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Research Article

Trajectory Planning Method for Mixed Vehicles Considering Traffic Stability and Fuel Consumption at the Signalized Intersection

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Traffic lights force vehicles to stop frequently at signalized intersections, which leads to excessive fuel consumption, higher emissions, and travel delays. To address these issues, this study develops a trajectory planning method for mixed vehicles at signalized intersections. First, we use the intelligent driver car-following model to analyze the string stability of traffic flow upstream of the intersection. Second, we propose a mixed-vehicle trajectory planning method based on a trigonometric model that considers prefixed traffic signals. The proposed method employs the proportional-integral-derivative (PID) model controller to simulate the trajectory when connected vehicles (equipped with internet access) follow the optimal advisory speed. Essentially, only connected vehicle trajectories need to be controlled because normal vehicles simply follow the connected vehicles according to the Intelligent Driver Model (IDM). The IDM model aims to minimize traffic oscillation and ensure that all vehicles pass the signalized intersection without stopping. The results of a MATLAB simulation indicate that the proposed method can reduce fuel consumption and NO_x, HC, CO_2, and CO concentrations by 17%, 22.8%, 17.8%, 17%, and 16.9% respectively when the connected vehicle market penetration is 50 percent.

1. Introduction

Urban traffic flow is frequently interrupted by sharp acceleration and deceleration of vehicles at signalized intersections. Such stop-and-go traffic caused by unsafe driving behaviours not only influences the stability of the traffic flow but also leads to high crash risks. Furthermore, vehicle fuel consumption and emissions are dramatically increased when vehicles slow down or idle, and excessive travel delays become more likely.

With increasing technological developments, a vast array of intelligent transportation methods has been proposed to solve this problem. For example, Rakha & Kamalanathsharma [1] used such methods to improve fuel consumption efficiency when vehicles approach a signalized intersection. Liu et al. [2] proposed a method enabling autonomous vehicles to pass through an intersection without idling; this method was implemented by establishing an intersection management system that assigns reasonable priorities for all present vehicles. Yang et al. [3] proposed an eco-driving algorithm that instructs a driver how to pass through an intersection smoothly without stop-and-go behavior. Yao et al. [4] designed a trajectory smoothing method based on individual variable speed limits with location optimization in coordination with prefixed signals. Qu et al. [5] proposed a new method based on the weighted least square that can describe the speed-density or flow-density relationship of the empirical data precisely. Furthermore, he applies a new calibration approach to produce the random traffic flow fundamental diagrams [6], the ensuing experiment indicated that the proposed approach could fit the real speed-density data and derive the speed distributions according to the different given densities. Zhou et al. [7] investigated the characteristic of the oscillation at the signalized intersection, proposed a data driven car following model; this model had a high accuracy under oscillation and could distinguish the attribution of the drivers. Ding et al. [8] designed a method to predict the potential for yellow-light or red-light-running and determine a balance between algorithm efficiency and computational time. Levin and Rey [9] developed a reservation-based intersection control protocol and improved its applicability to situations with a large number of vehicles. Li et al. [10] proposed

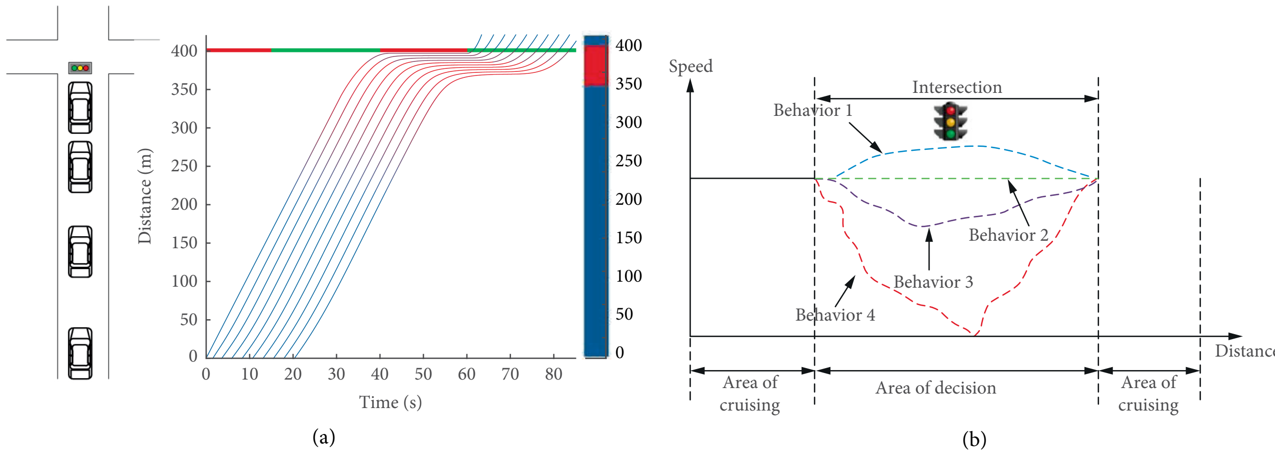


Figure 1: Illustration of (a) a congested signalized intersection and (b) different unsafe driving behaviours.

a trajectory planning method based on shooting heuristic [11, 12] with a piecewise function; the simulation revealed that the model could decrease the stop-and-go movements. In order to solve the connected infrastructure design problem, 2 linear models were proposed by Li et al. [13]. The set of tests showed that the presented model had a better performance. Xu et al. [14] presented a new energy consumption index and generalized regression neural network respectively to describe the relationship between the truck fuel consumption and drivers' driving behavior. The experiment indicated that the two models could predict the fuel consumption accurately. Amir el al. [15] developed a mixed traffic speed harmonization model with connected autonomous vehicles and conducted sensitivity analysis and multi-lance scenario test. Bichiou and Rakha [16] transferred the best vehicle speed problem at an intersection into a constrained optimal problem. Lee and Park [17] proposed a connected-vehicle sensing algorithm for an intersection based on vehicle-to-infrastructure (V2I) technology. Jiang et al. [18] built an optimal control method for a signalized intersection, which obtains optimal speed through the minimum principle. Jing et al. [19] proposes a new approach to reduce oscillation and fuel consumption. Ma et al. [20] reviewed a number of speed harmonization methods and investigated performance of the methods in real traffic. Stacy el al. [21] proposed new method of the freeway speed harmonization experiment based on the Internet of Vehicle; the real experiment of the I-66 freeway indicated it can reduce the traffic oscillation. Guo et al. [22] proposed a joint optimization of vehicle trajectories and intersection controllers, two different strategies were investigated to control trajectory and control intersection, respectively. Hale et al. [23] compared different signal timing optimization methods and observed the best performance in the heuristic algorithm based on V2I technology. Zhao et al. [24] proposed a connected-vehicle control strategy for a signalized intersection that employs model prediction control to guide connected vehicles. Finally, Feng et al. [25] proposed a joint control method of vehicle trajectories and traffic signals for connected and automated vehicles at a signalized intersection.

However, none of these studies acknowledge the very likely scenario that a driver may not follow the recommended speed strategy because of a complex real-life environment. Therefore, this study attempts to solve previous problems related to trajectory planning at signalized intersections. First, we utilize the Intelligent Driver Model (IDM), a type of car-following model [26], to theoretically evaluate traffic oscillations at signalized intersections. Second, we propose a mixed-vehicle trajectory planning method (MVTPM). This method provides an optimal advisory speed strategy based on current vehicle status and signal phase and timing information (SPaT), then uses a proportional-integral-derivative (PID) controller to simulate trajectories when drivers follow the speed strategy. This study has the following four objectives: (1) to analyze traffic oscillation at signalized intersections; (2) to simplify the vehicle speed control strategy to only consider a few key variables; (3) to use the PID controller to simulate trajectories; and (4) to perform numerical experiments to verify the efficiency of the proposed MVTPM.

This study assumes that there is no delay in data transmission loss between communication in the control area, overtaking, and lane changing behavior is also not considered. Although these assumptions are too ideal to replicate transportation environments, the results from this ideal scenario can reveal potential connected and automated vehicle (CAV) technologies for improving existing problems in traffic systems.

This paper is organized as follows. Section 2 describes the key problems in the research field. Section 3 introduces the proposed mixed-vehicle trajectory planning method. Section 4 evaluates the passing efficiency and fuel consumption of the method using numerical experiments and Section 5 concludes this paper.

2. Problem Descriptions

2.1. Congested Signalized Intersection Problem. The traffic environment of a signalized intersection is very complex and characterized by vehicles idling while they wait to pass through the intersection, as shown in Figure 1(a). Many drivers do not know whether they can pass through the intersection before the traffic light turns red, so they perform unsafe driving behaviors, which leads to increased fuel consumption of vehicles at the intersection. These unsafe driving behaviors can be divided into four types: "Acceleration," "Cruise," "Deceleration," and "Idle" (Figure 1(b)). In addition, when the vehicle is running in idle, the engine speed is very low, so the fuel cannot be fully burned, discharging a large amount of carbon dioxide (CO_2), hydrocarbons (HC), carbon monoxide (CO), nitrogen oxide (NO_x), and other harmful and toxic gases, which can seriously affect the ecological environment.

In this study, the VT-Micro [27] fuel consumption model is employed to calculate the indexes of vehicle fuel consumption and emissions, such NO_x, HC, CO_2, and CO. To the best of our knowledge, there are some fuel consumption models, they are VT-Micro, Motor Vehicle Emission Simulator (MOVES), vehicle specific power (VSP) respectively. The MOVES model utilizes the concept of the VSP distributions to calculate fuel consumption and pollution. However, if we use default VSP distributions of the MOVES, it may lead to many mistakes. Besides, we just care about the performance of the proposed system and do not consider other nonrelevant factors, such as weather-related, vehicle-related, road-related etc. The VT-Micro use the vehicle's instantaneous speed and acceleration levels to estimate vehicle emissions, and our system's target is trajectory planning of the connected vehicle based on the acceleration and velocity, so it fits our requirement of the fuel consumption model. VT-Micro is a polynomial regression model, which is a function of acceleration and speed, and expressed as follows:

$$\mathrm{MOE} = \exp\left(\sum_{i}^{3}\sum_{j}^{3} k_{i,j} \times v \times a\right), \quad (1)$$

where $k_{i,j}$ is the correlation coefficient, v is the velocity, and a is the acceleration. However, this fuel consumption model cannot calculate CO_2, which is a key emission. Nevertheless, fuel consumption and CO_2 emissions are related [28] by the following function:

$$CO_2 = \alpha_1 v + \alpha_2 \mathrm{MOE}, \quad (2)$$

where α_1 and α_2 are coefficients and MOE is the fuel consumption.

The Xiaozhai intersection (Xi'an, China) is used as an example to illustrate the fuel consumption problem at a signalized intersection (Figure 2). The red-light time of the intersection is 100 s and the green-light time of the intersection is 40 s. First, we use a digital video camera on top of a bridge to count the number of idling cars (only considering straight-through vehicles) and calculate the fuel consumption. Second, we compare it with the fuel consumption of nonidling cars.

In the paper, we define the congested period is 08:00–09:00, one hour. During this time, the number of the passed vehicles is 1425. The traffic volume of the congested period is 1425/1 = 1425 pcu/h. We also define the uncongested period is 09:00–11:00, two hours. During this time, the number of the passed vehicles is 1478. The traffic volume of the uncongested is 1478/2 = 739 pcu/h. And we calculate the fuel consumption and pollution of the different period. According to Table 1, the fuel consumption and emissions were higher for the congested period, thus, this study aims to optimize vehicle speed and fuel consumption at a signalized intersection.

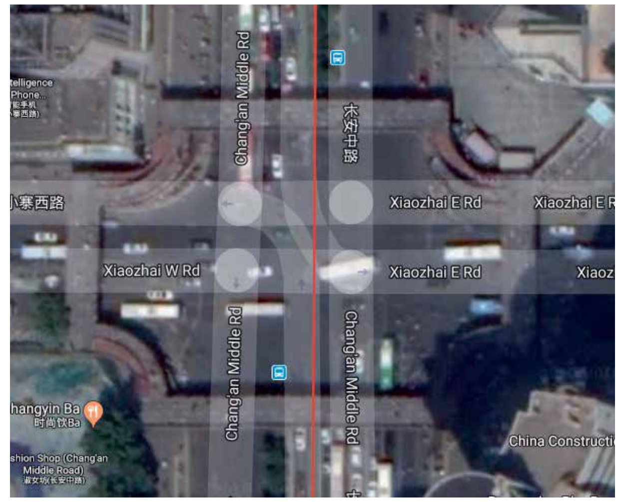


FIGURE 2: XiaoZhai Intersection in Xi'an, China.

2.2. The Stability Analysis Problem of the Car-Following Model. In general, the mathematical expression for the traditional car-following model is defined as

$$a_n = f(v_n, \Delta v_n, \Delta x_n), \quad (3)$$

where a_n, v_n, Δv_n, and Δx_n represent the nth vehicle's acceleration, speed, relative speed, and headway respectively. From Equation (3), we can first obtain the vehicle's acceleration then obtain the speed and distance using the integral to simulate the nth vehicle's trajectory.

There are two stability analyses in the car-following model: local stability and string stability. Local stability analysis mainly investigates the reaction of the vehicle to the fluctuation of the preceding vehicle's speed, focusing on local behaviour between the two vehicles. String stability analysis mainly investigates the influence of speed fluctuations of the head vehicle on the overall dynamic characteristics of the vehicle fleet. For example, the IDM car-following model is expressed as follows:

$$a_n = \alpha\left[1 - \left(\frac{v_n(t)}{v_0}\right)^4 - \left(\frac{s_n^*(t)}{s_n(t)}\right)^2\right], \quad (4)$$

where the nth vehicle's acceleration, max acceleration, instant speed at time t, desired speed, gap, and desired gap at time t is denoted as a_n, α ,$v_n(t)$, v_0, $s_n(t)$, and $s_n^*(t)$, respectively. The expression for the desired gap $s_n^*(t)$ is:

$$s_n^*(t) = s_0 + \max\left(0, T v_n + \frac{v_n(t)\Delta v_n(t)}{2\sqrt{\alpha\beta}}\right), \quad (5)$$

where s_0, T, $\Delta v_n(t)$, and β are the safe gap, reaction time, relative speed at time t, and comfort deceleration, respectively.

The local stability of the IDM car-following model is eternally stable [29]; thus, only the string stability of the IDM car-following model needs to be analysed. To linearize the nonlinear system with small perturbations at equilibrium time point h, we use a multivariate function first order Taylor

Table 1: Fuel consumption and emissions at XiaoZhai Intersection.

Intersection	Fuel (L)	CO_2 (kg)	CO (kg)	NO (kg)	HC (kg)
Congested	85.567	204.5861	1.0780	0.1396	0.0768
Uncongested	58.485	139.8460	0.7395	0.0785	0.0518

expansion to expand the acceleration function around the equilibrium point, as follows:

$$a(s, v, \Delta v) = a(s_h, v_h, \Delta v_h) + v_h'(v - v_h) + s_h'(s - s_h) + \Delta v_h'(\Delta v - \Delta v_h), \tag{6}$$

where s_h,v_h, and Δv_h denote the gap, speed, and relative speed at equilibrium point h; i.e.:, $v_h' = (\partial a/\partial v)|_h$, $s_h' = (\partial a/\partial s)|_h$, $\Delta v_h' = (\partial a/\partial \Delta v)|_h$. Furthermore, v_h', s_h', $\Delta v_h'$ are defined as below:

$$v_h' = \left.\frac{\partial a}{\partial v}\right|_h = -\alpha\left[\frac{4v_h^3}{v_0^4} + \frac{2T(s_0 + Tv_h)}{s_h^2}\right], \tag{7}$$

$$s_h' = \left.\frac{\partial a}{\partial s}\right|_h = \frac{2\alpha(s_0 + Tv_h)^2}{s_h^3}, \tag{8}$$

$$\Delta v_h' = \left.\frac{\partial a}{\partial \Delta v}\right|_h = \sqrt{\frac{\alpha}{\beta}}\frac{(s_0 + Tv_h)v_h}{s_h^2}. \tag{9}$$

The gap variation and speed variation are denoted as:

$$x_n = s_n - s_h, \tag{10}$$

$$y_n = v_n - v_h. \tag{11}$$

Thus, Equation (6) can be transformed into:

$$a(s, v, \Delta v) = a(s_h, v_h, \Delta v_h) + v_h' y_n + s_h' x_n + \Delta v_h'(y_{n-1} - y_n). \tag{12}$$

Then, we take the derivative with respect to x_n and y_n:

$$\dot{x}_n = \dot{v}_{n-1} - \dot{v}_n = \dot{y}_{n-1} - \dot{y}_n, \tag{13}$$

$$\dot{y}_n = s_h' x_n + (v_h' - \Delta v_h')y_n + \Delta v_h' y_{n-1}. \tag{14}$$

We consider the area upstream of the intersection as a linear system, where the transfer function is $G(i\omega)$, the speed disturbance of the first vehicle is $y_0 = e^{i\omega t}$, and the speed disturbance of the nth vehicle is $y_n = G^n(i\omega)e^{i\omega t}$. These terms are substituted into Equations (13) and (14):

$$-\omega^2 G^n(i\omega)e^{i\omega t} = s_h'\left(G^{n-1}(i\omega)e^{i\omega t} - G^n(i\omega)e^{i\omega t}\right) + (v_h' - \Delta v_h')i\omega G^n(i\omega)e^{i\omega t} = + \Delta v_h' i\omega G^{n-1}(i\omega)e^{i\omega t}, \tag{15}$$

$$G(i\omega) = \frac{s_n' + \Delta v_h' i\omega}{-\omega^2 + s_n' - i\omega(v_h' - \Delta v_h')}. \tag{16}$$

If the perturbation in a vehicle platoon is reduced rather than amplified, we have:

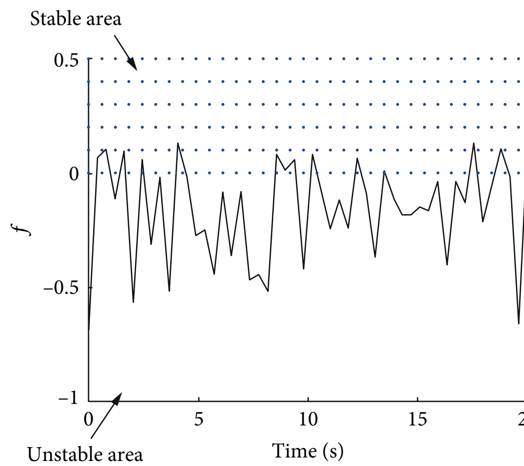


Figure 3: Traffic stability index for travel times of less than 20 s.

$$|G(i\omega)| = \frac{\sqrt{\omega^2(\Delta v_h')^2}}{\sqrt{(s_h' - \omega^2)^2 + \omega^2(v_h' - \Delta v_h')^2}} < 1. \tag{17}$$

Thus, the following inequation must be satisfied:

$$\omega^2(\Delta v_h')^2 + (s_h')^2 < (s_h' - \omega^2)^2 + \omega^2(v_h' - \Delta v_h')^2. \tag{18}$$

Because of the lower frequency, $\omega \to 0$, which is the stronger constraint on stability.

$$f = \frac{1}{2} - \frac{\Delta v_h'}{v_h'} - \frac{s_h'}{(v_h')^2} < 0. \tag{19}$$

Thus, the main reason for traffic flow instability is that the string stability is not satisfied. It is assumed that the vehicle speed fluctuates within a range of 8–12 m/s, and the gap fluctuates within a range of 20–30 m when the vehicle approaches the intersection. The instant velocity and gap of the vehicle are then substituted into Equation (19). The judgment expression is predominantly greater than zero, indicating that the traffic flow is in a stable state. However, as shown in Figure 3, the traffic flow is predominantly unstable for travel times of less than 20 s.

3. Mixed-Vehicle Trajectory Planning Method (MVTPM)

As shown in Figure 4, there are three units in the MVTPM: the input unit, control unit, and output unit. The input unit includes SPaT Information and vehicle status. SPaT information, obtained from DSRC Roadside, includes the signal phase and time. When a connected vehicle (i.e., a vehicle equipped with

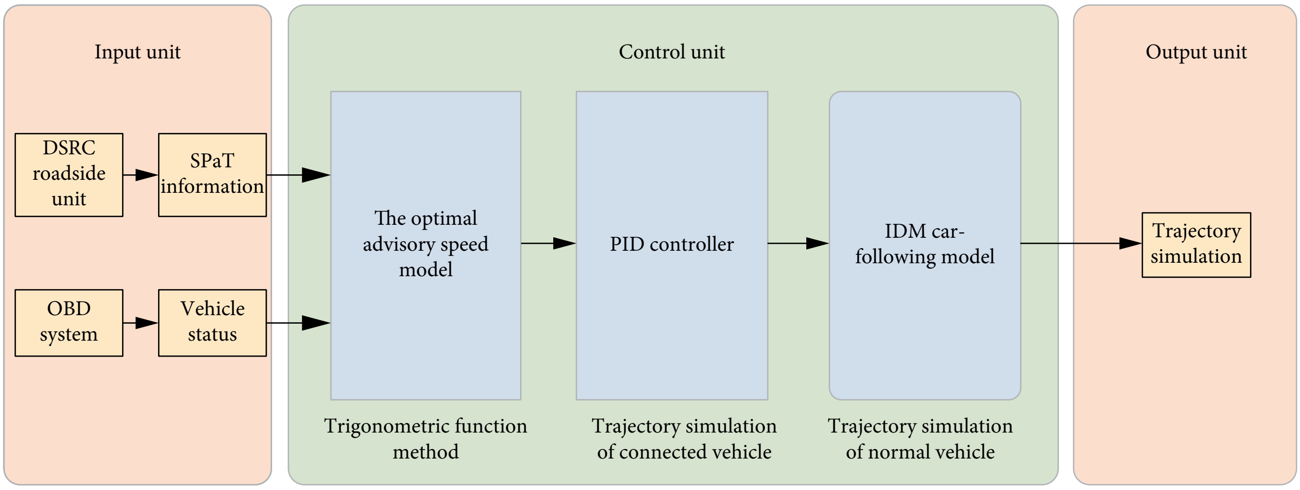


FIGURE 4: Mixed-vehicle trajectory planning method.

internet access) enters the control area upstream of the signalized intersection, the DSRC sends SPaT information to the connected vehicle. The vehicle status, which comes from OBD (on board diagnostics), includes vehicle current speed and acceleration and fuel consumption. The control unit includes three processes: the optimal advisory speed model is responsible for generating the optimal speed of connected vehicles based on the trigonometric function method; the PID controller simulates the trajectory when the driver follows the optimal speed advice; and the IDM car-following model simulates the trajectory followed by normal (nonconnected) vehicles. The output unit simulates the trajectory of mixed vehicles passing the signalized intersection from upstream to downstream.

3.1. Trigonometric Function Method. In this study, we employ the trigonometric function method to control the velocity of connected vehicles [30], which has many advantages such as smooth control and easy implementation. The trigonometric model is as follows:

$$v = \begin{cases} v1 = v_h - v_d\cos(st) & t = 0 \text{ to } \frac{\pi}{2s} \\ v2 = v_h - v_d\frac{s}{a}\cos\left(a\left(t - \frac{\pi}{2s} + \frac{\pi}{2a}\right)\right) & t = \frac{\pi}{2s} \text{ to } \left(\frac{\pi}{2a} + \frac{\pi}{2s}\right) \\ v3 = v_h + v_d\frac{s}{a} & t = \left(\frac{\pi}{2a} + \frac{\pi}{2s}\right) \text{ to } \frac{d}{v_a}, \end{cases} \tag{20}$$

where v_d denotes the speed difference, $v_d = |v_h - v_c|$, v_c represents the instantaneous speed of the connected vehicle entering the control area; v_h denotes the target maximum speed, the rate of change of acceleration in a different region is denoted as a, and s is the speed below the change in the deceleration rate of the target average speed.

The key to speed control is the velocity compensation mechanism, which states that the distance and time required for the connected vehicle to reach the intersection after entering the upstream control area is fixed. Thus, when the connected vehicle enters the control area slowly, the mechanism increases the distance by acceleration control. Similarly, when the connected vehicle enters the intersection rapidly, it decreases the distance by deceleration control. As shown in Figure 5, the enclosed area A should be equal to the sum of the enclosed area B1+B2.

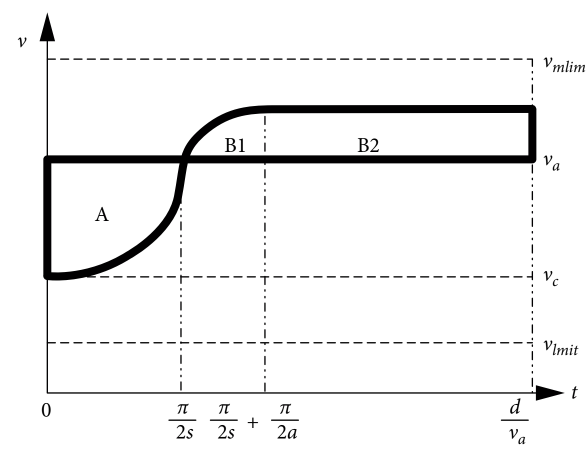


FIGURE 5: Schematic diagram of the distance compensation mechanism.

$$\begin{aligned} &\int_0^{\pi/2s}(v_h - v_d\cos(st))dt - \int_0^{\pi/2s} v_h dt \\ &= \int_{\pi/2s}^{\pi/2s+\pi/2a}\left(v_d\frac{s}{a}\cos\left(a\left(t - \frac{\pi}{2s} + \frac{pi}{2a}\right)\right) - v_a\right)dt \\ &\quad + \int_{\pi/2s+\pi/2a}^{d/v_a}\left(v_h + v_d\frac{s}{a}\right)dt - \int_{\pi/2s+\pi/2a}^{d/v_a} v_h dt. \end{aligned} \tag{21}$$

In Equation (21), a and s are the only unknown parameters so must be determined. The optimal solution of a and s is obtained by the following four limited conditions:

$$\begin{aligned} &\int_0^{\pi/2s}(v_h - v_d\cos(st))dt - \int_0^{\pi/2s} v_h dt \\ &= \int_{\pi/2s}^{\pi/2s+\pi/2a}\left(v_h - v_d\frac{s}{a}\cos\left(a\left(t - \frac{\pi}{2s} + \frac{pi}{2a}\right)\right)\right)dt \\ &+\int_{\pi/2s+\pi/2a}^{d/v_a}\left(v_h + v_d\frac{s}{a}\right)dt - \int_{\pi/2s}^{d/v_a} v_h dt \\ &\left|v_h sa\right| \le 10 \\ &v_d s \le 2 \\ &s = \max\{s\} s = [0, 1]. \end{aligned} \tag{22}$$

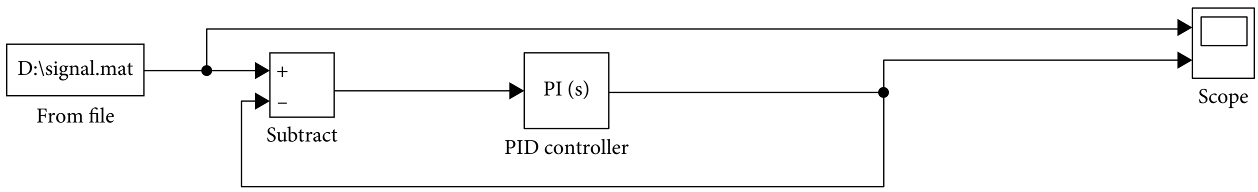


Figure 6: Diagram of the MATLAB Simulink program.

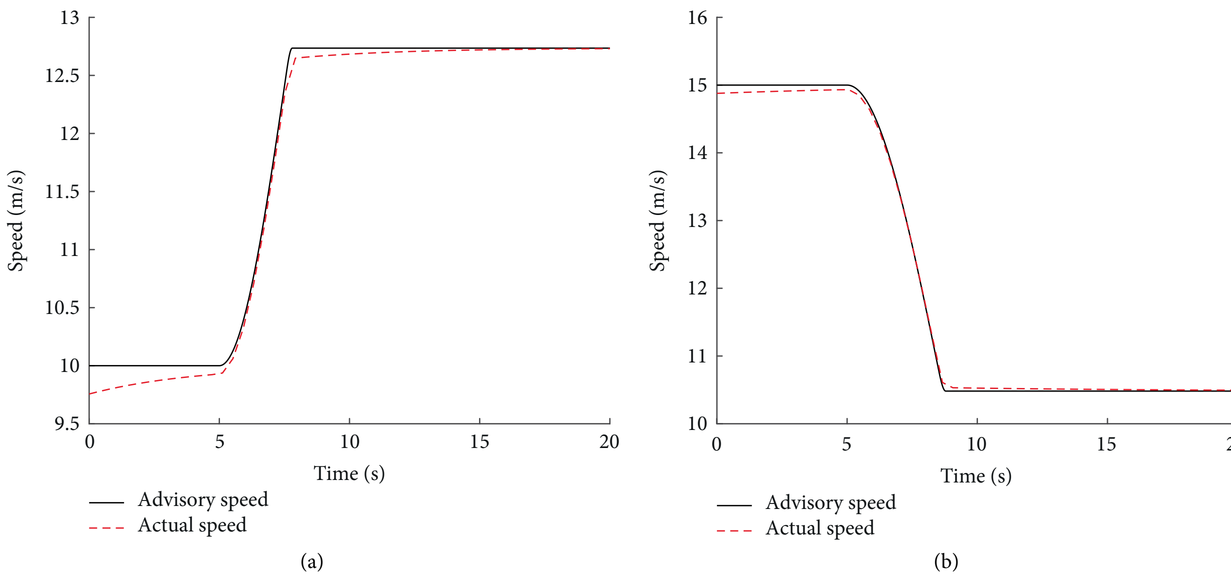


Figure 7: Driver response to optimal advisory speed: (a) acceleration and (b) deceleration.

The first condition is the distance compensation constraint mentioned above. The second and third conditions restrain the rate of change of acceleration and deceleration. According to economical connected-vehicle fuel consumption, in the fourth condition, fuel consumption is proportional to the speed control time; thus, the shorter the speed control time, the lower the fuel consumption. In trigonometric speed-guided expressions, variable s controls the speed-controlled completion; therefore, it must be qualified to the maximum value possible. In four constraints, there is only a and s unknown. To set s an initial value, use conditional one to obtain a, and then use the following three conditions to obtain the best solution.

3.2. Trajectory Planning of the Connected Vehicle. Many previous studies have focused on optimal speed control at a signalized intersection; however, few have considered the problem where a driver of a connected vehicle does not precisely follow the optimal advisory speed. The trajectory of a vehicle can vary enormously from the ideal trajectory because the speed control method is too complicated or impractical; therefore, it is necessary to determine the driver response to the optimal advisory speed. In this study, we use the PID model to simulate the driver's response when following the speed control method. The PID model is practical and simple, but its most important advantage is the clear physical meaning of the model parameters. That is, parameter P represents the time delay of the driver's behaviour, parameter I represents the accumulation of past errors, and parameter D represents the prediction of future errors. When the advisory speed is given, the driver pays more attention to the current difference; thus, we consider it a closed-loop model. Moreover, in order to simply the problem, we exclude the impact of the prediction of future errors and only consider the time delay and past errors. The diagram of the Simulink program in MATLAB is shown in Figure 6. Figure 7(a) shows the acceleration driver response, where the remaining green time is 20 s and the current velocity is 10 m/s. Figure 7(b) shows the deceleration driver response, where the remaining red light is 20 s and the current velocity is 15 m/s.

3.3. Trajectory Planning of the Normal Vehicle. The special conditions described by the co-existence of connected and normal vehicles in the traffic flow will continue for a long time in the foreseeable future. Therefore, it is necessary to improve the speed control algorithm to ensure that connected vehicles pass through the intersection without idling and normal vehicles follow connected vehicles through the intersection as much as possible. It is assumed here that the front vehicle of two vehicles in traffic flow is a connected vehicle and the rear vehicle is a normal vehicle. A mathematical model is established to describe their trajectory. Due to the trigonometric guidance method applied to the front vehicle, the driving trajectory model of the connected vehicle is obtained by the indefinite integral of the expression of the speed control function, as follows:

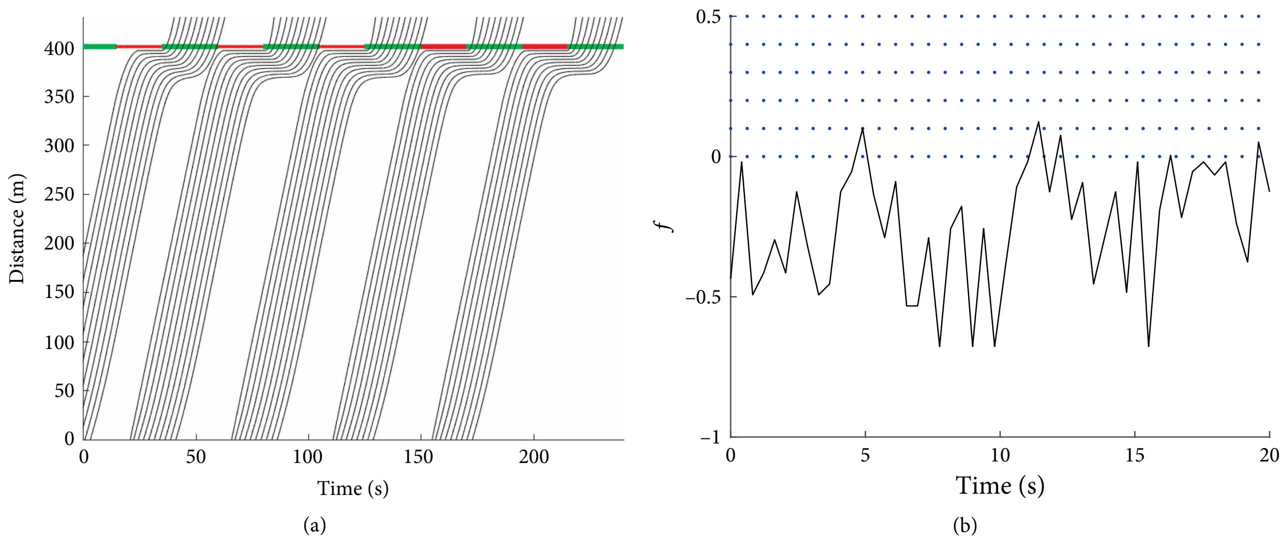


FIGURE 8: Effect of a connected vehicle penetration rate of 0% on (a) the spatial-temporal trajectories for normal (black) and connected (red) vehicles and the stability index of the mixed-vehicle queue.

TABLE 2: Parameters used in simulation of the MVTPM.

Basic parameters	Value
Speed control area length	400 m
Maximum speed limit of the section	72 km/h
Minimum speed limit of the section	36 km/h
Vehicle initial speed	54.0–64.8 km/h
Green light time	35 s
Red light time	25 s
Connected vehicle market penetration rate	0–100%
Maximum acceleration of connected vehicle	$2\,m/s^2$
Maximum deceleration of connected vehicle	$-2\,m/s^2$
Maximum acceleration of normal vehicle	$4\,m/s^2$
Maximum deceleration of normal vehicle	$-4\,m/s^2$
Following car response time	2.5 s

$$d = \begin{cases} d_1 = v_h t - \frac{v_d}{s}\sin(st) & t = 0 \text{ to } \frac{\pi}{2s} \\ d_2 = v_h t - \frac{s v_d}{a^2}\sin a\left(t - \frac{\pi}{2s} + \frac{\pi}{2a}\right) & \\ +vd\left(\frac{s}{a^2} + \frac{1}{s}\right)t & t = \frac{\pi}{2s} \text{ to } \left(\frac{\pi}{2a} + \frac{\pi}{2s}\right) \\ d_3 = v_h t + v_d \frac{s}{a} t & \\ +v_d\left(\frac{s}{a^2} - \frac{1}{s} - \frac{\pi s}{2a^2} - \frac{\pi}{2a}\right) & t = \left(\frac{\pi}{2a} + \frac{\pi}{2s}\right) \text{ to } \frac{d}{v_a}. \end{cases} \tag{23}$$

When the normal vehicle enters the control area, the instantaneous speed of the normal vehicle is v, the distance is zero, and the entering time is t. At the same time, the instantaneous velocity of the connected vehicle is $v_h - v_d\cos(st)$ and the distance is $v_h t - (v_d/s)\sin(st)$. The following form is established in the IDM expression:

$$\mathrm{a} = a_{\max}\left[1 - \left(\frac{v_c}{v_{\max}}\right)^4 - \left(\frac{s_0 + \max v_c t + v_c\left(v_h - v_d\cos(st)\right)/2\sqrt{a_{\max} b_{dec}}}{v_h t - (v_d/s)\sin(st)}\right)^2\right]. \tag{24}$$

The velocity and position of the rear car is as follows [31]:

$$v_{n+1}(t + \Delta t) = v_n(t) + \frac{1}{2}\left(a_{n+1}(t) + a_{n+1}(t + \Delta t)\right)\Delta t, \tag{25}$$

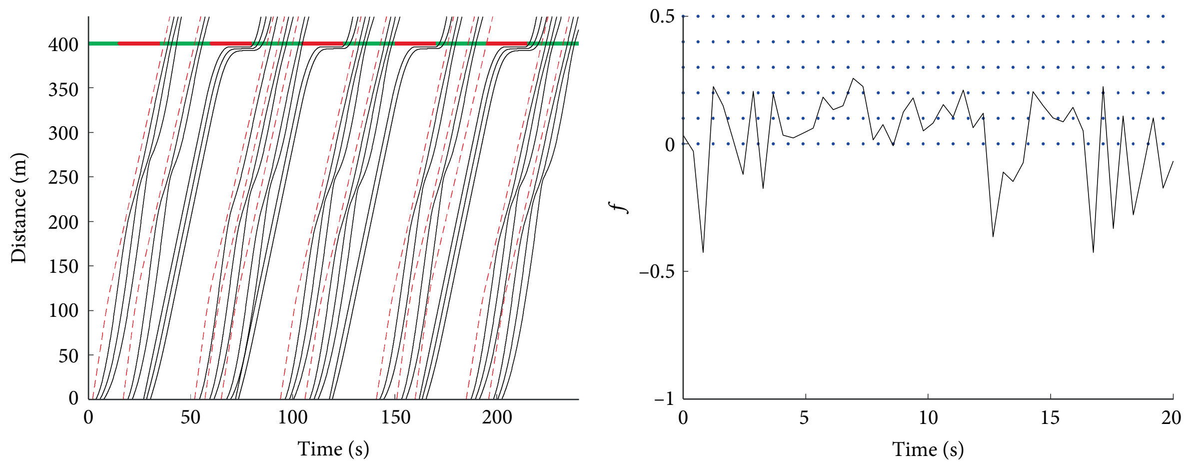
$$d_{n+1}(t + \Delta t) = d_{n+1}(t) + v_{n+1}(t)\Delta t + \frac{a_{n+1}(t)(\Delta t)^2}{2}. \tag{26}$$

4. Simulation Evaluation

In this section, we conduct numerical experiments to illustrate the efficacy of the proposed MVTPM at a hypothetical intersection. The MVTPM was evaluated with a one factor sensitivity analysis using connected vehicle market penetration rates (MPRs). We assume that approximately 48 vehicles pass through the intersection and calculate the fuel consumption and emission of these vehicles using VT-Micro. The simulation parameters as shown in Table 2.

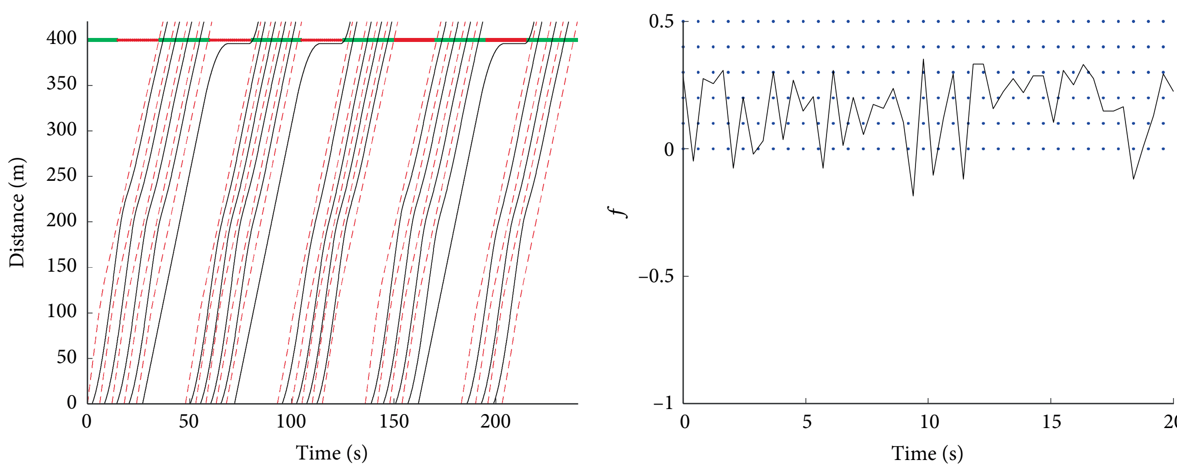
4.1. Trajectory Simulation. Figures 8–12 show the spatial-temporal trajectories for normal (black) and connected (red) vehicles with different connected vehicle market penetration rates and the stability index of the mixed-vehicle queue. Note that, at a lower penetration rate, many normal vehicles need to stop at the signalized intersection when the traffic light is red because there are not enough connected vehicles to guide them, leading to long waiting times and instability of the mixed-vehicle queue. However, with increasing market penetration of connected vehicles, the length of the queue is progressively shortened and the stability index increases to above zero.

4.2. Fuel Calculation. The proposed method is evaluated using the different MPRs, using the fuel consumption and concentrations of NO_x, HC, CO_2, and CO as measures of effectiveness. Table 3 confirms that the proposed MVTPM can optimize both objectives; i.e., reduce fuel consumption and vehicle emissions. Moreover, an increasing number of connected vehicles leads to further decreases in fuel consumption and emissions.



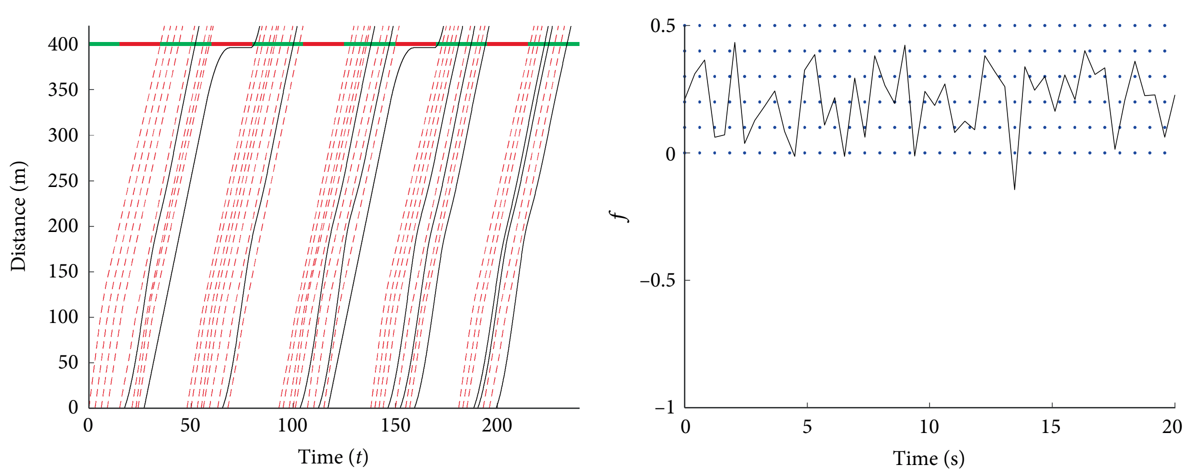
(a) (b)

Figure 9: Effect of a connected vehicle penetration rate of 25% on (a) the spatial-temporal trajectories for normal (black) and connected (red) vehicles and the stability index of the mixed-vehicle queue.



(a) (b)

Figure 10: Effect of a connected vehicle penetration rate of 50% on (a) the spatial-temporal trajectories for normal (black) and connected (red) vehicles and the stability index of the mixed-vehicle queue.



(a) (b)

Figure 11: Effect of a connected vehicle penetration rate of 75% on (a) the spatial-temporal trajectories for normal (black) and connected (red) vehicles and the stability index of the mixed-vehicle queue.

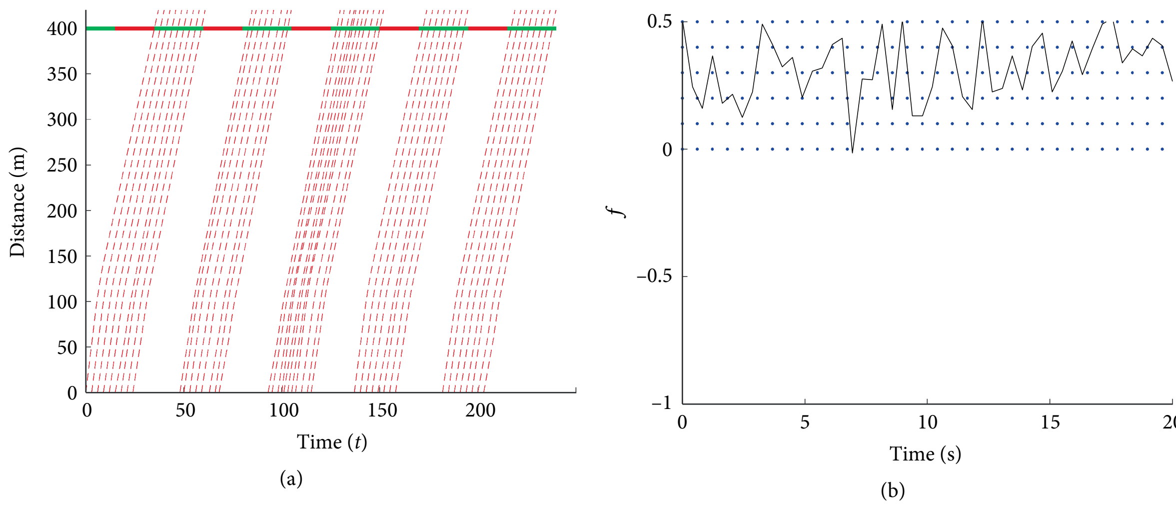


FIGURE 12: Effect of a connected vehicle penetration rate of 100% on (a) the spatial-temporal trajectories for normal (black) and connected (red) vehicles and the stability index of the mixed-vehicle queue.

TABLE 3: Fuel consumption and emissions of different MPRs.

Penetration rate	Fuel (L)	NO_x (mg)	HC (mg)	CO_2 (g)	CO (mg)
0%	2.88	4.70	2.59	6.89	36.31
25%	2.64	4.16	2.36	6.30	33.23
50%	2.39	3.63	2.13	5.72	30.16
75%	2.15	3.09	1.91	5.13	27.1
100%	1.9	2.55	1.68	4.54	24.02

5. Conclusion

This study analyzes the mechanism of traffic oscillations at signalized intersections from the perspective of string stability; then proposes a mixed-vehicle trajectory planning method using the triangular-function method to optimize the speed of connected vehicles in the mixed traffic flow. Moreover, the PID controller is used to simulate the trajectory of connected vehicles; and the IDM car-following model is used to optimize the speed of normal vehicles. MATLAB simulations reveal that the proposed trajectory planning method successfully reduces fuel consumption and emissions for the vehicles passing through a signalized intersection. When the market penetration rates of connected vehicles is 100%, the fuel consumption and NO_x, HC, CO_2, and CO concentrations decrease by 34%, 45.7%, 35.1%, 34.1%, and 33.9%, respectively. Future research should focus on improving fuel consumption, vehicle emissions, and vehicle passing efficiency at continuous signalized intersections in urban areas.

Data Availability

This research is all based on the simulation of the MATLAB, no data were used to support this study.

Conflicts of Interest

The authors declare that there is no conflicts of interest regarding the publication of this paper.

Authors' Contributions

The authors confirm contribution to the paper as follows: study conception and design: Shan Fang, Lan Yang; analysis and interpretation of results: Shan Fang, Shoucai Jing; draft manuscript preparation: Shan Fang, Tianqi Wang. All authors reviewed the results and approved the final version of the manuscript.

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