Research Article

Optimization of Aircraft Climb Trajectory considering Environmental Impact under RTA Constraints

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In order to realize the concept of air traffic sustainable operation, taking the aircraft climbing stage as an example, firstly, we establish the vertical trajectory model of aircraft climbing, analyze the change rule of aircraft performance parameters under different indicated airspeed, and establish the RTA and RHA constraint models according to the waypoint constraints. Then, considering the fuel economy and the greenhouse effect of pollutant emission, we establish a multiobjective model of aircraft flight parameter optimization, and, based on the multiobjective genetic algorithm, we establish an optimization model. Finally, we use B737-800 aircraft to carry out simulation experiments and find that, with the change of speed, fuel consumption and warming trend are different, and “objective weight, aircraft mass, flight distance, RTA time window, and wind” have different effects on the optimization results. The results show that this optimization method has a good compromise between fuel consumption and greenhouse effect by changing the weighting factor. By optimizing the flight parameters of the aircraft, it can effectively reduce the impact on the environment and provide theoretical support for the green flight of the aircraft.

1. Introduction

The climbing stage is an important stage in the flight of the aircraft, and the speed, flight time, fuel consumption, and climbing mode in the climbing flight performance parameters have important influence on the whole flight stage. Therefore, the research on the climbing stage of aircraft can clarify the mechanism of influence on aircraft climbing performance and provide theoretical support for optimizing aircraft fuel efficiency and improving environmental friendliness. In terms of departure trajectory optimization, Ho-Huu et al. [1] introduced a two-stage optimization framework to deal with the optimization and selection of aircraft departure trajectories as well as the flight allocation among these routes, so as to minimize the accumulated noise and fuel consumption. Visser et al. [2] developed an airport abatement tool for aircraft take-off noise based on noise models, geographic information systems, and dynamic trajectory optimization algorithms and validated it at Amsterdam Airport Schiphol using B737-300 aircraft. Prats

et al. [3] present a strategy for designing noise abatement procedures aimed at reducing the global annoyance perceived by the population living around the airports. A nonlinear multiobjective optimal control problem is implemented and numerically solved obtaining minimal annoyance trajectories. Torres et al. [4] comprehensively consider the impact of noise and NO_x and CO₂ emissions on the environment, establish a nonlinear multiobjective optimization, and solve to get the optimal departure trajectory of the aircraft. Visser et al. [5] proposed a multiphase/multicriteria trajectory optimization framework for city-pair, so that the aircraft can fly to optimum flight paths with the lowest possible noise and emissions. In Ho-Huu et al.'s work [6], in order to mitigate aircrafts negative impacts (e.g., noise and pollutant emission) on near-airport communities, a multiobjective optimization model involving noise and fuel consumption was proposed, and a multiobjective evolutionary algorithm based on decomposition (MOEA/D) was developed to solve the problem and optimize the departure trajectory of the aircraft. Then, they present a new

multiobjective optimization formulation for the design and allocation of optimal aircraft departure routes. In addition to the two conventional objectives of noise and fuel consumption, a new objective of aircraft flight frequency is introduced and solved by MOEA/D optimization algorithm. Finally, the reliability and applicability of the model are verified by an example [7].

In terms of approach trajectory optimization, Sang et al. [8] adopted the idea of Continuous Descent Operation (CDO) and the strategy of adjusting Top of Descent (TOD) to optimize the vertical flight trajectory of the aircraft for the approach of descent phase of the aircraft and finally realized the goal of reducing fuel consumption of the aircraft. In terms of cruise trajectory optimization, Jensen et al. [9] quantitatively analyzed the influence of different flight levels on flight fuel and optimized the flight level to reduce the fuel consumption of aircraft. Garcia et al. [10, 11] used the automatic control optimization algorithm to realize the optimization solution of aircraft fuel-saving trajectory. Four optimization methods were compared, and then two methods with better optimization performance were studied in depth; it was pointed out that the pseudospectral collocation methods could be used to solve the optimized trajectory quickly and effectively. In terms of meteorological factors, Patron et al. [12] studied and modeled the mechanism of the influence of wind on aircraft fuel consumption by combining horizontal and vertical tracks. In terms of combination optimization, Mendoza et al. [13, 14] studied the realization method of the optimal trajectory of aircraft. First, cost index was introduced to optimize the vertical fuel-saving trajectory by finding the optimal combination of altitude, surface speed, and Mach number. Then the dynamic weight graph is used to find the economic reference trajectories of the aircraft in horizontal and vertical directions. In terms of environmental protection, Rosenow et al. [15] analyzed the factors influencing the generation of contrails and studied the impact of contrails on global warming and then optimized the aircraft trajectory by reducing the generation of contrails. The above studies mainly optimize the flight trajectory of aircraft from the perspective of fuel economy, without considering the impact of airspace operation restrictions or the impact of engine exhaust on the environment.

The concept of required time of arrival (RTA) in trajectory based operation (TBO) can well solve the airspace operation limitation. It plans, manages, and optimizes the flight in the whole operation process strategically based on the time management, the information exchange between the air and the ground system, and the ability of the aircraft to accurately plan the flight trajectory in time and space and constructs the collision-free flight path between the aircraft, so as to avoid potential flight conflicts in airspace [16, 17]. Ramon and Adrian et al. [18, 19] studied CDO in the RTA mode and believed that this mode was a potential solution to reduce fuel consumption, pollutant emission, and noise impact in the airport area without reducing the airport capacity, and it was beneficial to allocate RTA before the aircraft began to descend. Higuchi et al. [20] used a three-parameter model to simulate the flight operation of aircraft

and optimized the aircraft in approach according to RTA constraints to improve the operation efficiency of busy airports. Vilardaga et al. [21] carried out quantitative tradeoffs in terms of fuel and time consumption to optimize the aircraft's 4D trajectory under the condition of meeting the constraints of RTA. Alejandro et al. [22] proposed a method that could provide an optimal combination of Mach Numbers for different waypoints to meet the time constraints required for aircraft during flight at a fixed altitude while taking account of aircraft fuel consumption. In the following year, Alejandro et al. [23] proposed an algorithm that satisfies the time of arrival constraint and has an economical vertical reference trajectory. Relative to the flight cost of the reference trajectory, the RTA constraint is satisfied and the flight cost of the optimized trajectory is reduced. However, the study did not take into account the environmental impact of emissions from aircraft engines, which deviates from NextGen's environmental goals.

The impact of aircraft on the environment is mainly the greenhouse effect caused by greenhouse gases emitted by engines, which ultimately affects global temperature changes. How to achieve fuel-efficient flight and reduce the impact of emissions on the environment while meeting the requirements of waypoint coordinated operation is an important part of realizing the new generation of air traffic and transportation system and ICAO Aviation System Block Upgrades (ASBU), as well as a problem that the civil aviation industry is concerned about and has to solve. In order to solve the deficiencies in the above studies, this paper considers the requirements of airspace operation restrictions, sets up RTA and required height of arrival (RHA) constraints on waypoints of aircraft in the climbing phase, and considers the temperature changes caused by aircraft emissions of greenhouse gases. The multiobjective optimization model of aircraft climbing stage under single waypoint constraint is established, and the genetic algorithm is designed to optimize aircraft flight parameters, and the sensitivity analysis of relevant influencing factors is carried out, hoping to provide theoretical support for the concept of green development of air transport.

2. Trajectory Optimization in the Climbing Phase

2.1. Problem Description. Route climb generally refers to the climb process in which an aircraft starts from an altitude of 1500 ft above the airport surface and climbs in a certain way and increases to the specified cruising altitude and cruising speed [24]. During the flight, the aircraft shall meet the requirements of waypoint collaborative constraint issued by the air traffic management system, coordinate the RTA constraint window and RHA constraint window of the aircraft at relevant positions, and construct the conflict-free flight path between the aircraft to avoid potential flight conflicts in the airspace. Therefore, the aircraft needs to plan the flight distance, flight altitude, and flight time in advance. This section first analyzes the departure vertical climb process of the aircraft, including the end of climb and the beginning of cruise level flight. Assuming that the aircraft

climbs according to the same indicated airspeed, the departure physical model of the aircraft is established, and the climb parameters and cruise parameters of the aircraft are calculated. Then, the flight parameters of the aircraft are optimized considering the fuel consumption of the aircraft and the impact of engine exhaust on the environment. Finally, the mathematical model of the optimization objective is described in detail, and the research framework of the problem is shown in Figure 1.

2.2. Vertical Section of Aircraft Climb. Due to the greater flight maneuverability of the aircraft below 10,000 feet, the aircraft is subject to more intervention by air traffic control (ATC), and the flight is less predictable. Therefore, this paper only considers the climbing phase of aircraft above 10,000 feet and the beginning of route cruise. The departure flight under the constraint of single waypoint starts from point A to the top of climb (TOC), and the aircraft climbs at the same indicated airspeed and then at the same speed to point B (with the same altitude). In the whole departure process, only point B has RTA and RHA constraints. As shown in Figure 2, the single waypoint constrained departure model of the aircraft, and R is the total flight range of the aircraft, which is composed of climb range r_c and cruise range r_{cr} .

Here, we introduce the idea of microelement to divide the departure trajectory into n_s segments. The flight parameters of each segment can be determined by the flight parameters of the intermediate trajectory point of the microelement, determined by the horizontal distance and flight height of the aircraft from point A. The horizontal flight distance between the aircraft and point A in the i -th ($i = 1, 2, \dots, n_s$) leg can be expressed as

$$r_{s,i} = \begin{cases} r_{s,i-1} + \Delta r, & r_c < r_{s,i-1} \leq R, \\ r_{s,i-1} + \frac{\Delta h \times (V_{T,i} + V_W)}{\gamma_{c,i}}, & r_{s,i-1} \leq r_c, \end{cases} \quad (1)$$

where i represents the serial number of the current segment; Δr and Δh represent the horizontal flight distance and climb height of the aircraft in a microsegment, respectively (unit: m); $V_{T,i}$ is the true airspeed (unit: kt); V_W is the effective wind speed (unit: kt); $\gamma_{c,i}$ is the climb rate (unit: m/s).

The flight altitude of the aircraft at the middle point of the i -th leg is

$$h_{s,i} = \begin{cases} H_c, & r_c < r_{s,i-1} \leq R, \\ h_{s,i-1} + \Delta h, & r_{s,i-1} \leq r_c. \end{cases} \quad (2)$$

The flight time for the aircraft to reach the middle point of the i -th leg is

$$t_{s,i} = \begin{cases} t_{s,i-1} + \frac{\Delta h}{\gamma_{c,i}}, & r_{s,i-1} \leq r_1, \\ t_{s,i-1} + \frac{\Delta r}{(V_{T,i} + V_{W,mps,i})}, & r_c < r_{s,i-1} \leq R, \end{cases} \quad (3)$$

where $V_{W,mps,i}$ is the wind speed (unit: m/s).

The aircraft uses the maximum continuous thrust to climb a certain altitude. When the flight speed is fixed, the climb range r_c and cruise range r_{cr} can be calculated. Suppose that the flight distance from TOC to point B is short; the fuel consumption of the aircraft in this process can be negligible compared with the total mass of the aircraft, and then the total masses of the aircraft can be approximately equal when the aircraft flies from TOC to point B. Note that i_{cr} is the last trajectory point in the cruise phase, and r_{cr} and r_c can be calculated as follows:

$$\begin{cases} r_c = \sum_{i=i_c+1}^{n_s} t_{s,i} \cdot (V_{T,i} + V_{W,mps,i}), \\ r_{cr} = R - r_c. \end{cases} \quad (4)$$

Δr and Δh are calculated as follows:

$$\begin{cases} \Delta r = \min\{\Delta r_0, r_{cr} - r_{s,i}\}, \\ \Delta h = \min\{\Delta h_0, H_c - h_{s,i}\}, \end{cases} \quad (5)$$

where Δr_0 and Δh_0 are used to represent the preset horizontal flight distance step and climb height step of the aircraft.

In this way, the flight distance, altitude, and time state parameters of the aircraft in the process of climbing under the constraint of single waypoint can be described by the following formulas:

$$\begin{cases} R_s = [r_{s,1} \ r_{s,2} \ \dots \ r_{s,n_s}], \\ H_s = [h_{s,1} \ h_{s,2} \ \dots \ h_{s,n_s}], \\ T_s = [t_{s,1} \ t_{s,2} \ \dots \ t_{s,n_s}]. \end{cases} \quad (6)$$

Single waypoint constraints include RHA constraints and RTA constraints. The RHA constraint of point B in Figure 2 is represented by the end height H_c ; the RTA constraint is presented in the form of time window, which can be represented as

$$T_{S,CON} = [t_{R,max} \ t_{R,min}]^T, \quad (7)$$

where $t_{R,max}$ and $t_{R,min}$ represent the maximum and minimum values of RTA window, respectively.

2.3. Aircraft Climb Parameters. Because the aircraft obeys the law of conservation of energy in the climbing process, we can use the full energy model to model and analyze the climbing process of the aircraft. The work of the external force on the aircraft is equal to the change of the mechanical energy of the aircraft. The forces on the longitudinal axis of the aircraft during the climb mainly include engine thrust T_c and drag D . The calculation formula is as follows:

$$(T_c - D)V_T = m_A g \frac{dh}{dt} + m_A V_T \frac{dV_T}{dt}. \quad (8)$$

The flight path of the aircraft is mainly changed by adjusting the position of the engine thrust throttle and elevator without considering the lifting device. According to (8), it is not difficult to find that any two of the three variables, thrust T_c , velocity V_T , and altitude change rate

FIGURE 1: Problem research framework.

FIGURE 2: Aircraft departure model constrained by single waypoint.

dh/dt , can determine another variable. The current research is generally to calculate the rate of altitude change with given speed and thrust. This is also the case in the climbing section. At this time, fix the throttle position of the aircraft and maintain the flight speed at the same indicated airspeed V_I or Mach number M as shown in the following equation:

$$(T_c - D)V_T = m_A g \frac{dh}{dt} + m_A V_T \left(\frac{dV_T}{dh} \right) \left(\frac{dh}{dt} \right). \quad (9)$$

By separating dh/dt to the left of the equation, we can get

$$\frac{dh}{dt} = \left[\frac{(T_c - D)V_T}{m_A g} \right] f\{M\}, \quad (10)$$

where $f\{M\} = [1 + (V_T/g)(dV_T/dh)]^{-1}$.

It is a function of Mach number M , which represents the ratio of the residual thrust used for climb to the residual thrust used for acceleration when climbing at a given speed. The specific calculation formula can be expressed as follows:

$$f\{M\} = \left\{ 1 + \frac{KRk_t}{2g} M^2 + \left(1 + \frac{K-1}{2} M^2 \right)^{(-1/K-1)} \left[\left(1 + \frac{K-1}{2} M^2 \right)^{(K/K-1)} - 1 \right] \right\}^{-1}. \quad (11)$$

In the process of aircraft climb, for certain climb height and flight distance, different engine thrust T_c , flight speed V_T , and climb rate γ_c will lead to different flight time and fuel consumption. For a given climbing altitude ΔH (m), the flight time t_c (s) and fuel consumption F_c (kg) of the climbing process can be obtained according to the principles of kinematics and dynamics:

$$\begin{cases} t_c = \frac{\Delta H}{\gamma_c}, \\ F_c = F_{f,c} \cdot t_c, \end{cases} \quad (12)$$

where $F_{f,c}$ is the climbing fuel flow (kg/s) and the climbing rate γ_c is calculated as follows:

$$\gamma_c = \frac{(T_c - D) \cdot V_T}{m_A g} f\{M\}. \quad (13)$$

The climbing fuel flow $F_{f,c}$ (kg/s) at maximum thrust is calculated as follows:

$$F_{f,c} = \eta \times Thr = C_{f,1} \cdot \left(1 + \frac{V_T}{C_{f,2}} \right) \cdot T_{\max \text{ climb}}, \quad (14)$$

where $C_{f,1}$ and $C_{f,2}$ are constants related to aircraft type and, for the maximum climb thrust $T_{\max \text{ climb}}$ of aircraft, the reader is referred to [19].

2.4. Aircraft Cruise Parameters. Without considering the turning of the aircraft, the aircraft can be regarded as a process of constant altitude in cruise flight. At this time, the aircraft can be regarded as a point-mass model. The gravity of the aircraft in the vertical axis direction is equal to the lift, and the engine thrust in the longitudinal axis direction is equal to the resistance of the aircraft. We have the following dynamic equations:

$$\begin{cases} m_A g = \frac{1}{2} \rho_H V_{T,mps}^2 S C_L, \\ T_{cr} = D = \frac{1}{2} \rho_H V_{T,mps}^2 S C_D, \end{cases} \quad (15)$$

where m_A is the mass of the aircraft (unit: kg); the acceleration of gravity $g = 9.8 \text{ (m/s}^2\text{)}$; $V_{T,mps}$ is the true speed (unit: m/s); S is the surface area of the wing (unit: m^2); C_L is the lift coefficient; T_{cr} is the engine thrust (unit: N); D is the resistance (unit: N); C_D is the resistance coefficient. C_D is a function of C_L , and the relationship between them is as follows:

$$C_D = C_{D0,CR} + C_{D2,CR} \times C_L^2, \quad (16)$$

where $C_{D0,CR}$ and $C_{D2,CR}$ are parameters related to aircraft type (see [25]).

Assuming that the aircraft flies at the same speed V_I (kt), V_T can be calculated by V_I :

$$\begin{cases} M = \left[\frac{2}{K-1} \left\{ \left[\frac{(1+(K-1/2)(V_I/a_0)^2)^{(K/K-1)} - 1}{\delta} \right] + 1 \right\}^{(K-1/K)} - 1 \right]^{(1/2)}, \\ V_T = M \cdot a_H, \end{cases} \quad (17)$$

where $\delta = P_H/P_0$ is the pressure ratio; sound speed $a_0 = 340.294 \text{ m/s}$.

In the process of cruise flight, for the given flight distance segment ΔR (m), according to the kinematics principle, the cruise flight time t_{cr} (s) and fuel consumption F_{cr} (kg) can be calculated. The specific calculation is as follows:

$$\begin{cases} t_{cr} = \frac{\Delta R}{V_{T,mps} + V_{W,mps}}, \\ F_{cr} = F_{f,cr} \cdot t_{cr}, \end{cases} \quad (18)$$

where $V_{W,mps}$ is the effective wind speed (unit: m/s); $F_{f,cr}$ is the cruise fuel flow (unit: kg/s):

$$F_{f,cr} = C_{f,1} \cdot \left(1 + \frac{V_T}{C_{f,2}} \right) \cdot T_{cr}. \quad (19)$$

3. Multiobjective Optimization Model

3.1. Establishment of Multiobjective Function. According to the research problem, we establish a multiobjective optimization model. The optimization objectives of aircraft climb parameters under single-point constraints include fuel consumption and temperature rise. The optimization model can be expressed as follows:

$$\begin{aligned} \min \quad & \{ \lambda_1 F_s + \lambda_2 T_{s,E} \} \\ \text{s.t.} \quad & \begin{cases} V_{I,\min} \leq V_I \leq V_{I,\max}, \\ h_{s,n_s} = H_{cr}, \\ t_{R,\min} \leq t_{s,n_s} \leq t_{R,\max}, \end{cases} \end{aligned} \quad (20)$$

where λ_1 and λ_2 represent the impact of fuel consumption objective and temperature rise objective on the optimization results.

(1) Fuel consumption calculation

When the flight meets the constraints of RHA and RTA, the fuel consumption F_s in the process of aircraft departure can be expressed as follows:

$$F_s = F_{s,c} + F_{s,cr} = \sum_{i=1}^{i_c} F_{f,c,i} \cdot t_{s,i} + \sum_{i=i_c+1}^{n_s} F_{f,cr,i} \cdot t_{s,i}, \quad (21)$$

where $F_{s,c}$ is the fuel consumption during climbing; $F_{s,cr}$ is the fuel consumption during cruise.

(2) The impact of aircraft flight on climate change

The development of air transport brings about huge economic benefits to the region and also impacts the global environment. According to the report on air activities issued by IPCC, there are five main substances that affect the global atmosphere: CO_2 , NO_x , contrail, water vapor, and cloud. The impact process is shown in Figure 3. The three emissions that have the greatest impact on global temperature change are CO_2 , NO_x , and contrail. Because the formation conditions of contrail are very strict, its impact cycle on temperature change is short, and the impact area is small [26], this paper focuses on the effects of CO_2 and NO_x emissions on the atmosphere, regardless of the role of contrail.

However, because different gases have different radiation properties and the greenhouse effect is not the same after gas emissions, how to evaluate the impact of engine emissions on climate change is particularly important. At present, the

FIGURE 3: Mechanism of the influence of engine emissions on the atmosphere.

main methods to study the impact of engine emissions on climate change are as follows: global warming potential (GWP), global temperature change potential (GTP), radiation intensity, and surface temperature change [27]. However, GWP only represents the integral effect of greenhouse gas radiative forcing, which cannot reflect the impact of short-lived gases on the climate. Compared with GWP, GTP can directly give the ratio of surface temperature change, which has a better evaluation effect. Therefore, it is widely used to characterize the impact of greenhouse gas emissions from aircraft engines on the environment [28]. GTP can be divided into two types: one is absolute pulse global temperature change potential (APGTP); the other is absolute sustained global temperature change potential (ASGTP). Because ASGTP represents the temperature effect of greenhouse gases under the condition of continuous emission, this paper studies the instantaneous emission of greenhouse gases during the climb process of aircraft, so APGTP is selected to characterize the impact of gas emissions on the temperature rise. $T_{AP,x}$ (n/kg) is used for APGTP, and the calculation formula is as follows:

$$T_{AP,x} = \frac{A_x (e^{-T_h/\alpha_x} - e^{-T_h/\tau})}{C (\tau^{-1} - \alpha_x^{-1})}, \quad (22)$$

where x is the type of emission gas (CO_2 and NO_x); A_x is the radiation forcing caused by the emission of 1 kg of greenhouse gas (unit: $\text{W} \cdot \text{m}^{-2} \cdot \text{kg}^{-1}$); T_h is the timescale of the influence of exhaust gas on temperature (unit: year); α_x is gas life cycle (unit: year); τ is the timescale of climate change response (unit: year); C is the environmental heat capacity of the climate system (unit: $\text{year} \cdot \text{W} \cdot \text{m}^{-2} \cdot \text{n}^{-1}$); the absolute values of the three “emissions per kilogram” indicators in different time ranges are given by comparing the climate impacts from well-mixed greenhouse gases and heterogeneous forcing [29, 30]. This paper studies the impact of air emissions on the environment over a period of 20 years.

The impact of greenhouse gas emissions on the environment can be expressed by the temperature rise, and the calculation formula is as follows:

$$\begin{cases} T_{s,E} = \sum_x E_x \cdot T_{AP,x}, \\ E_{\text{CO}_2} = F_s \cdot E_{I,\text{CO}_2}, \\ E_{\text{NO}_x} = F_s \cdot E_{I,\text{NO}_x}, \end{cases} \quad (23)$$

where E_x is the amount of emission. E_{I,CO_2} and E_{I,NO_x} represent the reference emission indexes of CO_2 and NO_x , respectively, where E_{I,NO_x} is obtained by linear interpolation of the emission index provided by the engine manufacturer and the corrected fuel flow in a log-log coordinate system; the actual fuel flow correction method is Boeing Method 2 (BM2). Please refer to [31] for detailed steps. The reference emission index of different emissions is obtained according to the ICAO engine exhaust emission database [32].

3.2. Constraints. Suppose that point B in Figure 2 is the convergence point in the process of aircraft departure. In order to avoid potential flight conflicts in the airspace, the air traffic management system regulates the trajectory of the aircraft by adjusting the flight speed of the aircraft. At this time, the constraint of single waypoint can be expressed as

$$\begin{cases} h_{s,n_i} = H_{cr}, \\ t_{R,\min} \leq t_{s,n_i} \leq t_{R,\max}, \end{cases} \quad (24)$$

where $[t_{R,\min}, t_{R,\max}]$ is the RTA constraint time window. When the aircraft flies according to the given RTA window, the flight speed matrix $V_{I,L}$ under the limitation of time window can be obtained, which is expressed as follows:

$$V_{I,L} = \begin{bmatrix} V_{I,L,\min} \\ V_{I,L,\max} \end{bmatrix}. \quad (25)$$

The flight speed under RTA restriction shall be as follows:

$$\begin{cases} V_{I,L,\min} \leq V_{I,\max}, \\ V_{I,L,\max} \geq V_{I,\min}. \end{cases} \quad (26)$$

4. Solution of Optimization Model

4.1. Fitness Function. According to the contents in Section 3.1, the optimization objectives of aircraft flight parameters include (1) minimum fuel consumption of aircraft and (2) minimum temperature rise. The final optimization objective is to make both objective functions as small as possible. Therefore, the fitness f_s can be set as follows:

$$f_s = \left(w_1 \frac{F_s}{F_{s,r}} + w_2 \frac{T_{s,E}}{T_{s,E,r}} \right)^{-1}, \quad (27)$$

where w_1 and w_2 are the weights of the two optimization objectives, meeting $w_1 + w_2 = 1$; F_s and $T_{s,E}$ refer to the individual fuel consumption and temperature changes in the iterative optimization process; $F_{s,r}$ and $T_{s,E,r}$ are the fuel consumption and temperature rise under reference conditions, respectively.

4.2. Climbing Trajectory Optimization Process. Genetic algorithm (GA) is a computational model to simulate the natural selection and genetic mechanism of Darwinian biological evolution. It is a method to search for the optimal solution by simulating the natural evolution. It was proposed by Professor John H. Holland of the University of Michigan in the United States [33]. GA is used to optimize the climb speed V_I of the aircraft, and binary encoding method with 8-bit encoding length is selected for encoding. Only a gene V_I is included on the chromosome, and the chromosome model is shown in Figure 4.

Set the population size as $N_{GA} = 10$; use roulette method to select parents; the selection probability is 0.5; carry out single-point crossing and mutation operation on the parents after the selection operation, the crossover probability is 0.9, and the mutation probability is 0.1. The individuals with the highest fitness value in the current generation are transferred to the next generation with a transfer probability of 1, which ensures that the optimization algorithm converges quickly and the parents will not get worse after cross mutation. Each generation of the optimization process is recorded as i . When the termination generation L_E is reached, the optimization ends, and the optimal individual, objective function, and other relevant parameters are retained. The optimization model of genetic algorithm is shown in Figure 5.

5. Simulation Case and Analysis

5.1. Simulation Condition Setting. This simulation case uses B737-800 aircraft for optimization and uses the optimization algorithm described above to optimize the climb trajectory of the aircraft. Assume that the aircraft is flying under standard atmospheric conditions, with temperature deviation $\Delta T = 0^\circ\text{C}$, calm wind, flight distance $R = 250$ km, initial mass of 65000 kg, initial climb height of 10000 ft, and TOC of 8400 m. The horizontal flight distance and climb altitude in a microsegment are $\Delta r_0 = 1000$ m, $\Delta h_0 = 1000$ ft, respectively, and the change range of the speed is 200–310 kt. According to this range, the flight time range $[t_{\min}, t_{\max}] = [660, 930]$, and the required arrival time constraint $[t_{R,\min}, t_{R,\max}] = [690, 890]$.

5.2. Simulation Results. First of all, in order to get the influence of flight speed on the optimization target, this paper tests the flight conditions at different speed and calculates the flight parameters and trajectories. The parameters corresponding to the lowest fuel consumption F_s and the corresponding temperature rise $T_{s,E}$ are shown in Table 1.

The influence of flight speed on flight trajectory is shown in Figure 6, and the step V_I is 20 kt. It can be seen from the

FIGURE 4: GA chromosome model with single waypoint constraint.

figure that the larger V_I is, the smoother the climbing slope is, and the shorter the level flight distance is.

When the target fuel consumption is the lowest, that is, $w_1 = 1, w_2 = 0$, the optimization result without RTA time constraint is shown in Figure 7. When the target temperature consumption is the lowest, that is, $w_1 = 1, w_2 = 0$, the optimization result without RTA time constraint is shown in Figure 8.

5.3. Sensitivity Analysis of Influencing Factors. Due to the volatility of the optimization results of genetic algorithm, in the follow-up study of the influencing factors, each case is carried out five times to eliminate the accidental error and find out the general influence rules of different influencing factors on the optimization results.

5.3.1. The Influence of Objective Weight on Optimization Results. In order to find the influence of multiobjective weight coefficient on the optimization results, the change of flight parameters is analyzed by increasing w_1 from 0 to 1 according to Step 0.1. According to Figure 9, with the increase of w_1 , the optimal flight speed of the aircraft decreases from 310 kt to 251 kt, and the fitness also shows a decreasing trend. In phase $w_1 \in [0, 0.6]$, the decreasing trend of adaptability is more significant than that of phase $w_1 \in [0.6, 1]$, and the change trends of fuel consumption and speed of the aircraft are basically the same, but the trend of temperature rise is opposite to that of the other three parameters. This is because the trends of fuel and temperature rise with speed V_I are different. When w_1 increases from 0 to 1, the corresponding fuel consumption will be reduced by about 6%, CO_2 emissions will be reduced by about 6%, NO_x will be reduced by about 17%, and the temperature rise will be increased by about 4%.

5.3.2. Influence of Aircraft Mass on Optimization Results. In order to study the influence of aircraft mass on the optimization results, the initial mass of the aircraft is changed from 60000 kg to 70000 kg, and the change step is 1000 kg to analyze the change of each flight parameter. According to Figure 10, with the increase of aircraft mass, the optimal flight speed of the aircraft increases from around 253 kt to around 280 kt, and the fitness also shows a decreasing trend. The fuel consumption of aircraft is basically consistent with the change trend of temperature rise. This is because, with the increase of mass, the aircraft needs more power to reach the same speed, so as to consume more fuel. At the same time, the emissions of CO_2 and NO_x also increase correspondingly, resulting in the increase of temperature rise. When the initial mass of the aircraft changes from 60000 kg to 70000 kg, the corresponding fuel

FIGURE 5: Genetic algorithm's optimization model.

TABLE I: Parameters corresponding to F_s and $T_{s,E}$.

	V_I (kt)	F_s (kg)	$T_{s,E}$ (10°C–12°C)	E_{NO_x} (kg)	E_{CO_2} (kg)
F_s minimum	251.56	1823.56	3.02	33.32	5680.38
$T_{s,E}$ minimum	309.57	1931.48	2.92	40.69	6016.56

FIGURE 6: Trajectory for different velocity.

FIGURE 7: Optimization results with minimum fuel consumption without RTA time constraint.

FIGURE 8: Continued.

FIGURE 8: Optimization results with minimum temperature rise without RTA time constraint.

FIGURE 9: Influence of objective weight on optimization results.

consumption increases by about 12%, CO_2 emissions increase by about 12%, and NO_x increases by about 26%, resulting in an increase of temperature rise by about 5%.

5.3.3. Influence of Flight Distance on Optimization Results. In order to study the influence of the flight distance on the optimization results, the horizontal flight distance of the

aircraft is changed from 200 km to 300 km, and the change step is 10 km to analyze the change of each flight parameter. According to Figure 11, with the increase of flight distance, the optimal flight speed of the aircraft decreases from 274 kt to 265 kt, and the fitness also shows a decreasing trend. The fuel consumption of aircraft is basically consistent with the change trend of temperature rise. This is because, with the increase of flight distance, the aircraft consumes more fuel,

FIGURE 10: Impact of aircraft mass on optimization results.

FIGURE 11: Continued.

FIGURE 11: Influence of flight distance on optimization results.

FIGURE 12: Effect of $t_{R,max}$ on optimization results.

and the emissions of CO_2 and NO_x also increase correspondingly, resulting in the increase of temperature rise. When the horizontal flight distance of the aircraft changes from 200 km to 300 km, the corresponding fuel consumption increases by about 35%, CO_2 emission increases by about 35%, and NO_x increases by about 24%, resulting in an increase of temperature rise by about 43%.

5.3.4. Influence of RTA Time Window on Optimization Results. In order to study the influence of RTA time window on optimization results, let $w_1 = w_2 = 0.5$, and analyze the

influence of the maximum value of RTA window on the optimization results. Fix $t_{R,min} = 690$ s, and increase $t_{R,max}$ from 710 s to 890 s, with a step of 20 s. According to Figure 12, with the increase of $t_{R,max}$, the fitness of the optimization results also shows a gradual increase trend and tends to be gentle at the end. The trend of fuel consumption of aircraft is basically the same as that of flight speed, showing a downward trend, and the downward trend is obvious in phase $[t_{R,min}, t_{R,max}] = [710, 850]$ s, and then there is an increasing trend. This is mainly because the flight speed of the aircraft is getting closer to the optimal flight speed as the time window moves backward. With the change

FIGURE 13: Effect of $t_{R,min}$ on optimization results.

of RTA time, temperature rise shows an upward trend first and then has a downward trend. This is mainly because the increase of $t_{R,max}$ makes the minimum value of the flight speed satisfy the RTA window increase, which further affects the fuel consumption of the aircraft and the change of the temperature rise, explaining that the change trends of the fuel consumption of the aircraft and the global total temperature are different with the change of the flight speed. With the increase of $t_{R,max}$ from 710s to 890s, the corresponding fuel consumption will be reduced by about 3%, CO_2 emissions will be reduced by about 3%, NO_x will be reduced by about 11%, and the temperature rise will be increased by about 1%.

Next, according to the value of $t_{R,min}$, the impact of optimization results is analyzed. As shown in Figure 13, the minimum value of RTA window has little effect on the optimized flight parameters in interval $t_{R,min} = [700, 850]$ s, but there is a significant reduction in $t_{R,min} = 870$ s. This is because when the RTA time window is satisfied, the larger the minimum value of the RTA window is, the smaller the corresponding maximum flight speed is. When $t_{R,min} = 870$ s, the corresponding maximum flight speed is about 263 kt. In conclusion, the influence of $t_{R,min}$ on each index is not obvious.

5.3.5. The Influence of Wind on Optimization Results. In order to analyze the influence of wind on the optimization

results, the wind speed is changed from -30 kt (upwind) to 30 kt (downwind) during the optimization test, and the change step is 5 kt. Set RTA window: $[t_{R,min}, t_{R,max}] = [690, 890]$ s, and set $w_1 = w_2 = 0.5$. According to Figure 14, with the increase of V_w , the fitness of optimization results shows an increasing trend, while the flight speed shows a decreasing trend. This is because the optimal speed is relatively fixed, and the change of wind speed will lead to the change of true airspeed. In the same way, the fuel consumption of aircraft and the temperature rise show a decreasing trend first and then have an increasing trend. This is mainly because the downwind or headwind will lead to the change of true airspeed under the same other conditions, which will affect the fuel consumption of the aircraft and the emissions of CO_2 and NO_x . When the wind speed changes from -30 kt to 30 kt, the corresponding fuel consumption is reduced by about 4%, CO_2 emissions are reduced by about 4%, NO_x is reduced by about 6%, and the temperature rise is reduced by about 2%.

5.3.6. Warming Index of Each Influencing Factor. Considering the fuel consumption and the corresponding temperature rise, the warming index is defined as the average temperature rise produced by consuming unit fuel. As shown in Figure 15, the warming index corresponding to "objective weight, aircraft mass, horizontal flight distance,

FIGURE 14: Effect of wind on optimization results.

FIGURE 15: Continued.

FIGURE 15: Warming index of each factor.

wind speed, and RTA time” is described. With the increase of w_1 from 0 to 1, the warming index increases by about 10%; with the change of aircraft initial mass from 60000 kg to 70000 kg, the warming index decreases by about 7%; with the change of aircraft horizontal flight distance from 200 km to 300 km, the warming index increases by about 6%; with the change of wind speed from -30 kt to 30 kt, the warming index increases by about 2%; with the change of $t_{R,min}$, the warming index does not change obviously; with the change of $t_{R,max}$ from 710 s at the beginning to 890 s, the warming index increases by 5%.

6. Conclusion

In this paper, the trajectory optimization problem of aircraft in departure climb phase is studied when the waypoint is constrained. At the same time, the temperature rise caused by the minimum fuel consumption and the emissions of CO_2 and NO_x is considered.

Firstly, the climbing trajectory model of the aircraft is established to analyze the flight parameter variation law of the aircraft at different speed; then the multiobjective optimization model is established for the optimization objective and the genetic algorithm is designed to solve it. Finally, the simulation test is carried out according to the optimized parameters. According to the research results, the following conclusions can be obtained:

- (1) In the process of aircraft climb, with the change of speed, the trend of temperature rise caused by fuel consumption and engine exhaust is different, so the temperature rise tradeoff between fuel consumption and exhaust can be realized by adjusting the size of w_1 and w_2 . When w_1 increases from 0 to 1, the corresponding fuel consumption will be reduced by about 6%, CO_2 emissions will be reduced by about 6%, NO_x will be reduced by about 17%, and the temperature rise will be increased by about 4%.
- (2) With the increase of aircraft mass, the fuel consumption and the temperature rise brought by engine exhaust are increasing. When the initial mass of the aircraft changes from 60000 kg to 70000 kg, the corresponding fuel consumption increases by about 12%, CO_2 emissions increase by about 12%, and NO_x increases by about 26%, resulting in an increase of temperature rise by about 5%.

- (3) When the horizontal flight distance of the aircraft changes from 200 km to 300 km, the corresponding fuel consumption increases by about 35%, CO_2 emission increases by about 35%, and NO_x increases by about 24%, resulting in an increase of temperature rise by about 43%.
- (4) The increase of the maximum time window of RTA will lead to the decrease of fuel consumption, while the temperature rise brought by exhaust gas shows a trend of first decreasing and then increasing. The minimum value of RTA window has little effect on the optimized flight parameters. With the increase of $t_{R,max}$ from 710s to 890s, the corresponding fuel consumption will be reduced by about 3%, CO_2 emissions will be reduced by about 3%, NO_x will be reduced by about 11%, and the temperature rise will be increased by about 1%.
- (5) Downwind will reduce the temperature rise caused by fuel consumption and engine exhaust, and upwind will bring adverse effects. When the wind speed changes from -30 kt to 30 kt, the corresponding fuel consumption is reduced by about 4%, CO_2 emissions are reduced by about 4%, NO_x is reduced by about 6%, and the temperature rise caused by this is reduced by about 2%.
- (6) The change rule of temperature rising index corresponding to different influencing factors is not the same. The environmental friendliness of the aircraft can be evaluated according to the warming index.

According to the research content and results of this paper, it can provide theoretical support for the realization of aircraft safety and green and coordinated operation in the future. The next step can be further studied from two aspects. First, the aircraft can be analyzed and studied when it is subject to multiple waypoint constraints. Secondly, the algorithm with higher precision is used to solve the model, which can improve the efficiency and precision of the method.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request. Most of the data of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Supplementary Materials

Supplementary materials associated with this article are simulation data and workspace data. (*Supplementary Materials*)

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