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Research Article

A Hybrid Machine Learning and Optimization Model to Minimize the Total Cost of BRT Brake Components

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Public transport is amongst critical infrastructures in modern cities, especially megacities, home to millions of people. The reliability of these systems is highly crucial for both citizens and service providers. If service providers overlook system reliability, a considerable amount of expenses will be wasted. Several factors such as vehicle failure, accident, lack of budget weather factors, and traffic congestion cause unreliability, among which vehicle failure plays a prominent role. The brake system is the most vulnerable and vital component of a public transportation bus. Brake reliability depends on driver's expertise, component quality, passenger loading, line situation, etc. Driver's expertise and components' quality are the most important factors for brake system reliability. This study aims to implement a hybrid machine learning and optimization model to minimize the total investment and reliability-related costs in a bus rapid transit (BRT) system. A regression analysis method is proposed to capture the main attributes of a joint brake system, including the level of education, training, and drivers' experience. The failure rate is modeled as a linear function of ETE and the quality of brake system subcomponents using a Lasso regression model. MILP optimization is then provided for optimizing the total expected costs for a bus rapid transit (BRT) system. Furthermore, a practical case is studied to investigate whether this optimization can reduce costs. The results confirm the efficiency of the hybrid optimization approach.

1. Introduction

Nowadays, cities are growing in size, and their populations are increasing rapidly. As citizens need to travel inside their cities more frequently, public transportation systems are getting ever-increasing importance in society. Many passengers travel by bus rapid transit (BRT), a left-side door bus operating in a fully separated lane. BRT reliability studies are pivotal because an interruption in such systems would result in passenger dissatisfaction and stakeholders would have to deal with vast economic losses. To overcome this challenge, the reliability of this transportation system is analyzed and then optimized. Reliability refers to the probability that a device performs its purpose adequately for the period intended under the operating conditions encountered [1]. A high level of reliability would be an excellent incentive for citizens to choose public transport [2]. Several works analyzed in detail in the next section aim to quantify and enhance the reliability of urban bus systems as a backbone to public transport.

There are several reasons for BRT system irregularity, including suboptimal scheduling, accident, bus failure, etc. Based on the analysis of historical data, the main reason for BRT irregularity and latency is bus failures, which is due to brake failure in most cases. Not only is brake failure the primary reason for bus failure, but also it completely interrupts the bus. The driver cannot even take the bus to the repair shop. Therefore, brake component reliability optimization is vital in enhancing overall reliability. However, system owners have limited financial resources; therefore, such enhancements should be constrained to available budgets and prospective future costs. To the best of our knowledge, this paper, for the first time, presents an analytical cost-benefit optimization for the brake system reliability considering the total costs imposed on the owners. At the first step, the brake failure rate is modeled as a function of subcomponent types and the education, training, and experience (ETE) indicators of the drivers. The former represents that a high-quality subcomponent lasts longer, while the latter represents the effect of driver skills in a better brake system. This is followed by modeling the primary investment, operation, and maintenance costs, including repair, replacement, HR training, and salary. Finally, convex mixed-integer linear programming (MILP) is provided to decide on the type of brake subcomponents of buses acquired for BRT lines and the ETE indicator of their drivers. The objective is to minimize the total cost, including investment, driver salaries, replacement costs, and economic loss due to bus interruptions and failures. To summarize, the main contributions of this paper are as follows:

(i) Modeling the brake failure rate based on subcomponents and associated drivers
(ii) Modeling various brake-related investments, operation, and maintenance costs
(iii) Optimizing subcomponents and driver planning to minimize total costs

The rest of this paper is organized as follows: related research materials are reviewed in Section 2; the general structure of the proposed approach is briefly introduced in Section 3; in Section 4, the failure rate is modeled as a function of the ETE indicator and the brake system quality; the formulation of the optimization problem is discussed in Section 5; a case study for a practical BRT system is presented in Section 6; results and Section 7 presents results and discussion, and finally, the conclusion is drawn in Section 8.

2. Related Works

Several types of research in the literature concentrated on the reliability of the public transportation system. Those works either modeled or proposed reliability enhancement solutions using online or offline methods. In the following sections, those research works are reviewed.

2.1. Reliability Modeling and Quantification. The first stage in reliability studies is modeling and quantification. Public transportation services are categorized into two groups: frequency-based and scheduling-based [3]. While scheduling-based services operate on predefined schedules, only headway time is of interest within the system management in frequency-based services. Moreover, passengers are divided into two types: commuters, who regularly travel for business or education purposes, and noncommuters, who use public transport for occasional travels in that specific pathas. Also, they analyzed the methods of computing headway and expected waiting time. Liu and Sinha [4] introduced three reliability metrics: "travel time reliability," "headway reliability," and "passenger wait time reliability." In [5], a set of reliability indicators from the viewpoint of customers are introduced. The latter expects that the indicators satisfy the four attributes "measurability," "ease of availability," "speed of availability," and "interpretability," in addition to being customer-oriented. In [6], the quality of service and transit reliability for older people (more than 65 years), counted as vulnerable users, are computed. A data-driven reliability study of public transportation for the Netherlands is presented in [7]. The automatic vehicle location (AVL) data are used for offline measuring time reliability in [8]. In [9], the percent of passengers receiving regular service (PPR) and percent of passengers receiving punctual service (PPP) using AVL data are computed. Three performance measures of punctuality index based on routes (PIR), deviation index based on stops (DIS), and evenness index based on stops (EIS) have been introduced and implemented for the Beijing transportation system in [10]. The probability that the public transport system performance is within the acceptable range for Beijing's transport system reliability is computed. The impact of ridership on the reliability of the public transportation system is modeled in [11]. A review of all influential factors in reliability, in addition to reliability metrics, is briefly discussed in [12]. It divides the factors into two groups: demand-side factors, including traffic flow, passenger route-wise demand, and directional flow at intersections, and supply-side factors, including facility design, accidents, driver behavior, traffic management scheme, vehicle breakdown, and weather. Next, the reliability of the Ahmedabad city is computed using gathered GPS data. A methodology for estimating the value of travel time reliability is presented in [13]. Bunker [14] presented a probabilistic reliability model for sections (the distance between stops). Although that model investigates the financial aspects of reliability, it does not discuss the improvement strategy.

2.2. Reliability Enhancement. Optimal reliability enhancement can substantially reduce system expenses. Moosavi et al. [15] categorized reliability enhancement strategies into three main groups: prioritizing, operational, and control. Prioritization approaches are those that give priority to the public within the city. Dedicating a separate lane to buses is an example of prioritization policies. Operational strategies include long-term accomplishments such as driver training and restructuring of bus routes (offline methods). Control strategies are real-time decisions such as skipping stops (online methods). The impacts of various control strategies on transportation reliability are then simulated. The paper does not model the economic aspects of reliability. Therefore, the approach does not give the stakeholder a vision for the financial benefits of reliability enhancement. An analytical control strategy optimization is suggested in [16]. This approach ensures the global optimality of final results. AVL data are utilized in [17–19] to identify routes that need assistance and reliability enhancement. Wang et al. [20] proposed a data-driven bus scheduling optimization to enhance the reliability of transportation systems. In [21], how headway variations cause an extra cost to passengers and how total cost (operator and user costs) will be optimized by bus stop placement and dispatching headway are discussed.

The papers mentioned above discussed the effect of path and control strategies on public transport reliability. However, for a BRT service in which a specific lane is dedicated to buses, the unwise path selection and suboptimal control strategies are not the primary cause of unreliability. These systems are highly reliable, especially in peak hours [22]. Breakdowns of buses are the main cause of interruptions and compromises to the service. A bus consists of several vulnerable components, of which brake systems are responsible for most interruptions. We checked the accuracy of this fact by comparing the reasons for a practical BRT system interruption. The historical data confirm that brake failure accounts for most of the buses' interruptions. Data analysis in [23] also confirms that the main factor that causes an urban bus being downtime in repair is the brake system failure. A fuzzy rule-based study of the Istanbul BRT system also indicates that the brake component is one of the vital components for retaining the reliability of the BRT system [24].

Furthermore, upon brake malfunctions, a severe risk is imposed on passengers and drivers. In this regard, the brake system functionality is also a key to safety [25]. Hence, analyzing the reliability of brake systems and their impact is an important subject.

Yusupov [25] presented a serial reliability model for brake systems. The reliability of the brake system is then computed based on the reliability of each subcomponent. A maximum likelihood estimation (MLE) method is presented in [26] for estimating the failure distribution of brake subcomponents. Moreover, the reliability of the brake system is computed using a fault tree. A Petri-net model for computing the reliability of mechatronic systems is presented in [27]. The critical reliability metrics, which are failure rate, mean time between failures, mean time to repair, and the brake system availability, are modeled in [28]. The shape of the brake piston ring is redesigned to improve the reliability in [29]. Yusupov et al. [30] first identified the subcomponents with the credible value of failure rates. Then, the relationship between these values and the brake component reliability was modeled. Finally, the method was simulated for the ABS brake system. None had studied the brake system reliability impact on the overall bus reliability.

To conclude, none of those above papers analyzed and optimized the brake system reliability as part of the whole. As a result, financial studies and the cost-benefit optimization for reliability enhancement are also missed. To fulfill this research gap, this paper presents a model for brake failure. The costs BRT systems endure due to brake failures are modeled, and the BRT system's total costs related to brake failures are optimized.

3. Proposed Methodology: Big Picture

To enhance the reliability of the brake components, first, the influential factor should be identified, as depicted in Figure 1. The drivers' expertise and subcomponents' quality are discussed in Section 4. Experts can provide an approximately precise score for drivers and subcomponents. The relationship between the failure rate and these scores can be modeled with machine learning (ML). Increasing the score for these factors inevitably causes a decrease in the failure rate. However, this increase requires extra investment either in components or salary and training costs. Due to limitations in available budget, the total cost, covering reliability enhancement budget and interruption, and operation and management (O&M) costs should be optimized. Since O&M cost computation requires every subcomponent failure and replacement cost, the decomposition overall failure rate to subcomponent failure rate is necessary.

4. Failure Rate Model

To model the brake system failure rate, the features contributing failure rate value should be extracted. These are the features that must be modeled and qualified. The main reasons for brake system failure are low-quality brake components and careless drivers. Therefore, the features are the driver expertise score and the quality of the brake system. A machine learning model is then trained to estimate the failure rate based on these two features. These features are brake quality scores and drivers' expertise scores. Obviously, the better the quality of the brake and the more skillful the driver are, the less the failure rate of the brake component is. The following sections introduce both the features and model. This model is exploited inside our optimization problem in Section 5.

4.1. Brake Quality Score. Subcomponent types have a significant contribution to the failure rates of a component. The better the subcomponents, the longer the component survives. For example, the brake system is composed of several subcomponents, four of which are responsible for most failures: pedal, retarder, ABS, and pad. Each falls within one of the following quality bands: A (highest quality), B, and C (lowest quality), with scores of 15, 10, and 5, respectively. In the end, the sum of the subcomponent scores is scaled between 0 and 100.

4.2. Driver Expertise Score. Highly skilled drivers can better maintain and manage the brake system, and therefore, this can be regarded as an influential factor in calculating the failure rate. This paper introduces the ETE indicators representing the level of education, annual training hours, and total years of driving experience. To calculate ETE, the score of education level and experience are calculated according to Table 1. The values in Table 1 are based on the filled surveys. The training score is then calculated according to (1). In this formula, $h_{\min}$ is the minimum hour of required training and S_h is the coefficient of training hour in ETE score. Finally, the sum of these three scores is scaled between 0 and 100.

$$\text{training\_score} = S_h * (h - h_{\min}). \tag{1}$$

Figure 1: The general structure of the brake reliability-related cost minimization approach.

Table 1: Score table for education and years of experience.

Education level	Under high school	High school diploma	Associate degree	Bachelor	Master	Ph.D.
Score	2	3	4	6	8	10
Years of experience	0–5	5–10	10–15	15–20	20–25	25–30
Score	2	3	4	6	8	10

4.3. Failure Rate Model. Failure rate (f) is estimated as a function of ETE and brake quality score (Q) as shown in the following equation:

$$\hat{f} = g(\text{ETE}, Q). \tag{2}$$

There is no analytical formula that relates ETE and Q to the failure rate. Thus, a data-driven model is used instead. The g function is estimated using machine learning methods. Machine learning includes various models such as linear regression, decision tree, and artificial neural network. In this paper, the *Lasso* method, which fits a linear function to an input-output relationship [31], is employed to model failure. The learner minimizes the mean square error between the actual and predicted output. To regularize the coefficients and prevent overfitting, a term of the first-order norm of the coefficients is added to the objective function according to (3) [31]. This trained linear failure rate model is later used in MILP cost optimization.

$$\text{Min}\left\{\frac{1}{M}\sum_{m=1}^{M}\left[f_m-\left(\beta_0+\beta_1*ETE_m+\beta_2*Q_m\right)\right]^2+\lambda*\left(\left|\beta_0\right|+\left|\beta_1\right|+\left|\beta_2\right|\right)\right\}, \tag{3}$$

where β_1 and β_2 are the coefficients of ETE and Q in the linear fitted function, and λ is the regularization factor, which is a hyperparameter. Hyperparameters should be assigned a value before the training task. M is the total number of samples, and m is the index of samples. This optimization is solved via the scikit-learn package in the python programming language.

The brake component failure rate can be approximately decomposed into subcomponent failure rates by multiplying the failure rate with a fraction of the failure rate of that subcomponent.

5. Modeling and Optimizing Total Cost

This section presents the mathematical optimization model to minimize the investment and reliability-related costs in a BRT system under the risk of braking failure. The main idea is to optimize brakes and ETE factors before operating new buses in a BRT system. It is expected that the optimized operating plan could significantly improve the reliability of the operations and reduces the total cost of the BRT system. To do so, the main pillars of costs and constraints must first be identified. The total cost has four pillars:

(i) Investment cost (IC): the amount of money used to buy subcomponents, subject to budget availability.

(ii) Human resource cost (HRC): driver salaries and training costs, depending on driver education and experience.

(iii) Outage cost (OC): cost of an interruption in bus operations due to failure in brake components. This would undoubtedly incur costs as fewer passengers are served.

(iii) Replacement cost (RC): the cost of replacing a failed or damaged brake subcomponent with a new one.

The objective function is the sum of the investment, human resource, outage, and replacement costs, as asserted in equation (4). In the following sections, details of computing each cost and associated constraints are explained.

$$\text{Min}\{\text{IC}+\text{HRC}+\text{RC}+\text{OC}\}. \tag{4}$$

5.1. Investment Cost. The investment cost is the sum of subcomponent costs. Referring back to Section 4, the four subcomponents pedal, retarder, ABS, and pad are responsible for the majority of brake failures. The indices of p, r, a, and d are used as notations for the mentioned elements. The set of pedal types is symbolized as $\boldsymbol{P}$, retarder types as $\boldsymbol{R}$, ABS types as $\boldsymbol{A}$, and pad types as $\boldsymbol{D}$. Equation (5) represents the investment cost for N buses. The symbol |.| in this equation and successive equations refers to the size of the set. The binary variable $B(i,e)$ indicates whether a subcomponent of type e is bought for bus i. The parameter C (.) is the cost of subcomponents.

$$\text{IC}=\sum_{i=1}^{N}\left(\sum_{p=1}^{|\mathbf{P}|}B(i,p)*C(p)+\sum_{a=1}^{|\mathbf{A}|}B(i,a)*C(a)+\sum_{r=1}^{|\mathbf{R}|}B(i,r)*C(r)+\sum_{d=1}^{|\mathbf{D}|}B(i,d)*C(d)\right). \tag{5}$$

Since only one subcomponent type can be installed in a bus, constraints (6)–(9) should be satisfied.

$$\sum_{p=1}^{|\mathbf{P}|}B(i,p)=1 \quad \forall i, \tag{6}$$

$$\sum_{a=1}^{|\mathbf{A}|}B(i,a)=1 \quad \forall i, \tag{7}$$

$$\sum_{r=1}^{\mathbf{R}}B(i,r)=1 \quad \forall i, \tag{8}$$

$$\sum_{d=1}^{|\mathbf{D}|}B(i,d)=1 \quad \forall i, \tag{9}$$

The supplier can provide a limited quantity for each type of subcomponent. The situation is asserted in (10)–(13). The parameter Max_e is the maximum number of subcomponents (of general type e) that can be supplied.

$$\sum_{i=1}^{N}B(i,p)\le \text{Max}_p \quad \forall p, \tag{10}$$

$$\sum_{i=1}^{N}B(i,a)\le \text{Max}_a \quad \forall a, \tag{11}$$

$$\sum_{i=1}^{N}B(i,r)\le \text{Max}_r \quad \forall r, \tag{12}$$

$$\sum_{i=1}^{N}B(i,d)\le \text{Max}_d \quad \forall d, \tag{13}$$

There is a limited amount of investment budget as stated in (14):

$$\mathrm{IC} \leq \mathrm{IC}_{\max}. \tag{14}$$

5.2. Human Resource Cost. Driver salaries and training during Y years constitute the total human resource cost. Equation (15) formalizes this fact for N buses. This formula neglects the fixed HR costs. In this equation, ed is the index of education that belongs to set $\boldsymbol{ED}$ = {Under high school, High school diploma, Associate degree, Bachelor, Master, Ph.D.}. The binary variable $EDU_{i,ed}$ indicates whether the level of education of the i^{th} bus driver is equal to ed. The index x represents the index of experience level. It can take quantitative values of Table 1. The set of these values is denoted as $\boldsymbol{X}$. $EXP_{i,x}$ is a binary variable, which equals one if the driver of the i^{th} bus has an experience level of x. The continuous variable h_i is the total training hours of the i^{th} bus driver. C_h is the annual cost per hour of training. $C(ed)$ and $C(x)$ are the additional monthly income that system owners should pay to a driver with an education level of ed and experience of x.

$$\mathrm{HRC} = Y.\left\{\sum_{i=1}^{N}\left[C_h * \left(h_i - h_{\min}\right) + 12\sum_{e\,d=1}^{|E\,D|}\mathrm{EDU}_{i,e\,d} * C(e\,d) + 12\sum_{x=1}^{|X|}\mathrm{EXP}_{i,x} * C(x)s\right]\right\}. \tag{15}$$

Constraint (16) asserts there is a lower and upper band for training hours.

$$h_{\min} \leq h_i \leq h_{\max} \quad \forall i. \tag{16}$$

Each driver has only one specific level of education and experience. This fact is mathematically modeled in (17) and (18).

$$\sum_{e\,d=1}^{|\mathbf{ED}|}\mathrm{EDU}_{i,e\,d} = 1 \quad \forall i, \tag{17}$$

$$\sum_{exp=1}^{|\mathbf{X}|}\mathrm{EXP}_{i,x} = 1 \quad \forall i. \tag{18}$$

Since regulations and policies limit the number of employed drivers who possess a specific level of education, constraint (19) sets the maximum number of drivers within each level of education. In this equation, Max_{ed} is the maximum number of drivers with an education level of ed that policies allow to hire.

$$\sum_{i=1}^{N}\mathrm{EDU}_{i,e\,d} \leq \mathrm{Max}_{e\,d} \quad \forall \mathrm{ed}. \tag{19}$$

The transportation service company would prefer not to dedicate a tremendous amount of money to HR. Therefore, the human resource cost is bounded as represented in constraint (20).

$$\mathrm{HRC} \leq \mathrm{HRC}_{\max}. \tag{20}$$

5.3. Outage Cost. Equation (21) states that the outage cost is the multiplication of total duration years (Y), the brake system failure rate of bus i (f_i), the average time a bus stays in a repair shop due to brake failure (μ), and the bus interruption cost per hour (I_i).

$$\mathrm{OC} = \sum_{i=1}^{N}\left(Y * f_i * \mu * I_i\right). \tag{21}$$

The failure rate is estimated with a linear model, as discussed in Section 4. It is stated in (22):

$$f_i = \beta_0 + \beta_1 * \mathrm{ETE}_i + \beta_2 * Q_i \quad \forall i. \tag{22}$$

ETE and Q, introduced in more detail in Section 4, are calculated through equations (23) and (24). In these equations, S_x and S_{ed} are the scores of experiences and education for the experience level of x and education level of ed according to Table 1 in Section 4. According to Table 1, the maximum ETE and Q scores are 40 and 60. To scale these scores between 0 and 100, they are multiplied by ratios 100/40 and 100/60. These two coefficients can change if a different scoring schema is used.

$$\mathrm{ETE}_i = \frac{100}{40}\left[\sum_{e\,d=1}^{|\mathbf{ED}|} S_{e\,d} * \mathrm{EDU}_{i,e\,d} + \sum_{x=1}^{|\mathbf{X}|} S_x * \mathrm{EXP}_{i,x} + S_h * \left(h_i - h_{min}\right)\right] \quad \forall i, \tag{23}$$

$$Q_i = \frac{100}{60}\left[\sum_{p=1}^{|\mathbf{P}|} S_p * B(i,p) + \sum_{a=1}^{|\mathbf{A}|} S_a * B(i,a) + \sum_{r=1}^{|\mathbf{R}|} S_r * B(i,r) + \sum_{d=1}^{|\mathbf{D}|} S_d * B(i,d)\right] \quad \forall i. \tag{24}$$

The total available ETE is limited due to issues such as a limited number of high-quality candidates. Similarly, the total quality of the brake system is limited. Constraints (25) and (26) restate this fact.

$$\sum_{i=1}^{N} \mathrm{ETE}_i \leq \mathrm{SUM\_ETE}_{\max} \quad \forall i, \tag{25}$$

$$\sum_{i=1}^{N} \mathrm{Q}_i \leq \mathrm{SUM\_Q}_{\max} \quad \forall i. \tag{26}$$

5.4. Replacement Cost. Replacement cost is the sum of the expected cost of replacing each subcomponent after it fails. This fact is mathematically asserted in equation (27). The replacement cost for each component equals the cost of a single component multiplied by the expected damages over Y years. It can be rewritten as (28)–(31).

$$\mathrm{RC} = \sum_{i=1}^{N} \left[\sum_{p=1}^{|\mathbf{P}|} \mathrm{RC}_{i,p} + \sum_{a=1}^{|\mathbf{A}|} \mathrm{RC}_{i,a} + \sum_{r=1}^{|R|} \mathrm{RC}_{i,r} + \sum_{d=1}^{|\mathbf{D}|} \mathrm{RC}_{i,d} \right] \quad \forall i, \tag{27}$$

$$\mathrm{RC}_{i,p} = Y * f_i * v_P * B(i,p) * C(p) \quad \forall i, p, \tag{28}$$

$$\mathrm{RC}_{i,a} = Y * f_{i.} * v_A * B(i,a) * C(a) \quad \forall i, a, \tag{29}$$

$$\mathrm{RC}_{i,r} = Y * f_i * v_R * B(i,r) * C(r) \quad \forall i, r, \tag{30}$$

$$\mathrm{RC}_{i,d} = Y * f_i * v_D * B(i,d) * C(d) \quad \forall i, d. \tag{31}$$

In (28), the parameter vP is the relative failure frequency of the pedal. This parameter is approximated by analyzing historical data. It can be estimated using historical data. Other variables inside (28)–(31) are introduced in previous sections. In (28), the multiplication of the continuous variables f_i and $B(i,p)$ is nonlinear. The same happens in (29)–(31) for ABS, retarder, and pad, respectively. To linearize these equations, a conversion, introduced in [32], is used. According to this conversion, equation (32) is linearized by replacing it with (33) and (34) [32]. This conversion is applied to (28)–(31) for them to linearize.

$$\text{multiplication} = \text{binary} * \text{continuous}, \tag{32}$$

$$0 \leq \text{multiplication} \leq \text{binary} * \text{continuous}_{\max}, \tag{33}$$

$$\begin{aligned} &\text{continuous} + (\text{binary} - 1) * \text{continuous}_{\max} \\ &\quad \leq \text{multiplication} \leq \text{continuous}. \end{aligned} \tag{34}$$

To summarize, the optimization problem is modeled as a mixed-integer linear program (MILP) with the objective function of equation (4) and constraints (5)–(27) and linearized (28)–(31). Decision variables are the types of subcomponents chosen for bus brakes and driver education and experience and training.

6. Case Study

To verify the efficiency of the method, a real case study of a BRT service is presented. The first BRT system in Tehran, Iran, was initiated in 2007. Currently, ten routes are operating in Tehran. Buses operate in specially dedicated routes in which other vehicles are not allowed. Moreover, in the case of a junction, BRT buses have priority. Additional routes are planned and added as required. The data for brake system failures, subcomponent types, and drivers of 183 buses were collected. However, costs were modified for security reasons. This case will decide the types of subcomponents and ETEs for ten buses planned to be exploited in three BRT lines for 20 years. Subcomponent prices are shown in Table 2. Recall from Section 4 that types A, B, and C components have 15, 10, and 5 scores, respectively. Due to the supplier limitations, no more than two subcomponents of type A can be provided. The average repair times for each subcomponent and relative failure frequencies are listed in Table 3. The brake system average repair time is the average repair time for each subcomponent weighted by the relative failure frequencies. Each hour of training per year costs 6.7 USD. The maximum training hours per year for each driver is 120 hours. The salaries of drivers are listed in Table 4. The company's policy allows a maximum of one Ph.D. driver, two masters, and three holding any other degrees.

The transportation company has a budget of 1,800,000 USD for driver salaries and training over 20 years. Similarly, no more than 9000 USD is available for the brake system of these ten buses. Each hour of bus interruption costs 201.289 USD for line 1, 196.2 USD for line 2, and 150 USD for line 3. Therefore, the maximum total score of 900 is considered available for both subcomponents and driver ETEs.

7. Results and Discussion

As demonstrated in Table 5, the training is at the maximum possible level. It is mainly because training is relatively cheaper. Subcomponents of type A are only installed at buses 4 and 5, belonging to line 1. On the contrary, buses 9 and 10 possess subcomponents of less quality. This is because the interruptions in line 3 result in lower outage costs. In the optimal strategy, the salary and brake investment costs are 1,799,427.795 and 8,970.461 USD, close to their maximum values. The total expected replacement and outage costs are 2,857,811.418 and 5,690,168.350 USD, respectively. Therefore, the total cost is 10,356,378.86 USD.

7.1. Sensitivity Analysis. The transport company may hypothesize whether increasing the brake system investment or training and salaries would lower total costs over 20 years. To investigate this, several cases of sensitivity analysis are performed. First, the effect of brake investment limitation is investigated. Next, an analysis is performed to identify whether an enhancement in the HR cost limitation would change the optimal total cost. It is assumed that the sum of brake investment and HR costs is constant. Finally, the effects on the total expected cost are evaluated.

7.1.1. Brake Investment Limitations. If constraint (14), which limits the brake investment cost, is omitted, the total cost would be 10,064,524.85 USD, and the

Table 2: Price of subcomponents (USD).

Subcomponent	Type A	Type B	Type C
Retarder	247.678	201.238	154.798
Pad	77.399	68.111	61.919
ABS	773.99	619.19	495.35
Pedal	120.3591	100.399	20.8235

Table 3: Repair times and relative failure frequencies.

Subcomponent	Repair time (hr)	Relative failure frequency
Retarder	2.5785	0.1766
Pad	2.8197	0.2470
ABS	0.783	0.3114
Pedal	4.258033	0.2649

Table 4: Salary per month of drivers (USD).

Education level	Under high school	High school diploma	Associate degree	Bachelor	Master	Ph.D.
Salary	544.892	557.276	603.715	650.155	743.034	1083.591
Years of experience	0–5	5–10	10–15	15–20	20–25	25–30
Salary	80.49535	100.7430	140.9907	182.9102	241.9814	322.4767

Table 5: Results.

Bus #	Edu.	Training	Exp.	ETE	Retarder	Pad	ABS	Pedal	Q
1	High school diploma	120	5–10	65.0	B	B	C	B	58.33
2	Bachelor	120	5–10	72.5	B	B	C	B	58.33
3	Associate degree	120	5–10	67.5	B	B	C	B	58.33
4	Bachelor	120	5–10	72.5	A	A	B	A	91.66
5	Bachelor	120	5–10	72.5	A	A	B	A	91.66
6	High school diploma	120	5–10	65.0	B	B	C	B	58.33
7	Under high school	120	5–10	62.5	B	B	C	B	58.33
8	High school diploma	120	5–10	65.0	B	B	C	B	58.33
9	Under high school	120	0–5	60.0	B	B	C	B	58.33
10	Under high school	120	0–5	60.0	B	B	C	C	50.00

investment cost would be 10350.356 USD, which is 1350.365 USD more than the current investment budget. Therefore, if the transportation service provides an additional 1350.365 USD financial resources for investing in the brake system, the total cost would decrease by 291,854.01 USD. Figure 2 depicts the effect of changing the brake investment cost on the total expected cost over 20 years.

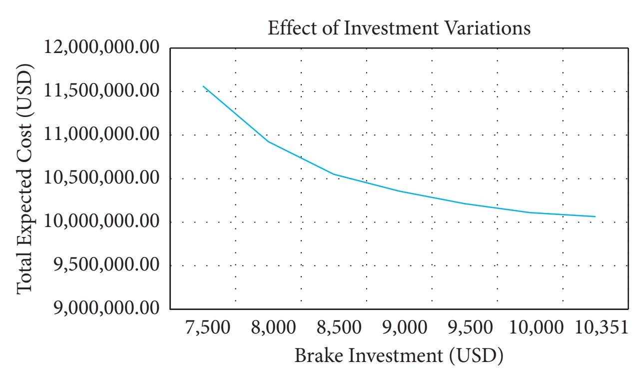

Figure 2: Brake investment sensitivity analysis.

7.1.2. HR Cost Limitations. Similar to the procedure used in the previous section, constraint (20) is omitted to analyze the effects of HR cost limitations. In this case, the HR cost would be 2587994.854 USD, and the total expected cost would be 9275496.72 USD. Therefore, an increase of 787,994.854 USD in the HR cost would result in 1,080,882.14 USD benefits in the total expected cost. Since the interval of spending HR and total expected costs are almost simultaneous, the BRT service can revise its policy based on the results of this optimization. The variation of total expected cost during 20 years versus the HR cost during the same interval is depicted in Figure 3.

7.1.3. HR Cost Limitation and Brake Investment Budget Joint Analysis. As seen in previous sections, an increase in HR limitation or brake investment would decrease the expected cost. If the transport company questions whether decreasing one limitation in favor of the other would reduce the expected cost, another sensitivity analysis should be performed. Table 6 provides more insights into this question. It

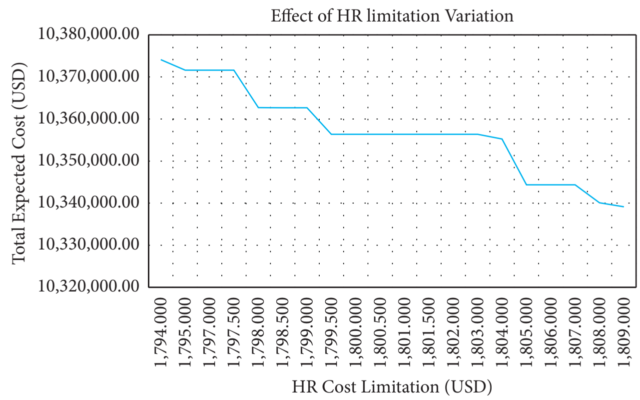

Figure 3: HR cost sensitivity analysis.

Table 6: HR cost and brake investment limitations trade-off.

Case #	Brake investment limitation	HR cost limitation	Total cost
1	8,000	1,801,000	10,924,940.31
2	8,500	1,800,500	10,551,420.18
3 (Base case)	9,000	1,800,000	10,356,378.86
4	9,500	1,799,500	10,213,323.76
5	10,000	1,799,000	10,118,665.99

Table 7: Related notations.

	Indexes		
a	Index of ABS types	p	Index of pedal type
d	Index of pad types	r	Index of retarder types
ed	Index of level of education	x	Index of level of experience
i	Index of buses		
	Sets		
$\boldsymbol{A}$	Set of ABS types	$\boldsymbol{P}$	Set of pedal type
$\boldsymbol{D}$	Set of pad types	$\boldsymbol{R}$	Set of retarder types
$\boldsymbol{ED}$	Set of the of education	$\boldsymbol{X}$	Set of levels of experience
	Parameters		
β_0	Constant value in failure rate model	I_i	Cost of an hour of interruption of bus i
β_1	Coefficient of ETE in failure rate model	Max_e	Maximum available of item e (e could be a subcomponent type or education level or experience level)
β_2	Coefficient of Q in failure rate model	S_e	The score of item e (e could be a subcomponent type or education level, or experience level)
M	Number of samples for failure rate modeling	v_g	The relative failure rate of item e (e could be a pedal, pad, retarder, or ABS)
N	Number of buses	Y	Total years of planning
$C(e)$	Cost of item e (e could be a subcomponent type or education level or experience level)		
C_h	The annual cost of an hour of training	μ	Average buses' brake repair time
	Variables		
$B(i,e)$	A binary variable indicating whether item e is bought for bus i (e could be a pedal, pad, retarder, or ABS)	HRC	Human resource cost
$EDU_{i,ed}$	Binary variable indicating whether education level of the driver of bus i is ed	IC	Investment cost
ETE_i	Education, training, and experience score of bus i's driver	OC	Outage cost
$EXP_{i,x}$	Binary variable indicating whether the experience level of the driver of bus i is x	RC	Replacement cost
f_i	The failure rate of bus i	Q_i	Brake of i^{th} bus's quality score

can be observed that increasing brake investment limitation while keeping the sum of brake investment and HR cost constant would decrease the total expected cost. Notice that HR cost is spent in the broader interval of time. Therefore, supplying the financial resources for HR costs is easier.

8. Conclusions

This paper presented a joint brake system and driver employment and training optimization for buses in BRT systems. The objective function was to minimize the brake reliability-related costs plus investment costs. It has been observed that both qualities of the brake system subcomponents and driver ETE (education, training, and experience) indices are influential factors for the failure rate and, in consequence, the total expected cost. However, there are limited financial resources for these two factors, which should be modeled. Also, overspending on these two factors may put an unnecessary extra cost on the shoulders of service providers. Therefore, sensitivity analysis and optimization should be performed. A case study has been presented and analyzed to verify the efficiency of the method. The results assert that better subcomponents and drivers should be dedicated to bus lines with more interruption costs per hour. It has also been shown that if enough budgets are provided for brake systems, the total expected cost will decrease noticeably.

Furthermore, sufficient spending for the ETE would reduce costs. Providing a budget for the brake system is a challenging task. However, the ETE expenses are spread over many years; therefore, they are more practical to provide. Moreover, the saved money, which should have been expended as interruption losses, can be dedicated to ETE. The results have been presented to the abovementioned practical BRT system owner. After analyzing the strategy, they agreed to implicate HR employment, training, and subcomponent supply results in their planning and operations programs.

Nevertheless, considering the role of other factors, including seasonal factors and loading, in brake system reliability results in a more precise cost modeling and optimization in practice. For the future stream of research, co-optimizing the total expected brake-related expenses with repair staff employment is suggested. The optimal number of repair staff is employed to decrease expected outage durations and expected outage cost in consequence.

9. Summary of Notions

Table 7 contains a summary of all indices, variables, and parameters that have been mentioned throughout this paper.

Data Availability

The bus failure data and driver specifications are not publicly published due to the safety and security of third parties.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this study.

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