Research Article

Stochastic Risk Assessment with a Lagrangian Solution for the Optimal Cost Allocation in High-Speed Rail Networks

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In large-scale high-speed rail networks (HSRNs), the occurrence of occasional malfunctions or accidents is unavoidable. The key issue considered in this study is the optimal allocation of the maintenance costs, based on the stochastic risk assessment for HSRNs. Inspired by the theoretical risk evaluation methods in the complex network, three major factors, including the local effects, global effects, and component self-effects are considered in the process of assessing the impact on the network components (nodes or lines). By introducing the component failure occurrence probability, which is considered to be an exponential function changing with the component maintenance costs, a feasible stochastic risk assessment model of the HSRNs together with the component impact assessment is proposed that can better unify the impact assessment of both the high-speed rail stations and railways. An optimal allocation algorithm based on a Lagrangian relaxation approach is designed. Correspondingly, the optimal cost allocation scheme can be determined using the algorithm to eliminate the various HSRN risks under the given costs. Furthermore, a real-world case study of the HSRNs in eastern China is illustrated. Compared with the genetic algorithm, the simulation shows that the approach can solve the optimal cost allocation problem to more effectively reduce the risks of large-scale HSRNs in practice.

1. Introduction

Currently, high-speed rail (HSR) is considered as one of the prevailing intercity travel modes, providing brand-new and high-quality travel services for various passengers. Noteworthy progress has been made in the construction of HSR infrastructures worldwide [1, 2]. Furthermore, an increasing number of HSR lines are located in the same region, leading to the implementation of large-scale high-speed rail networks (HSRNs), for instance, in Western Europe, Japan, and China [2, 3].

Compared with common rail networks, HSRNs are vulnerable to most stochastic contingencies. In such a large-scale HSRN, certain risks are unavoidable [4–7]. For example, the 7.23 China “Yongwen” railway accident, which

occurred in 2011, was caused by an equipment failure, and it caused severe damage to the regional HSRN [8]. Thus, realizing the risk assessment to ensure the reliability and robustness in HSRNs is still a challenging issue [4, 9, 10], and it is essential to analyze both the research objects and methods.

From the perspective of research objects, the existing research mainly focused on four aspects, namely, the road networks [11–14], rail networks [15–18], aviation networks [19, 20], and urban rail networks [21–23]. Additionally, several other interesting research objects have also been considered, such as the vulnerability of rural road networks [24, 25], resilience of freight transportation networks [26], public transport networks [10, 14, 27], and effectiveness of information network computing [28, 29].

HSRNs, which are special rail transportation networks with high-speed trains mainly for passenger transportation, always consist of numerous mechanical and electronic equipment. For HSRNs, Fiondella et al. [30] took the lead in adopting a game-theoretic risk assessment to study the impact of defending the network lines against their vulnerabilities, and they further explored the optimal assignment of the finite defensive resources to the railways. Subsequently, Zhang et al. [6] performed a detailed research on the structural vulnerability when a network was subjected to two different malicious attacks. Although the risk assessment of the HSRNs is considerably different from that of the other transportation networks, only a few studies have focused on this aspect.

In terms of the mainstream methods, many risk assessment methods for the transportation networks have been presented. In principle, these assessment methods mainly consist of four types, namely, complex network methods [6, 17, 22, 23, 31], linear or integer programming methods [9, 20, 26, 32], sequence of critical nodes or lines [27, 33, 34], and data survey and analysis [35–37]. Furthermore, certain novel studies in this field have been reported. For instance, Cox et al. [38] presented the operational metrics to aid decision makers in rendering more informed judgments pertaining to resource allocation and the design of a portfolio of security and recovery strategies. Zhang et al. [39] proposed an analysis framework for the vulnerability of a network of networks subjected to failures. However, most of the aforementioned methods are partially inapplicable to the risk assessment of the HSRNs due to the specific characteristics of the HSRNs.

The failure probability of each component in a network is significantly different. However, for the sake of feasibility, the prevalent risk assessment methods are inclined to directly remove the target nodes (or lines) from a transportation network instead of considering a stochastic failure probability of a component (node or line). Additionally, for the sake of convenience, these studies concentrated on either the node or line failures to conduct risk assessments based on the classical graph theory, and the nodes and lines in one network as a whole were not considered. In the HSRNs, the nodes and lines are interrelated and interact with each other.

Thus, we attempt to simultaneously discuss the impact assessment of each component in the HSRNs and the risk assessment of the entire network. We effectively combine the risk assessment with the maintenance cost allocation as a united approach. In summary, we try to make the following three contributions to both the risk assessment and safeguard cost allocation scheme in the HSRNs:

- (1) For a new application field (HSRN), on the basis of the specific characteristics of HSR stations and railways, a united risk assessment model is formulated (see Section 3). Introducing the optimization theory and complex network methods, the assessment model better unifies the impact assessment of the nodes and lines.

- (2) Three impact factors in the HSRNs, including the local effects, global effects, and component self-effects, are considered simultaneously to formulate the new impact assessment model, in order to solve the issue more precisely (see Sections 3.1 and 3.2).
- (3) For this HSRN risk assessment model, a comprehensive HSRN maintenance cost allocation scheme can be developed by using a Lagrangian relaxation algorithm designed to solve the assessment model to minimize the network risk (see Section 4).

The remainder of this paper is organized as follows. In Section 2, the major features of the HSRNs, pertaining to the risk assessment are analyzed. Section 3 formulates the risk assessment model for HSRNs, including the impact assessment of the nodes and lines and the risk assessment of the entire HSRNs. Section 4 describes the Lagrangian multiplier optimization algorithm for solving the proposed unified risk assessment model and explains the cost allocation approach. In Section 5, a real-world numerical case is provided to illustrate the proposed methodology. Finally, the conclusion of the research is summarized in Section 6.

2. Problem Statement

Numerous assiduous works have addressed the network risk assessment in recent years. In general, unlike the network risk assessments of aviation, highway, and subway networks, the risk assessment of HSRNs is characterized as being increasingly complex. The network risk assessments of aviation and subway networks are mainly focused on the investigation of the node failures (as the number of line failures is less), while that of the road networks is based mainly on the analysis of the line failures (as the node effects are insignificant). Such a research approach is in full compliance with reality. Due to the features of high speed, high density, high technology, and large scale in the HSRNs, the risk assessment framework of the HSRNs should be different from that of the conventional rail networks [6, 7, 40, 41].

The two elements, namely, stations (nodes) and lines (links) interact with each other in the HSRNs. The station failures affect the normal operation of the adjacent lines; similarly, the line faults, to a certain extent, restrict the normal functioning of the stations on both the ends. In addition, high-speed rail stations and lines are likely to be faulty and suffer from attacks. In other words, the HSRN risk assessment must consider both the stations and the lines. However, the traditional approaches that only consider the influence of either the station or the line fault do not reflect the actual situation. Furthermore, the HSRN risk assessment needs to consider the network topology (star, ring, mesh, etc.). Different network topology structures involve varying degrees of risk; for instance, the risk of the random topology structure is lower than that of the small-world network structure [42–45].

Considering the component (nodes or lines) failures, we should focus more on both the relative position of the component in the network and the importance of the

component. For a specific node, the failure involves three types of influences: (1) local effects: the lines and nodes directly adjacent to the node are affected; (2) global effects: these effects influence the entire HSRNs; and (3) node self-effects: the node itself cannot work properly. However, the overall impact of the line failure is similar to that of the node failure. The former studies on the risk assessment of the traffic network, which focused on either local or global effects, were incomplete and did not conform to the actual situation. For example, if a failure occurs in the Sendai or Nagoya Station in central Japan, the global effects will be greater than the local effects. In contrast, if a failure occurs in the Albany or New Jersey Station in the eastern United States, the local effects will be significantly greater than the global effects.

The heterogeneity needs to be punctiliously considered due to the difference in the scales and levels between the various stations in the HSRNs when analyzing the effects of a node. For example, the Urumqi and Turpan stations located in the Xinjiang province of China are not far from each other; however, the impacts of the Urumqi Station are significantly greater than those of the Turpan Station. However, for simplifying the model, most of the previous investigations inappropriately regarded all the nodes as homogeneous. However, the analysis of the line effects must also take into account the length of a line and other factors. Moreover, the influence of a failure, regardless of the type of influence, in certain major hub stations in the HSRNs, such as in Germany's Frankfurt and Colin stations and France's Paris and Lille stations, can be extremely critical.

Consequently, it is still a realistic challenge to formulate an optimal maintenance cost allocation approach to ensure the safe operation in the HSRNs. A higher maintenance cost for a component (nodes or lines) in a network corresponds to stronger component robustness. Our study addresses how to accurately allocate the limited total maintenance costs to promote the safety and robustness of the HSRNs.

3. Risk Assessment Model for the HSRNs

The proposed HSRN risk assessment model involves two parts: (1) the impact assessment of the HSRNs, including the line between the HSR stations and the connecting stations, and (2) the risk assessment of the entire HSRN. For clarity, we define the influence caused by the failure of the HSR station (or line) as the impact and the failure occurrence probability of the entire HSRN as the network risk.

3.1. Impact Assessment of the HSRNs. The high-speed railway network consists of HSR stations and lines connecting stations. Therefore, it is necessary to establish the corresponding high-speed railway network model first and then make the corresponding impact assessment according to the overall characteristics of its stations and connecting stations.

$G(V, E, A)$ represents an HSRN composed of a limited number of nodes, in which $V = \{v_i | i = 1, 2, \dots, n\}$ represents the node set and $E = \{e_k | k = 1, 2, \dots, m\}$ represents

the line set; v_i represents a node in the network; and e_k represents a line in the network. A represents an adjacent matrix to describe the relationships between the nodes in the network, and $A = \{a_{i,j} | a_{i,j} = 0, 1\}$. If v_i and v_j are directly adjacent, $a_{i,j} = 1$; otherwise, $a_{i,j} = 0$ [46, 47].

3.1.1. Impact Assessment of the HSR Stations. To assess the impact of an HSR station comprehensively and accurately, all the three factors including the local effects, global effects, and component self-effects should be taken into account altogether, as shown in Figure 1. The degree centrality can be adopted to measure the local effects of the stations [48], betweenness centrality is used to measure the global effects [49], and the station grade is used to measure the node self-effects. As indicated in Figure 1, if node 7 fails, the local effects directly influence the nodes 4, 5, 6, 8, and 9, as well as all the HSR lines linked with node 7; the global effects correspond to an indirect influence on all the nodes in the entire network, and the node self-effects lead to the failure of node 7.

D_i^v represents the degree centrality of v_i , which can be calculated using the following equation:

$$D_i^v = \frac{1}{n-1} \sum_{j=1}^n a_{i,j}. \quad (1)$$

$D_i \in [0, 1]$ is a standardized value considering the network scale, with a greater D_i indicating a higher level of degree centrality of v_i . B_i^v represents the betweenness centrality of v_i , which can be calculated using the following equation:

$$B_i^v = \frac{2}{(n-2)(n-1)} \sum_{s=1, s \neq i}^n \sum_{t=1, t \neq i}^n \frac{\delta_{s,t}^i}{\delta_{s,t}}, \quad (2)$$

where $\delta_{s,t}$ represents the total of all the shortest paths $d_{s,t}$ from v_s to v_t and $\delta_{s,t}^i$ is the number of all these shortest paths (among $\delta_{s,t}$ shortest paths) through the given node v_i . The betweenness centrality is the ratio of the shortest paths in all the node pairs through the given node in the network, reflecting the transfer and cohesive function of the given node in the entire network. A greater B_i^v indicates a higher level of betweenness centrality of v_i .

C_i^v represents the self-effects of v_i , with a greater C_i^v indicating a larger scale of v_i . The value of C_i^v can be achieved based on the station class of station v_i , where $c_i^v = \{1, 2, 3, \dots\}$, as shown in the following equation:

$$C_i^v = \frac{c_i^v}{c_{\max}^v}, \quad (3)$$

where $c_{\max}^v$ represents the maximum value of all the station classes; through normalization processing, $C_i^v \in (0, 1]$.

Considering all the three factors mentioned above, we can define the impact H_i^v of HSR station v_i , as shown in the following equation:

$$H_i^v = \beta_1 D_i^v + \beta_2 B_i^v + \beta_3 C_i^v, \quad (4)$$

FIGURE 1: Diagram of component impact assessment.

where β_1, β_2 , and β_3 are the weight coefficients, reflecting the importance of each factor, and $\beta_1 + \beta_2 + \beta_3 = 1$. Thus, $H_i^v \in [0, 1]$.

3.1.2. Impact Assessment of the HSR Lines. The impact of the HSR line depends on the two impacts of the stations on both sides of the line. H_k^l represents the impact of line l_k , which can be calculated using the following equation:

$$H_k^l = \frac{1}{2(n-1)} \left(\frac{H_{i1}^{v,k}}{D_{i1}^{v,k}} + \frac{H_{i2}^{v,k}}{D_{i2}^{v,k}} \right), \quad (5)$$

where $D_{i1}^{v,k}$ represents the degree centrality of v_{i1} linked with line l_k and $H_{i1}^{v,k}$ represents the impact of v_{i1} linked with line l_k . It can be seen that $(n-1)D_{i1}^{v,k}$ represents the degree of v_{i1} and $H_k^l \in [0, 1]$.

3.2. Risk Assessment of the Complete HSRNs. If the failure occurrence probability of node v_i and line l_k is p_i^v and p_k^l , respectively, the corresponding impact of v_i and l_k is expected to be E_i^v and E_k^l , as shown in equations (6) and (7), respectively:

$$E_i^v = p_i^v \cdot H_i^v + (1 - p_i^v) \cdot 0 \quad (6)$$

$$E_k^l = p_k^l \cdot H_k^l + (1 - p_k^l) \cdot 0 \quad (7)$$

where p_i^v and p_k^l denote the nonlinear function of the maintenance costs for the corresponding node and unit length line, respectively. For a station, a larger amount of maintenance costs invested in a station corresponds to a lower occurrence probability of the failure station. For different stations, to reduce the same risk probability, a larger station requires the investment of higher maintenance costs. Similarly, for a line, a larger amount of maintenance costs invested in the unit length of the certain line corresponds to a lower failure probability of the section. Without loss of generality, we assume that the

occurrence failure probability of the station and the maintenance costs accord with the exponential function and that of the line and the maintenance costs in the unit length is the same as that of the station [32]. Thus, the formula can be constructed as follows:

$$p_i^v = e^{-\theta \cdot (g_i^v / C_i^v)}, \quad (8)$$

$$p_k^l = e^{-\varphi \cdot (g_k^l / u_k)}, \quad (9)$$

where g_i^v is the maintenance cost invested in station (node) i and g_k^l is the maintenance cost invested in line k ; the coefficients θ and φ represent the change trend of the exponential function, as shown in Figure 2. According to equations (8) and (9), the relationship between p_i^v and the two independent variables (g_i^v and C_i^v) is as shown in Figure 3(a), and the relationship between p_k^l and the two independent variables (g_k^l and u_k) is as shown in Figure 3(b).

Next, the risk R of the entire network $G(V, L, A)$ can be defined as the sum of the expected values of the impacts of the network elements (nodes and lines), representing the relative probability of the failure occurring in the entire network, as shown in the following equation:

$$R = \sum_{i=1}^n E_i^v + \sum_{k=1}^m E_k^l = \sum_{i=1}^n (p_i^v \cdot H_i^v) + \sum_{k=1}^m (p_k^l \cdot H_k^l). \quad (10)$$

4. Optimization Algorithm for the Risk Minimization Costs

Under the constraint of the limited total maintenance costs, the allocation of the maintenance costs to minimize the risk R in the entire network is a realistic optimizing problem. If the total maintenance cost of the entire network is g^m , the optimization model can be constructed as shown in the following equation:

FIGURE 2: Variation tendency of the exponential function with the parameters.

FIGURE 3: Variation tendency of the failure probability with the independent variables. (a) Station failure probability. (b) Line failure probability.

$$\begin{aligned} \min R(g_i^v, g_k^l) &= \sum_{i=1}^n \left(e^{-\theta \cdot (g_i^v / C_i^v)} \cdot H_i^v \right) + \sum_{k=1}^m \left(e^{-\varphi \cdot (g_k^l / u_k)} \cdot H_k^l \right), \\ \text{s.t. } \begin{cases} \sum_{i=1}^n g_i^v + \sum_{k=1}^m g_k^l = g^m, \\ 0 \leq g_i^v \leq g^m, \\ 0 \leq g_k^l \leq g^m. \end{cases} \end{aligned} \quad (11)$$

The Lagrangian multipliers [50–52] λ can be adopted to solve the special nonlinear optimization problem, such as that defined in equation (11), to acquire the optimal solution. The Lagrangian optimization function Φ can be constructed as shown in the following equation:

$$\Phi(g_i^v, g_k^l, \lambda) = \sum_{i=1}^n \left(e^{-\theta \cdot (g_i^v / C_i^v)} \cdot H_i^v \right) + \sum_{k=1}^m \left(e^{-\varphi \cdot (g_k^l / u_k)} \cdot H_k^l \right) - \lambda \left(\sum_{i=1}^n g_i^v + \sum_{k=1}^m g_k^l - g^m \right). \quad (12)$$

The expression is differentiated with respect to g_i^v , g_k^l , and λ , as shown in equations (13), (14), and (15), respectively:

$$\frac{\partial \Phi(g_i^v, g_k^l, \lambda)}{\partial g_i^v} = -\frac{\theta}{C_i^v} H_i^v e^{-(\theta / C_i^v) \cdot g_i^v} - \lambda = 0, \quad (13)$$

$$\frac{\partial \Phi(g_i^v, g_k^l, \lambda)}{\partial g_k^l} = -\frac{\varphi}{u_k} H_k^l e^{-(\varphi / u_k) \cdot g_k^l} - \lambda = 0, \quad (14)$$

FIGURE 4: Diagram of the HSRN in Mid-Eastern China.

$$\frac{\partial \Phi(g_i^v, g_k^l, \lambda)}{\partial \lambda} = \sum_{i=1}^n g_i^v + \sum_{k=1}^m g_k^l - g^m = 0. \quad (15)$$

To solve equations (13) and (14), the results of equations (13) and (14) can be expressed as shown in equations (16) and (17), respectively:

$$g_i^v = -\frac{C_i^v}{\theta} \left(\ln \left(\frac{C_i^v}{\theta H_i^v} \right) + \ln(-\lambda) \right), \quad (16)$$

$$g_k^l = -\frac{u_k}{\varphi} \left(\ln \left(\frac{u_k}{\varphi H_k^l} \right) + \ln(-\lambda) \right). \quad (17)$$

Next, equations (16) and (17) can be substituted into equation (15), from which the following equation is derived:

$$\sum_{i=1}^n \left(\frac{C_i^v}{\theta} \left(\ln \left(\frac{\theta H_i^v}{C_i^v} \right) - \ln(-\lambda) \right) \right) + \sum_{k=1}^m \left(\frac{u_k}{\varphi} \left(\ln \left(\frac{\varphi H_k^l}{u_k} \right) - \ln(-\lambda) \right) \right) = g^m. \quad (18)$$

Furthermore, the following equation can be derived from equation (18):

$$\ln(-\lambda) = \frac{\sum_{i=1}^n ((C_i^v/\theta) \ln(\theta H_i^v/C_i^v)) + \sum_{k=1}^m (u_k/\varphi \ln(\varphi H_k^l/u_k)) - g^m}{\sum_{i=1}^n (C_i^v/\theta) + \sum_{k=1}^m (u_k/\varphi)}. \quad (19)$$

According to equation (19), $\ln(-\lambda)$ can be substituted into equations (16) and (17) to calculate g_i^v and g_k^l , respectively. Therefore, we can obtain the optimal maintenance cost allocation of each station and line in the network, as shown in equations (20) and (21), respectively:

$$g_i^v = -\frac{C_i^v}{\theta} \left(\ln \left(\frac{C_i^v}{\theta H_i^v} \right) + \frac{\sum_{i=1}^n ((C_i^v/\theta) \ln(\theta H_i^v/C_i^v)) + \sum_{k=1}^m (u_k/\varphi \ln(\varphi H_k^l/u_k)) - g^m}{\sum_{i=1}^n (C_i^v/\theta) + \sum_{k=1}^m (u_k/\varphi)} \right), \quad (20)$$

$$g_k^l = -\frac{u_k}{\varphi} \left(\ln \left(\frac{u_k}{\varphi H_k^l} \right) + \frac{\sum_{i=1}^n ((C_i^v/\theta) \ln(\theta H_i^v/C_i^v)) + \sum_{k=1}^m (u_k/\varphi \ln(\varphi H_k^l/u_k)) - g^m}{\sum_{i=1}^n (C_i^v/\theta) + \sum_{k=1}^m (u_k/\varphi)} \right). \quad (21)$$

TABLE 1: Parameter values used in the simulation.

Parameter	θ	φ	β_1	β_2	β_3	c
Value	0.001	0.0015	0.2	0.5	0.3	1,2,3,4

TABLE 2: Station indexes and impact.

No.	Name	Station class	Degree	Degree centrality	Betweenness centrality	Station scale	Impact (H)
1	Si'an	2	1	0.042	0	0.750	0.233
2	Chengchow	2	3	0.125	0.217	0.750	0.359
3	Shangqiu	4	2	0.083	0.188	0.250	0.186
4	Hsuchow	3	2	0.083	0.246	0.500	0.290
5	Huainan	4	2	0.083	0.174	0.250	0.179
6	Bengbu	3	3	0.125	0.406	0.500	0.378
7	Xinyang	4	2	0.083	0.159	0.250	0.171
8	Ho-fei	2	4	0.167	0.543	0.750	0.530
9	Nanking	2	4	0.167	0.623	0.750	0.570
10	Chinkiang	4	2	0.083	0.188	0.250	0.186
11	Soochow	2	2	0.083	0.094	0.750	0.289
12	Yueyang	4	2	0.083	0.275	0.250	0.229
13	Wuhan	2	4	0.167	0.565	0.750	0.541
14	Chaohu	3	2	0.083	0.072	0.500	0.203
15	Huzhou	4	2	0.083	0.312	0.250	0.247
16	Shanghai	1	2	0.083	0.058	1	0.346
17	Changsha	2	3	0.125	0.261	0.750	0.380
18	Nanchang	2	3	0.125	0.152	0.750	0.326
19	Shangrao	3	4	0.167	0.217	0.500	0.292
20	Hangzhou	2	4	0.167	0.449	0.750	0.483
21	Ningbo	3	2	0.083	0.217	0.500	0.275
22	Kwangchow	2	2	0.083	0.109	0.750	0.296
23	Shamchun	2	2	0.083	0.022	0.750	0.253
24	Hockchew	2	3	0.125	0.065	0.750	0.283
25	Yujeu	3	2	0.083	0.094	0.500	0.214

5. Examples and Results

We selected the HSRNs in Mid-Eastern China, 2018, as the test object to establish the network model according to the appropriate abstraction on the actual HSRN structure. The aim of the simulation was to illustrate and validate the proposed optimization model of the risk assessment and maintenance cost allocation. The test network consisted of 25 HSR stations and 64 HSR lines, and the corresponding station names and line lengths are shown in Figure 4.

The parameters used in the simulation are listed in Table 1. These values are not actual values but serve to illustrate the abovementioned approach. The shortest path can be solved using the Floyd algorithm.

On the basis of the proposed risk assessment model and cost allocation optimization algorithm, the 25 test stations and 64 lines in the network were evaluated. The approach could obtain the basic attribute information and impact values of these stations and lines, as given in Tables 2 and 3.

From Table 2, it can be seen that the attribute values vary considerably for different nodes. Among these nodes, the degree centrality of nodes 8, 9, 13, 19, and 20 is higher, while the betweenness centrality of nodes 6, 8, 9, 13, and 20 is higher. Overall, nodes 6, 8, 9, 13, and 20 are more important in the network because their impact values are higher.

From Table 3, it can be seen that the impact values of the lines are also considerably different. Among these lines, the impact values of lines 1, 2, 27, 38, 39, 51, etc., are larger, while those of lines 45, 46, 47, 49, 60, etc., are smaller.

Taking the total maintenance costs (20000, 35000, and 50000, respectively) as an example, the allocation scheme of the optimal maintenance costs among the various stations and lines is described in detail in Figure 5. Figure 5(a) shows the maintenance cost allocation scheme among different stations, and Figure 5(b) shows the corresponding scheme among different lines.

Based on the impact values of the stations and the lines and the optimization algorithm for the risk minimization costs, the corresponding input allocation is completed. The model can achieve the specific allocation of maintenance input for each station and line under the goal of minimum maintenance failure risk when the input of high-speed railway network maintenance is limited. The calculated maintenance strategy is the optimal allocation scheme in network maintenance with a global optimal solution provided by the model. When the maintenance cost is fixed, the maximum effect of the investment can be achieved. Therefore, it can provide auxiliary support for the high-speed railway management department to make a reasonable routine maintenance plan.

TABLE 3: Line indexes and impact.

No.	V1	V2	Distance	Impact (H)	No.	V1	V2	Distance	Impact (H)
1	1	2	481	0.249	33	13	18	369	0.172
2	2	1	481	0.249	34	14	8	91.9	0.165
3	2	3	221	0.150	35	14	19	548	0.123
4	2	7	331	0.145	36	15	9	216	0.188
5	3	2	221	0.150	37	15	20	82.3	0.173
6	3	4	173	0.168	38	16	11	106	0.224
7	4	3	173	0.168	39	16	20	177	0.207
8	4	6	194	0.191	40	17	12	155	0.170
9	5	6	56.8	0.152	41	17	18	395	0.166
10	5	8	105	0.157	42	17	22	703	0.194
11	6	4	194	0.191	43	18	13	369	0.172
12	6	5	56.8	0.152	44	18	17	395	0.166
13	6	9	207	0.189	45	18	19	261	0.128
14	7	2	331	0.145	46	19	14	548	0.123
15	7	13	211	0.156	47	19	18	261	0.128
16	8	5	105	0.157	48	19	20	336	0.137
17	8	9	171	0.194	49	19	24	453	0.118
18	8	13	389	0.189	50	20	15	82.3	0.173
19	8	14	91.9	0.165	51	20	16	177	0.207
20	9	6	207	0.189	52	20	19	336	0.137
21	9	8	171	0.194	53	20	21	154	0.182
22	9	10	67.7	0.166	54	21	20	154	0.182
23	9	15	216	0.188	55	21	25	277	0.173
24	10	9	67.7	0.166	56	22	17	703	0.194
25	10	11	167	0.168	57	22	23	136	0.194
26	11	10	167	0.168	58	23	22	136	0.194
27	11	16	106	0.224	59	23	24	837	0.156
28	12	13	233	0.176	60	24	19	453	0.118
29	12	17	155	0.170	61	24	23	837	0.156
30	13	7	211	0.156	62	24	25	320	0.142
31	13	8	389	0.189	63	25	21	277	0.173
32	13	12	233	0.176	64	25	24	320	0.142

(a)

FIGURE 5: Continued.

FIGURE 5: Illustration of the optimal cost allocation scheme. (a) Optimal cost allocation scheme for all the stations. (b) Optimal cost allocation scheme among all lines.

FIGURE 6: Illustration of the optimal cost allocation scheme.

Figure 6 illustrates the nonlinear numerical relationship between the entire network risk and total maintenance costs. To demonstrate the advantages of the proposed optimization algorithm, we compared this comprehensive optimization

allocation approach with those of the genetic algorithm (GA) and average allocation method, in which the total maintenance costs are distributed evenly among all the stations and lines. As seen from the figure, irrespective of the

TABLE 4: Comparison of the three optimization allocation schemes.

	$\sum_i g_i^v$	$\sum_k g_k^l$	Risk	CPU time (s)
Average allocation	7022.470	17977.530	6.749	0.031
Optimal allocation	11373.810	13626.190	6.022	0.187
GA allocation	7087.190	17912.810	6.729	605.780

type of allocation scheme, with an increase in the total maintenance costs, the risk of the entire network is significantly reduced. A comparison of the three allocation schemes indicates that the proposed optimization allocation scheme exhibits notable advantages, and it is better than the other two allocation schemes in various situations. When the total maintenance costs are higher than 100000, the network risk corresponding to the optimal allocation scheme is close to zero.

The calculation results of this example indicate that the risk assessment model and the maintenance cost allocation algorithm proposed in this paper can help realize the risk assessment and cost allocation in various conditions to determine the optimal maintenance cost allocation scheme. In addition, the total cost of 25000 was considered as an example to illustrate the concrete computation process of the three optimization allocation schemes, as presented in Table 4.

6. Conclusions

This paper proposes an effective approach to integrate the HSRN risk assessment and maintenance cost allocation for the actual situation of the HSRNs. The actual test results show that the comprehensive optimization approach can be suitably applied for the HSRN risk assessment. Under the conditions of the given limited maintenance costs, the approach can solve the optimization allocation of the maintenance costs to minimize the entire HSRN risk in practice. In addition, the approach can also be used to measure the total amount of maintenance costs in a certain range of network risk control.

In the specific usage of the approach to allocate maintenance costs, the model parameters must be calibrated according to the actual situation in each region to ensure that the approach has a high fitting degree. The example provided in this paper is aimed at illustrating the feasibility and effectiveness of the approach. In reality, certain other risk factors related to the HSRNs exist. The next step of our research will be to introduce several important risk factors into the proposed assessment model to make the model and algorithm more precise and practical.

Data Availability

The data used to support the findings of this study are owned by the Chinese Railway Company.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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