Urban Rail Transit System Network Reliability Analysis Based on a Coupled Map Lattice Model

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During the last twenty years, the complex network modeling approach has been introduced to assess the reliability of rail transit networks, in which the dynamic performance involving passenger flows have attracted more attentions during operation stages recently. This paper proposes the passenger-flow-weighted network reliability evaluation indexes, to assess the impact of passenger flows on network reliability. The reliability performances of the rail transit network and passenger-flow-weighted one are analyzed from the perspective of a complex network. The actual passenger flow weight of urban transit network nodes was obtained from the Shanghai Metro public transportation card data, which were used to assess the reliability of the passenger-flow-weighted network. Furthermore, the dynamic model of the Shanghai urban rail transit network was constructed based on the coupled map lattice (CML) model. Then, the processes of cascading failure caused by network nodes under different destructive situations were simulated, to measure the changes of passenger-flow-weighted network reliability during the processes. The results indicate that when the scale of network damage attains 50%, the reliability of the passenger-flow-weighted network approaches zero. Consequently, taking countermeasures during the initial stage of network cascading may effectively prevent the disturbances from spreading in the network. The results of the paper could provide guidelines for operation management, as well as identify the unreliable stations within passenger-flow-weighted networks.

1. Introduction

With the rapid development of urbanization, rail transit lines of megacities have been extended into a network undertaking large-scale urban commuter passengers. Metro networks not only improve the efficiency of the transit system but also expand the risk of fault propagation. Although the topological network of the urban transit system is relatively simple, as one typical social network, the passenger-flow-weighted one is relatively complex [1]. Therefore, from the perspective of a complex network, we establish the network dynamics model of the rail transit system, to analyze the cascading failure process and the changes of passenger-flow-weighted reliability will assist the metro management agency to improve the capacity of passenger transportation and effectively prevent large-scale burst accidents.

Many studies have been devoted to the cascading failure and passenger-flow-weighted reliability from different perspectives in transportation networks [2, 3]. Previous research mainly concentrated on the network vulnerability, accessibility, and other characteristics, while few studies
focus on the passenger-flow-weighted network reliability and cascading failure process of the subway network. Latora and Marchiori [4] investigated the Boston metro network and verified that the subway network has small world characteristics, from which the concepts of network efficiency and connectivity index were proposed. Scale-free networks behave differently under random and intentional attacks [5, 6]. Random attacks may merely cause the network to be slightly affected, while an intentional attack seriously affects the network which is the beginning of network robustness research. Sun and Guan [7] set up the Shanghai network model and proposed indicators to measure the vulnerability of the network from the line perspective. They found that circular lines usually have the highest value of vulnerability of the network from the line perspective. They also found that the heterogeneity and vulnerability of the Beijing subway network vary over time when passenger flow is taken into consideration [11]. From the perspective of passenger ridership, Chen et al. [12] utilized the binomial logit model (BNL) to estimate mode choices and distinguish the relationship between metro and taxi as substitutable, complementary, and extended types. The temporal and spatial characteristics of passenger trips and the unbalanced features of line entry and exit by mining the AFC card data of Shanghai Metro are explored as an application of passenger data [13]. The research evaluated the travel reliability of passengers based on travel time index. For the network cascading failure, the impact of vertex failures on the probability of trip failure in a number of transit topological networks is analyzed by an absorbing Markov chain model [14]. The absorbing Markov chain model also measures the unreliable probability of a missing transit line due to insufficient capacity. The results indicated that the hub and spoke graph is the most reliable type for random vertex failures. For link failure, a normative approach for transportation network reliability based on game theory is proposed to analyze the weak points of the network and the performance in case of link failure [15]. The reliability estimates were divided into two parts, one is the probability to encounter a failure and the other is the probability to arrive at a destination within an acceptable travel time. The two methods analyze the network reliability from the perspective of probability.

The reliability evaluation methods above have rather good performance compared with traditional methods. However, the change of network reliability during dynamic processes has still not been mentioned. In addition, the network cascading failure behavior is another important issue. This study simulates the processes of cascading failures caused by network nodes based on a coupled map lattice (CML) model and measures the passenger-flow-weighted network reliability during cascading failure processes. The CML model is a common dynamic network simulation method, which describes the continuous changes of the chaotic state. Crucitti et al. [16] simulated the cascading failure of power grid and Internet, utilizing the efficiency index to measure the changes of network function. They found that the breakdown of a single node is sufficient to collapse the efficiency of the entire system if the node is among the ones with the largest loading. Cui et al. [17] applied the CML model to cascading failure of small-world networks and found that a larger mean node degree (referring to the number of edges directly connected with the node within the network) can delay the cascading failure process. Cui et al. [18] modified the original CML model and proposed a sequential cascading failure model with edge disturbance, in which the application of the CML model is extended to edge elements. Zhang et al. [19] proposed an improved CML model to simulate the urban road traffic network of Beijing, in which the cascading failures were tested using different attack strategies.

To sum up, the network cascading failure process has been investigated through different disrupted node selection strategies, measuring the reliability of the network through the change of these properties by network efficiency and invulnerability during the process. However, the passenger-flow-weighted reliability is still less involved, which has a large influence on the rail transit network. In this paper, based on the actual metro passenger flow data, improved reliability indexes involving passenger flow parameter were proposed, to better reflect the field network reliability situations. The remainder of the paper is structured as follows: Section 2 describes the definitions and methodology for the urban metro system network reliability analysis used, in which the dynamic state modeling of metro stations based on coupled map lattice is proposed. An empirical study on network cascading failure process of the Shanghai Metro system is conducted. The cascading failure processes under different conditions are investigated. Finally, conclusions and recommendations are provided in Section 4.

2. Definitions and Methodology

Reliability concept reflects the function of the network during the processes of cascading failure. As a metro network is generally composed of multiple nodes and connected edges between nodes, network reliability can be considered as the sum of the reliability of all nodes.
Therefore, measuring reliability changes need to utilize other function indicators. Network topology has a large impact on reliability, but in the real world, due to the different distribution and routes of passenger flows. Reliability analysis from the perspective of passenger flow weight has more practical significance [20]. We try to combine topological index indicators with passenger flow weight to obtain real metro network reliability.

2.1. Reliability Measure Indexes

2.1.1. Betweenness and Passenger-Flow-Weighted Betweenness. Betweenness refers to the number of shortest paths passing through the node for the shortest paths between all nodes within the network [21], which is often used to evaluate the centrality of nodes in an unweighted topological network. However, the node betweenness only considers the factor of network topology, ignoring the passenger flow factor. However, nodes with large betweenness values in an unweighted network may not be practically significant if incorporating the real passenger flow weight [3]. By combining the betweenness centrality of nodes with the actual passenger flow weight, the passenger-flow-weighted betweenness index of node passenger flow can be obtained with the mathematical expressions as follows:

\[
C_B(v) = \sum_{s,t\in V} \frac{2\theta(s,t|v)}{(N-1)(N-2)\theta(s,t)},
\]

\[
C_B^W(v) = W_v C_B(v) = W_v \cdot \sum_{s,t\in V} \frac{2\theta(s,t|v)}{(N-1)(N-2)\theta(s,t)},
\]

\[
W_v = \frac{N Q_v}{\sum_{i=1}^{N} Q_i},
\]

\[
C(V) = \frac{1}{N} \sum_{i=1}^{N} C_B^W(v),
\]

where \(C_B(v)\) denotes the betweenness value of node \(v\) after normalization; \(C_B^W(v)\) denotes the passenger-flow-weighted betweenness value of node \(v\); \(N\) denotes the number of all nodes in the network; \(\theta(s,t)\) denotes the total number of shortest paths from nodes \(s\) to \(t\); \(\theta(s,t|v)\) denotes the number of shortest paths from \(s\) to \(t\) passing through node \(v\); \(Q_i\) denotes the total amount of passengers passing through node \(i\) in one day; \(Q_v\) denotes the total amount of passengers passing through node \(v\) in one day; and \(W_v\) denotes the ratio of passenger flow at node \(v\) and the average node passenger flow; and \(C(V)\) denotes the passenger-flow-weighted betweenness value of network \(V\).

2.1.2. Entropy and Passenger-Flow-Weighted Entropy. Entropy is applied for describing the chaotic degree of the system. The higher entropy value is related to the higher chaos degree of the system. A network topology entropy index as calculated in equation (2) is introduced to measure the uniformity of the unweighted network as follows:

\[
D_v = \frac{k_v}{\sum_{i=1}^{N} k_i},
\]

\[
E = -\sum_{v=1}^{N} D_v \ln D_v,
\]

where \(D_v\) represents the proportion of node degree \(v\) to all nodes; \(k_v\) represents the degree of node \(i\); \(N\) denotes the number of all nodes in the network; and \(E\) represents the value of network topology entropy; Furthermore, the node passenger flow intensity was introduced to measure the equilibrium degree of the served passenger flow intensity of network nodes. Then, the entropies of network passenger flow intensity after normalization are as follows:

\[
G_j(V) = \frac{G - G_{\min}}{G_{\max} - G_{\min}} = -\sum_{v=1}^{N} I_v \ln I_v = -\sum_{v=1}^{N} I_v \log I_v,
\]

\[
G = \sum_{v=1}^{N} I_v \ln I_v,
\]

\[
I_v = \frac{Q_v}{\sum_{i=1}^{N} Q_i},
\]

where \(G_{\min} = 0\) denotes the theoretical minimum entropy of the network; \(G_{\max} = \ln N\) denotes the theoretical maximum entropy of the network; \(G\) denotes the passenger-flow-weighted entropy of the network; \(N\) denotes the number of all nodes in the network; \(Q_i\) denotes the total amount of passengers passing through node \(i\) in one day; \(Q_v\) denotes the total amount of passengers passing through node \(v\) in one day; and \(I_v\) denotes the proportion of passenger flow passing through node \(v\) to the total passenger flow passing through all nodes.

Thus, based on the concept, the proposed passenger-flow-weighted reliability considers the passenger flow factor by indexes of betweenness and entropy.

2.2. Dynamic State Modeling of Metro Stations Based on CML. Being widely used in modeling complex dynamic systems, the CML model defines the coupling relationship between adjacent network nodes and describes the spatiotemporal chaotic state changes of the nodes. This means that the CML state value of node is solely determined by the previous time state of the node and the previous time state of the adjacent nodes. The expressions are defined as follows:

\[
X_i(t+1) = (1 - \epsilon) f(X_i(t)) + \epsilon \sum_{j=1,j\neq i}^{N} \frac{a_{ij} f(X_j(t))}{k(i)},
\]

where \(X_i(t)\) is the CML state of node \(i\) at time \(t\), \(\epsilon\) is the coupling coefficient, and the larger coupling coefficient is always related to the higher mutual influence of the nodes. \(a_{ij}\) is the corresponding row \(i\) and column \(j\) element within the
adjacency matrix $A = (a_{ij})_{N \times N}$, reflecting the network connection information, $a_{ij} = 1$ denotes that node $i$ is directly connected to node $j$; otherwise, $a_{ij} = 0$. $f(x) = 4x(1 - x)$ is the chaotic logistic mapping function ($x \in [0, 1]$), and $X_i(t+1)$ is obtained from $X_i(t)$ through chaotic mapping function and other mathematical operations. When $0 < X_i(t) < 1$, node $i$ is in the healthy state. During investigating the cascading failure processes of the network system caused by node $i$, exerting an external disturbance $R \geq 1$ at time step $m$, the node fails, which can be expressed as follows:

$$X_i(t + 1) = \left(1 - \varepsilon\right)f\left(X_i(t)\right) + \varepsilon \sum_{j=1,j \neq i}^{N} a_{ij}f\left(X_j(t)\right) + R.$$  \hspace{1cm} (5)

The failed node $i$ will be removed from the network at the next time step $m + 1$. Due to the coupling mechanism of the CML model, the CML state of adjacent nodes of $i$ is affected and the cascading failure of the network is induced.

3. Empirical Study for the Network Cascading Failure Process

3.1. Passenger Flow Data Process and Analyses. As the necessary parameter of weighted reliability indexes, the actual passenger flow is an essential part extracted from the card record provided by Shanghai Public Transport Card Company Limited. We selected the data of August 29, 2016 (Monday). During the day, about 2 million passengers and 5 million trips were obtained after the data process. Considering that most metro passengers tend to choose the shortest routes and high cost of subway transfer time, the Dijkstra shortest path algorithm [22] is applied to assign 5 million trips to the Shanghai network according to the corresponding OD of each passenger. After assignment, the amounts of daily passenger flow of each network node were obtained and used as the parameter of reliability indexes.

3.2. CML State of Network Nodes. Although the states of nodes are decided by the CML model, the dynamic state of different nodes is different due to their respective connection relationship within the network. Therefore, each node has a special CML state value $X_i(t)$ at any time step.

3.2.1. Changes of CML State Values of Network Nodes without Disturbance $R$. As the CML model describes the relationship between adjacent nodes in the network, any single node is controlled by the model. At the initial time $t = 0$, the state value $X_i(0)$ of all nodes in the network is set to a random value between (0, 1), and the states at subsequent time steps are calculated according to formula (4). If not disturbed, the node state value is always within the range of (0, 1). Taking the Dongchuan Road subway station node of Metro Line 5 as an example, the CML states of each time step of the Dongchuan Road station are affected by the CML state of adjacent Jianchuan Road and Jiangchuan Road stations. The change of the CML state value without $R$ interference is shown in Figure 1.

3.2.2. Changes of CML State Values of Network Nodes with Disturbance $R$. By selecting node $S$ randomly and exerting disturbance $R \geq 1$ at time step $m$, the node will fail and be removed from the network at the next time step, and the node status value is set to 0 at the subsequent time step. Taking the node of the Dongchuan Road Station as an example, by exerting the interference $R = 1$ at the 51st time step, the change of the CML status is as shown in Figure 2.

As presented, after adding disturbance $R = 1$ at time step $t$, the network node fails and the CML state value of Dongchuan Road, $X(t)$, exceeds the critical value 1. Then, the CML value of the following step $X(t + 1)$ was set to 0, and the rest of the steps are followed.

3.3. Cascading Failure Processes and Passenger-Flow-Weighted Reliability Analysis of the Shanghai Network. Under the conditions of different node degrees, coupling coefficients $\varepsilon$, and disturbances $R$, the cascading failure process of the rail transit network system behaves differently, and the corresponding network reliability state changes are also different. A small $R$ disturbance cannot lead to cascading failure of the network, while the large coupling coefficient $\varepsilon$ leads to too fast propagation speed, and the node degree mainly describes the impact of node importance on network cascading failure.

3.3.1. Influence of the Node Degree on Passenger-Flow-Weighted Reliability. The higher degree value of the node generally indicates higher importance within the network, which tends to be located in the central area of the network. To investigate the influences of nodes with different degrees on the cascading failure process of network, four stations, Pudong International Airport (degree value = 1), Jianchuan Road (degree = 2), Lancun Road (degree = 3), and South Xizang Road (degree = 4), were selected randomly. With the coupling coefficient $\varepsilon = 0.2$, by exerting the same disturbance $R = 4$, the cascading failure process caused by network nodes is as shown in Figure 3.
Figure 3 compares the cascading failure processes of the metro network caused by nodes with different degrees, while Figures 4(a) and 4(b) measure the changes of passenger-flow-weighted network betweenness and entropy during the processes of different cascading failures. Under the same disturbance $R$ and coupling coefficient $\varepsilon$, nodes with larger degrees tend to cause cascading failure processes faster than nodes with smaller degrees.

When time step $t \leq 5$, the cascading failure processes are in the initial stage, and the disturbance $R$ can only propagate to the adjacent nodes of the failure node in a relatively slow speed. During the initial stage, the passenger-flow-weighted betweenness and entropy of the network are slightly affected, which indicates that the overall function of the network was not affected. When $t > 5$, the speed of cascading failure begins to accelerate because the disturbance $R$ spreads to the center area of the network and other lines through the transfer nodes. Eventually, all nodes were infected and failed, and the passenger-weighted betweenness and entropy indexes decrease to 0.

At the acceleration stage, the passenger-flow-weighted entropy declines constantly, indicating that the passenger flow chaos is declining. Different from the passenger-flow-weighted entropy, the weighted betweenness rapidly decreases to 0. As the disturbance $R$ spread to nodes in the central area of the network, which undertakes large amounts of passenger flow, the connectivity function of the network is seriously affected. It was found that the node degree value mainly affects the speed of cascading failure processes.

3.3.2. Influence of Disturbance $R$ on Passenger-Flow-Weighted Reliability. Disturbance $R$ mainly refers to the size of node failure, during which the network system shows different situations from reliability to collapse under different $R$ values. The network will not cause cascading failure if $R$ value is small, but when $R$ value is large, the network will collapse continuously. To highlight the influence of different $R$ values on the network cascading failure processes, the node with the largest degree value is selected. Consequently, the Century Avenue Station (degree = 6) is selected and exerts different disturbance values ($R = 2.6$ to 4.4), and the network cascading failure processes is shown in Figure 5.

Figure 5 describes the range and speed of cascading failure processes caused by the Century Avenue Station under different disturbances ($R = 2.6$–4.4). The disturbance $R$ factor mainly affects the scale of the network. Under the same coupling coefficient $\varepsilon$, nodes with larger disturbance $R$ tend to cause larger scale cascading failures. When $R \leq 3.6$, it is not enough to cause consecutive network collapse, only part nodes fail. However, when $R > 3.6$, diagrams of the cascaded failure processes of different $R$ values merely coincide to each other, which indicates that the speed of network cascading failure attains the maximum. Nodes affected by disturbance $R$ at each time step are all infected and become ineffectual.

Figures 6(a) and 6(b) present the changes of network passenger-flow-weighted betweenness and passenger-flow-weighted entropy reliability during the process of cascading failure, respectively. During the process of cascading failure
caused by different $R$ values, the reliability of passenger-flow-weighted entropy decreases continuously. However, the passenger-flow-weighted betweenness declines rapidly. For the Century Avenue Station, disturbance $R \leq 3.6$ is the critical value. Once exceeding the critical value, the scale of network cascading failure attains the maximum. In general, $R$ disturbance mainly affects the scale of network cascading failure, and the network has another critical value leading to the disturbance spread.

3.3.3. Influence of Coupling Coefficient $\varepsilon$ on Passenger-Flow-Weighted Reliability. The coupling coefficient $\varepsilon$ measures the strength of the interaction between adjacent nodes. In general, the value of $\varepsilon$ affects the disturbance spread ability, in which a large $\varepsilon$ indicates that the interaction between nodes is strong and the disturbance spread ability is strong. Conversely, a smaller $\varepsilon$ indicates weak interaction and disturbance spread ability. By randomly selecting the Pudong Avenue Station with a degree value of 2 and exerting $R = 4$ under different values of $\varepsilon$ from 0.1 to 0.7, the cascading failure process and the changes of reliability are as shown in Figure 7.

Figure 7 describes the range and speed of the cascading failure process of the Pudong Avenue node under different $\varepsilon$ coupling coefficients with $R = 4$. When $\varepsilon \leq 0.2$, merely a part of the network nodes fail, which will not lead to continuous network collapse. When $\varepsilon > 0.3$, the cascading failure
diagrams in Figure 7 coincide with each other, indicating the network cascading failure process attains the maximum speed. When $\epsilon = 0.3$, all networks are damaged, but the damage speed is different from the conditions of $\epsilon > 0.3$.

Figures 8(a) and 8(b) present the changes of network passenger-flow-weighted betweenness and passenger-flow-weighted entropy reliability during the process of cascading failure, respectively. The passenger-flow-weighted betweenness decreases faster compared to the passenger-flow-weighted entropy, which demonstrates the overall connectivity of the network decreases faster.

In general, the coupling coefficient $\epsilon$ affects the scale and speed of network cascading failure by affecting the size of $R$ in the disturbance spread processes.
4. Conclusions and Recommendations

Although urban rail transit network reliability theory has been developed for many years, the passenger-flow-weighted reliability of the metro network system are relatively unexplored. The passenger-flow-weighted reliability index is essential for metro management to identify the unreliable nodes in the network. The cascading processes analysis also provides a new perspective for the planning of metro networks. This study assumed that passengers in an urban metro travel according to the shortest path, which is close to the real travel habits of the passenger under large-scale data. The main contributions of the paper are concluded as follows:

(i) Constructing the passenger-flow-weighted reliability evaluation index of the network: by utilizing the Shanghai public transport card data, the field actual passenger flow weight was taken as the influencing factor, considering the impact of the real passenger flow weight on the network reliability. Eventually, the passenger flow was proved to have a significant impact on the network reliability.

(ii) Establishing the dynamic model of the Shanghai Metro network based on CML model, thus to investigate the influence of different parameters on the cascading failure processes: the node degree and disturbance $R$, respectively, affect the speed and scale of cascading failure processes, in which the coupling coefficient affects both the scale and speed of cascading failure processes.

(iii) Measuring the changes of passenger-flow-weighted reliability of the Shanghai Metro network during cascading failure processes: the passenger-flow-weighted betweenness index decreases faster than the entropy index, which reaches zero when the scale of cascading failure is more than half, indicating rapid decline of the network connectivity.

Data Availability

The data will be made available upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors’ Contributions

The authors confirm contribution to the paper as follows: Shaojie Wu, Yan Zhu, Ning Li, and Xingju Wang were involved in study conception and design; Yan Zhu, Yizeng Wang, and Daniel (Jian) Sun collected data; Yizeng Wang, Ning Li, and Xingju Wang carried out model simulation; Yan Zhu, Ning Li, and Daniel (Jian) Sun analysed and interpreted the simulation results; and Shaojie Wu, Yan Zhu, and Daniel (Jian) Sun prepared the draft manuscript. All authors reviewed the results and approved the final version of the manuscript.

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