

# Research Article **Driver's Anticipation and Memory Driving Car-Following Model**

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We developed a new car-following model to investigate the effects of driver anticipation and driver memory on traffic flow. The changes of headway, relative velocity, and driver memory to the vehicle in front are introduced as factors of driver's anticipation behavior. Linear and nonlinear stability analyses are both applied to study the linear and nonlinear stability conditions of the new model. Through nonlinear analysis a modified Korteweg-de Vries (mKdV) equation was constructed to describe traffic flow near the traffic near the critical point. Numerical simulation shows that the stability of traffic flow can be effectively enhanced by the effect of driver anticipation and memory. The starting and breaking process of vehicles passing through the signalized intersection considering anticipation and driver memory are presented. All results demonstrate that the AMD model exhibit a greater stability as compared to existing car-following models.

## 1. Introduction

How, two successive vehicles interact with each other on a road, has been studied since the 1950s. Milestone carfollowing models include the first version of linear models [1], the first version of nonlinear models [2–4], the carfollowing model based on space headway [5], intelligent driver models (IDM) and its extensions considering both the optimal velocity and space headway [6–10], and some other models [11–24]. Anticipation has been proposed as early as 2006 by Treiber et al.

The Optimal Velocity (OV) model by Bando et al. [25] uses only a few parameters to reveal the complex dynamic characteristics of traffic flow, such as traffic flow instability, traffic congestion, and the formation of stop-and-go waves. It was created to overcome the problem of excessively high acceleration and unrealistic deceleration observed in Newell's model. However, due to its dependency on the following distance, the OV model and many of its derivatives produce not only high decelerations but also unrealistically high accelerations. This even applies to the AMD model, see Figure 1, dotted line (accelerations above  $4 \text{ m/s}^2$  are unrealistic except for 4-wheel-drive sports or electric vehicles).

To overcome the dilemma of unrealistic deceleration in OV model, Helbing and Tilch [26] developed Generalized Force (GF) model in Table 1 by adding velocity difference to the OV model. In the GF model, the optimal velocity function is specified by  $V(s_n(t)) = V_1 + V_2 \tanh (C_1(s_n(t) - lc) - C_2)$ with the optimal parameter values:  $V_1 = 6.75$  m/s,  $V_2 = 7.91$  m/s,  $C_1 = 0.13 \text{ m}^{-1}$ ,  $C_2 = 1.57$ , lc = 5 m and the Heaviside function is unity when the velocity of the leading vehicle is lower than that of the following vehicle, and zero otherwise. Several other extensions of the OV model have been suggested to depict more characteristics of traffic flow by considering the relative velocity between the leading and following vehicles [27-30]. Jiang et al. [31] put forward Full Velocity Differences (FVD) model, which uses both negative and positive velocity differences to handle unreasonable high acceleration rate and deceleration rate in GF model. Ge et al. [32] extend a car-following model by taking into account the relative velocity of leading and following  $\Delta v_{n+1}$  and  $\Delta v_n$  on single lane highway and obtained two velocity difference (TVD) model. These models exclusively depend on the current states between the following vehicle and the leading vehicle at time *t* without taking into account driver anticipation and driver memory. Anticipation has been proposed as early as 2006 [33].



FIGURE 1: Acceleration of unobstructed leading vehicle and its following vehicles in AMD model.

Zheng et al. [34] came up with an anticipation driving (AD) model based on the FVD model for analyzing the effect of driver's response to upstream traffic stimuli and the stability of traffic flow. In the AD model, the optimal velocity is expressed as  $V(s_n(t) + k\Delta v_n(t)) = V_1 + V_2 \tanh (C_1(s_n(t) + k\Delta v_n(t)) - lc) - C_2,$ which is extended from the GF model where the optimal velocity is expressed as  $V(s_n(t))$ . Driver's visual angle was considered by Zhou [35] with an improved velocity model. Peng and Cheng [36] substituted anticipation optimal velocity with optimal velocity to develop an extended model based on FVD model, then analyzed the impact of the anticipation term on traffic flow stability. Tian et al. [37] introduced a velocity anticipation to construct an accident model for avoiding accident under special braking situation. Song et al. [38] improved an optimal velocity model by introducing traffic jerk and full velocity difference. A multi-anticipative model was constructed by to describe the drivers' forecast impact on traffic flow. Kang et al. [33] considered the individual driving style and included forecast and response delay behavior of driver in the car-following model. These models consider only driver anticipation but ignore driver memory.

Zhang [40] established driver's memory by considering human tendency to resist sudden changes of velocity and take into account the velocity in previous and next time. Put forward an extended lattice model of traffic flow with consideration of driver's memory. Yu and Shi [41] derived an improved car-following model to study the effects of multiple velocity difference changes with short-term driver memory on the stability and fuel economy of traffic flow based on FVD model as an effective factor on driver's anticipation behavior.

Table 1 lists an optimal velocity family of car-following models. Based on the aforementioned review, it is clear that driver anticipation and driver memory have not been both taken into account in existing car-following models. Driver memory of previous traffic information may have substantial influence on driver's car-following behavior. In this study we propose a new model, namely, the driver's anticipation and memory driving (AMD) car-following model to consider the effect of both driver memory and driver anticipation.

The remainder of the paper is organized as follows. In Section 2, we present a new car-following model. In Section 3, linear stability analysis of the new model is conducted to study the existence and stability of traveling wave solutions using analytical method. In Section 4 nonlinear stability analysis is conducted. In Section 5 we carry out numerical simulations of the new car-following model for different scenarios. Concluding remarks are given in Section 6.

#### 2. A New Car-Following Model

Based on the AD model, we propose the following anticipation-memory driving (AMD) model

$$\frac{dv_n(t)}{dt} = a\{V(s_n(t) + k\Delta v_n(t)) + \beta[V(s_n(t-m)) - v_n(t-m)] - v_n(t)\} + \lambda\Delta v_n(t),$$
(1)

where k is the forecast time step;  $k\Delta v_n(t)$  is the difference between the estimated future space headway for a time horizon k and the actual space headway;  $\beta$  is a dimensionless parameter describing the sensitivity of driver memory for the previous traffic information; m is the memory step with a unit in second;  $V(s_n(t-m))$  is the optimal velocity at previous time t - m;  $v_n(t - m)$  is the actual velocity at previous time t - m. The newly introduced term to the AD model is  $\beta[V(s_n(t-m))-v_n(t-m)]$ , a driver memory term expressed in terms of the proportional difference between the optimal velocity and the actual velocity at previous time t - m. Although there may be different function forms and different influencing factors to reflect the way that drivers' car-following behavior is affected by driver's memory, here we only adopt a linear function for its simplicity.

Clearly, the driver memory term plays the role of feedback for the car-following behavior. Different drivers may exhibit different values of  $\beta$  and the same driver may exhibit different

Model	Equations	β	k(s)	$\lambda(s^{-1})$	References
Optimal velocity (OV) model	$dv_n(t)/dt = a[V(s_n(t)) - v_n(t)]$		I		Bando et al. [25]
Generalized-force (GF) model	$d\nu_n(t)/dt = a[V(s_n(t)) - \nu_n(t)] + \lambda\Delta\nu_n(t)H(-\Delta\nu_n(t))$	I	I	0.5	Helbing and Tilch [26]
Full-velocity-differences (FVD) model	$dv_n(t)/dt = a[V(s_n(t)) - v_n(t)] + \lambda \Delta v_n(t)$		I	0.5	Jiang et al. [31]
Two-velocity-differences (TVD) model	$d\nu_n(t)/dt = a[V(s_n(t)) - \nu_n(t)] + \lambda[p\Delta\nu_n(t) - (1-p)\Delta\nu_{n+1}(t)]$	Ι	I	0.5	Ge et al. [32]
Anticipation-driving (AD) model	$d\nu_n(t)/dt = a\{V[s_n(t) + k\Delta\nu_n(t)] - \nu_n(t)\} + \lambda\Delta\nu_n(t)$ $V(s_n(t)) = V_1 + V_2 \tanh(C_1s_n(t) - lc) - C_2$	I	0.1, 0.2, 0.3	0.5	Zheng et al. [34]
Anticipation-memory- driving (AMD) model	$dv_n(t)/dt = a\{V(s_n(t) + k\Delta v_n(t)) + \beta[V(s_n(t-m)) - v_n(t-m)] - v_n(t)\} + \lambda\Delta v_n(t)$	0.1	0.1, 0.2, 0.3	0.5	This paper

TABLE 1: Optimal Velocity family of car-following models as a special case of AMD model.

values of  $\beta$  at different time. When  $\beta = 0, k = 0, \lambda = 0$ , the AMD model degrades to the OV model [25]. When  $\beta = 0, k = 0$ , the AMD model degrades to the FVD model [31]. When  $\beta = 0$ , the AMD model degrades to the AD model [34]. Therefore, the OV model, FVD model and AD model are all special cases of the AMD model.

#### 3. Linear Stability Analysis

First carried out the linear stability analysis of the GHR model proposed by Bando et al. [25]. Applied an analytical analysis of the stability of the multi-regime car-following model through a numerical simulation. Details of the stability analysis method for a general car-following model are given by and reference there-in. In this section the linear stability analysis is applied to the new car-following model. We derive the stability conditions of the AMD model and investigate the conditions influencing the long-wave length instabilities of traffic flow.

Suppose that all vehicles are distributed with an identical space headway b (i.e., a constant) and move uniformly with the optimal velocity V(b). The steady-state solution of (1) without traffic jam can be written as

$$x_n^0(t) = bn + V(b)t,$$
 (2)

where *b* is the headway defined by b = L/N, *N*, and *L* are the total numbers of cars and the road length, respectively.

Assume that  $y_n(t)$  stands for a small perturbation from the steady state  $x_n^0(t)$ . we have

$$x_n(t) = x_n^0(t) + y_n(t).$$
 (3)

Substitute (2) and (3) into (1). Make a Taylor expansion of the variables. Neglect the higher order terms and  $y_n(t)$ . We have

$$y_n'' = a \{ V'(b) \Delta y_n + kV'(b) \Delta y_n' + \beta [V'(b) \Delta y_n'(t-m) - y_n(t-m)] - y_n' \} + \lambda \Delta y_n',$$
(4)

where  $V'(b) = dV(\Delta x_n)/d\Delta x_n\|_{\Delta x_n=b}$ ,  $\Delta y_n(t) = y_{n+1}(t) - y_n(t)$ , and  $s_n = \Delta x_n$ .

According to the method of polar perturbation expanding  $y_n(t)$  into a Fourier series as an orthonormal set, i.e.,  $y_n(t) \propto e^{(i\alpha_k n+zt)}$ , (4) can be rewritten in terms of z.

$$(1 - a\beta m)z^{2} + (a + a\beta - akV'(b)(\exp(ik) - 1) + \beta amV'(b) \cdot (\exp(ik) - 1) - \lambda(\exp(ik) - 1))z - (1 + \beta)aV'(b)(\exp(ik) - 1).$$
(5)

Equation (5) is a polynomial and its root can be found by the method of zeros. Expanding  $z = z_1(ik) + z_2(ik)^2 + \cdots$  into (4), the first and second order terms of *ik* are, respectively, deduced as follows.

$$z_{1} = V'(b),$$

$$z_{2} = \left(\frac{2akV'(b) - 2\beta amV'(b) + 2\lambda + (1+\beta)a}{2a(1+\beta)}\right)V'(b)$$

$$-\frac{(1 - a\beta m)(V'(b))}{a(1+\beta)},$$
(6)

The uniform flow will remain stable provided that  $z_2$  is a positive value. Otherwise, the uniformly steady-state traffic



FIGURE 2: The neutral stability line in the headway-sensitivity space.

flow becomes unstable. Thus the neutral stability curve is given by.

$$V'(b) = \frac{\lambda}{((1 - 2a\beta m) + ak)} + \frac{(1 + \beta)a}{2((1 - 2\beta am) + ak)}.$$
 (7)

Consequently, the stable traffic flow is derived as

$$V'(b) < \frac{\lambda}{((1 - 2a\beta m) + ak)} + \frac{(1 + \beta)a}{2((1 - 2\beta am) + ak)}.$$
 (8)

As  $\beta = 0, k = 0$ , the stable condition of the AMD model degrades to the stable condition of the FVD model.

Figure 2 shows the neutral stable curves in the headway-sensitivity space  $(\Delta x, a)$  for the FVD model AD and the AMD model with  $\beta = 0.1, m = 1, \lambda = 0.5, k = 0.1$  The neutral stability curves for the FVD model, AD model and the AMD model are indicated in Figure 2. In this figure the existing apex denotes the critical point  $(h_c, a_c)$  and  $h_c$  is the critical headway. The area below the neutral stability line shows the unstable region where density waves appear in traffic flow. The region above the neutral stability line corresponds to stable traffic. Clearly, the stable region of the AMD model is bigger than that of the FVD model and AD model because the critical points of the AMD model are significantly below those of the FVD and AD model. This plot indicates that the region of stability increases by considering leading vehicle's movement at the previous moment.

#### 4. Numerical Study

In this section we conduct numerical analysis of traffic phenomenon using different car-following models. The focus is on how the driver's anticipation and driver's memory simultaneously affect the following vehicle's velocity and acceleration. Here, we presume all vehicles being identical. In all scenarios a = 0.41,  $\beta = 0.1$ ,  $\lambda = 0.5$ ,  $V_1 = 6.75$  m/s,  $V_2 = 7.91$  m/s,  $C_1 = 0.13$ m<sup>-1</sup>,  $C_2 = 1.57$ , lc = 5m Also, three hypothetical



FIGURE 3: A platoon of eleven vehicles proceed as the traffic light turns from red to green.

values 0.1, 0.2, and 0.3 are used for parameter *k* and 1, 2, and 3 used for parameter m. In reality, the actual value of *k* can be obtained from calibration of observed traffic data.

4.1. *The Start-Up Process.* To explore the simultaneous effect of the driver's anticipation and memory on traffic behavior during the starting process, the same scenario as in Jiang et al. [31] is set up here in Figure 3.

Suppose that eleven cars are waiting in front of a red traffic light and with identical space headway of 7.4 m. Each car's velocity is zero at time t < 0. At time t = 0s the traffic light turns from red to green and all cars begin to move. First the leading car starts and then following cars move gradually. Consider a leading and a following pair of vehicles. The velocity of the leader is defined by  $v_{\text{leader}} = v_0(t)$ . The follower duplicates the leader's velocity but with some delay time  $v_{\text{follower}} = v_0(t - \delta t)$ , in which  $\delta t$  is the delay time of vehicle motion. From the time delay of vehicle motion, we can further estimate the kinematic wave speed (i.e., disturbance propagation speed) at jam density,  $c_p$ , which is defined by  $c_i = 7.4/\delta t$ .

Based on the AMD model, we simulate the motion of eleven vehicles with the parameters m = 1s,  $\beta = 0.1$ . The simulation results are shown in Figures 4(a)-4(d) and Table 2.

A straightforward observation is that the following vehicles can duplicate the behavior of the velocities of the leading vehicles but with some delay time  $\delta t$ .

Del Castillo and Benitez [42] observed that the kinematic wave speed  $c_j$  is between 17 and 23 km/h. Bando et al. [5] observed empirically that  $\delta t$  is of the order of 1 *s*. According to Table 2, these two parameters of the AMD model are, respectively,  $\delta t = 1.2$  s and  $c_j = 20.98$  km/h, both falling into the typical range of empirical observation.

From Figure 4 and Table 2, we can further compare the delay time  $\delta t$  and the kinematic wave speed  $c_j$  of the AMD model with those of the other car-following models. Clearly,  $\delta t$  and  $c_j$  are respectively shorter and higher than those of the OV model, FVD model, and AD model. It means that the delay time can reduce and increase the start-up velocity by considering space headway at next moment and previous traffic information, which contributes to an increased transportation capacity and efficiency in intersections.

Figure 1 depicts the start-up acceleration process of the 1<sup>st</sup> to the 11<sup>th</sup> vehicles according to the AMD model. The leading vehicle, the first following vehicle, and other following vehicles are displayed by a dashed red line, a dashed blue line, and solid blue lines, respectively. The acceleration of the leading and following vehicles falls into a range of the empirical acceleration (0, 4) m/s<sup>-2</sup> observed by Helbing and Tilch [26]. The maximum value of acceleration of our model is also lower than

that of the AD model [31]. It means that the driver starts up earlier when the traffic light changes to green without having a higher acceleration due to considering space headway at next moment and previous vehicle motion. That is, vehicles behaving according to the AMD model accelerate more quickly than the AD model, but do not generate an unrealistic high acceleration observed in the OV model.

4.2. Traffic Flow Evolution with an Initial Small Perturbation. In this subsection we investigate the effect of an initial small perturbation to traffic flow according to the AMD model. Suppose N = 100 vehicles are uniformly running on a circuit road with a length of L = 1500 m under a periodic boundary condition as shown in Figure 5. The initial state is set in (9).

We first explore the impact of driver anticipation and memory on traffic flow stability with an initial small perturbation.

$$x_{1}(0) = 1m,$$
  

$$x_{n}(0) = \frac{n-1}{N} \qquad n \neq 1, n = 2, 3, 4...100,$$
  

$$v_{n}(0) = V\left(\frac{L}{N}\right) = V_{1} + V_{2} \tanh\left(C_{1}\left(\frac{L}{N}\right) - lc\right) - C_{2}.$$
(9)

with the optimal parameter values:  $V_1 = 6.75$  m/s,  $V_2 = 7.91$  m/s,  $C_1 = 0.13$  m<sup>-1</sup>,  $C_2 = 1.57$ , lc = 5 m.

Figure 6 shows the snapshots of velocity distributions of all vehicles simulated according to different car-following models (i.e., different driver behaviors) and different values of m at the time of t = 300 s, t = 1000 s, t = 2000 s, respectively. The OV model exhibits negative velocity at some moments, which is apparently unrealistic.

According to Figure 6, the homogeneous traffic flow evolves to congestion, which corresponds to stop-and-go traffic, as time increases from t = 0 s to t = 2000 s according to the OV, FVD, and AD models. On the contrary, according to the AMD model, the stop-and-go traffic does not appear until very late at t = 2000 s and only becomes visible for AMD model with m = 1 and m = 2. Furthermore, the velocity of the OV, FVD and AD models fluctuates much more widely than that of the AMD model for all times. It illustrates that the effect of driver memory plays an important role in traffic flow stability. It provides a behavioral mechanism for modulating traffic flow fluctuation.

In Figure 6(a) traffic is almost stable around the  $v_0 = 3.95$  m/s at t = 300 s, which is a little less than the initial velocity  $v_0 = 4.669 \text{ (m/s)}$ . It means that by considering space headway at next moments (i.e., driver anticipation) and previous traffic information (i.e., driver memory), drivers end up with lowering their initial velocity a little, to increase the distance to the adjacent vehicle to avoid crash. In Figures 6(b) and 6(c) the velocity fluctuation decreases gradually with the increasing value of *m*, for k = 0.1 The amplitude of the AMD model is minimal when m = 3. The result means that the deviation between the expected velocity and the actual velocity decreases when the memory time *m* increases as feedback for the driver, and a driver can anticipate his expected velocity more realistically according to the real traffic situation with memory of his driving situation during 3 previous moments. Therefore, the traffic stability is improved with the AMD model.



FIGURE 4: Motion of eleven vehicles starting up from a red traffic signal. (a) The OV model, (b) The FVD model, (c) The AD model, and (d) The AMD model.

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Model	a (1/s)	$\lambda$ (1/s)	β	K (S)	<i>ot</i> (s)	$c_j (\text{km/n})$
OV	0.41	_	0	_	2.4	11.1
FVD	0.41	0.5	0	_	1.4	19.03
AD	0.41	0.5	0	0.1	1.34	19.88
AMD	0.41	0.5	0.1	0.1	1.27	20.98

To further investigate that driver anticipation and driver memory both can improve traffic stability; we study the evolution of small perturbation for the AD and AMD models. In Figure 7 the vehicle's velocity is snapshotted at t = 1000 s and t = 2000 s to explore how the forecast time k and driver memory time m affect the traffic stability.

In Figures 7(a) and 7(b) the fluctuation of the AD and AMD models drop with the increasing value of *k* in both models. Figure 7(a) shows that with the increase of *k* in the AMD model, traffic fluctuation dies out at t = 1000 s in comparison with the AD model. Figure 7(b) exhibits that the amplitude of vehicles changes smoothly around the initial velocity  $v_0 = 4.669$  (m/s) by increasing *k* and the velocity of vehicles maintain near  $v_0 = 4.669$  (m/s) at t = 2000 s. From Figure 7(b) we can find that the stability of traffic flow improves most when both driver anticipation and driver memory are considered.



FIGURE 5: A platoon of one hundred vehicles running a circular road with length of 1500 m.

The motion of the vehicles eventually begin to transit from homogenous phase to stop-and-go phase, which form "hysteresis loops" after a sufficiently long time [31]. Here, we



FIGURE 6: Snapshots of velocities of all vehicles according to different car-following models in k = 0.1 at different times. (a) t = 300s, (b) t = 1000 s, and (c) t = 2000 s.



FIGURE 7: Snapshots of velocities of 100 vehicles simulated by AD and AMD models with different values of k (m = 1 in the AMD model). (a) t = 1000s, and (b) t = 2000s.

chose the 30th vehicle as the subject vehicle to compare the hysteresis loop obtained from the AMD model and the AD model. As can be seen from Figure 8, when the congestion becomes stationary after enough time, the motion of 30th vehicle begins to form the "hysteresis loop". It is obvious that the hysteresis loop of the AMD model is much smaller than



FIGURE 8: Hysteresis loops for FVD model and AMD model with m = 1, k = 0.1. (a) The AD model, and (b) The AMD model.



FIGURE 9: Motions of 11 vehicles during the braking process according to the FVD, AD, and AMD models. (a) FVD model, (b) AD model, and (c) AMD model.

that of the AD model. This indicates the stability of traffic flow can greatly enhance by considering the anticipation driving and driving memory. So the AMD model is superior in terms of traffic flow stability to the FVD model.

It can be seen from Figure 8 that the minimum headways (Point *H*) of both models are smaller than the safe headway,

which is 15 m according to Jiang et al. [31]. It means that it takes a longer time to observe the stop-and-go traffic in the AMD model than in the AD model.

4.3. Braking Process. In this subsection traffic arrival process on a single lane roadway with a traffic light using the AMD



FIGURE 10: Acceleration evolutions of single vehicle during braking process.

model is carried out to explore the influences of driving anticipation and driving memory on the following vehicles' velocity and acceleration.

The hypothetical initial conditions are given as follows. When t < 0 the traffic light is green and 11 vehicles are running with a uniform velocity of 4.66 (m/s). All vehicles are distributed uniformly with headway of 15 m. The distance between the platoon leading vehicle and the stopping line is 10 m. The red light is assumed to be a virtual standing vehicle of extension zero at the stopping line as noted in Treiber and Kesting [10]. At time t = 0 the traffic light shifts to red and the platoon-leading vehicle immediately breaks, and the following vehicles duplicate the leading vehicle's velocity with a delay time and begins to slow down gradually. All vehicles finally stop in a column behind the stopping line.

The velocities' evolutions of 11 vehicles during the braking process when the platoon leader start braking (t = 0) till that time all vehicles stop in a column behind the stop line simulated by the FVD, AD, and the AMD models are illustrated in Figures 9(a)–9(c). All following vehicles can duplicate the leading vehicles' velocities but with some delay, and finally stop behind the stopping line. The delays of vehicles' motion simulated by the AMD, AD model, and FVD model are 1.27 s, 1.34 s, and 1.4s respectively. Clearly, the AMD model corresponds to a shorter delay time than the AD model and FVD model.

Figure 10 depicts the simulation of accelerations' evolutions of the 4th and 8th vehicles due to the traffic signal using the FVD model, AD model, and the AMD model, respectively. The acceleration of the leading and following vehicles falls into the ranges of empirical deceleration  $(-3 \text{ m/s}^2, 4 \text{ m/s}^2)$ , which was observed by Helbing and Tilch [26] from real driving behaviors. From Figure 10 we can see that the distance between the AMD model's curves and FVD model become larger when decreasing the number of vehicles. From Figure 10 it can be found that the curves of the AMD model are lower than for those of AD model and FVD model. It shows that by considering driver's negative velocities and excessive acceleration won't appear, and also a driver can start braking process faster and gentler to reduce the velocity, which contributes to the improved safety and fuel consumption.

# 5. Concluding Remarks

Existing car-following model in the literature lack in considering driver's anticipation and driver memory in the same model. For this reason we developed the AMD model and conducted numerical analysis to investigate the evolution of small perturbation in traffic flow according to the AMD model. The results show that driver memory significantly affects the evolution of small perturbation in traffic flow. Considering driver memory, improves the stability of traffic flow. Considering driver memory can increase the safety and the efficiency of traffic operation by optimizing traffic light time at signalized intersections. The results of starting and braking process demonstrate that the AMD model can more successfully anticipate the delay time of vehicle motion and the kinematic wave velocity at jam density.

# Abbreviations

$x_n(t)$ :	The position of the <i>n</i> th vehicle at time <i>t</i> ;
$v_n(t)$ :	The velocity of the <i>n</i> th vehicle at time <i>t</i> ;
$v_n(t-m)$ :	The velocity of the <i>n</i> th vehicle at previous time
	t-m;
$s_n(t)$ :	The space headway between vehicles $n$ and $n + 1$
	at time $t, s_n(t) = \Delta x_n(t) = x_{n+1}(t) - x_n(t);$
$s_n(t-m)$ :	The space headway between vehicles $n$ and $n + 1$
	at time $t - m$ ;
$\Delta v_n(t)$ :	The relative velocity between following and
11	leading vehicle, $\Delta v_n(t) = v_{n+1}(t) - v_n(t);$
$k\Delta v_n(t)$ :	The difference between the estimated future
	space headway for a time horizon $k$ and the
	actual space headway;
$V(\cdot)$ :	The optimal velocity function;
a:	The sensitivity of the driver given by the inverse
	of the delay time of vehicle motion $\tau$ , namely,
	$a = 1/\tau;$
<i>H</i> :	The Heaviside function;
h:	The headway used; in Equation (13);
λ:	The sensitive constant;
<i>p</i> :	The weight;
$\overline{k}$ :	The forecast time;
m:	The memory time;
β:	The sensitivity of driver memory for the
	previous traffic information;
$\delta t$ :	The delay time of vehicle motion;
<i>C</i> <sub>i</sub> :	The kinematic wave speed (i.e., disturbance
,	propagation speed) at jam density, equal to the
	quotient of the headway divided by the delay
	time of vehicle motion $\tau$ ; $c_i = h/\tau$ ;
с:	The propagation speed of the kink wave.
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#### **Data Availability**

The data published in this paper can be available and accessible from the corresponding author upon written official request with a legitimate justification.

# **Additional Points**

*Highlights.* (i) Proposed a driver anticipation and memory driving (AMD) car-following model. (ii) AMD model takes into account the effect of driver memory of previous traffic information. (iii) AMD model exhibits higher stability than many existing car-following model.

## **Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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