A Timetable Optimization Model for Urban Rail Transit with Express/Local Mode

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1. Introduction

Urban rail transit system has been adopted as a main kind of rapid transit in big cities. With the development of urban scale, the central city becomes oversaturated, which leads to the increasing of long-distance travels between urban centers and suburbs. Accordingly, a new type of special train operation mode is utilized to fulfill the demand of passengers, e.g., the express/local mode. The express/local mode is a combination of express and local trains, which means that some trains will provide regular services at every station while some trains will only provide special service at specific stations. An express train can overtake a local train at station if the station is equipped with overtaking conditions. Cities, e.g., Tokyo, Guangzhou, and Shanghai, are already using this mode to increase the satisfaction of travelers.

Similar to the regular stop mode, the timetable adjustment and optimization of express/local mode aim at determining a property schedule for a set of trains so that they can follow some operational requirements [1]. It is also beneficial for efficient train operations and satisfactory traveling services. In express/local mode, the express trains stop less than the local trains, which means that express trains could have less accelerations and decelerations. Comparatively, the travel time of express train is less than that in the local mode. Thus, many long-distance travelers would prefer to wait for the next express train. Due to the special operating mode of express train and uncertainty in passenger route selection, the number of boarding passengers is hard to calculate as well as the passenger waiting time. The solutions to this problem include adjusting headway time, dwelling time, train departure time, train arrival time, and train running time [2]. A variety of models have been constructed to optimize the train timetable in recent literatures. The objective functions of these studies can be divided into four categories.

The first category takes the satisfaction of passengers into account, which is to reduce the total time spent by passengers. Note that the time mainly involves the total passenger travel time [3], the total delay time [4], the total passenger interchange time [5], or the total passenger waiting time at platforms [6]. For instance, Niu et al. [7] aimed at reducing total passenger travel time and developed a nonlinear optimization model to solve the transit-
scheduling problem. This model considered oversaturated conditions, i.e., train capacity was set to be limited. Later, they constructed integer programming models [8] in skip-stop operating mode with given predetermined train skip-stop plans and origin-to-destination (O-D) ridership data. Sun et al. [9] formulated three optimization models to design demand-sensitive timetables by reducing passenger waiting time at platforms. They also pointed out that the model without considering train capacity may offer optimal temporal train configuration.

The aim of the second category is to reduce the operating cost. Many studies convert the train operating cost into train energy consumption. For example, Zhou et al. [10] optimized the train timetable by adjusting the running time during the trips. The train traction energy could be reduced by this means. Yang et al. [11, 12] proposed energy saving models by adjusting the train timetable, which aimed to improve the utilization of recovery energy. These energy saving models indicated that the more energy that was regenerated from braking phase, the more recovery energy that could be utilized for accelerating phase.

The third category considers both the satisfaction of passenger and the reduction of the operation cost. For instance, Yang et al. [13] formulated an optimization model, which could reduce energy consumption and travel time. Sun et al. [14] considered the time-variant characteristics of passenger demand at each station and developed a bi-objective timetable optimization model. Wang et al. proposed an event-driven model [15], which included departure events, arrival events, and passenger arrival rates change events. The model also took the transfer time of passengers into account. Then, they focused on the skip-stop operating mode and constructed a bi-objective timetable optimization model [16]. To solve this model, a new iterative convex programming approach was proposed.

The last category concentrates on other factors to establish the optimization model, e.g., Robenek et al. [17] established a timetable optimization model under elastic passenger demand. The model considered the actual choices of the passengers.

However, these studies of transit-scheduling problem mainly focus on the standard stop mode or the skip-stop mode not the express/local mode. There have been some recent research studies on express/local mode. For instance, Luo et al. [18] proposed an integer programming model to determine the stopping patterns for express trains. The aim of this model was to reduce the total passenger travel time. Yang et al. [19] developed a model that could be utilized in the condition of cross-line express operating mode. The results indicated that the total trip time could be greatly optimized by adjusting the train stopping patterns. Nevertheless, these models paid much attention to determining the train stopping patterns and offered an even schedule. An even headway timetable may lead to longer passenger waiting time because some passengers may not board the coming train and have to wait at platforms [7].

Compared with standard stop mode, the express/local mode is more challenging. And the literatures on the timetable optimization for the metro lines with this special mode are so limited. Gao et al. [20] proposed a timetable optimization model incorporating the passenger route choices. However, there are still some shortages can be improved. One is that the model can be used only when the ratio of express/local is 1:1; the other is that the headway between local trains is set to be constant.

Among all of the objective functions mentioned in the previous section, passenger waiting time (PWT) at platforms visually reflects the unplanned travel time of passengers, which is also the main factor causing a decrease in passenger satisfaction. Accordingly, this study aims at reducing the total PWT and develops a timetable optimization model in the condition of express/local mode. The remainder of this paper is organized as follows. Section 2 concretely describes the express/local mode and develops an optimization model. Then, a generic algorithm is used to solve the model. In Section 3, a case based on real-world data from Guangzhou Metro Line 14 of China is presented. Finally, a brief conclusion and some suggestions for the future research are provided in Section 4.

2. Formulation of Optimization Model

In this section, a brief description and some assumptions of express/local mode are made. And then, this study formulates a timetable optimization model to adjust the headway and train dwelling time. Headway is the time interval between the departure of two adjacent trains at the first station.

2.1. Problem Statement. On the metro line with express/local mode, all stations can be divided into two categories, i.e., “minor station” and “major station.” The stations that allow express train to stop and serve the passengers are called major stations. As for the other stations, express trains will pass them without stopping. By this means, the total travel time of express train can be reduced. And these stations are called minor stations. Figure 1 shows the illustration for express/local mode. In the sketch map, solid circle represents the train stopping patterns while squares represent different kinds of stations. Minor stations are marked as solid square, while major stations are marked as hollow square. Besides, the overtaking stations, which equip with overtaking facilities, are marked as square with shadows. Note that the overtaking stations are the places that allow the express trains to overtake the local trains. When overtaking event happens, the local train in front will stop at the overtaking station until the next express train passes the station successfully. If the overtaking station is a major station, both express and local trains will stop to serve the passengers, and it may spend more time to finish the overtaking event. Thus, overtaking stations are usually chosen from the minor stations. Furthermore, the express/local mode is more suitable for the metro lines that have moderate passenger volume and service frequency [21]. Therefore, oversaturated situation is not considered in this study, i.e., all of the passengers can board the train they want. It needs to be pointed out that the express trains do reduce the train
running time. On the other hand, they increase the waiting time for passengers at minor stations. For simplicity, we notate the ratio of express/local train as 1: N (N is a positive integer and is known in advance). On most of the metro lines with express/local mode, the ratio of express trains to local trains is less than 1. Recent research studies of express/local mode discussed the case where the ratio is equal to 1, i.e., N = 1 [7, 18–20].

And other assumptions are made to facilitate the model formulation as follows:

1. Passengers with the same O-D have the steady arrival rate during the time horizon \( t_k \). And the real-world O-D data can be obtained in advance.
2. There is only one overtaking station on the metro line, i.e., an express train can only overtake a local train once during the trip (see Figure 2(a)), and the case shown in Figure 2(b) will not occur.
3. To ensure balanced service for passengers waiting at major and minor stations, departure mode of the express and local trains should be a cyclic pattern (see Figure 3). Between two express trains, there will be N local trains departing from the first station. Every cycle consists of an express train and N local trains. We define such cycle as “train departure cycle” (TDC), which will be used in the following sections.

The decision variables of this model are the headway and the dwelling time. And the objective is total PWT. All of the parameters and their definitions are given in Table 1.

2.2. Objective Functions. Figure 3 shows the time-distance diagram for express/local mode. Train 3 represents the express train in the TDC of our study; station \( z \) represents the overtaking station. \( Z_- \) represents the set of stations before the overtaking station, i.e., stations 1, 2, \ldots, \( z - 1 \), while \( Z_+ \) represents the rest of the stations on express/local line, i.e., stations \( z, z + 1, \ldots, n \).

2.2.1. Calculation of Departure Interval. A TDC ends when the last train in this cycle departs via the first station. At the same time, the next cycle starts. For visual illustration, we take the train cycle in Figure 3 as an example. Train 0 departs via station 1 at \( t_{d_0} \). As for the cycle which includes train 1, train 2, \ldots, and train \( N + 1 \), \( t_{d_0} \) is the beginning time.

Furthermore, departure time \( t_{d_i} \) and arrival time \( t_{a_i} \) at different stations can be calculated by headway, dwelling time \( d_i \), and train running time \( r_i \), i.e.,

\[
\begin{align*}
\text{td}_1^0 &= 0, \\
\text{td}_i^k &= \begin{cases} 
\text{td}_i^0 + \sum_{v=1}^{k} h_v, & i = 1, \\
\text{td}_i^0 + \sum_{v=1}^{k} h_v + \sum_{u=1}^{i-1} t_u + \sum_{u=2}^{i} d_u, & i = 2, 3, \ldots, n,
\end{cases} \\
\text{ta}_i^k &= \text{td}_i^k - d_i^k, & i = 2, 3, \ldots, n; k = 1, 2, \ldots, N + 1, \\
d_i^k &= \begin{cases} 
d_{i,v}, & k \in K_e, \\
d_{i,v}, & i \in I_c; k \in K_e, \\
0, & i \in I_j; k \in K_e
\end{cases}
\end{align*}
\]

In formula (4), \( d_{i,v} \) and \( d_{i,v} \) represent the dwelling time of local trains and express trains at station \( i \), respectively. Note that the express train will not stop at minor stations. Thus, the dwelling time at minor station is equal to 0 (see Figure 4). In other words, the arrival time and the departure time of express train are the same according to formula (3).
analyze different kinds of situations, this study constructs an optimization model with five cases as follows:

**Case 1.** This represents a kind of local train. During the train running process, the train in front of this kind of trains is a local train. The train behind this kind of trains is also a local train, i.e., train 5, train 6, ..., train \( N + 1 \) in Figure 3.

**Case 2.** This represents a kind of local train. Before the train arrives at the overtaking station, it runs between two local trains. After the train passes the overtaking station, it runs between a local train and an express train, i.e., train 1 in Figure 3.

**Case 3.** This represents a kind of local train. The train can be overtaken by an express train at overtaking station, i.e., train 2 in Figure 3.

**Case 4.** This represents the express train, i.e., train 3 in Figure 3.

**Case 5.** This represents a kind of local train. Before the train arrives at the overtaking station, it runs between an express train and a local train. After the train passes the overtaking station, it runs between two local trains, i.e., train 4 in Figure 3.

According to assumption (3), one train departure cycle only has one express train. Therefore, the train in Case 2, Case 3, Case 4, or Case 5 is unique. Note that if the ratio is different, the number of cases may be different. If the ratio is 1 : 1, 1 : 2, or 1 : 3, the timetable can also be optimized by our idea, but the number of cases in a TDC may be different (see Figure 5). Figures 5(a)–5(d) show four situations when the ratio is different. When the ratio is less than 1 : 3, i.e., \( N \geq 3 \), a TDC includes all five kinds of cases (see Figure 5(d)). When the ratio is equal to 1 : 3, only Case 1 is not in a TDC (see Figure 5(c)). When the ratio is 1 : 2, a TDC includes three cases of train (see Figure 5(c)). When the ratio is 1 : 1, a train departure cycle only includes two cases of train, i.e., Case 3.
Table 1: List of parameters and variables.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{ij} )</td>
<td>Number of passengers who arrive at station ( i ) and want to reach station ( j ) before train ( k ) leaves</td>
</tr>
<tr>
<td>( \lambda_{ij} )</td>
<td>Arrival rate of passengers who arrive at station ( i ) and want to reach station ( j )</td>
</tr>
<tr>
<td>( W_{ij} )</td>
<td>Number of passengers who choose to wait for the next express train at station ( i ) and want to reach station ( j )</td>
</tr>
<tr>
<td>( Q_{ij} )</td>
<td>Number of passengers who want to reach station ( j ) but left by express train at station ( i )</td>
</tr>
<tr>
<td>( X )</td>
<td>Set of stations, ( X = {1, 2, \ldots, n} ), where 1 is the first station and ( n ) is the terminal station</td>
</tr>
<tr>
<td>( N )</td>
<td>The number of local trains in a train departure cycle (TDC)</td>
</tr>
<tr>
<td>( L_i ); ( I_i )</td>
<td>Set of major stations; set of minor stations</td>
</tr>
<tr>
<td>( K_e, K_l )</td>
<td>Set of express trains; set of local trains</td>
</tr>
<tr>
<td>( z )</td>
<td>Index of overtaking station (overtaking station belongs to minor station)</td>
</tr>
<tr>
<td>( i )</td>
<td>Index of station. Passengers start the travel at station ( i )</td>
</tr>
<tr>
<td>( j )</td>
<td>Index of station. Passengers end the travel at station ( j )</td>
</tr>
<tr>
<td>( k )</td>
<td>Index of train</td>
</tr>
<tr>
<td>( Z_s )</td>
<td>Set of stations including the overtaking station and the stations after overtaking station, ( Z_s = {z, z + 1, \ldots, n} )</td>
</tr>
<tr>
<td>( Z_m )</td>
<td>Set of stations before overtaking station, ( Z_m = {1, 2, \ldots, z - 1} )</td>
</tr>
<tr>
<td>( t_h )</td>
<td>The time horizon. O-D data are counted during this time horizon</td>
</tr>
<tr>
<td>( H )</td>
<td>Sum of all headways in a train departure cycle (TDC)</td>
</tr>
<tr>
<td>( \Delta t_{ik}^k )</td>
<td>Departure interval between train ( k - 1 ) and train ( k ) at station ( i )</td>
</tr>
<tr>
<td>( h_k )</td>
<td>Headway between train ( k - 1 ) and train ( k )</td>
</tr>
<tr>
<td>( T_{sij} )</td>
<td>Total waiting time of passengers who want to reach station ( j ) till they board the train ( k ) at station ( i )</td>
</tr>
<tr>
<td>( w_{ij} )</td>
<td>The percent of passengers at station ( i ) who choose to wait for the next express train to reach station ( j )</td>
</tr>
<tr>
<td>( q_i, q_j )</td>
<td>Minimum arrival interval between express train and local train at overtaking station</td>
</tr>
<tr>
<td>( q_i, q_j )</td>
<td>Minimum departure interval between express train and local train at overtaking station</td>
</tr>
<tr>
<td>([h_{min}, h_{max}])</td>
<td>Upper and lower boundaries of departure interval between two adjacent trains</td>
</tr>
</tbody>
</table>

**Case 1.** These trains will not be affected by the express train during the running process, which makes easier to calculate the departure interval at different stations. Obviously, \( \Delta t_{ik}^k \) is equal to the headway \( h_k \). If train \( k \) is in Case 1, we have

\[
\Delta t_{ik}^k = \Delta t_{ik}^k - \Delta t_{i(k-1)}^k, \quad i = 1, 2, \ldots, n; \text{ train } k \text{ is in Case 1.}
\]

**Case 2.** The same as Case 1, departure interval in Case 2 is equal to headway and can be calculated by formula (5).

**Case 3.** Before this local train arrives at overtaking station, \( \Delta t_{ik}^k \) is the same as that in Case 1 and Case 2. While arriving at the overtaking station, the local train needs to wait for extra time till the express train passes the overtaking station. After the local train is overtaken by the express train, the sequence of train changes. Departure interval at major station is between this train and the express train. However, at minor station, the express train will not stop and departure interval is between this train and the local train in Case 2. Thus, if train \( k \) is in Case 3, \( \Delta t_{ik}^k \) can be formulated as follows:

![Illustration for train departure time and arrival time in express/local mode.](image-url)

and Case 4 (see Figure 5(a)). In fact, all kinds of situations mentioned can be modelled according to our method. Considering the applicability of the timetable optimization model, we give the formulas with all five cases.

Figure 6 is a train timetable, which illustrates different departure intervals in a TDC. Then, we give the formulations of departure interval in different cases.
Case 4. Note that express train will not stop at minor stations, and we only need to calculate the departure interval when station $i$ is a major station. As shown in Figure 6, if $i \in Z_+$, the departure interval is between express train 3 and the local train 1 (see $\Delta t^k$ in Figure 6). Thus, if train $k$ is in Case 4, $\Delta t^k_i$ can be written as

$$\Delta t^k_i = \begin{cases} td^k_i - td^{k-1}_i, & i \in Z_-, \\ td^k_i - td^{k-1}_i, & i \in Z_+; i \in I_o, \\ td^k_i - td^{k+1}_i, & i \in Z_+; i \in I_e, \end{cases}$$

(6)

Case 5. Before this local train arrives at overtaking station, an express train runs in front of this local train
and departure intervals are different at different stations. Similar to Case 3, $\Delta t^k_i$ can be formulated as

$$\Delta t^k_i \equiv \begin{cases} td^k_i - td^k_{i-2}, & i \in Z_-, i \in I_s, \\ td^k_i - td^k_{i-1}, & i \in Z_-; i \in I_e, \\ td^k_i - td^k_{i-2}, & i \in Z_+, \\ \end{cases} \quad i = 1, 2, \ldots, n; \text{ train } k \text{ is in Case 5.} \quad (8)$$

2.2.2. Calculation of Total PWT. Based on the real-world Automated Fare Collection (AFC) data, different O-D flows of passengers within $t_h$ are obtained. The number of passengers arriving at station $i$ and want to reach station $j$ is marked as $P^k_{ij}$. Station $j$ is the destination of the trip, i.e., $j = i + 1, i + 2, \ldots, n$. Note that $P^k_{ij}$ should be rounded to an integer. According to the first assumption, arrival rate of passengers with the same O-D is steady. Thus, $P^k_{ij}$ can be calculated by O-D data and departure interval as follows:

$$P^k_{ij} = \left[ \frac{\lambda_{ij} \cdot \Delta t^k_i}{t_h} \right], \quad i = 1, 2, \ldots, n - 1; k = 1, 2, \ldots, N + 1. \quad (9)$$

With two types of train service provided, passengers have more than one route choices to reach the destination, e.g., some passengers are fully sensitive to the time spent during the journey. They would like to take express trains to save time, even they have to wait longer. Others prefer to take the upcoming train, i.e., the first train subsequent to their arrival, rather than transfer or wait. Furthermore, the choice behaviour can be influenced by the nature of origin and destination. Considering the psychological factors of passengers, all possible route selections are shown in Table 2. Among the 9 kinds of route selections, categories 1, 2, 4, 6, and 7 are common in daily life, i.e., passengers take the upcoming train to complete their travels. While categories 3, 5, and 7 involve transfer behaviour. Category 9 represents the case that the coming train is a local train and the train behind the local train is an express train. Passengers choose to wait longer till the express train comes. For convenience, passengers are generally unwilling to transfer to express/local mode and proposed a formulation to estimate the percent of these passengers.

$$\omega_{ij} = \frac{j - i}{n}, \quad (10)$$

where $\omega_{ij}$ indicates that if the travel distance is long, passengers would like to wait for the next express train. Then, $W_{ij}$ can be calculated by following expressions:
Next, we give the method to calculate the total PWT in different cases.

**Case 1.** The train departure interval is equal to the headway in this case; all passengers get on the upcoming train without route selections. Since the arrival rate is steady, the passenger number profile at station is a straight line in Figure 7. Figure 7(a) means that all the passengers can get on the upcoming train. \( t_x \) and \( t_y \) represent the departure time interval. \( P \) is the number of passengers arriving at station. Obviously, the total PWT can be obtained by calculating the area of each triangle. Then, the total PWT of Case 1 is

\[
T_{s_{ij}}^k = \frac{P_{ij}^k \Delta t_i^k}{2}, \quad i = 1, 2, \ldots, n; \text{train } k \text{ is in Case 1.}
\]

**Case 2.** Before the express train overtakes a local train, the calculation of waiting time is similar to formula (16). After that happens, a group of passengers may choose to wait for the next express train and will not get on the train in Case 2. As a result, the total waiting time is

\[
P + Q \text{ in Figure 7(b). Thus, we calculate these two kinds of PWT, respectively. The waiting time of passengers who arrive at station } i \text{ and want to reach station } j \text{ is } (P_{ij}^k \Delta t_i^k)/2, \text{ which can be obtained by calculating the area of green triangle in Figure 7(b). And the total waiting time of passengers left by express train is } ((Q_{ij} \Delta t_{ij}^k)/2) + Q_{ij} \Delta t_i^k, \text{ which can be obtained by calculating the area of yellow trapezoid.}
\]

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**Table 2: Passenger route selections under different O-D.**

<table>
<thead>
<tr>
<th>O-D</th>
<th>Minor station</th>
<th>Major station</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minor station</td>
<td>(1) Local train</td>
<td>(2) Local train → express train</td>
</tr>
<tr>
<td>Major station</td>
<td>(4) Local train</td>
<td>(6) Local train</td>
</tr>
<tr>
<td></td>
<td>(5) Express train → local train</td>
<td>(7) Local train → express train</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(8) Upcoming express train</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(9) Wait for the next express train</td>
</tr>
</tbody>
</table>
Then, the total waiting time in Case 3 can be formulated as

\[
T_{s_{ij}}^k = \begin{cases} 
\frac{(P_{ij}^k - W_{ij,3}) \Delta t_i^k}{2}, & i \in Z_-, \\
\frac{P_{ij}^k \Delta t_i^k}{2}, & i \in Z_+; i \in I_s, \quad i = 1, 2, \ldots, n; \text{train } k \text{ is in Case 3.} \\
\frac{P_{ij}^k \Delta t_i^k}{2} + \frac{Q_{ij} \Delta t_i^{k+1}}{2} + Q_{ij} \Delta t_i^k, & i \in Z_+; i \in I_e,
\end{cases}
\]  

Case 4. In Case 4, train \( k \) is an express train. Those passengers who choose to wait for express train in Cases 2 and 3 will board this train. Besides, the method to calculate the PWT is similar to the situation in Figure 7(b). The total waiting time in Case 4 can be formulated by following expression, i.e.,

\[
T_{s_{ij}}^k = \begin{cases} 
\frac{(P_{ij} - Q_{ij}) \Delta t_i^k}{2} + \frac{W_{ij,3} \Delta t_i^{k-1}}{2} + W_{ij,3} \Delta t_i^k, & i \in Z_-; i \in I_e, \\
\frac{(P_{ij}^k - Q_{ij}) \Delta t_i^k}{2} + \frac{W_{ij,2} \Delta t_i^{k-2}}{2} + W_{ij,2} \Delta t_i^k, & i \in Z_+; i \in I_e,
\end{cases}
\]  

, train \( k \) is in Case 4.
Case 5. Before passing the overtaking station, the train in Case 5 runs behind the express train, so \( Q_{ij} \) should also be taken into consideration. After the train passes the overtaking station, the number of passengers will not be affected by express train. As a result, the formulation is

\[
T \leq \sum_{j=1}^{n} \sum_{i=1}^{n-1} \left( \sum_{k \in \text{Case 1}} T_{s_{ij}}^k + \sum_{k \in \text{Case 2}} T_{s_{ij}}^k + \sum_{k \in \text{Case 3}} T_{s_{ij}}^k + \sum_{k \in \text{Case 5}} T_{s_{ij}}^k \right) + \sum_{k \in \text{Case 4}} \sum_{i, i' \leq j} \sum_{k \in \text{Case 5}} T_{s_{ij}}^k.
\]

2.3. Constraints. First, departure interval \( \Delta t_i^k \) need to be constrained as follows:

\[
h_{\text{min}} \leq \Delta t_i^k \leq h_{\text{max}}.
\]

Here, \( h_{\text{min}} \) represents the permitted time interval between two adjacent trains. \( h_{\text{max}} \) describes the passengers’ tolerance limit for waiting.

The dwelling time at different stations should also be limited. To ensure the quality of metro service, dwelling at stations should not be less than \( d_{\text{min}} \). Besides, dwelling time of local trains should not be longer than \( d_{\text{max}} \), especially at the overtaking station. Otherwise, passengers in local trains will complain. Express train is utilized to reduce the total travel time. Thus, the dwelling time of express train should not be too long, and \( d_{\text{max}} \) is the upper bound of it.

\[
d_{\text{min}} \leq d_{i,e} \leq d_{\text{max}}, \quad i \in I_e,
\]

\[
d_{\text{min}} \leq d_{i,s} \leq d_{\text{max}},
\]

When a train arrives at a station, the time interval between this train and the train behind it also needs to be considered carefully (see Figure 8). Then, the following constraints are required:

\[
\Delta t_i^k - d_{i,s} \geq \theta_{\text{safe}}.
\]

Note that some expressions of waiting time are the same in formula (20), but the result may differ. When \( i \in Z_\ast \) and \( i \in I_e \) in Case 5, \( \Delta t_i^k \) represents the departure interval between the train in Case 3 and the train in Case 5. As for \( i \in Z_\ast \), the calculation of \( \Delta t_i^k \) changes (see formula (8)). Moreover, number of arrival passengers \( P_{ij}^k \) changes as well.

In short, total PWT is

\[
T = \sum_{j=1}^{n} \sum_{i=1}^{n} \left( \sum_{k \in \text{Case 1}} T_{s_{ij}}^k + \sum_{k \in \text{Case 2}} T_{s_{ij}}^k + \sum_{k \in \text{Case 3}} T_{s_{ij}}^k + \sum_{k \in \text{Case 5}} T_{s_{ij}}^k \right) + \sum_{k \in \text{Case 4}} \sum_{i, i' \leq j} \sum_{k \in \text{Case 5}} T_{s_{ij}}^k.
\]

The time period \( H \) includes all of the headways. \( H \) is related with the number of trains operating on the express/local line per hour. In this study, \( H \) is given in advance. The constraint of headway is

\[
h_1 + h_2 + \cdots + h_{N+1} = H.
\]

To ensure that express train overtakes local train at overtaking station, departure interval and arrival interval between these two trains at overtaking station should also be limited (see Figure 8) and we use \( k^* \) to represent the express train. Then, we have following constraints:

\[
da^* - ta^*_{k^* - 1} \geq \theta_a,
\]

\[
td^*_{k^*} - da^* \geq \theta_d.
\]

\( \theta_a \) and \( \theta_d \) are set according to the safe interval in the overtaking station.

Finally, the timetable optimization problem with express/local mode can be formulated as

\[
\begin{align*}
\text{min} & \quad T \\
\text{s.t.} & \quad \text{constraints (22) – (28)}
\end{align*}
\]

In formula (29), \( T \) represents the total PWT in a TDC.

2.4. Problem Solving and Algorithm. A genetic algorithm (GA) is used to solve this optimization problem. GA is a method for searching an optimal solution by simulating the natural evolution process.
In this study, headway and the dwelling time are set to be the decision variables, i.e., the vector containing \((h_1, h_2, \ldots, h_{N+1}, d_{1x}, d_{3x}, \ldots, d_{nx}, d_{3x}, d_{3,3}, \ldots, d_{nx})\) is processed with binary coding making up the genes of chromosome. The operation process is as follows:

(i) **Step 1.** Initialize the parameters: Set the maximum number of population size \(\text{pop}\_\text{size}\) and randomly generate individuals, then set the maximum evolution generation number \(\text{max}\_\text{generation}\), and set the evolution generation \(u\) counter to 1.  

(ii) **Step 2.** Coding and initial solution: Variables are coded to compose the genes of chromosome with random initial values. If variables satisfy the constraints, turn step 3. If not, the initial solution should be regenerated.

(iii) **Step 3.** Calculation and selection: Calculate the fitness values \(T_u\) for all chromosomes. Select the chromosomes by spinning the roulette wheel, e.g., if \(T_u \leq T_{u-1}\), this generation of chromosomes consists of elitists, which should be kept as the current best solution. If \(T_u > T_{u-1}\), then give up on choosing this generation,

(iv) **Step 4.** Reproduce: The current chromosomes produce the next generation of individuals through crossover and mutation operations. If every individual satisfies the constraints, turn step 5. If not, reproduce the individual.

(v) **Step 5.** If \(u\) is equal to \(\text{max}\_\text{generation}\), stop the cycle and obtain the optimal solution. If not, return to Step 3.

### 3. Case Study

#### 3.1. Experiment Description

Guangzhou Metro Line 14 is 76.3 kilometers long and has 22 stations, and express/local mode is used to meet the needs of long-distance travelers. The upstream direction of the line is chosen for model verification. Figure 9 shows the sketch map of the line. The major stations are JW, XH, CCT, and DF, while the others are minor stations. Particularly, station ZLT is the overtaking station. The ratio of express/local train is 1:4. Train 3 is an express train; train 1, train 2, train 4, and train 5 are local trains. All of the cases, i.e., Case 1, 2, \ldots, 5, are needed to be considered. Train running time and original dwelling time are shown in Tables 3 and 4, respectively. Note that when an express train is overtaking a local train, the dwelling time of the local train increases to 180s. The other parameters are set as \(\alpha = 13, \ H = 1380\ s, \ z = 6, \ h_{\text{min}} = 100\ s, \ h_{\text{max}} = 600\ s, \ d_{\text{min}} = 35\ s, \ de_{\text{max}} = 60\ s, \ ds_{\text{max}} = 180\ s, \ t_h = 1800\ s, \ \theta_{\text{safe}} = 65\ s, \ \theta_{a} = 65\ s, \ \text{and} \ \theta_{d} = 65\ s\). Table 5 presents the passenger flow \(\lambda_{ij}\) within \(t_h\).

#### 3.2. Results and Discussion

The result of the GA is shown in Figure 10. The total passenger waiting time at platforms is reduced by about 9.3%. Optimized headways and dwelling time are displayed in Tables 6 and 7, respectively. The optimized train timetable is shown in Figure 11.

To better analyze the results, the PWTs at different stations under original timetable and optimized timetable are shown together (as shown in Table 8). Apparently, although the reduction of PWT is not significant at some of the stations, e.g., stations ZLT, ML, SG, CC, and CCT, the total PWT reduces a lot. Among all of the stations, JW has the highest reduction in PWT, which is up to 118448 s. Note that JW also has the highest passenger flow according to Table 5. It could be inferred that the optimization model can reduce the PWT, especially in the stations with large passenger flow.

Different from the standard stop mode, the express/local mode may cause more PWT at platforms. The reason is that express train will leave a large number of passengers according to formulas (15), (18), and (20). Likely, those who choose to board the next express train also need to wait for extra time (see formulas (13), (14), and (19)). Specifically, we display the changes of PWT between these two passenger groups in Table 9. First, PWT of these two groups is reduced by 23.0% and 20.6%, respectively. Second, the number of passengers left by express train reduces by 15.4%, which improves the comfort during the trip.
Figure 9: Sketch map of Guangzhou Metro Line 14.

Table 3: Train running time between each station.

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<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>$r_j^k (s)$</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Local</td>
<td>230</td>
<td>156</td>
<td>171</td>
<td>256</td>
<td>220</td>
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<td>227</td>
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Table 4: Original train dwelling time at different stations.

<table>
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<th>Station</th>
<th>BD</th>
<th>XL</th>
<th>TH</th>
<th>ZL</th>
<th>ZLT</th>
<th>ML</th>
<th>XH</th>
<th>TP</th>
<th>SG</th>
<th>CC</th>
<th>CCT</th>
<th>DF</th>
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<tbody>
<tr>
<td>$d_j^k (s)$</td>
<td></td>
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<td></td>
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<td>Local</td>
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<td>35</td>
<td>35</td>
<td>35</td>
<td>35/180</td>
<td>35</td>
<td>40</td>
<td>35</td>
<td>35</td>
<td>35</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Express</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>45</td>
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<td>45</td>
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</tbody>
</table>

Table 5: Passenger flow within $t_h$.

<table>
<thead>
<tr>
<th>$\lambda_{ij}$</th>
<th>JW</th>
<th>BD</th>
<th>XL</th>
<th>TH</th>
<th>ZL</th>
<th>ZLT</th>
<th>ML</th>
<th>XH</th>
<th>TP</th>
<th>SG</th>
<th>CC</th>
<th>CCT</th>
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<td>392</td>
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<td>319</td>
<td>160</td>
<td>1814</td>
<td>230</td>
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<td>153</td>
<td>2772</td>
<td>722</td>
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<tr>
<td>BD</td>
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<td>—</td>
<td>169</td>
<td>194</td>
<td>225</td>
<td>149</td>
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<td>101</td>
<td>50</td>
<td>47</td>
<td>31</td>
<td>119</td>
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<tr>
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<td>—</td>
<td>250</td>
<td>101</td>
<td>263</td>
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<td>76</td>
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<td>—</td>
<td>101</td>
<td>131</td>
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<td>124</td>
<td>81</td>
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<td>101</td>
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<td>47</td>
<td>338</td>
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<td>32</td>
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<td>700</td>
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Table 6: Original and optimized headways.

<table>
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<th></th>
<th>$h_1$ (s)</th>
<th>$h_2$ (s)</th>
<th>$h_3$ (s)</th>
<th>$h_4$ (s)</th>
<th>$h_5$ (s)</th>
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<tr>
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<td>399</td>
<td>333</td>
<td>190</td>
<td>162</td>
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<tr>
<td>Optimized</td>
<td>289</td>
<td>354</td>
<td>299</td>
<td>140</td>
<td>298</td>
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</tbody>
</table>

Table 7: Optimized train dwelling time at different stations.

<table>
<thead>
<tr>
<th>Station</th>
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<th>TH</th>
<th>ZL</th>
<th>ZLT</th>
<th>ML</th>
<th>XH</th>
<th>TP</th>
<th>SG</th>
<th>CC</th>
<th>CCT</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d^i_t$ (s)</td>
<td>Local</td>
<td>35</td>
<td>35</td>
<td>35</td>
<td>35/180</td>
<td>35</td>
<td>35</td>
<td>35</td>
<td>35</td>
<td>35</td>
<td>35</td>
<td>40</td>
</tr>
<tr>
<td>Express</td>
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</tbody>
</table>

Table 8: Changes of PWT at different stations.

<table>
<thead>
<tr>
<th>Station</th>
<th>JW</th>
<th>BD</th>
<th>XL</th>
<th>TH</th>
<th>ZL</th>
<th>ZLT</th>
<th>ML</th>
<th>XH</th>
<th>TP</th>
<th>SG</th>
<th>CC</th>
<th>CCT</th>
<th>Total</th>
</tr>
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<tbody>
<tr>
<td>Original (s)</td>
<td>1280972</td>
<td>178461</td>
<td>156598</td>
<td>110684</td>
<td>117243</td>
<td>89810</td>
<td>97548</td>
<td>193320</td>
<td>130708</td>
<td>61071</td>
<td>29844</td>
<td>19107</td>
<td>2465366</td>
</tr>
<tr>
<td>Optimized (s)</td>
<td>1162524</td>
<td>160043</td>
<td>140436</td>
<td>99261</td>
<td>105143</td>
<td>82500</td>
<td>89608</td>
<td>174418</td>
<td>120070</td>
<td>56100</td>
<td>27415</td>
<td>18455</td>
<td>2235973</td>
</tr>
<tr>
<td>Difference (s)</td>
<td>$-118448$</td>
<td>$-18418$</td>
<td>$-16162$</td>
<td>$-11423$</td>
<td>$-12100$</td>
<td>$-7310$</td>
<td>$-7940$</td>
<td>$-18902$</td>
<td>$-10638$</td>
<td>$-4971$</td>
<td>$-2429$</td>
<td>$-652$</td>
<td>$-229393$ (9.3%)</td>
</tr>
</tbody>
</table>
### 4. Conclusions

This study develops a timetable optimization model in metro line with express/local mode, which can reduce the total passenger waiting time at platforms. The main contributions include (1) the optimization model is designed for the operation metro line with express/local mode. (2) Passenger route selection is considered in the model. The case study of Guangzhou metro line 14 shows that this model can reduce the total passenger waiting time at platforms by 9.3% in comparison with the original timetable. Specifically, the model can also reduce the waiting time of those who are left by the express train.

Furthermore, there are still some improvements could be done in future work, e.g., oversaturated situation can be considered in the model. In addition, the passenger flow is assumed to be static in this study. Thus, an improved model based on dynamic passenger flow can be further studied in future work.

### Data Availability

The data used in this study all come from Guangzhou Metro Group, e.g., parameters about train operation, O-D data, dwelling time at different stations, train running time during the trip, and headway. They are all listed in the paper. Note that the O-D data are collected from the real-world Automated Fare Collection (AFC) data. AFC data include private information of passengers and cannot be shared due to the Confidentiality Agreement.

### Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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### References


