the two-lane cellular automata model (STCA) by introducing lane-changing rules [7]. STCA model can well describe lane-changing process with simple evolution rules. Using cellular automata model, the asymmetric lane-changing behavior between the fast lane and the slow lane and effects of lane-changing rules on multilane highway traffic were discussed, respectively [8, 9].

Some researchers adopted game theory to study lane-changing behavior. Kesting et al. first introduced game theory into lane-changing study and established a game model considering acceleration as the payoff of lane-changing [10]. Based on automated driving systems, Wang et al. put forward an approach to generate optimal lane-changing strategies while obeying safety and comfort requirements, including strategic overtaking, cooperative merging, and selecting a safe gap [11]. Also, game theory was used to analyze the influence of vehicle aggregation situations and driver's aggressiveness on the lane-changing behavior [12, 13].

Probabilistic methods were also employed to study lane-changing behavior. Li et al. proposed a lane-changing intention recognition algorithm combining with the hidden Markov model and Bayesian filtering techniques, which has been proved to have high recognition accuracy [14]. Lee et al. established an exponential probability model to study the relationship between lane-changing probability and relative gap as well as relative velocity [15]. In addition, some researchers took driver's risk perception and uncertainties of surrounding vehicles into consideration while building probabilistic models [16–18].

The lane-changing model based on utility selection was firstly proposed by Ahmed et al. [19]. The model takes individual vehicle as the research object and generates lane-changing demand by evaluating the utility (satisfaction degree) of driving on each lane. On this basis, Toledo et al. further integrated mandatory and discretionary lane-changing behavior into a utility model [20]. Later, through real vehicle experiments, Sun and Elefteriadou considered driver factors and established utility-based lane-changing models in various scenarios [21].

The next generation simulation program (NGSIM), cosponsored by the U.S. Department of Transportation and the Federal Highway Administration, greatly promotes the research on highway lane-changing behavior by reducing the difficulty of obtaining actual data. Talebpour et al. constructed a lane-changing simulation framework, which was verified by NGSIM data to have higher accuracy than the basic gap acceptance model [22]. By using NGSIM data, Woo et al. and Vechione et al. evaluated the performance of a dynamic potential energy model on lane-changing behavior prediction and the effects of decision variables on discretionary lane-changing as well as mandatory lane-changing, respectively [23, 24].

In recent years, the great popularity of artificial intelligence has led to some lane-changing behavior studies based on it. The neural network model can predict lane-changing behavior more accurately than the multinomial logit model [25]. It was adopted to explore the relationship between the driver's lane-changing intention and lane-changing acceptance [26] and to analyze traveling heading angle as well as acceleration during lane-changing [27]. Recently, Balal et al. built a binary decision model of discretionary lane-changing based on fuzzy inference system to predict driver's lane-changing intention and the choice of the timing of lane-changing [28].

In the existing studies about lane-changing behavior, most of the lane-changing models were built based on the assumption of perfect rationality, which assume that drivers can accurately perceive the utility of lane-changing or lane-keeping and make the choice with maximum utility. However, studies have shown that experimental results based on perfect rationality deviate from the actual traffic behavior. Because while facing risk in decision-making, people cannot make accurate quantitative analysis objectively, but rely on subjective perception [29]. Therefore, a method that can reflect the driver's perception error and risk attitude is needed to describe the driver's decision-making behavior more accurately.

2.2. The Application of CPT in Transportation Research. Simon came up with the concept of bounded rationality, which has been widely accepted [5]. It is believed that when faced with decision-making, people's perception, judgment, and prediction abilities are limited. Based on this basic idea, Tversky and Kahneman proposed the prospect theory and developed it into CPT [29, 30].

CPT was mostly used to investigate travel behavior such as route choice and travel mode choice. It has not been applied to the study of driving behavior until recent years. Hamdar et al. conducted a series of studies on car-following behavior based on CPT [31–34]. In 2008, Hamdar et al. established a car-following model that captures risk-taking behavior under uncertainty and reflects drivers' cognition of driving conditions of surrounding vehicles. In this study, gains and losses are assumed as the increases and decreases of velocity, respectively. In the subsequent studies, Hamdar et al. improved the previous car-following model through correlating subjective utilities with acceleration and deceleration and probed the effects of stochastic acceleration and lane-changing behavior on travel time reliability. In addition, the acceleration model was extended to explore the driver's perception of dynamical driving environment (i.e., different weather conditions and road geometry configurations) and the way to execute acceleration maneuvers accordingly. Chow et al. used CPT to investigate driver's choice behavior between high-occupancy-vehicle lanes and mixed-use lanes and calibrated CPT parameters with the genetic algorithm [35]. Taking the lane choice behavior between general purpose lanes and managed lanes as research objects, Huang et al. compared the difference between the performances of CPT and expected utility theory (EUT) in travel time estimation [36] and explored the influence of gender, age, and income on the probability weighting for risky travel times [37].

At present, CPT has been rarely applied to driving behavior study, especially for lane-changing behavior. Several scholars used CPT to analyze the risky travel time differences of lanes with different characteristics and functions (such as management lane and general lane) and discussed the driver's choice behavior of lanes with diverse attributes. However, these studies in essence belong to the category of route choice research and fail to reflect the relationship between the stochastic lane-changing behavior among common lanes with the same attributes and the surrounding traffic conditions.

Through applying CPT, a free lane-changing decision-making model based on dynamic reference point is established in this paper. The measurement criteria of collision risk and gains are unified by introducing safety spacings, which eliminates the uncertainty of collision risk in previous studies. In addition, the driver risk factor is introduced to assess the influence of surrounding vehicles' dynamic running states on discretionary lane-changing, which enriches the lane-changing decision-making studies under the framework of bounded rationality. CPT can well describe the driver group's perception error and risk attitude, but it also has a limitation to reflect the characteristic differences among individuals.

3. Minimum Safety Spacing for Discretionary Lane-Changing

3.1. Discretionary Lane-Changing Process. Now, presume a scenario of discretionary lane-changing, as shown in Figure 1. Vehicle *M* (the merging vehicle) is going to drive from the originating to the destination lane. Vehicle *Ld* is the leading vehicle in the destination lane, and vehicle *Fd* is the following vehicle in the destination lane. While vehicle *Lo* is the leading vehicle in originating lane and vehicle *Fo* is the following vehicle in originating lane. Vehicle *M* needs to drive into the gap between vehicle *Ld* and vehicle *Fd* with a certain longitudinal and lateral acceleration in order to implement to lane-changing process. Driving velocities of vehicle *Ld* and *Fd* and the longitudinal spacings between vehicle *M* and *Ld*, *M*, and *Fd* are mainly considered to evaluate the potential risks of lane-changing for vehicle *M*. Vehicle *M* can easily control the distance from vehicle *Lo*, while the distance from vehicle *Fo* is mainly adjusted by vehicle *Fo*. Then, we can consider that there is no need for vehicle *M* to notice vehicle *Fo* during lane-changing. Therefore, the minimum longitudinal safety spacings between vehicle *M* and *Ld*, *M* and *Fd* are mainly analyzed while evaluating the risk of lane-changing.

$t=0$ is defined as the moment when vehicle *M* starts to accelerate laterally, that is, the moment when the lane-changing starts. $t=T$ is the moment when vehicle *M* stopped laterally accelerating after entering the destination lane, that is, the end time of the lane-changing. And, T is the lane-changing duration.

Shladover et al. proposed that the trajectory of the merging vehicle during the lane-changing can be described by a sinusoidal function [38]. The lateral acceleration of vehicle *M* is defined as $a_{\mathrm{lat}}(t)$, which can be expressed by

$$a_{\mathrm{lat}}(t)=\frac{2\pi D}{T^2}\times\sin\left(\frac{2\pi t}{T}\right),\quad t\in[0,T]. \tag{1}$$

By integrating equation (1) two times, the lateral displacement $y(t)$ of vehicle *M* can be obtained as follows:

$$y(t)=-\frac{D}{2\pi}\times\sin\left(\frac{2\pi t}{T}\right)+\frac{Dt}{T},\quad t\in[0,T], \tag{2}$$

where D is the lateral displacement of vehicle *M* when the lane-changing is completed. During the lane-changing, vehicle *M* needs to move from the centerline of the originating lane to the centerline of the destination lane. Therefore, D can be considered as the width of a complete lane.

The continuous process of lane-changing is divided into two phases. Phase I is from the moment $t=0$ to the moment $t=t_C$ when the central point of vehicle *M* coincides with the lane boundary line. At the end of phase I, the lateral displacement of vehicle *M* reaches $D/2$, and then we can get $t=T/2$ according to equation (2). Phase II starts from the moment $t=t_C$ to the moment $t=T$. The process of discretionary lane-changing is shown in Figure 2:

The vehicles' following relationship will change during the process of lane-changing. In phase I, vehicle *M* follows vehicle *Lo* and vehicle *Ld* at the same time. The driver of vehicle *M* will pay attention to the leading vehicle *Lo* in the originating lane and the leading vehicle *Ld* in the destination

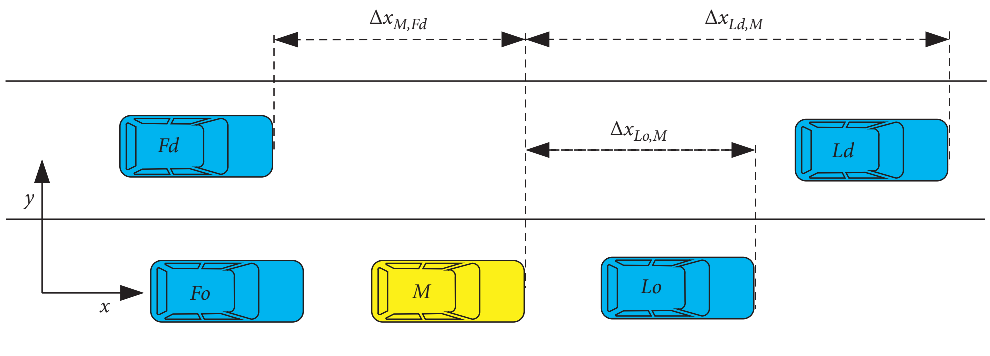


Figure 1: Pre-lane-changing configuration showing position of the involved vehicles.

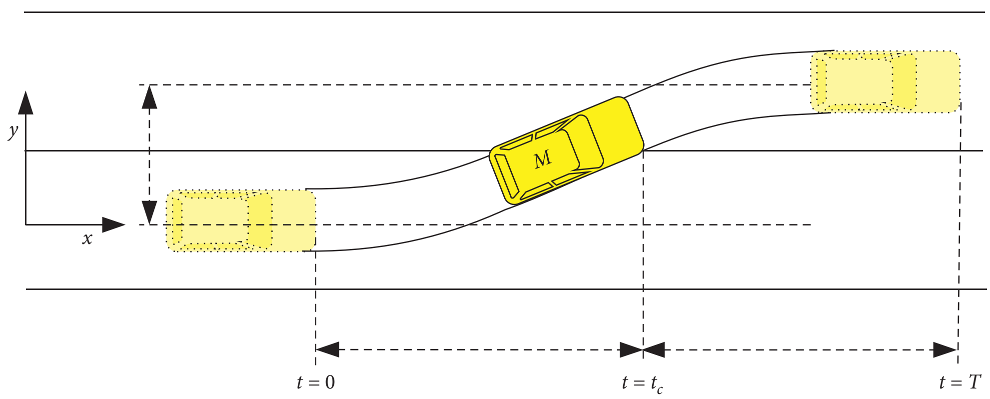


Figure 2: Discretionary lane-changing process.

lane simultaneously. So, the longitudinal acceleration of vehicle M is affected by both of vehicle Lo and Ld. Vehicle Fd shifts its attention from vehicle Ld to vehicle M when perceives the lane-changing intention of vehicle M. And, the leading vehicle of vehicle Fd changes from vehicle Ld to vehicle M. So, the acceleration of vehicle Fd is assumed to be only affected by vehicle M.

In phase II, vehicle M has already entered the destination lane and has been driving following vehicle Ld. In this stage, the acceleration of vehicle M is only affected by vehicle Ld. At the same time, vehicle Fd continually drives following vehicle M, and the acceleration of vehicle Fd is still only affected by vehicle M.

Vehicle Lo and vehicle Ld are both leading vehicles and will not be affected by the following vehicle M. Considering that the entire lane-changing process generally lasts for only a few seconds, for the convenience of study, this paper considers that vehicle Lo and vehicle Ld moves at the constant velocity $v_{Lo}(0)$ and $v_{Ld}(0)$, respectively. That is, the longitudinal acceleration of vehicle Lo and vehicle Ld are both 0.

3.2. Full Velocity Difference Model. In previous studies on driving behavior, the mainstream car-following models only considered the factor of spacing, but ignored the influence of relative velocity on acceleration. To make up for this defect, Jiang et al. [39] proposed the full velocity difference model (FVDM), which can exactly explain the phenomenon that when the velocity of the leading vehicle is greater than the following vehicle, the following vehicle will not slow down, even if the spacing between them is very small. FVDM can be expressed as follows:

$$\begin{aligned} a_{i,i+1}(t) &= \kappa\left[V\left(\Delta x_{i,i+1}(t)\right) - v_{i+1}(t)\right] + \varepsilon\Delta v_{i,i+1}(t), \\ V\left(\Delta x_{i,i+1}(t)\right) &= V_1 + V_2 \tanh\left[C_1\left(\Delta x_{i,i+1}(t) - l_i\right) - C_2\right], \end{aligned} \tag{3}$$

where $a_{i,i+1}(t)$ is the acceleration of the following vehicle $i+1$ behind the leading vehicle i at the moment t, $\kappa[V(\Delta x_{i,i+1}(t)) - v_{i+1}(t)]$ represents the influence of spacing on the acceleration of following vehicle, $\varepsilon\Delta v_{i,i+1}(t)$ represents the influence of relative velocity on the acceleration of following vehicle, $\Delta x_{i,i+1}(t) = x_i(t) - x_{i+1}(t)$ is the spacing between the following vehicle $i+1$ and the leading vehicle i at the moment t, $\Delta v_{i,i+1}(t) = v_i(t) - v_{i+1}(t)$ is the relative velocity of vehicle $i+1$ and vehicle i at the moment t, and $v_{i+1}(t)$ is the velocity of vehicle $i+1$ at the moment t. l_i is the length of vehicle i; κ and ε are driver's response sensitivity coefficients. $V(\cdot)$ is optional velocity function. C_1 represents the jam density. C_2 is the adjustment factor. V_1 and V_2 are the parameters related to velocity.

In this paper, FVDM is applied to lane-changing scene to study the acceleration changes of vehicle M and vehicle Fd.

3.3. Longitude Acceleration of Vehicle M. The driver of vehicle M has a limited perceptive sensitivity and needs a reaction time Δt. It is assumed that from $t = 0$, the driver updates the perception of the involved vehicles' running status every Δt. The driver adjusts the longitudinal acceleration according to the latest perceived driving status and drives with the latest updated longitudinal acceleration for Δt until the next time when longitudinal acceleration updates. The natural numbers N_1 and N_2 are assumed to satisfy the following relationships:

$$\begin{gathered} N_1 \Delta t \leq t_C < (N_1+1)\Delta t, \\ N_2 \Delta t \leq T < (N_2+1)\Delta t. \end{gathered} \tag{4}$$

3.3.1. Lane-Changing Phase I. In phase I, vehicle M drives following both the vehicle Lo and Ld at the same time. When FVDM is extended to the double-anticipative car-following situation, $a_{M1}(i \cdot \Delta t)$, the longitudinal acceleration of vehicle M at the moment $t = i \cdot \Delta t$, can be calculated by

$$\begin{aligned} a_{M1}(i \cdot \Delta t) &= \mu(i \cdot \Delta t) \cdot a_{Lo,M}(i \cdot \Delta t) + (1-\mu(i \cdot \Delta t)) \cdot a_{Ld,M} \\ &\quad \cdot (i \cdot \Delta t), \quad i = 0, 1, 2, \ldots, N_1. \end{aligned} \tag{5}$$

$\mu(i \cdot \Delta t)$ is the influence degree coefficient of vehicle Lo on the longitudinal acceleration of vehicle M. While $1-\mu(i \cdot \Delta t)$ is the influence degree coefficient of vehicle Ld on the longitudinal acceleration of vehicle M. In phase I, with lateral displacement of vehicle M increasing, the driver's attention to the vehicle Lo gradually decreases, while the attention to the vehicle Ld gradually increases. Therefore, during the lane-changing, $\mu(i \cdot \Delta t)$ gradually decreases from 1 to 0, and $1-\mu(i \cdot \Delta t)$ gradually increases from 0 to 1 accordingly. In phase I, the lateral displacement of vehicle M gradually increases from 0 to$D/2$, and the relationship between $\mu(i \cdot \Delta t)$ and the lateral displacement $y(i \cdot \Delta t)$ of vehicle M is as follows:

$$\mu(i \cdot \Delta t) = 1 - \frac{2y(i \cdot \Delta t)}{D}, \quad i = 0, 1, 2, \ldots, N_1. \tag{6}$$

Combining equations (2) and (6), it can be rewritten as

$$\mu(i \cdot \Delta t) = \frac{1}{\pi}\sin\left(\frac{2\pi i \cdot \Delta t}{T}\right) - \frac{2i \cdot \Delta t}{T} + 1, \quad i = 0, 1, 2, \ldots, N_1. \tag{7}$$

According to FVDM, to calculate $a_{M1}(i \cdot \Delta t)$, variates $v_M(i \cdot \Delta t)$, $\Delta x_{Ld,M}(i \cdot \Delta t)$, and $\Delta x_{Lo,M}(i \cdot \Delta t)$ are necessary, which can be obtained as follows:

$$v_M(i \cdot \Delta t) = v_M(0) + \sum_{j=0}^{i-1} a_{M1}(j \cdot \Delta t) \times \Delta t, \quad i = 0, 1, 2, \ldots, N_1, \tag{8}$$

$$\begin{aligned} \Delta x_{Ld,M}(i \cdot \Delta t) &= \Delta x_{Ld,M}(0) + \sum_{j=0}^{i-1}\left[\int_0^{\Delta t}\int_0^{\varphi} -a_{M1}(j \cdot \Delta t)\,d\tau\,d\varphi \right. \\ &\quad \left. + \left(v_{Ld}(0) - v_M(j \cdot \Delta t)\right) \times \Delta t\right], \\ &\quad i = 0, 1, 2, \ldots, N_1, \end{aligned} \tag{9}$$

$$\begin{aligned} \Delta x_{Lo,M}(i \cdot \Delta t) &= \Delta x_{Lo,M}(0) \\ &\quad + \sum_{j=0}^{i-1}\left[\int_0^{\Delta t}\int_0^{\varphi} -a_{M1}(j \cdot \Delta t)\,d\tau\,d\varphi + \left(v_{Lo}(0) \right.\right. \\ &\quad \left.\left. - v_M(j \cdot \Delta t)\right) \times \Delta t\right], \quad i = 0, 1, 2, \ldots, N_1, \end{aligned} \tag{10}$$

where $v_M(0)$ is the initial longitudinal velocity of vehicle M. $\Delta x_{Lo,M}(i \cdot \Delta t)$ and $\Delta x_{Ld,M}(i \cdot \Delta t)$ are the initial space headways between vehicle M and Lo, vehicle M and Ld, respectively.

3.3.2. Lane-Changing Phase II. At the beginning of phase II, $t = t_C$, vehicle M has already entered the destination lane and has been driving following vehicle Ld. Every time interval Δt, vehicle M adjusts the longitudinal acceleration according to the relative velocity and the longitudinal spacing between vehicle Ld and itself. In phase II, $a_{M2}(i \cdot \Delta t)$, the longitudinal acceleration of vehicle M at the moment $i \cdot \Delta t$, can be obtained as follows:

$$a_{M2}(i \cdot \Delta t) = a_{Ld,M}(i \cdot \Delta t), \quad i = N_1+1, N_1+2, \ldots, N_2. \tag{11}$$

Similarly, to calculate $a_{M2}(i \cdot \Delta t)$, variates $v_M(i \cdot \Delta t)$ and $\Delta x_{Ld,M}(i \cdot \Delta t)$ are necessary, which can be obtained as follows:

$$\begin{aligned} v_M(i \cdot \Delta t) &= v_M(N_1 \Delta t) + \sum_{j=N_1}^{i-1} a_{M2}(j \cdot \Delta t) \times \Delta t, \\ &\quad i = N_1+1, N_1+2, \ldots, N_2, \end{aligned} \tag{12}$$

$$\begin{aligned} \Delta x_{Ld,M}(i \cdot \Delta t) &= \Delta x_{Ld,M}(N_1 \Delta t) + \sum_{j=N_1}^{i-1}\left[\int_0^{\Delta t}\int_0^{\varphi} \right. \\ &\quad - a_{M2}(j \cdot \Delta t)\,d\tau\,d\varphi \\ &\quad \left. + \left(v_{Ld}(0) - v_M(j \cdot \Delta t)\right) \times \Delta t\right], \\ &\quad i = N_1+1, N_1+2, \ldots, N_2. \end{aligned} \tag{13}$$

3.4. Longitude Acceleration of Vehicle Fd. In the same way, vehicle Fd updates its acceleration following the similar principle as vehicle M does. That is, considering the same reaction time Δt of vehicle Fd, the longitudinal acceleration is updated every time interval Δt. The acceleration of the vehicle Fd is only influenced by vehicle M. Thus, during the lane-changing process, the longitudinal acceleration $a_{Fd}(i \cdot \Delta t)$ of vehicle Fd can be calculated by

$$a_{Fd}(i \cdot \Delta t) = a_{M,Fd}(i \cdot \Delta t), \quad i = 0, 1, 2, \ldots, N_2. \tag{14}$$

According to FVDM, to calculate $a_{Fd}(i \cdot \Delta t)$, variates $v_{Fd}(i \cdot \Delta t)$ and $\Delta x_{M,Fd}(i \cdot \Delta t)$ are necessary, which can be obtained as follows:

$$v_{Fd}(i\cdot\Delta t)=v_{Fd}(0)+\sum_{j=0}^{i-1}a_{Fd}(j\cdot\Delta t)\times\Delta t,\quad i=0,1,2,\ldots,N_2, \tag{15}$$

$$\Delta x_{M,Fd}(i\cdot\Delta t)=\begin{cases}\Delta x_{M,Fd}(0)+\sum_{j=0}^{i-1}\left[\int_0^{\Delta t}\int_0^{\varphi}a_{M1}(j\cdot\Delta t)-a_{Fd}(j\cdot\Delta t)\mathrm{d}\tau\,\mathrm{d}\varphi+\left(v_M(j\cdot\Delta t)-v_{Fd}(j\cdot\Delta t)\right)\times\Delta t\right],\\ \quad i=1,2,\ldots,N_1,\\ \Delta x_{M,Fd}(N_1\Delta t)+\sum_{j=N_1}^{i-1}\left[\int_0^{\Delta t}\int_0^{\varphi}\left(a_{M2}(j\cdot\Delta t)-a_{Fd}(j\cdot\Delta t)\right)\mathrm{d}\tau\,\mathrm{d}\varphi+\left(v_M(j\cdot\Delta t)-v_{Fd}(j\cdot\Delta t)\right)\times\Delta t\right],\\ \quad i=N_1+1,N_1+2,\ldots,N_2,\end{cases} \tag{16}$$

where $v_{Fd}(0)$ is the initial longitudinal velocity of vehicle *Fd*. $\Delta x_{M,Fd}(0)$ is the initial longitudinal space headway between vehicle *M* and *Fd*.

3.5. Minimum Safety Spacing. Jula et al. proposed that vehicle *M* should consider the safety spacing between each related vehicle and itself during the lane-changing [40]. In order to prevent from colliding, it is only necessary to ensure that both $\mathrm{Sr}_{Ld,M}(t)$ and $\mathrm{Sr}_{M,Fd}(t)$ are more than 0 at any moment within the interval $[t_C,T]$. That is, $\mathrm{Sr}_{Ld,M}(t)$ and $\mathrm{Sr}_{M,Fd}(t)$ should satisfy the following equations:

$$\mathrm{Sr}_{Ld,M}(t)=\left[\mathrm{Sr}_{Ld,M}(0)+\int_0^t\int_0^{\varphi}a_{Ld}(\tau)-a_M(\tau)\mathrm{d}\tau\,\mathrm{d}\varphi+\left(v_{Ld}(0)-v_M(0)\right)\times t\right]>0,\quad\forall t\in[t_C,T], \tag{17}$$

$$\mathrm{Sr}_{M,Fd}(t)=\left[\mathrm{Sr}_{M,Fd}(0)+\int_0^t\int_0^{\varphi}a_M(\tau)-a_{Fd}(\tau)\mathrm{d}\tau\,\mathrm{d}\varphi+\left(v_M(0)-v_{Fd}(0)\right)\times t\right]>0,\quad\forall t\in[t_C,T], \tag{18}$$

where $a_{Ld}(\tau)$, $a_M(\tau)$, and $a_{Fd}(\tau)$ are the accelerations of the vehicle *Ld*, *M*, and *Fd* during the lane-changing process, respectively. $v_{Ld}(0)$, $v_M(0)$, and $v_{Fd}(0)$ are the initial velocities of vehicles *Ld*, *M*, and *Fd*, respectively. $\mathrm{Sr}_{Ld,M}(0)$ is the initial longitudinal spacing between the vehicle *M* and *Ld*. $\mathrm{Sr}_{M,Fd}(0)$ is the initial longitudinal spacing between vehicles *Fd* and *M*.

Under the condition that equations (17) and (18) are satisfied, the minimum values of $\mathrm{Sr}_{Ld,M}(0)$ and $\mathrm{Sr}_{M,Fd}(0)$ are, respectively, denoted as the initial minimum safety spacings that vehicle *M* needs to keep with vehicle *Ld* and vehicle *Fd* before lane-changing, which can be marked as $\mathrm{MSS}(Ld,M)$ and $\mathrm{MSS}(M,Fd)$, and they can be calculated according to the following equations:

$$\mathrm{MSS}(Ld,M)=\max\left[\int_0^t\int_0^{\varphi}a_M(\tau)-a_{Ld}(\tau)\mathrm{d}\tau\,\mathrm{d}\varphi+\left(v_M(0)-v_{Ld}(0)\right)\times t\right],\quad\forall t\in[t_C,T], \tag{19}$$

$$\mathrm{MSS}(M,Fd)=\max\left[\int_0^t\int_0^{\varphi}a_{Fd}(\tau)-a_M(\tau)\mathrm{d}\tau\,\mathrm{d}\varphi+\left(v_{Fd}(0)-v_M(0)\right)\times t\right],\quad\forall t\in[t_C,T]. \tag{20}$$

Therefore, to calculate $\mathrm{MSS}(Ld,M)$ and $\mathrm{MSS}(M,Fd)$, the longitudinal accelerations of vehicles *Ld*, *M*, and *Fd* should be obtained first. According to Sections 3.3 and 3.4, equations (19) and (20) can be rewritten as

$$\mathrm{MSS}(Ld,M)=\max\left(\sum_{j=0}^{N_1-1}\left(\int_0^{\Delta t}\int_0^{\varphi}a_{M1}(j\cdot\Delta t)\mathrm{d}\tau\,\mathrm{d}\varphi+\left(v_M(j\Delta t)-v_{Ld}(0)\right)\times\Delta t\right)+\sum_{j=N_1}^{i-1}\left(\int_0^{\Delta t}\int_0^{\varphi}a_{M2}(j\cdot\Delta t)\mathrm{d}\tau\,\mathrm{d}\varphi+\left(v_M(j\cdot\Delta t)-v_{Ld}(0)\right)\times\Delta t\right)\right),\quad i=N_1+1,N_1+2,\ldots,N_2, \tag{21}$$

$$\mathrm{MSS}(M,Fd)=\max\left(\sum_{j=0}^{N_1-1}\left(\int_0^{\Delta t}\int_0^{\varphi}a_{Fd}(j\cdot\Delta t)-a_{M1}(j\cdot\Delta t)\mathrm{d}\tau\,\mathrm{d}\varphi+\left(v_{Fd}(j\cdot\Delta t)-v_M\cdot(j\cdot\Delta t)\right)\times\Delta t\right)+\sum_{j=N_1}^{i-1}\left(\int_0^{\Delta t}\int_0^{\varphi}a_{Fd}(j\cdot\Delta t)-a_{M2}(j\cdot\Delta t)\mathrm{d}\tau\,\mathrm{d}\varphi+\left(v_{Fd}(j\cdot\Delta t)-v_M\cdot(j\cdot\Delta t)\right)\times\Delta t\right)\right),\quad i=N_1+1,N_1+2,\ldots,N_2. \tag{22}$$

According to the above analysis, when the initial states of vehicle *M*, *Ld*, *Lo*, and *Fd* are fixed, $\mathrm{MSS}(Ld,M)$ and $\mathrm{MSS}(M,Fd)$ are determined by T (that is, the lane-changing duration) and vary with T.

4. Minimum Safety Spacing for Lane-Keeping

If vehicle M does not make the decision of lane-changing, but continues to keep driving following vehicle Lo in the originating lane, there is also a minimum safety spacing for vehicles M and Lo. If the initial spacing between vehicles M and Lo is bigger than the minimum safety spacing for lane-keeping, vehicle M has surplus space for accelerating; if not, vehicle M will decelerate to ensure safety and finally keep a safety spacing with vehicle Lo. Such a scenario is assumed: vehicle Lo suddenly brakes until completely stopping, and the maximum braking deceleration of vehicle Lo is as the same as vehicle M. In order to avoid the collision, vehicles M and Lo need to keep a minimum safety spacing $\mathrm{MSS}(Lo, M)$ between themselves, which can be described as follows:

$$\mathrm{MSS}(Lo, M) = v_M(0)\, t_d + \frac{v_M^2(0) - v_{Lo}^2(0)}{2a_{\max}}, \quad (23)$$

where $a_{\max}$ is the maximum braking deceleration generally valued as $6\,\mathrm{m/s^2}$~$8\,\mathrm{m/s^2}$ [41]. Because different types of vehicle have different power performances, it is impossible to determine the specific maximum braking deceleration of each vehicle. Therefore, $a_{\max}$ in this paper takes $7\,\mathrm{m/s^2}$, the empirical value of maximum braking deceleration of small cars on the dry pavement. t_d is the braking delay time of vehicle M, that is, the time interval from the moment vehicle Lo starts braking to the moment vehicle M starts braking. It is related to driver characteristics and vehicle performance and is generally valued at 1.2 s~2.0 s in other similar studies [41]. It is hard to observe the braking delay time of each vehicle, so t_d in this paper takes 1.5 s, the empirical value under the same environment in congeneric studies.

5. Modeling of Discretionary Lane-Changing Decision-Making Behavior

5.1. Functions of CPT. CPT believes that the decision-maker is boundedly rational [30]. The decision-maker processes and judges the utility relying on the value function and the probability weight function. CPT considers the decision-maker's characteristics of bounded rationality as follows: (1) decision-makers make judgments and choices by predicting the potential gains and losses of different options. The gains and losses depend on reference points. (2) Different people have different sensitivity of gains and losses. When faced with gains, people tend to be risk-averse. When faced with losses, people tend to be risk-seeking. (3) While making decisions, people tend to overestimate the small probability event and underestimate the large probability event.

The above characteristics can be described by the value function $\Phi(x_i)$ and the probability weight function $w(p_i)$ as follows:

$$\Phi(x_i) = \begin{cases} (x_i - x_0)^{\alpha}, & x_i - x_0 \geq 0, \\ -\lambda(-x_i + x_0)^{\beta}, & x_i - x_0 < 0, \end{cases} \quad (24)$$

$$w(p_i) = \frac{p_i^{\gamma}}{\left(p_i^{\gamma} + (1 - p_i)^{\gamma}\right)^{1/\gamma}}, \quad (25)$$

where x_0 is the reference point and x_i is the possible utility of an option. When $x_i \geq x_0$, x_i represents a gain, and when $x_i < x_0$, x_i represents a loss. p_i is the probability corresponding to x_i. α, β, λ, and γ are related parameters. Among them, α and β represent decision-makers' attitude to risk. The values of α and β are between 0 and 1. And, the larger α and β are, the more inclined the decision-maker is to pursue risk. λ represents the sensitivity of the decision-maker to the losses. γ reflects the deviation degree between the probability perceived by the decision-maker and the actual probability.

The shapes of $\Phi(x_i)$ and $w(p_i)$ are shown in Figure 3.

While applying CPT, the possible utilities are sorted from small to large and divided into $n+1$ gains (the utilities equal to or larger than the reference point) and m losses (the utilities less than the reference point). The possible utilities are expressed as $x_{-m}, \ldots, x_1, x_0, \ldots, x_n$, and the corresponding probabilities are $p_{-m}, \ldots, p_{-1}, p_0, \ldots, p_n$. Therefore, the cumulative prospect value $\mathrm{CPV}(x, p)$ is calculated as follows:

$$\pi^{+}(p_i) = w(p_i + \ldots + p_n) - w(p_{i+1} + \ldots + p_n), \quad 0 \leq i \leq n, \quad (26)$$

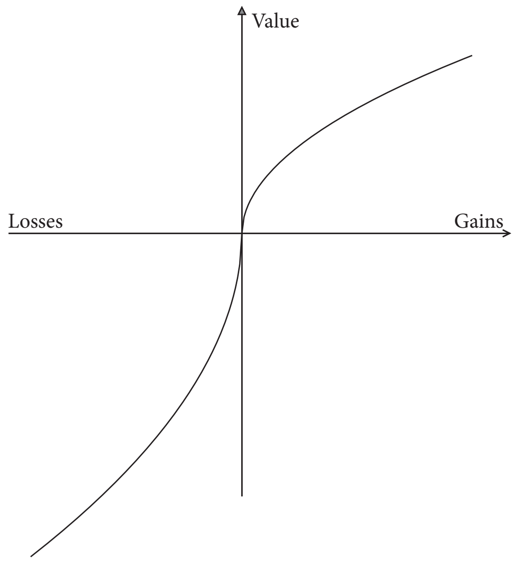
$$\pi^{-}(p_i) = w(p_{-m} + \ldots + p_{-i}) - w(p_{-m} + \ldots + p_{-i-1}), \quad -m \leq -i < 0, \quad (27)$$

$$\mathrm{CPV}(x, p) = \sum_{i=0}^{n} \Phi(x_i)\pi^{+}(p_i) + \sum_{i=-m}^{-1} \Phi(x_i)\pi^{-}(p_i). \quad (28)$$

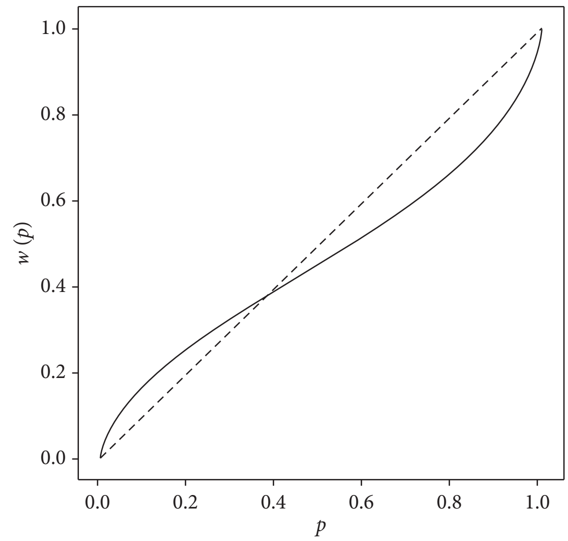
The decision-maker will choose the option with the largest prospect cumulative value.

5.2. Model Establishment. The main basis for the driver to make decisions is the accelerating space obtained from lane-changing or lane-keeping. The accelerating space depends on the longitudinal spacings between the objective vehicle and related vehicles. Regardless of lane-changing or lane-keeping, if the longitudinal spacing is bigger than the corresponding minimum safety spacing, the driver will obtain a gain; otherwise, the driver will obtain a loss. Therefore, whether to change lanes becomes a risk decision-making behavior, which meets the applicable conditions of CPT.

From the above analysis, it is known that whether the accelerating space can be obtained is the criterion for judging gains or losses. The utilities of the lane-changing and the lane-keeping are defined as the corresponding initial longitudinal spacings between vehicle M and the related



(a)



(b)

Figure 3: (a) Value function. (b) Probability weighing function.

vehicles. And, the reference points are the dynamic minimum safety spacings corresponding to the lane-changing duration.

5.2.1. The Duration of Discretionary Lane-Changing. In the actual driving situation, drivers cannot accurately perceive the duration of discretionary lane-changing that has not yet happened. They can only predict it based on the previous experience of lane-changing in the same driving environment. Therefore, the duration of discretionary lane-changing is not a definite value, but subjects to a probability distribution. It is assumed that the duration of discretionary lane-changing follows a normal distribution: $T \sim N(\mu_0 \sigma^2)$. The significant distribution range of lane-changing duration is determined under a confidence of 99.8%, assuming that it is $[T_1, T_2]$. The minimum perceived time interval is denoted as Δt, and the range of lane-changing duration $[T_1, T_2]$ is divided into $n+m+1$ subintervals. Since Δt is small, the mean value $\overline{T}_i = 1/2\,(T_1 + i \cdot \Delta t + T_1 + (i+1) \cdot \Delta t)$ of each subinterval $[T_1 + i \cdot \Delta t, T_1 + (i+1) \cdot \Delta t)$ (i from 0 to $n+m$) is used to represent any value within the subinterval. Then, the duration of discretionary lane-changing T has been discretized, and there are $n+m+1$ values in total. The probability of each optional value of T can be expressed by the following probability distribution function:

$$
\begin{aligned}
P\left(T = \overline{T}_i\right) &= P\left(T_1 + i \cdot \Delta t \le T < T_1 + (i+1) \cdot \Delta t\right) \\
&= \int_{T_1 + i \cdot \Delta t}^{T_1 + (i+1) \cdot \Delta t} \frac{1}{\sigma\sqrt{2\pi}} e^{-(t-\mu_0)^2/2\sigma^2} \mathrm{d}t, \quad i = 0, 1, 2, \ldots, n+m.
\end{aligned}
\tag{29}
$$

5.2.2. The Cumulative Prospect Value of Lane-Changing. At the initial moment of lane-changing, there is a longitudinal spacing and corresponding minimum safety spacing between vehicles M and Ld, M and Fd, so the utility of lane-changing consists of front-utility and rear-utility. Front-utility is defined as the difference between the initial front longitudinal spacing $\mathrm{Sr}_{Ld,M}(0)$ and the front minimum safety spacing $\mathrm{MSS}(Ld, M)$. Rear-utility is defined as the difference between the initial rear longitudinal spacing $\mathrm{Sr}_{M,Fd}(0)$ and the rear minimum safety spacing $\mathrm{MSS}(M, Fd)$. The corresponding front-subjective utility $\Phi_{Ld,M}$ and rear-subjective utility $\Phi_{M,Fd}$ can be calculated by

$$
\Phi_{Ld,M} = \begin{cases} \left(\mathrm{Sr}_{Ld,M}(0) - \mathrm{MSS}(Ld, M)\right)^{\alpha}, & \mathrm{Sr}_{Ld,M}(0) - \mathrm{MSS}(Ld, M) \ge 0, \\ -\lambda\left(-Sr_{Ld,M}(0) + \mathrm{MSS}(Ld, M)\right)^{\beta}, & \mathrm{Sr}_{Ld,M}(0) - \mathrm{MSS}(Ld, M) < 0, \end{cases} \tag{30}
$$

$$
\Phi_{M,Fd} = \begin{cases} \left(\mathrm{Sr}_{M,Fd}(0) - \mathrm{MSS}(M, Fd)\right)^{\alpha}, & \mathrm{Sr}_{M,Fd}(0) - \mathrm{MSS}(M, Fd) \ge 0, \\ -\lambda\left(-\mathrm{Sr}_{M,Fd}(0) + \mathrm{MSS}(M, Fd)\right)^{\beta}, & \mathrm{Sr}_{M,Fd}(0) - \mathrm{MSS}(M, Fd) < 0. \end{cases} \tag{31}
$$

The total subjective utility of lane-changing is composed of front-subjective utility and rear-subjective utility and is calculated by

$$
\Phi_{Ld,M,Fd} = \theta\,\Phi_{Ld,M} + (1-\theta)\Phi_{M,Fd}, \tag{32}
$$

where the value of the parameter θ is between 0 and 1, reflecting the driver's attention degree on front-subjective utility. The larger value of θ indicates that when the accelerating space (front-utility) provided by lane-changing is larger, the driver shows a stronger tendency to change lanes for pursuing higher velocity. Therefore, θ represents the risk attitude of drivers. Larger θ denotes that, the driver is less sensitive to the risk of collision with vehicle Fd during lane-changing, which reflects the driver's risk-seeking tendency when faced with gains. Smaller θ denotes that, the driver pays more attention to safety rather than velocity benefit which reflects the driver's risk-aversion tendency when faced with gains.

$\mathrm{MSS}(Ld, M)$ and $\mathrm{MSS}(Ld, M)$ are related to the lane-changing duration T, so the reference points of front-utility and rear-utility are dynamic reference points changing with T.

According to previous analysis, the lane-changing duration T may take $n + m + 1$ possible values. For each possible value $\overline{T}_i$, the driver may obtain a gain or a loss. Each front-subjective utility and rear-subjective utility of lane-changing has the same probability as the corresponding lane-changing duration. That is, the probability of front-subjective utility and rear-subjective utility can be expressed as follows:

$$P\left(\Phi_{Ld,M}\left(\overline{T}_i\right)\right) = P\left(T = \overline{T}_i\right), \quad i = 0, 1, 2, \ldots, n + m, \tag{33}$$

$$P\left(\Phi_{M,Fd}\left(\overline{T}_i\right)\right) = P\left(T = \overline{T}_i\right), \quad i = 0, 1, 2, \ldots, n + m. \tag{34}$$

The total subjective utility of lane-changing and the corresponding probability when T takes different values can be calculated. The total subjective utility of lane-changing can be divided into m negative values and $n + 1$ nonnegative values from small to large: $\Phi_{-m}, \ldots, \Phi_{-1}, \Phi_0, \ldots, \Phi_n$. And, the corresponding probabilities are $p_{-m}, \ldots, p_{-1}, p_0, \ldots, p_n$. Based on equations (26)~(28), the cumulative prospect value of lane-changing can be calculated by

$$\mathrm{CPV}_C = \sum_{i=0}^{n} \Phi_i \pi^+ \left(p_i\right) + \sum_{i=-m}^{-1} \Phi_i \pi^- \left(p_i\right). \tag{35}$$

5.2.3. The Cumulative Prospect Value of Lane-Keeping. If vehicle M decides to keep following vehicle Lo in the originating lane, the utility of lane-keeping is the difference between the initial spacing $\mathrm{Sr}_{Lo,M}(0)$, and the minimum safety spacing for lane-keeping $\mathrm{MSS}(Lo, M)$. Then, the subjective utility of lane-keeping can be calculated by

$$\Phi_{Lo,M} = \begin{cases} \left(\mathrm{Sr}_{Lo,M}(0) - \mathrm{MSS}(Lo, M)\right)^{\alpha}, & \mathrm{Sr}_{Lo,M}(0) - \mathrm{MSS}(Lo, M) \geq 0, \\ -\lambda\left(-\mathrm{Sr}_{Lo,M}(0) + \mathrm{MSS}(Lo, M)\right)^{\beta}, & Sr_{Lo,M}(0) - \mathrm{MSS}(Lo, M) < 0. \end{cases} \tag{36}$$

$\mathrm{MSS}(Lo, M)$ is determined by the initial velocity of vehicle M and Lo. Therefore, the subjective utility of lane-keeping can be regarded as a certain value with probability 1, and the cumulative prospect value of lane-keeping CPV_K is equal to the subjective utility, that is,

$$\mathrm{CPV}_K = \Phi_{Lo,M}. \tag{37}$$

6. Numerical Analysis and Model Assessment

6.1. Data Collection. In order to obtain discretionary lane-changing data effectively, a two-way six-lane urban expressway, the southern section of the second ring road in Xi'an, Shaanxi province, China, was selected as the survey site. The selected road section with no large vehicles entering is about 1300 m in length. The driving behavior in the east-to-west way was observed, where the middle lane and the inside lane are both straight-going lanes and the outside lane is the straight-right-lane. In order to exclude the mandatory lane-changing behavior, the vehicles that move from the middle lane to the inside lane or move from the outside lane to the middle lane are recorded as discretionary lane-changing vehicles. Video shooting was carried out on the pedestrian overpass above the selected road section for several times with good weather and high visibility in December 2018. And, the valid camera shooting range is about 150 m long.

After the video recording, Simi Motion, a video image processing software was used to analyze the video frame by frame (sampling frequency is 30 fps) to track the vehicle's location at every moment. The first-order Butterworth filter and symmetric exponential moving average method (SEMA) were, respectively, used to filter velocity noise and random errors of lateral position. The vehicles in the middle lane or the outside lane were considered target vehicles. And, left-lane-changing vehicles were recorded as lane-changing samples while non-lane-changing vehicles were recorded as lane-keeping samples. Each sample recorded the velocity and the position of the target vehicle, the front vehicle, the left front vehicle, and the left rear vehicle. In addition, the lane-changing sample also contained the duration of lane-changing of the target vehicle. Finally, 940 samples were obtained, including 401 discretionary lane-changing samples and 539 lane-keeping samples. Descriptive statistics of lane-changing samples and lane-keeping samples are shown in Table 1.

6.2. Distribution of Discretionary Lane-Changing Duration. 401 observed samples of discretionary lane-changing are used to analyze the lane-changing duration. One-sample Kolmogorov–Smirnov test indicates that the data obey normal distribution: $T \sim N\,(5.48, 0.75)$, as shown in Table 2 and Figure 4.

Table 1: Descriptive statistics of samples.

Variable	Lane-changing		Lane-keeping	
	Mean	S.D.	Mean	S.D.
v_M	10.50	1.11	10.69	1.45
v_{Lo}	10.04	1.38	11.13	1.62
v_{Ld}	11.38	1.42	11.45	1.80
v_{Fd}	11.70	1.83	11.99	1.93
$\Delta x_{Lo,M}$	16.88	4.78	21.81	5.91
$\Delta x_{Ld,M}$	43.39	12.06	28.33	10.10
$\Delta x_{M,Fd}$	19.67	6.70	12.89	4.86

Table 2: One-sample Kolmogorov–Smirnov test.

Number		401
Normal parameters	Mean	5.477
	Std. Deviation	0.748
Most extreme differences	Absolute	0.034
	Positive	0.027
	Negative	−0.034
Kolmogorov–smirnov Z		0.673
Asymp. sig. (2-tailed)		0.755

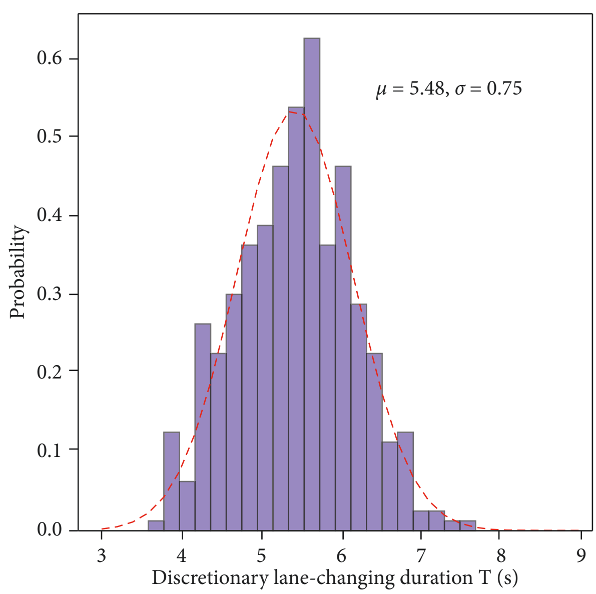


Figure 4: The distribution of discretionary lane-changing duration.

According to statistics, the 99.8% confidence interval of lane-changing duration is $[\mu_0 - 3.01\sigma,\ \mu_0 + 3.01\sigma]$, that is, $T \in [3.22, 7.73]$. Assuming $\Delta t = 0.5$ s, interval $[3.22, 7.73]$ can be divided into 9 subintervals by Δt. The mean value of each subinterval is used to represent any value within the subinterval, and then the optional values of lane-changing duration are 3.47 s, 3.97 s, 4.47 s, 4.97 s, 5.47 s, 5.97 s, 6.47 s, 6.97 s, and 7.47 s, respectively. The probability of each lane-changing can be calculated by equation (27). Furthermore, the probability of each subjective utility of lane-changing is the same as the probability of corresponding lane-changing duration according to equations (33) and (34). Then, the cumulative prospect value of lane-changing and lane-keeping can be calculated.

6.3. Parameters Calibration

6.3.1. Parameters Calibration of FVDM. This part will calibrate the parameters of FVDM according to the data extracted from the video. The optional velocity function $V(\Delta x_{i,i+1}(t))$ in FVDM represents the optimal velocity of the vehicle $i+1$ when the space headway between the leading vehicle i and the following vehicle $i+1$ is $\Delta x_{i,i+1}(t)$. When the traffic density reaches the maximum, that is, the jam density C_1, the average space headway reaches the minimum. In this case, vehicles can hardly move, so the optimal velocity of vehicle $i+1$ is 0, that is,

$$V\left(\Delta x_{\text{jam}}\right) = V_1 + V_2 \tanh\left[C_1\left(\Delta x_{\text{jam}} - l_i\right) - C_2\right] = 0. \tag{38}$$

When $\Delta x_{i,i+1}(t) \longrightarrow \propto$, vehicle $i+1$ can run at free velocity, so the optimized speed is equal to free velocity v_{free}, that is,

$$\lim_{\Delta x \longrightarrow \propto} V(\Delta x) = V_1 + V_2 = v_{\text{free}}. \tag{39}$$

In this study, the observation objects are small cars, and l_i takes 5 m. According to the observed data, the average minimum space headway $\Delta x_{\text{jam}} = 7.69$ m, so $C_1 = (1/\Delta x_{\text{jam}}) = 0.13\ \text{m}^{-1}$. The speed limit of the survey section is 70 km/h (19.44 m/s); therefore, the free speed $v_{\text{free}} = 19.44$ m/s. According to equations (38) and (39), V_1 and V_2 can be expressed by the equation with C_2. Therefore, the next step is to calibrate κ, ε, and C_2 in FVDM.

In this part, the least square method was adopted. The relevant data of vehicle M and Fd in 401 lane-changing samples is used to estimate the parameters, and the root-mean-squared error (RMSE) is shown as follows:

$$\text{RMSE}(\kappa, \varepsilon, C_2) = \sqrt{\frac{1}{802}\sum_{i=1}^{401}\left[\left(v_{i,M}(T_i) - v_{i,M}^{\text{real}}(T_i)\right)^2 + \left(v_{i,Fd}(T_i) - v_{i,Fd}^{\text{real}}(T_i)\right)^2\right]}, \tag{40}$$

where $v_{i,M}(T_i)$ and $v_{i,Fd}(T_i)$, respectively, represent the calculated velocity of vehicle M and vehicle Fd at the end of lane-changing T_i in sample i. $v_{i,M}^{\text{real}}(T_i)$ and $v_{i,Fd}^{\text{real}}(T_i)$, respectively, represent the actual velocity of vehicle M and vehicle Fd at the end of lane-changing in sample i. When $\text{RMSE}(\kappa, \varepsilon, C_2)$ reaches the minimum, the corresponding parameter values are shown in Table 3.

Table 3: Estimated value and empirical value of the parameters in FVDM.

Parameters	Estimated value	Empirical value [39]
κ	$0.62\,\mathrm{s}^{-1}$	$0.41\,\mathrm{s}^{-1}$
ε	$0.44\,\mathrm{s}^{-1}$	$0.5\,\mathrm{s}^{-1}$
V_1	8.55 m/s	6.75 m/s
V_2	10.89 m/s	7.91 m/s
C_1	$0.13\,\mathrm{m}^{-1}$	$0.13\,\mathrm{m}^{-1}$
C_2	1.42	1.57

Table 4: Estimated value and empirical value of the parameters in CPT.

Parameters		Estimated value	Empirical value [30]
α		0.58	0.88
β		0.83	0.88
λ		1.94	2.25
γ	Gain	0.71	0.61
	Loss	0.86	0.69
θ		0.68	—

6.3.2. Parameters Calibration of CPT. To calibrate the parameters of CTP, 940 samples are randomly divided into two data sets. Data set I contains 174 lane-changing samples and 255 lane-keeping samples; data set II contains 227 lane-changing samples and 284 lane-keeping samples. Samples in data set I were used to calibrate parameters, while samples in data set II were used to evaluate the model accuracy.

The lane-changing samples in data set I were numbered from 1 to 174, and the lane-keeping samples were numbered from 175 to 429. And, the RMSE between the results predicted by model and drivers' actual decision-making is shown in equations (41) and (42), and the least square method was used to estimate the parameters.

$$\mathrm{RMSE}(\alpha,\beta,\lambda,\gamma,\theta)=\sqrt{\frac{1}{429}\left[\sum_{i=1}^{174}(R_i-1)^2+\sum_{i=175}^{429}R_i^2\right]}, \tag{41}$$

$$R_i=\begin{cases}1, & \mathrm{CPV}_{C,i}>\mathrm{CPV}_{K,i},\\ 0, & \mathrm{CPV}_{C,i}\le \mathrm{CPV}_{K,i},\end{cases} \tag{42}$$

where $\mathrm{CPV}_{C,i}$ and $\mathrm{CPV}_{K,i}$ represents the cumulative prospect value of lane-changing and lane-keeping for the sample numbered i, respectively.

When $\mathrm{RMSE}(\alpha,\beta,\lambda,\gamma,\theta)$ reaches the minimum, the corresponding parameter values are obtained as shown in Table 4.

$\alpha<\beta$ represents that decision-makers are more sensitive to losses than gains, and $\lambda>1$ indicates that decision-makers tend to be loss-averse. $\gamma<1$ represents that decision-makers tend to overestimate small probabilities and underestimate large ones. $\theta>0.5$ shows that drivers pay more attention to the front longitudinal spacing than the rear longitudinal spacing during lane-changing.

6.4. Model Evaluation

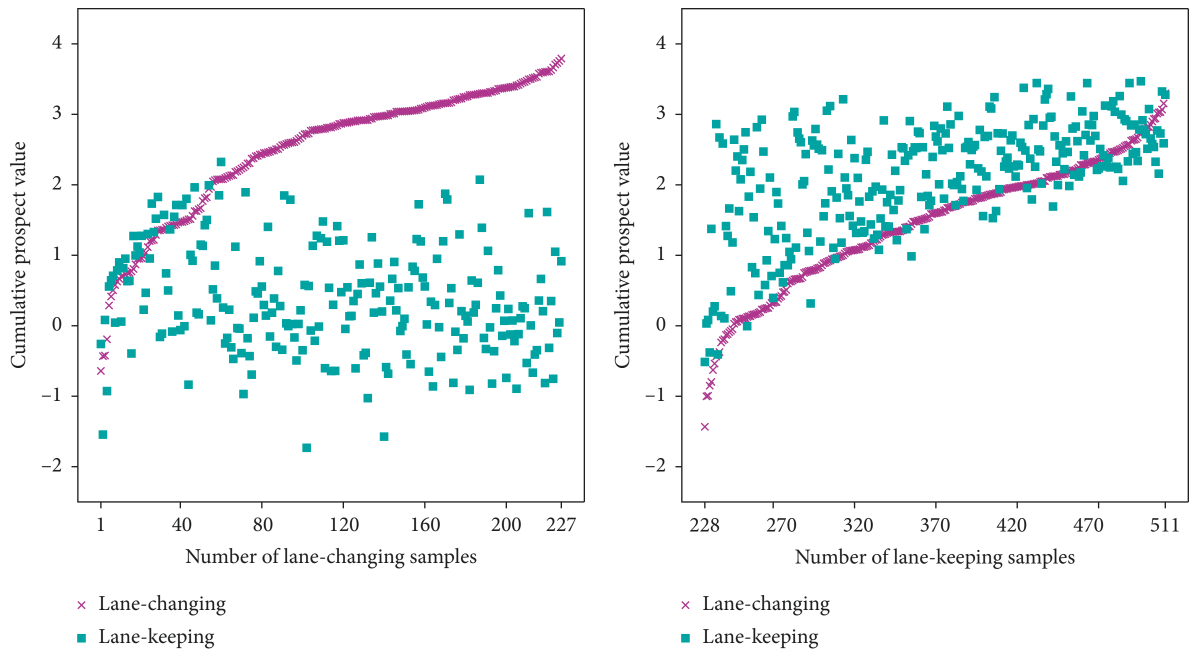
6.4.1. Prediction Based on CPT. Samples in data set II were used to evaluate the accuracy of the calibrated model. Firstly, the cumulative prospect values of both lane-changing and lane-keeping of all samples in data set II are calculated. Then, the lane-changing samples and lane-keeping samples in data set II are numbered, respectively, in the ascending order according to the cumulative prospect value of lane-changing from small to large. Cumulative prospect values of lane-changing samples (no. 1–227) and lane-keeping samples (no. 228–511) are shown in Figure 5.

As can be seen from Figure 5(a), most of the cumulative prospect value of lane-changing is located above those of lane-keeping, that is, most of the lane-changing samples in data set II are predicted to change lanes, and only a few of them are predicted to keep the lane, which is consistent with most of the actual travel behavior. In Figure 5(b), the cumulative prospect values of lane-keeping for most lane-keeping samples are above those of lane-changing, showing that the results predicted by the calibrated model are mostly consistent with drivers' actual behaviors. In addition, it is found that most of the prospect values of lane-keeping for lane-changing samples do not exceed 2 in Figure 5(a). It indicates that there exists a sufficient accelerating space for vehicle M in the current lane when the cumulative prospect value of lane-keeping reaches a certain level. This makes it unnecessary for vehicle M to change lanes, even if the cumulative prospect value of lane-changing is much larger than that of lane-keeping (as shown in the upper right area of Figure 5(b)).

As can be seen intuitively from Figure 5, the results generated from the model do not deviate too much from the drivers' actual decision-makings. Numerical comparison of predicted results and observed samples is shown in Table 5.

As shown in Table 5, the matching rates are over 84%, indicating that the model has a high accuracy in predicting drivers' decision-making behaviors. Whether in data set I or data set II, the matching rate on lane-keeping samples is lower than that on lane-changing samples. According to the decision-making model, when the cumulative prospect value of lane-changing is equal to or slightly greater than that of lane-keeping, drivers will choose to change lanes. However, in actual driving situation, drivers may not change lanes for pursuing a relatively small gain because there is a certain operation cost of lane-changing. These may explain why the matching rate on lane-keeping samples is lower than that on lane-changing samples.

6.4.2. Comparison with Random Utility Model. In order to compare the prediction performance of CPT-based model with that of perfect rationality-based model, classic random utility model (RUM) is also considered. And, the utility functions based on RUM are shown as follows:



(a) (b)

FIGURE 5: The distribution of cumulative prospect values of lane-changing and lane-keeping (a) for lane-changing samples and (b) for lane-keeping samples.

TABLE 5: Estimated results in data set I and data set II.

Samples	Lane-changing			Lane-keeping		
	Estimated number	Observed number	Matching rate (%)	Estimated number	Observed number	Matching rate (%)
Data set I	156	174	89.96	224	255	87.84
Data set II	200	227	88.11	241	284	84.86

$$U_K = V_K + \xi_K = \alpha_1 + \beta_1\left(Sr_{Lo,M}(0) - \mathrm{MSS}(Lo, M)\right) + \xi_K, \tag{43}$$

$$U_C = V_C + \xi_C = \sum_{i=0}^{n+m} P\left(T = \overline{T}_i\right)\left[\beta_2\left(\mathrm{Sr}_{Ld,M}(0) - \mathrm{MSS}(Ld, M)_{\overline{T}_i}\right) + \beta_3\left(\mathrm{Sr}_{M,Fd}(0) - \mathrm{MSS}(M, Fd)_{\overline{T}_i}\right)\right] + \xi_C, \tag{44}$$

where V_K and V_C, respectively, represent the observable utility of lane-keeping and lane-changing; ξ_K and ξ_C are the random error of lane-keeping and lane-changing, respectively; $\alpha_1, \beta_1, \beta_2$, and β_3 are coefficients, which are calibrated as −0.122, 0.146, 0.097, and 0.049, respectively. Assuming that ξ_K and ξ_C are independent and obey Gumbel distribution, the probability of lane-changing is shown as follows:

$$P_C = P\left(U_C > U_K\right) = \frac{1}{1 + e^{V_K - V_C}}. \tag{45}$$

Data set II was used to evaluate the prediction performance of RUM. Take the lane-changing probability calculated through RUM of each sample as the classification threshold, which determines the prediction error. And, the prediction result of RUM in the case with the smallest prediction error was compared with that of CPT, as shown in Table 6.

Take lane-changing as positive alternative and lane-keeping as negative alternative. Then, the true positive rate (TPR) = 200/227 = 0.8811 and the false positive rate (FPR) = 43/284 = 0.1514 of CPT-based model. Similarly, the TPR and FPR of RUM are 0.8282 and 0.2535, respectively. TPR of CPT based model is slightly larger than that of RUM, while FPR of CPT-based model is significantly smaller than that of RUM. This indicates that compared with CPT, RUM is more likely to overestimate the possibility of lane-changing. The receiver operating characteristic (ROC) curves of CPT and RUM based on data set II are shown in Figure 6.

The area under curve (AUC) of CPT-based model is 0.865, bigger than AUC of RUM, 0.772. Table 6 and Figure 6 indicate that CPT-based model performs better on prediction than RUM. It proves that the bounded rationality hypothesis of CPT is more consistent with the driver's actual decision-making behavior.

6.5. Sensitivity Analysis. In order to study the relationship between the driving states of involved vehicles and drivers'

Table 6: Prediction confusion matrix of data set II based on CPT and RUM.

		Estimated number via CPT		Estimated number via RUM	
		Lane-changing	Lane-keeping	Lane-changing	Lane-keeping
Observed number	Lane-changing	200	27	188	39
	Lane-keeping	43	241	72	212

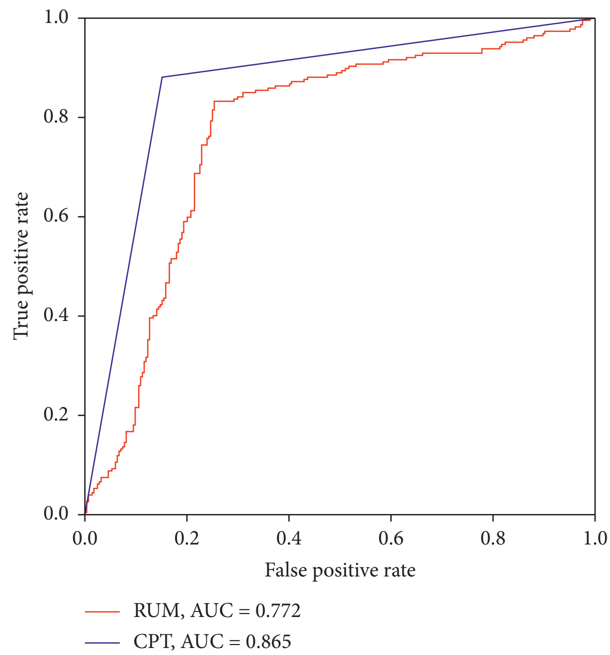


Figure 6: Receiver operating characteristic curves.

decision-makings, some decision variables' effects on the cumulative prospect values are analyzed.

6.5.1. *Effects of v_M, $\Delta x_{Lo,M}$, and $\Delta x_{Lo,M}$ on the Cumulative Prospect Value of Lane-Keeping.*

Influence of $\Delta v_{Lo,M}$ and $\Delta x_{Lo,M}$ when $v_M = 10$ m/s and influence of v_M and $\Delta x_{Lo,M}$ when $\Delta v_{Lo,M} = 3$ m/s are analyzed, respectively, as shown in Figure 7.

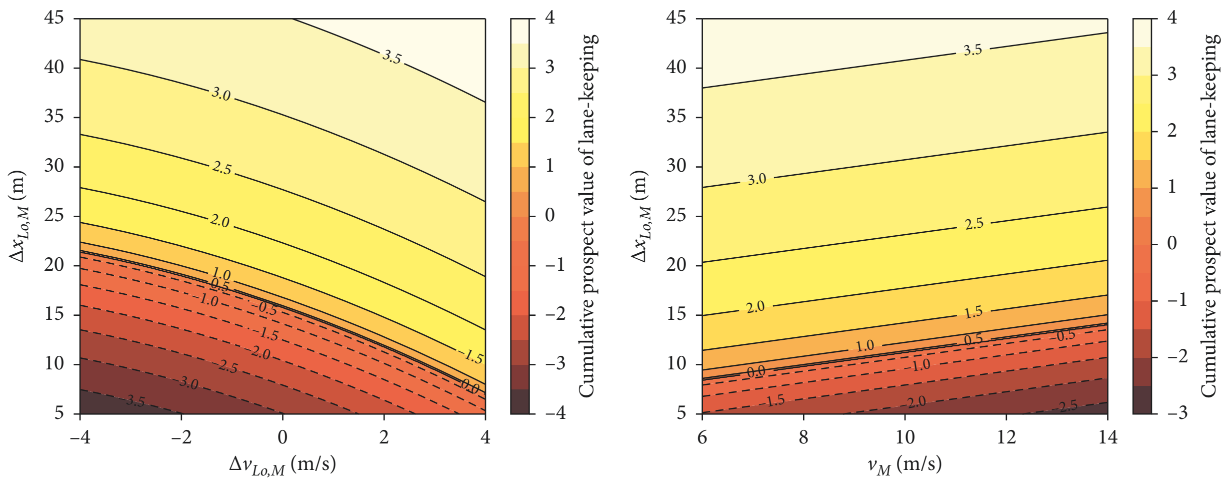
The series of curves in Figure 7 are contour lines, where each point on the same curve represents the equal cumulative prospect value of lane-keeping. Obviously, Figure 7(a) shows that the cumulative prospect value of lane-keeping increases significantly with the increase of $\Delta v_{Lo,M}$ and $\Delta x_{Lo,M}$. In addition, the farther away from the contour line valued 0, the sparser it is, indicating that the driver's sensitivity to gains or losses decreases gradually with the increase of gains or losses. This reflects the actual driving situation: when $\Delta v_{Lo,M}$ or $\Delta x_{Lo,M}$ is large enough, vehicle M can obtain sufficient accelerating space, and its intention to follow vehicle Lo will not significantly become stronger with the increase of $\Delta v_{Lo,M}$ or $\Delta x_{Lo,M}$.

Figure 7(b) demonstrates that the prospect value of lane-keeping increases with $\Delta x_{Lo,M}$ increasing, which is consistent with the conclusion shown in Figure 7(a). However, when $\Delta v_{Lo,M}$ and $\Delta x_{Lo,M}$ are fixed, the prospect value of the lane-keeping presents a downward trend with the increase of v_M. Because a greater v_M will lead to larger minimum safety spacing for lane-keeping, which will bring about less gains, the relatively flat contour lines in Figure 7(b) indicates that the prospect value of the lane-keeping is not significantly affected by v_M.

6.5.2. *Effects of $\Delta x_{Ld,M}$, $\Delta x_{M,Fd}$, $\Delta v_{Ld,M}$, and $\Delta v_{M,Fd}$ on the Cumulative Prospect Value of Lane-Changing.*

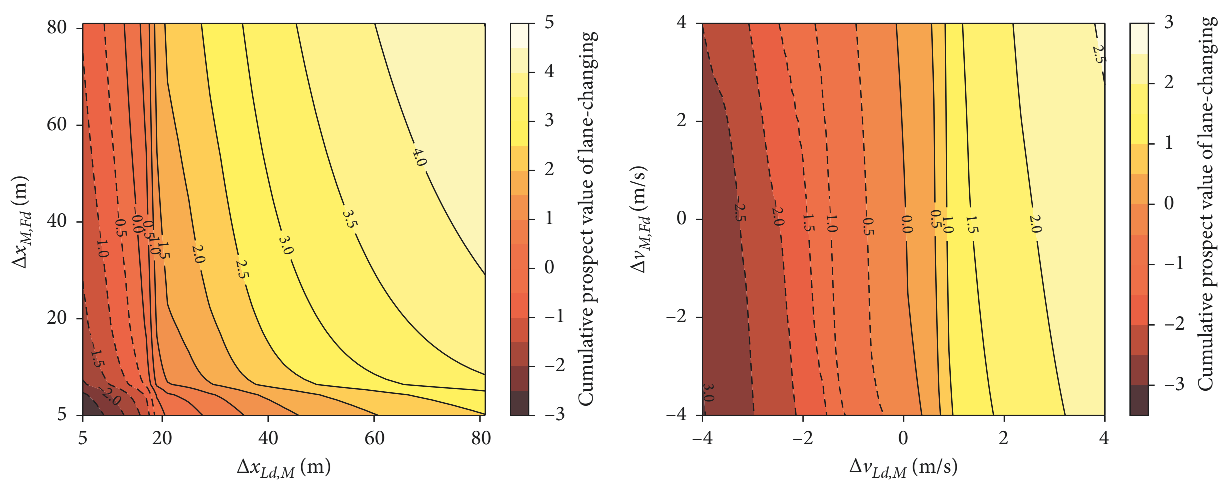
Influence of $\Delta x_{Ld,M}$ and $\Delta x_{M,Fd}$ on cumulative prospect value of lane-changing when $v_M = v_{Ld} = v_{Lo} = v_{Fd} = 10$ m/s and $\Delta x_{Lo,M} = 20$ m and influence of $\Delta v_{Ld,M}$ and $\Delta v_{M,Fd}$ when $v_M = 10$ m/s and $\Delta x_{Lo,M} = \Delta x_{Ld,M} = \Delta x_{M,Fd} = 20$ m are analyzed, respectively, as shown in Figure 8.

In Figure 8(a), the cumulative prospect value of lane-changing increases with $\Delta x_{Ld,M}$ and $\Delta x_{M,Fd}$ increasing, and $\Delta x_{Ld,M}$ has a significant impact on it. In addition, it can be found that the left area and lower area in Figure 8(a) are darker in color, indicating that the cumulative prospect value of lane-changing is much smaller than that of other areas. Especially in the left area of Figure 8(a), steep contour lines indicate that when $\Delta x_{Ld,M}$ is very small, the gain of lane-changing keeps almost unchanged or even present as a loss even if $\Delta v_{M,Fd}$ increases continuously. Further, the sensitivity of cumulative prospect value of lane-changing to $\Delta x_{M,Fd}$ decreases sharply with the increase of $\Delta x_{M,Fd}$. Because the intention of vehicle M to



(a) (b)

FIGURE 7: Contour plots of the cumulative prospect value of lane-keeping (a) as function of $\Delta v_{Lo,M}$ and $\Delta x_{Lo,M}$ for $v_M = 10$ m/s and (b) as function of v_M and $\Delta x_{Lo,M}$ for $\Delta v_{Lo,M} = 3$ m/s.



(a) (b)

FIGURE 8: Contour plots of the cumulative prospect value of lane-changing (a) as function of $\Delta x_{Ld,M}$ and $\Delta x_{M,Fd}$ for $v_M = v_{Ld} = v_{Lo} = v_{Fd} = 10$ m/s and (b) as function of $\Delta v_{Ld,M}$ and $\Delta v_{M,Fd}$ for $\Delta x_{Lo,M} = \Delta x_{Ld,M} = \Delta x_{M,Fd} = 20$ m.

change lanes will not substantially become stronger with the increase of $\Delta x_{M,Fd}$ when the minimum safety space for lane-changing is met.

In Figure 8(b), it can be observed that the cumulative prospect value of lane-changing increases with $\Delta v_{Ld,M}$ and $\Delta v_{M,Fd}$ increasing. And, the steep contour lines show that the cumulative prospect value of lane-changing is very sensitive to $\Delta v_{Ld,M}$. That means, the greater the v_{Ld} is, the greater the increment of velocity that vehicle M will obtain after lane-changing and the stronger the intention of vehicle M to change lanes. However, $\Delta v_{M,Fd}$ has little effect on the intention of vehicle M to change lanes. Therefore, the relative velocity between vehicle M and vehicle Ld is one of the decisive factors for the driver's decision-making.

7. Conclusion

Based on previous studies, this paper studies the driver's decision-making behavior of discretionary lane-changing under the framework of bounded rationality and achieves the following results:

(1) The dynamic car-following relationship between the involved vehicles during discretionary lane-changing is analyzed. A method to calculate the acceleration during lane-changing is proposed through applying a car-following model, thus the minimum safety spacing for lane-changing is derived.

(2) Considering the unpredictability of lane-changing duration, the minimum safety spacing for lane-

changing is put forward as the dynamic reference point for decision-making, and the decision-making model of lane-changing with the accelerating space as its utility is established based on CPT. The risk attitude parameter θ is introduced into the model to represent the driver's attention degree to velocity and safety during the process of lane-changing.

(3) 940 samples were collected and randomly divided into two data sets. Data set I was used to calibrate parameters, and data set II was used to evaluate the accuracy of the model. The result indicates that CPT-based model performs better in decision-making behavior prediction than RUM. It is also found that when the cumulative prospect value of lane-keeping reaches a certain level, vehicle M will not make lane-changing even if the cumulative prospect value of lane-changing is much larger than that of lane-keeping, which verifies the driver's bounded rationality.

(4) The effects of the driving states of involved vehicles on the cumulative prospect value of lane-keeping and lane-changing are analyzed, respectively. It is found that the cumulative prospect value of lane-keeping is sensitive to $\Delta v_{Lo,M}$ and $\Delta x_{Lo,M}$, but not sensitive to v_M. While the cumulative prospect value of lane-changing is sensitive to the changes of $\Delta x_{Ld,M}$ and $\Delta v_{Ld,M}$, but not sensitive to $\Delta v_{M,Fd}$. Moreover, the sensitivity of the cumulative prospect value of lane-changing to $\Delta x_{M,Fd}$ decreases sharply with the increase of $\Delta x_{M,Fd}$.

The current lane-changing decision-making models are mostly established in the framework of perfect rationality, which cannot accurately reflect the limitation of driver's perception and the difference in attitudes towards losses and gains. This study considers the characteristics of the driver's bounded rationality and better simulates the driver's actual decision-making behavior, which will enrich the research on microscopic traffic behavior. But there are also some shortcomings as follows:

(1) The driver's risk attitude parameter θ introduced into the study reflects the overall attributes to risk of the driver group, but fail to well present the attitude's differences between individual drivers.

(2) The operation cost of lane-changing was not considered in the model.

The above shortcomings lead to a certain deviation between the prediction results and the observation results, which will be improved in further studies.

Data Availability

The data used to support the findings of this study are included within the supplementary information file.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This project was supported by the Fundamental Research Funds for the Central Universities of China (grant nos. 300102218410 and 300102218521). The authors thank all participants for their time and all colleagues in the Traffic Engineering Department, School of Highway, Chang'an University.

Supplementary Materials

The supplementary materials of the manuscript contain the initial running states of the involved vehicles in collected samples. (*Supplementary Materials*)

References

[1] P. G. Gipps, "A model for the structure of lane-changing decisions," *Transportation Research Part B: Methodological*, vol. 20, no. 5, pp. 403–414, 1986.

[2] Q. Yang and H. N. Koutsopoulos, "A microscopic traffic simulator for evaluation of dynamic traffic management systems," *Transportation Research Part C: Emerging Technologies*, vol. 4, no. 3, pp. 113–129, 1996.

[3] P. Hidas, "Modelling lane changing and merging in microscopic traffic simulation," *Transportation Research Part C: Emerging Technologies*, vol. 10, no. 5-6, pp. 351–371, 2002.

[4] A. Kondyli, I. Soria, A. Duret, and L. Elefteriadou, "Sensitivity analysis of CORSIM with respect to the process of freeway flow breakdown at bottleneck locations," *Simulation Modelling Practice and Theory*, vol. 22, pp. 197–206, 2012.

[5] H. A. Simon, "A behavioral model of rational choice," *The Quarterly Journal of Economics*, vol. 69, no. 1, pp. 99–118, 1955.

[6] K. Nagel and M. Schreckenberg, "A cellular automaton model for freeway traffic," *Journal de physique I*, vol. 2, no. 12, pp. 2221–2229, 1992.

[7] D. Chowdhury, D. E. Wolf, and M. Schreckenberg, "Particle hopping models for two-lane traffic with two kinds of vehicles: effects of lane-changing rules," *Physica A: Statistical Mechanics and Its Applications*, vol. 235, no. 3-4, pp. 417–439, 1997.

[8] E. Fukuda, J. Tanimoto, Y. Iwamura, K. Nakamura, and M. Akimoto, "Field measurement analysis to validate lane-changing behavior in a cellular automaton model," *Physical Review E*, vol. 94, no. 5, p. 52209, 2016.

[9] T. Zhang and Y.-C. Lin, "Hybrid multilane models for highway traffic," *The European Physical Journal B*, vol. 91, no. 11, p. 274, 2018.

[10] A. Kesting, M. Treiber, and D. Helbing, "General lane-changing model MOBIL for car-following models," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1999, no. 1, pp. 86–94, 2007.

[11] M. Wang, S. P. Hoogendoorn, W. Daamen, B. van Arem, and R. Happee, "Game theoretic approach for predictive lane-changing and car-following control," *Transportation Research Part C: Emerging Technologies*, vol. 58, pp. 73–92, 2015.

[12] X. Wang, J. Zhang, Y. Liu, W. Yunyun, F. Wang, and J. Wang, "The drivers' lane selection model based on mixed fuzzy many-person multi-objective non-cooperative game," *Journal of Intelligent and Fuzzy Systems*, vol. 32, no. 6, pp. 4235–4246, 2017.

[13] H. Yu, H. E. Tseng, and R. Langari, "A human-like game theory-based controller for automatic lane changing," *Transportation Research Part C: Emerging Technologies*, vol. 88, pp. 140–158, 2018.

[14] K. Li, X. Wang, Y. Xu, and J. Wang, "Lane changing intention recognition based on speech recognition models," *Transportation Research Part C: Emerging Technologies*, vol. 69, pp. 497–514, 2016.

[15] J. Lee, M. Park, and H. Yeo, "A probability model for discretionary lane changes in highways," *KSCE Journal of Civil Engineering*, vol. 20, no. 7, pp. 2938–2946, 2016.

[16] F. Yan, M. Eilers, A. Lüdtke, and M. Baumann, "Developing a model of driver's uncertainty in lane change situations for trustworthy lane change decision aid systems," in *Proceedings of the 2016 IEEE Intelligent Vehicles Symposium (IV)*, pp. 406–411, IEEE, Madrid, Spain, June 2016.

[17] H. Zhou and M. Itoh, "How does a driver perceive risk when making decision of lane-changing?," *IFAC-PapersOnLine*, vol. 49, no. 19, pp. 60–65, 2016.

[18] J. Suh, H. Chae, and K. Yi, "Stochastic model-predictive control for lane change decision of automated driving vehicles," *IEEE Transactions on Vehicular Technology*, vol. 67, no. 6, pp. 4771–4782, 2018.

[19] K. Ahmed, M. Ben-Akiva, H. Koutsopoulos, and R. Mishalani, "Models of freeway lane changing and gap acceptance behavior," *Transportation and Traffic Theory*, vol. 13, pp. 501–515, 1996.

[20] T. Toledo, H. N. Koutsopoulos, and M. E. Ben-Akiva, "Modeling integrated lane-changing behavior," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1857, no. 1, pp. 30–38, 2003.

[21] D. J. Sun and L. Elefteriadou, "Lane-changing behavior on urban streets: an "in-vehicle" field experiment-based study," *Computer-Aided Civil and Infrastructure Engineering*, vol. 27, no. 7, pp. 525–542, 2012.

[22] A. Talebpour, H. S. Mahmassani, and S. H. Hamdar, "Modeling lane-changing behavior in a connected environment: a game theory approach," *Transportation Research Procedia*, vol. 7, pp. 420–440, 2015.

[23] H. Woo, Y. Ji, H. Kono et al., "Dynamic potential-model-based feature for lane change prediction," in *Proceedings of the 2016 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*, pp. 838–843, IEEE, Budapest, Hungary, October 2016.

[24] M. Vechione, E. Balal, and R. L. Cheu, "Comparisons of mandatory and discretionary lane changing behavior on freeways," *International Journal of Transportation Science and Technology*, vol. 7, no. 2, pp. 124–136, 2018.

[25] J. Zheng, K. Suzuki, and M. Fujita, "Predicting driver's lane-changing decisions using a neural network model," *Simulation Modelling Practice and Theory*, vol. 42, pp. 73–83, 2014.

[26] A. Díaz-Álvarez, M. Clavijo, F. Jiménez, E. Talavera, and F. Serradilla, "Modelling the human lane-change execution behaviour through multilayer perceptrons and convolutional neural networks," *Transportation Research Part F: Traffic Psychology and Behaviour*, vol. 56, pp. 134–148, 2018.

[27] J. Tang, F. Liu, W. Zhang, R. Ke, and Y. Zou, "Lane-changes prediction based on adaptive fuzzy neural network," *Expert Systems with Applications*, vol. 91, pp. 452–463, 2018.

[28] E. Balal, R. L. Cheu, and T. Sarkodie-Gyan, "A binary decision model for discretionary lane changing move based on fuzzy inference system," *Transportation Research Part C: Emerging Technologies*, vol. 67, pp. 47–61, 2016.

[29] A. Tversky and D. Kahneman, "Prospect theory: an analysis of decision under risk," *Econometrica*, vol. 47, no. 2, pp. 263–292, 1979.

[30] A. Tversky and D. Kahneman, "Advances in prospect theory: cumulative representation of uncertainty," *Journal of Risk and Uncertainty*, vol. 5, no. 4, pp. 297–323, 1992.

[31] S. H. Hamdar, M. Treiber, H. S. Mahmassani, and A. Kesting, "Modeling driver behavior as sequential risk-taking task," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 2088, no. 1, pp. 208–217, 2008.

[32] S. H. Hamdar, H. S. Mahmassani, and M. Treiber, "From behavioral psychology to acceleration modeling: calibration, validation, and exploration of drivers' cognitive and safety parameters in a risk-taking environment," *Transportation Research Part B: Methodological*, vol. 78, pp. 32–53, 2015.

[33] S. H. Hamdar, A. Talebpour, and J. Dong, "Travel time reliability versus safety: a stochastic hazard-based modeling approach," *IEEE Transactions on Intelligent Transportation Systems*, vol. 16, no. 1, pp. 264–273, 2015.

[34] S. H. Hamdar, L. Qin, and A. Talebpour, "Weather and road geometry impact on longitudinal driving behavior: exploratory analysis using an empirically supported acceleration modeling framework," *Transportation Research Part C: Emerging Technologies*, vol. 67, pp. 193–213, 2016.

[35] J. Y. J. Chow, G. Lee, and I. Yang, "Genetic algorithm to estimate cumulative prospect theory parameters for selection of high-occupancy-vehicle lane," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 2157, no. 1, pp. 71–77, 2010.

[36] C. Huang, M. Burris, and W. D. Shaw, "Models of transportation choice with risk: an application to managed lanes," *Transportation Letters*, vol. 9, no. 5, pp. 276–295, 2017.

[37] C. Huang, M. Burris, and W. D. Shaw, "Differences in probability weighting for individual travelers: a managed lane choice application," *Transportation*, vol. 44, no. 2, pp. 375–393, 2017.

[38] S. E. Shladover, C. A. Desoer, J. K. Hedrick et al., "Automated vehicle control developments in the PATH program," *IEEE Transactions on Vehicular Technology*, vol. 40, no. 1, pp. 114–130, 1991.

[39] R. Jiang, Q. Wu, and Z. Zhu, "Full velocity difference model for a car-following theory," *Physical Review E*, vol. 64, no. 1, p. 17101, 2001.

[40] H. Jula, E. B. Kosmatopoulos, and P. A. Ioannou, "Collision avoidance analysis for lane changing and merging," *IEEE Transactions on Vehicular Technology*, vol. 49, no. 6, pp. 2295–2308, 2000.

[41] Q. Lou and L. Xu, "Car-following model based on the minimum safety distance and simulation study," *Science Technology and Engineering*, vol. 10, no. 2, pp. 569–573, 2010.