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# Research on Platoon Dispersion Delay of Traffic Flow considering Coordinated Control

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To improve the traffic capacity and reduce the delay of signalized intersections, the delay of coordinated control intersections is studied. Based on the freedom and randomness of the speed change and considering the delay problem caused by the discrete behavior, the authors deduced a new delay model. Firstly, by analyzing the kinematic behavior of traffic flow under coordinated control, it is found that traffic flow reaches the downstream intersection in two different forms. The two forms were as follows: the tail vehicles of discrete traffic flow were truncated and the front vehicles of discrete traffic flow were stopped. Then, the authors deduced the new delay model by analyzing the two conditions. Finally, the delay of the two cases is analyzed, which can be used as the basis for setting the phase difference between coordinated control intersections. The correctness of the model is verified by designing two example coordinated control intersections under unsaturated flow with MATLAB. Results show that the discrete traffic flows will have different impacts on delay or traffic efficiency when they arrive at the downstream intersection in different forms. Through the analysis of the delay of vehicles, when the green split is less than 0.64, the tail truncation delay is greater than the front truncation delay. When the green split is greater than or equal to 0.64, the opposite is true. The phase difference of upstream and downstream intersections can be optimized and coordinated according to the goal that vehicles can smoothly pass through the coordinated control intersection or ensure the minimum delay, so as to give full play to the space-time utilization of the coordinated control intersection.

## 1. Introduction

In urban roads, due to the existence of signal intersections, the continuous traffic flow is divided into discrete states. At the same time, due to the influence of the external environment, the driving speed of each vehicle in the traffic flow is different, resulting in the gradual increase of headway and the gradual dispersion of queuing vehicles leaving the stop line. This phenomenon of changing the traffic flow from a dense state to decentralized state is termed as traffic flow dispersion [1]. Traffic flow dispersion is caused by traffic signals, which will also affect the delay of signalized intersections [2]. Traffic flow dispersion mainly studies the relationship between the flow of upstream and downstream intersections and can better co-ordinate and control the intersections [3, 4].

As a part of the coordinated control system, the study of discrete traffic flow model can greatly reduce vehicle delay,

queue length, and so on [5]. Many traffic control systems use the classical Robertson model to predict vehicle arrival [4]. In the classical Robertson model, the basic assumption is that the vehicle travel time follows the moving geometric distribution. Therefore, many scholars propose new models based on this assumption to reduce vehicle delay. At present, the research on traffic flow dispersion mainly focuses on two aspects: one is to establish a new model based on the traditional Robertson model and considering the parameters under different conditions; the second is to study the discrete model of traffic flow under mixed distribution.

However, there are two limitations in the current research process. Firstly, some studies analyze the relationship between traffic flow dispersion and delay but focus on the shunting coordination between traffic flows and do not analyze the kinematic characteristics of traffic flow. Secondly, there is little analysis on the delay problem of

coordinated control intersection, which can not provide a relevant reference for traffic management and control. Therefore, this study combines the kinematic characteristics of traffic flow and coordinated control to analyze the delay of discrete traffic flow.

The remainder of this paper is organized as follows. The literature review is presented in Section 2. Section 3 is the analysis of vehicle dispersion behavior under coordinated control. Analysis of the tail truncation of discrete traffic flow and analysis of front stop of discrete traffic flow are in Sections 4 and 5. Section 6 is the comparison and analysis of two operating conditions of discrete traffic flow. Example verification is in Sections 7, and Section 8 closes the paper with conclusions.

## 2. Literature Review

As an important part of signal control system, the relationship between traffic flow dispersion and delay needs to be further studied. A lot of research has been carried out on traffic flow dispersion and intersection delay. As early as 1956, Pacey proposed the Pacey method based on the assumption that the vehicle speed obeyed the normal distribution [6]. Potts proposed a new discrete model based on the Pacey method and considering the characteristics of traffic density [1]. Later, due to the demand for signal control optimization for traffic prediction, Robertson [4] proposed a traffic flow discrete model with travel time conforming to the moving geometric distribution function from the perspective of traffic flow, so as to analyze the degree of vehicle dispersion, making the calculation of the model simpler and widely used. Through data fitting, Polus A found that the normal distribution and lognormal distribution can better fit the distribution of vehicle travel time than the moving geometric distribution [7].

Many kinds of research focus on the dispersion of vehicle flow from the perspective of optimization; For example, Forde and Daniel [8] evaluate the performance of the HCM platoon dispersion model under friction and no friction conditions and prove the HCM platoon dispersion model performs well under no friction traffic condition, but not under friction traffic condition. Zhang et al. [9] studied a new macrotraffic model considering the continuous delayed optimal flow. Ke et al. [10] presented an approach to distribute the gated flow based on the queue lengths or experienced delay at the gated signalized junctions, which can improve the overall network performance. Hao et al. [11] proposed a model considering delay minimizing coordinated MPC with max pressure control, which reduces the growth of distant downstream queues. Calle-Laguna et al. [12] enhanced the Webster model and developed new formulations to compute the optimum cycle length, considering vehicle delay, fuel consumption, and tailpipe emissions.

Some scholars carry out system optimization from the perspective of relevant parameters of vehicle flow discrete model [13–18]. Qi et al. [14] presented a model that effectively decreases delays considering mixed traffic flow of Human-Driven Vehicles (HDVs) and Connected and Autonomous Vehicles (CAVs). Wu et al. [15] proposed an empirical optimization method to minimize the control delay by optimizing the length of contraflow lanes and the offset between adjacent intersections. Mohamed et al. [17] analyzed the platoon dispersion characteristics under heterogeneous and laneless traffic conditions. Some researchers also optimize the model parameters in different situations by combining discrete model with coordinated control; Nagatani [19] presented the stochastic model of vehicular traffic controlled by signals, which show that the traffic dispersion depends highly on the cycle time and the strength of split's irregularity. Tobita and Nagatani [5] presented control of the unbalanced two-route system by the green-wave strategy of signals and derived the relationship between the mean tour time and the offset time. Although many studies [20–24] have proposed some new methods to calibrate the parameters of the classic Robertson model under different traffic conditions or select new technologies for research, it is necessary to further analyze the influence of vehicle kinematic behavior and delay on coordinated control intersection.

## 3. Analysis of Vehicle Dispersion Behavior under Coordinated Control

Hai and Tao [25] proposed that the queuing and delay of traffic flow passing through the downstream intersection have nothing to do with the form of platoon dispersion model of the traffic flow, but only with the traffic flow and average travel time at the exit of the upstream intersection. However, in the coordinated control of intersections, due to the discrete phenomenon of traffic flow, the headway becomes larger when the traffic flows from the upstream intersection to the downstream intersection. Therefore, it is not possible to pass downstream intersections at a saturated flow rate, which will reduce the traffic efficiency through downstream intersections and also affect the delay. This study will conduct an in-depth discussion on this situation.

The influence of two adjacent intersection signals on vehicle flow dispersion behavior is considered. Assuming that the green signal ratio of the two intersection signals is the same, the duration of red light is $r$, and the duration of the green light is $g$, ignoring the green light interval time, the cycle length is $C$, and the green light start time difference is $t_0$. Suppose the distance between upstream intersection $n$ and downstream intersection $n + 1$ is $x$, the starting speed of queued vehicles is $v$, the traffic flow on the road section is in equilibrium at the initial moment, the intersection $n$ stop line is located at $x_0$, and the signal light changes from green to red when $t = 0$. The schematic diagram of the intersection is shown in Figure 1.

When the green light at intersection $n$ is on, the queuing vehicles pass the stop line at the saturation flow rate $S$ and drive to the downstream intersection. The discrete behavior occurs at section $x$. The discrete traffic flow is at the free-flow speed $v_f$. When it reaches the intersection $n + 1$, according to the Pacey method, the arrival rate is as follows:
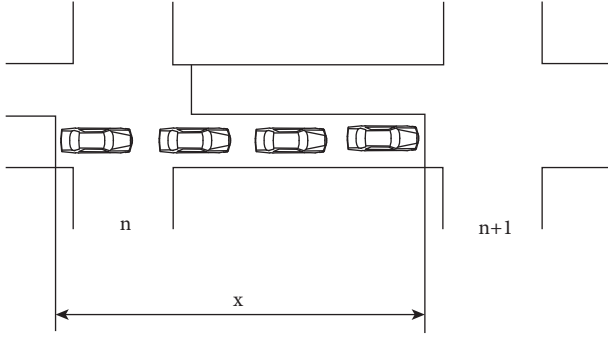
FIGURE 1: Traffic flow platoon dispersion.

$$q_2(j) = \sum_{i=1}^{j} q_1(i) f(j - i), \tag{1}$$

where $q_2(j)$ is the traffic flow rate at the stop line of the downstream intersection in the time period $j$; $q_1(i)$ represents the exit flow rate of the traffic flow at the upstream stop line section in the time period $i$; $f(j - i)$ represents the vehicle probability distribution function with travel time $(j - i)$ from the upstream stop line section to a downstream section is a transformed normal distribution function.

Because of $q_1(i) = S$, $q_2(j) < S$. If the phase difference between intersection $n$ and intersection $n + 1$ is $x/v_f$, when the vehicle reaches the downstream intersection $n + 1$, it just catches the green light, and the traffic flow passes through the downstream intersection at the arrival rate $q_2(j)$; that is, it does not pass the intersection $n + 1$ at the saturation flow rate. The tail of the discrete traffic flow will be cut off at the intersection $n + 1$, which will reduce the efficiency of traffic; that is, the number of vehicles passing by at the same time is relatively small.

If you want to improve the traffic efficiency of intersection $n + 1$, that is, the traffic flow can pass through the intersection $n + 1$ at the saturation flow rate, we need to stop the front of the traffic flow; that is, let the front car stop at the red light at the intersection $n + 1$, wait for the rear car to arrive, and start to pass the intersection again, but this will increase the delay of the traffic flow. In summary, the discrete traffic flow from the upstream intersection to the downstream intersection will cause two situations; namely, the front part of the traffic flow is blocked and the tail part is cut off. The two conditions are analyzed and studied separately below.

## 4. Analysis of the Tail Truncation of Discrete Traffic Flow

The traffic flow disperses from the upstream intersection to the downstream intersection. Under the action of coordination control, the vehicles in front of the discrete traffic flow just catch up with the green light when they arrive at the downstream intersection. However, because the discrete traffic flow cannot pass at the saturated flow rate, some of the vehicles behind cannot pass during the green light and will be stopped by the red light. This phenomenon is called the tail truncation of discrete traffic.

As mentioned above, the cycle length of intersection $n$ and intersection $n + 1$ is $C$, the green light duration is $g$, the red light duration is $r$, the saturation flow rate of traffic flow at intersection $n$ is $S$, and the arrival rate of intersection $n + 1$ is

$$
\begin{aligned}
q_2(j) &= \sum_{i=1}^{j} q_1(i) f(j - i) \\
&= \sum_{i=1}^{j} S f(j - i).
\end{aligned}
\tag{2}
$$

The phase difference between intersection $n$ and intersection $n + 1$ is $x/v_f$. At this time, the flow of traffic leaving intersection $n$ just catches the green light when it reaches intersection $n + 1$, and the traffic flow will pass through intersection $n + 1$ at flow rate $q_2(j)$. Since $q_2(j) < S$, the traffic flow passes through the intersection $n + 1$ at a departure flow rate lower than the saturation flow rate $S$.

This will lead to a reduction in the efficiency of traffic flow through the intersection; the number of vehicles leaving the intersection at the same time will be reduced. This part of the reduction should be the green light duration multiplied by the difference between the saturated flow rate and the discrete arrival flow rate; that is,

$$
\begin{aligned}
m &= g \times S - g \times q_2(j) \\
&= g \times (S - q_2(j)),
\end{aligned}
\tag{3}
$$

where $m$ represents the number of vehicles passing through the intersection caused by the discrete arrival flow rate of less than the saturation flow rate. Other parameters are the same as above.

Then we can deduce the following formula:

$$
\begin{aligned}
m &= g \times (S - q_2(j)) \\
&= g \times S \left( 1 - \sum_{i=1}^{j} f(j - i) \right).
\end{aligned}
\tag{4}
$$

The traffic flow leaving the upstream intersection shows discrete behavior when heading to the section of the downstream intersection. If the phase difference between the two intersections is $x/v_f$, then the front traffic flow just catches the green light when it reaches the downstream intersection and can pass the downstream intersection smoothly. At this time, the departure flow rate $q_2(j)$ of the traffic flow is less than the saturated flow rate $S$, which will cause the vehicles behind the traffic flow to be unable to use the green light period of this cycle to pass through the intersection. They must wait for the red light to pass in the next green light period; that is, the tail of the discrete traffic flow is truncated. Without considering the loss before and after, the queue length and delay time diagram of traffic flow are shown in Figure 2.

Line BF represents the saturation flow rate $S$; line AF represents the departure flow rate $q_2(j)$; $t1$ is the time when the tail truncated vehicle arrives at the downstream intersection; line BP is parallel to the ordinate; line AF and line BP intersect at point $E$; the number of vehicles coming from the

upstream intersection is $Sg$. Due to the discrete behavior of traffic flow, vehicles pass through the downstream intersection at the departure flow rate $q_2(j)$. The number of vehicles passing through the downstream intersection during the green light period is $q_2(j)g$, and the remaining vehicles queue up at the stop line to wait for the next green light; line AI and line EJ are parallel to the abscissa.

Triangle BFE is the delay caused by the truncation of the tail of discrete traffic flow. The headway between vehicles becomes longer and cannot pass the intersection at a saturated flow rate, resulting in reduced traffic efficiency. Its value is the triangular BFE area, which is

$$T_1 = \frac{1}{2}\overline{BE} \times g, \tag{5}$$

where $T_1$ is time loss caused by the truncation of the tail of discrete traffic; $\overline{BE}$ represents the number of vehicles with the tail truncated; and $g$ is intersection green time.

It can be seen from Figure 2 that vehicles with their tails cut off need to wait in line for the next green light to pass through the intersection, and the delay $\Delta D_1$ incurred during this period is the area of the trapezoid AEJI, that is,

$$\Delta D_1 = \frac{1}{2}\left(\overline{AI} + \overline{EJ}\right) \times \overline{BE}. \tag{6}$$

It can be seen from formula (3) that

$$\begin{aligned}\overline{BE} &= m, \\ \overline{BE} &= g \times (S - q_2(j)).\end{aligned} \tag{7}$$

So the time loss $T_1$ is

$$\begin{aligned}T_1 &= \frac{1}{2}\overline{BE} \times g \\ &= \frac{1}{2}g^2 \times (S - q_2(j)).\end{aligned} \tag{8}$$

Also,

$$\frac{\overline{AH}}{t_1} = q_2(j). \tag{9}$$

Therefore, the delay $\Delta D_1$ caused by vehicles with truncated tail is as follows:

$$\Delta D_1 = \frac{1}{2}m \times \left(2r - \frac{m}{q_2(j)}\right). \tag{10}$$

## 5. Analysis of Front Stop of Discrete Traffic Flow

If you want to improve the efficiency of the intersection and the traffic flow to pass through the stop line at a saturated flow rate, the vehicles in front of the traffic flow must meet the red light when they arrive at the downstream intersection, so that the vehicles queue up and the time headway between the vehicles is compressed, so that the traffic flow can pass the downstream intersection at the saturation flow rate; this situation is called the front stop of discrete traffic flow. When the traffic flow passes through the downstream

intersection at a saturated flow rate, the number of vehicles passing at the same time is the largest, which makes the traffic efficiency the highest. In this case, the front part of the traffic flow needs to stop, which will lead to delay. The following is a detailed analysis of this situation.

Similarly, suppose the cycle length of intersection $n$ and intersection $n + 1$ is $C$, the green light duration is $g$, the red light duration is $r$, and the saturation flow rate of intersection $n$ is $S$. Without considering the loss before and after, the specific queue length and delay time are shown in Figure 3.

In Figure 3, the line AF represents the saturation flow rate of the traffic flow; the line AO represents the arrival rate of the traffic flow. Because the traffic flow has discrete movement, the arrival rate of the intersection $n + 1$ is $q_2(j)$, and the time for the preceding vehicle to stop and wait for the red light is $t_2$. The delay $\Delta D_2$ caused by the front of the discrete traffic flow being stopped is the area of the triangular AOF, namely,

$$\Delta D_2 = \frac{1}{2}\overline{OF} \times \overline{AE}. \tag{11}$$

From Figure 3, the following formula is derived:

$$\frac{\overline{AE}}{g + t_2} = q_2(j),$$

$$\frac{\overline{AE}}{g} = S, \tag{12}$$

$$t_2 = \frac{g(S - q_2(j))}{q_2(j)},$$

$$\begin{aligned}\Delta D_2 &= \frac{1}{2}Sgt_2 \\ &= \frac{1}{2}\frac{Sg^2(S - q_2(j))}{q_2(j)} \\ &= \frac{1}{2}\frac{Sgm}{q_2(j)}.\end{aligned} \tag{13}$$

## 6. Comparison and Analysis of Two Operating Conditions of Discrete Traffic Flow

In the setting process of signal coordination control, the arrival of discrete traffic flow can be controlled by changing the phase difference of two adjacent intersections. If the traffic flow reaches the downstream intersection, it just catches the green light and passes the downstream intersection smoothly. At this time, due to the discrete behavior, the headway of the vehicle in the traffic flow increases; that is, the departure flow rate is lower than the saturation flow rate, the traffic efficiency of the traffic flow passing through the downstream intersection will be reduced, and the vehicles at the end of the traffic flow will be truncated at the intersection. If you want to improve the efficiency of the traffic flow through the downstream intersection, you need to let the vehicles in the front of the traffic flow meet the red light
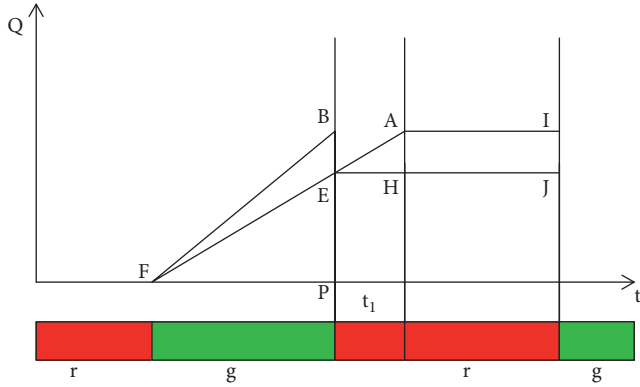
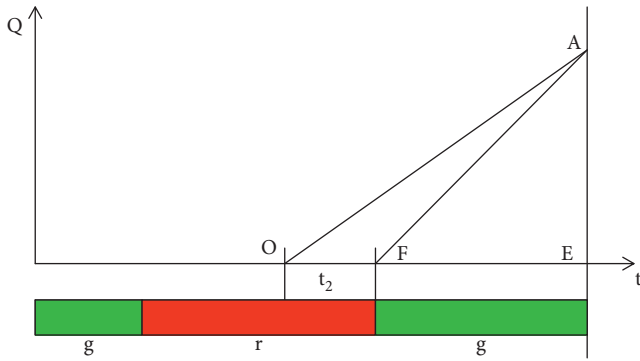FIGURE 2: Queue length and delay time with the tail vehicles of discrete traffic flows were truncated.



FIGURE 3: Queue length and delay time with the front vehicles of discrete traffic flows were stopped.

at the stop line of the downstream intersection, wait for the subsequent vehicles to arrive, and queue, and compress the headway between the vehicles, so that the traffic flow can pass through the downstream intersection at a saturated flow rate.

The following is a comparative analysis of the delay caused by two cases of discrete traffic flow tail cutoff and front stop.

(1) Suppose $\Delta D_1 = \Delta D_2$; from equations (10) and (13), we can know

$$\frac{1}{2} m \left( 2r - \frac{m}{q_2(j)} \right) = \frac{1}{2} \frac{Sgm}{q_2(j)},$$

$$\frac{2q_2(j)}{2s + q_2(j)} = \frac{g}{C}. \tag{14}$$

(2) If $\Delta D_1 < \Delta D_2$, in the same way, we can get

$$\frac{2q_2(j)}{2s + q_2(j)} < \frac{g}{C}. \tag{15}$$

(3) If $\Delta D_1 > \Delta D_2$, we can get

$$\frac{2q_2(j)}{2s + q_2(j)} > \frac{g}{C}. \tag{16}$$

In summary, when the value of green signal ratio is different, three kinds of relations between the delay caused by the tail truncation of discrete traffic flow and the delay caused by the front truncation of discrete traffic flow can be obtained:

(1) When the relationship between the arrival rate and the green signal ratio is $2q_2(j)/(2S + q_2(j)) = g/C$, the delay caused by the tail truncation of discrete traffic flow is equal to the delay caused by the front truncation of discrete traffic flow.

(2) When the relationship between the arrival rate and the green signal ratio is $2q_2(j)/(2S + q_2(j)) < g/C$, the results show that the delay caused by the tail truncation of discrete traffic flow is less than that caused by the front truncation of discrete traffic flow.

(3) When the relationship between the arrival rate and the green signal ratio is $2q_2(j)/(2S + q_2(j)) > g/C$, the delay caused by the tail truncation of discrete traffic flow is greater than that caused by the front truncation of discrete traffic flow.

In the first and second cases, the tail cutoff delay is less than or equal to the front stop delay, and the discrete traffic flow can not only pass through the intersection with the green wave but also make the delay value smaller. Therefore, when $2q_2(j)/(2S + q_2(j)) \leq g/C$, the phase difference $\Delta g$ of adjacent intersections can be set as $x/v_f$, that is, $\Delta g = x/v_f$.

In the third case, the tail cutoff delay is greater than the front stop delay. If the vehicle can smoothly pass through each intersection under coordinated control as an important setting parameter, the phase difference of adjacent intersections is set as $\Delta g = x/v_f$; if the delay is small as an important setting parameter, the phase difference $\Delta g$ of adjacent intersections is set as $t_1 + x/v_f$, that is,

$$\Delta g = \frac{g(S - q_2(j))}{q_2(j)} + \frac{x}{v_f}. \tag{17}$$

## 7. Example Verification

In this section, MATLAB is used to analyze these two delays. Two examples are designed to make the following assumptions: the distance between two adjacent intersections is 1000 m, the signal period of the intersection is 90s, the speed of the vehicle between the intersections is 60 km/h, and the saturated flow at the upstream intersection is $S = 1800\ veh/h$ and $S = 1200\ veh/h$, respectively.

On the premise that the signal period remains the same, if the green signal ratio adopts different values, the relationship between the delay $\Delta D_1$ caused by the truncated traffic flow at the tail and the delay $\Delta D_2$ caused by the stopped traffic flow at the front is compared as shown in Table 1.

Under different circumstances, the tail truncation delay and the front stop delay are made into a line chart, as shown in Figure 4.

It can be seen from Figure 4 that $a$ showed the saturated flow at the upstream intersection is $S = 1800\ veh/h$ and $b$

TABLE 1: Two delay values under different green split.

| Signal period (S) | Duration of green light (s) | $\Delta D_1$ | $\Delta D_2$ | $\Delta D_1$ | $\Delta D_2$ |
|---|---|---|---|---|---|
| 90 | 10 | 25.09410352 | 1.680896478 | 16.7294 | 1.1206 |
| 90 | 15 | 35.19923292 | 3.782017076 | 23.46616 | 2.52134 |
| 90 | 20 | 43.67641409 | 6.723585912 | 29.11761 | 4.48239 |
| 90 | 25 | 50.52564701 | 10.50560299 | 33.68376 | 7.00374 |
| 90 | 30 | 55.7469317 | 15.1280683 | 37.16462 | 10.08538 |
| 90 | 35 | 59.34026814 | 20.59098186 | 39.56018 | 13.72732 |
| 90 | 40 | 61.30565635 | 26.89434365 | 40.87044 | 17.92956 |
| 90 | 45 | 61.64309632 | 34.03815368 | 41.0954 | 22.6921 |
| 90 | 50 | 60.35258805 | 42.02241195 | 40.23506 | 28.01494 |
| 90 | 55 | 57.43413154 | 50.84711846 | 38.28942 | 33.89808 |
| 90 | 60 | 52.88772679 | 60.51227321 | 35.25848 | 40.34152 |
| 90 | 65 | 46.7133738 | 71.0178762 | 31.14225 | 47.34525 |
| 90 | 70 | 38.91107257 | 82.36392743 | 25.94072 | 54.90928 |
| 90 | 75 | 29.48082311 | 94.55042689 | 19.65388 | 63.03362 |
| 90 | 80 | 18.4226254 | 107.5773746 | 12.28175 | 71.71825 |



(a)



(b)
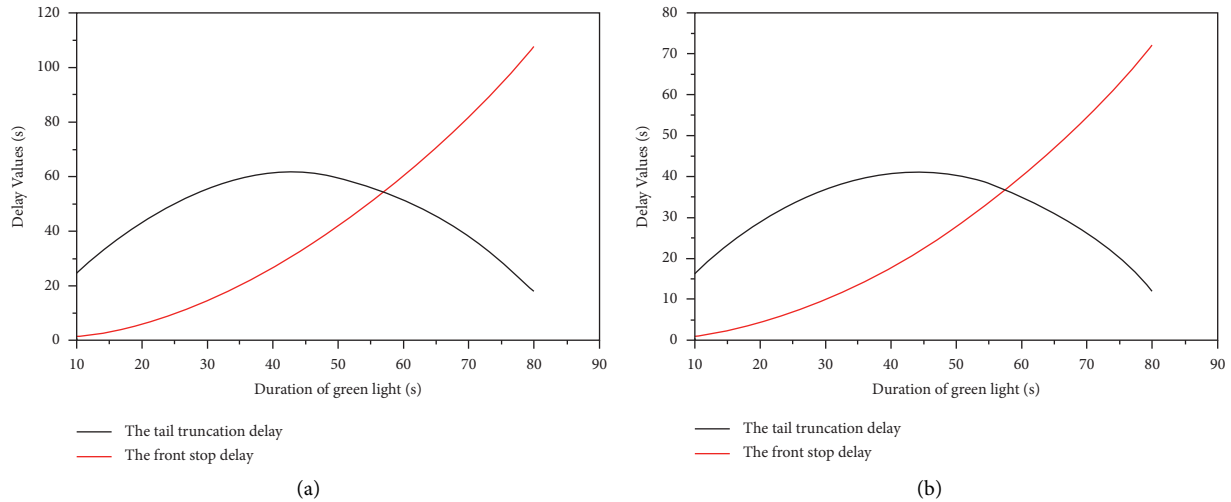
FIGURE 4: Line graphs of the front vehicles of discrete traffic flows were stopped and the tail vehicles of discrete traffic flows were truncated.

showed the saturated flow at the upstream intersection is $S = 1200$ $veh/h$. Through the comparative analysis of $a$, $b$ two diagrams, it is found that no matter how the saturated flow at upstream intersection changes, the two delay values are equal when the green signal ratio is around 2/3; when the green signal ratio is less than 2/3, the tail truncation delay is greater than the front stop delay, then the front truncation can be used for discrete traffic flow, and the phase difference can take the value of (17); when the green signal ratio is greater than or equal to 2/3, the tail truncation delay is less than or equal to the front stop delay, then the tail truncation can be used for discrete traffic flow, and the phase difference can be $\Delta g = x/v_f$ to control the traffic flow.

## 8. Conclusions

Based on the analysis of discrete kinematics behavior of traffic flow, this paper concludes that there are two situations when the discrete traffic flow arrives at the downstream

intersection from the upstream intersection, that is, the front stop of discrete traffic flow and the tail cut-off discrete traffic flow. The delay of these two situations is deeply studied, and different delay models are obtained.

Through the analysis of the examples, the two delays are different under different green signal ratios. When the green signal ratio is about 2/3, the two delay values are equal. In different green signal ratios, the two adjacent intersections adopt different phase differences, which can minimize the total delay and maximize the traffic efficiency of the traffic flow. This can control the discrete traffic flow finely, and it is more conducive to the improvement of the overall operational efficiency.

Considering the dispersion of traffic flow, combined with different green signal ratios, the phase difference of upstream and downstream intersections can be coordinated according to different objectives, so as to make the traffic flow pass with maximum efficiency and then provide a new theoretical basis for signal coordination control.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Authors' Contributions

Yan Liu, Kuo Zhang, and Li-jie Wang conceptualized the study, wrote the original draft, and validated the data and were responsible for methodology; Yan Liu performed formal analysis; Yan Liu and Kuo Zhang visualized the study.

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