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Research Article

Cruising for Parking with Autonomous and Conventional Vehicles

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Parking is a cumbersome part of auto travel because travelers have to search for a spot and walk from that spot to their final destination. This conventional method of parking will change with the arrival of autonomous vehicles (AV). In the near future, users of AVs get dropped off at their final destination and the occupant-free AVs search for the nearest and most convenient parking spot. Hence, individuals no longer bear the discomfort of cruising for parking while sitting in their vehicle. This paper quantifies the impact of AVs on parking occupancy and traffic flow on a corridor that connects a home zone to a downtown zone. The model considers a heterogeneous group of AVs and conventional vehicles (CV) and captures their parking behavior as they try to minimize their generalized travel costs. Insights are obtained from applying the model to two case studies with uniform and linear parking supply along the corridor. We show that (i) CVs park closer to the downtown zone in order to minimize their walking distance, whereas AVs park farther away from the downtown zone to minimize their parking search time, (ii) AVs experience a lower search time than CVs, and (iii) higher AV penetration rates reduce travel costs for both AVs and CVs.

1. Introduction

There are close to one billion parking spaces in the U.S. which is roughly four times more than the existing number of passenger cars and light-duty trucks in the country [1]. This abundance of parking does not imply that parking is ample and easy to find everywhere. Lack of available parking in business districts of major cities makes drivers search for a long time to find a coveted spot that is close enough to their final destination. A worldwide parking survey of 20 cities shows that drivers spend an average of 20 minutes in search of parking [2]. Cruising for parking adversely affects traffic as well. A study of downtown parking shows that between 8% and 74% of traffic belongs to vehicles that are cruising for parking [3].

Resolving the parking dilemma has been a challenge for city planners for decades and the solution is not yet clear. On the one hand, cities cannot afford to increase parking supply because parking facilities take up valuable land that can otherwise be allocated to road capacity (in the case of on-street parking) or real-estate (in the case of off-street parking). On the other hand, decreasing parking supply further exacerbates cruising for parking and leads to longer search times. As a result, cities have resorted to managing parking demand instead of supply using pricing strategies [4–6], parking permits [7–9], and parking time restrictions [10, 11]. Parking demand management strategies, however, have drawbacks as well. Parking pricing, for instance, causes inequality between a heterogeneous group of travelers, and parking permits require proper enforcement measures which can be costly.

Autonomous vehicles (AV) have great potential to resolve many of the current parking problems. With the proliferation of AVs, passengers will no longer need to park close to their final destination. Instead, passengers get dropped off by the AVs at their final destination, and the occupant-free AVs drive to park at the nearest or most affordable parking spot. Figure 1 illustrates the round-trip home-to-work journey (i.e., travel tour) of a *conventional vehicle* (CV) driver and an

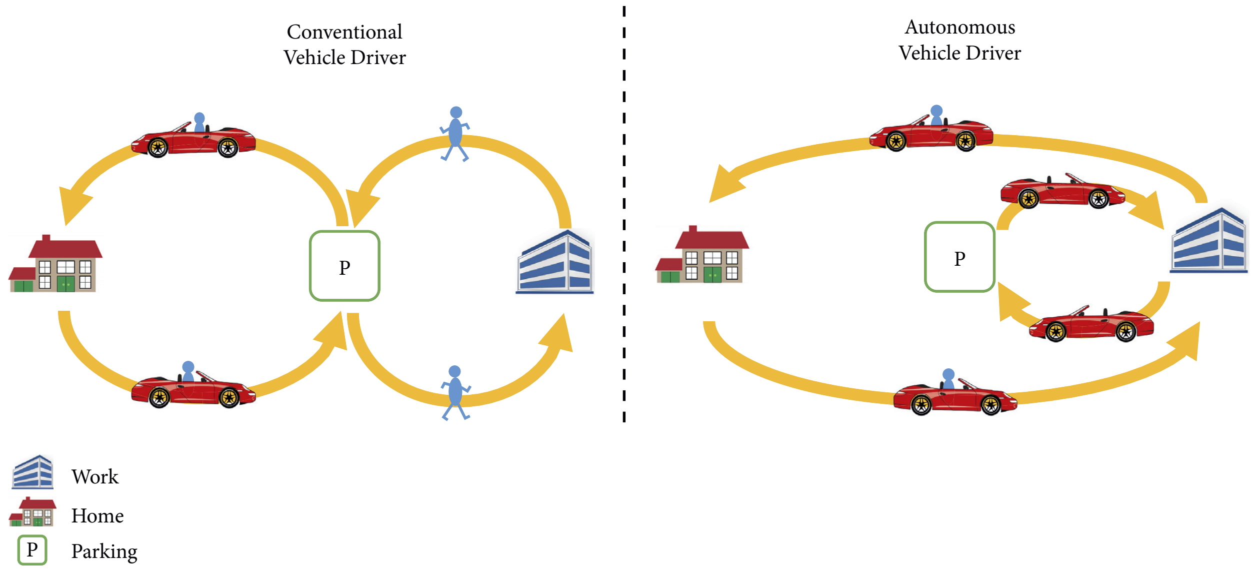


Figure 1: Parking pattern of autonomous and conventional vehicle drivers.

AV passenger. As illustrated, the CV driver first finds a parking spot and then walks from that spot to her workplace, whereas the AV passenger gets dropped off right at her workplace without having to walk or search for parking while in the vehicle.

Motivated by the impact of AV parking, cities are partnering with car manufacturers to rethink parking in major urban areas. Audi, for instance, is planning to implement a parking pilot in Somerville, Boston, for AVs. The pilot is estimated to save up to 62% in parking space (or equivalently $100 million USD in real-estate value) in the district of Assembly Row where the heart of the project lies [12]. With AVs each taking two square meters less parking space than CVs, city planners are able to pack more AVs into each parking facility. It is anticipated that AV parking will influence traffic as well. Audi estimates that the transformation of on-street parking spots into lanes of traffic will reduce congestion by 20–50% because of the higher road capacity, reduction of tailbacks at intersections, and fewer vehicles searching for parking [12].

The emerging pattern of AV parking (as shown in Figure 1) has advantages for urban planners, traffic operators, AV passengers, and CV drivers. Urban planners no longer need to allocate as much space to parking in areas where renting costs are high and land is valuable. Thus, space can be used more efficiently with improved aesthetics since large parking lots are not a pleasant sight. Traffic operators benefit when the on-street parking spots are transformed into pickup and drop-off locations for AV passengers. AV passengers benefit because (i) they no longer have to find a parking spot and (ii) they do not have to walk from their parking spot to their final destination. Lastly, CV drivers benefit from AV parking because of lower competition for spots that are closer to major activity centers such as central business districts. That is, as AVs park farther away, CVs have the advantage of finding vacant spots at a lower search time and closer to their final destination.

Although AVs promise a modern way of parking, there are still many remaining questions to be answered about new parking patterns, impact on traffic, cruising time, land use, and role of AV penetration rates. While it is hypothesized that AVs park farther from their final destination, it is not yet clear how far away they are willing to park. Parking too close to the destination may still create some competition with CVs and parking too far increases travel costs. The second question is whether AVs increase traffic congestion by introducing additional legs to the tour of each vehicle or if they reduce traffic congestion because of their potentially more efficient use of road capacity. Third, as AVs park farther away, parking land use may change so that more parking facilities are developed farther away from downtown cores where land is more affordable and suitable for building parking facilities. Motivated to answer these questions, we present an equilibrium model to quantify the impact of AVs on parking facilities and roads.

The model is generic and is sensitive to several key parameters including parking supply, AV road occupancy, AV parking occupancy, and value-of-time of drivers. By varying these parameters, we are able to find analytical insights from a case study with a uniform parking capacity across a corridor. For other complex parking supply structures, we present a discretization method that divides the parking supply into segments for easier analysis.

The remainder of this paper is organized as follows. We present a literature review of existing studies on models of parking behavior in Section 2. We present the model is Section 3. An extension of the model is presented in Section 4 to find the optimal parking land use in a city. We provide analytical results on a case study with uniform parking supply distribution in Section 5. Additional insights are provided in Section 6 from numerical experiments. We present the conclusions of this study in Section 7 along with directions for future research.

2. Literature Review

In this section, we review the existing studies related to the impact of AVs on urban parking patterns. We advocate the importance of AV parking in Section 2.1, and we present a review of relevant parking studies in Section 2.2.

2.1. Autonomous Vehicles. Autonomous vehicles are now in the testing phase for many car manufacturers (including Audi, Ford, GM, Toyota, Nissan, Volvo, Volkswagen, BMW, and Cadilac) and technology investors. Google, among the key investors in the automated driving technology, has tested AVs over more than 2 million miles in four cities: Mountain View (since 2009), Austin (since 2015), Metro Pheonix (since 2016), and Kirkland (since 2016) [13]. Successfully completing the testing phase, AVs are anticipated to be available to the public on a mass scale by 2025 [14]. As AVs are an inevitable reality, a number of studies have investigated their impact in terms of fuel economy [15], induced traffic [16], willingness-to-pay for AVs [17], traffic flow [18–22], safety [23, 24], intersection control [25, 26], and use of AVs as shared fleet between a group of users [27–30]. In a recent survey, Fagnant and Kockelman [14] argue that AVs will change parking in two ways. First, parking patterns may change as AVs are able to self-park in less-expensive facilities, and second, parking facilities will be relocated from central business districts to areas with less-expensive renting costs. Motivated by the impact of AV parking, we provide a synopsis of the relevant literature on parking and we discuss the application of existing parking models on AVs.

Provision of Level-5 autonomy is now pursued by many car manufacturers including Tesla, Audi, Ford, Toyota, Nissan, Volvo, Volkswagen, BMW, and Cadilac, among many others. The advanced technology used in the vehicles includes *light detection and ranging systems*, sensors, software, and additional computing power. These autonomy components can cost more than \$30,000 USD (even up to \$100,000 USD for military uses) [31], but they are expected to become more affordable as AVs become available to the public on a mass scale. Hensley [32] estimates that, 15 years after the commercialization of AVs, their cost can drop from a \$10,000 USD mark-up (i.e., the additional payment for autonomy technology) to a \$3,000 USD mark-up. Other studies have more optimistic mark-up estimates of \$1,000–\$1,500.

2.2. Parking Models. There are many studies on practical aspects of parking (see [33], for a thorough review) including parking pricing [4, 34, 35], cruising for parking [36, 37], enforcement [38], parking competition [39, 40], optimal parking control strategies [4, 41], and parking for commercial vehicles [42–45]. The existing literature is not explicitly applicable to AV parking because AVs do not have the same parking pattern as CVs. Nevertheless, the common ground between AVs and CVs is that they both need to cruise to find appropriate parking. Although there is strong evidence that CVs have to search for parking, we argue that AVs also have to search when dispatched by vehicle owners to a parking facility. In such cases, the search time is interpreted as the waiting time of an AV for an empty spot at a full parking facility.

The literature on cruising for parking is abundant. Arnott and Inci [36] analyze the economic impacts of cruising for parking in a "bathtub" model that considers the influence of cruising-for-parking vehicles on traffic congestion. They show that it is always efficient to increase the parking price to the point where cruising for parking is eliminated while on-street parking is unsaturated. Arnott and Inci [46] investigate the stability of equilibrium solutions in a parking model with cruising for parking. Arnott and Rowse [47] extend the model of Arnott and Inci [36] with the assumption that demand is completely inelastic to simplify the computational complexity of the model. In an effort to relax the parking space contiguity assumption in Arnott and Inci [36], Levy et al. [48] compare the results of an analytical parking model PARKANALYST with a geo-simulation model PARKAGENT and show that parking heterogeneity (i.e., distinction between on-street, off-street, paid, and free parking) becomes critically important when the occupancy rate is above 92%.

Cruising for parking is analyzed at a network scale as well. The existing models consider cruising cost to be the Lagrange multiplier associated with the parking capacity constraint at each parking node. The Lagrange multiplier is nonzero when the parking constraint is binding which indicates the next vehicle that enters the parking facility has to incur a cruising cost [49, 50]. We use a similar strategy by imposing a constraint that restricts parking occupancy to be lower than (or equal to) parking supply at each facility. With this assumption, we estimate the parking cruising cost at each parking facility.

3. Equilibrium Model

3.1. Problem Setting. Consider a city with two zones connected with a corridor of length D (km), as shown in Figure 2(a). A demand of V (vehicles per hour) leaves the "home zone" and heads for the "downtown zone." A ratio of r of the vehicles is autonomous and the remaining $1-r$ are conventional vehicles; the AV and CV demands are Vr and $V(1-r)$ (vehicles per hour), respectively. This setup is similar to the morning commute problem, except (i) the model is static and (ii) there is no bottleneck.

The vehicles can park anywhere along the corridor as long as there is available parking. Consider point X on the corridor located x (km) away from the downtown zone, as shown in Figure 2(a). A CV that parks at point X drives $D-x$ to reach point X and then walks (or use any other access mode) x to reach the downtown zone. Hence, the travel cost of a CV that parks at X is

$$C_{cv}(x) = 2(D-x)t + 2xw, \tag{1}$$

where t is the driving cost per km, w is the walk (access) cost per km, and the factor 2 accounts for the return trip. Naturally, $w > t$ because CV owners would otherwise walk from the home zone to the downtown zone instead of driving.

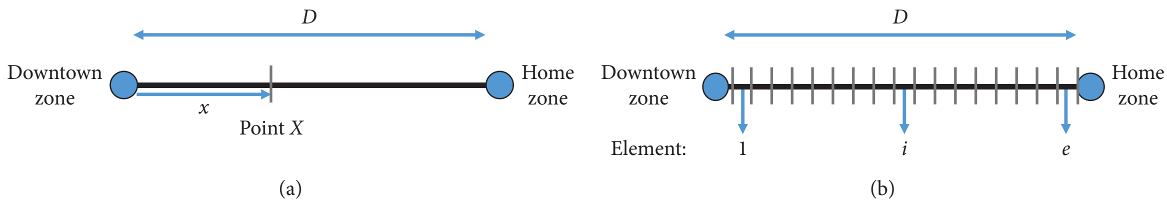


FIGURE 2: (a) Two zone city and (b) two zone city with the corridor divided into e elements.

The AV parking pattern is distinct from regular vehicles. AVs drive D to drop off their occupants downtown and then drive back x to park at point X. On the return trip, AVs drive x to the downtown zone to pick up their occupants and then drive D back to the home zone. Hence, the travel cost of an AV that parks at X is

$$C_{\mathrm{av}}(x) = 2Dt + 2x\bar{t}, \tag{2}$$

where $\bar{t}$ is the travel cost per km of an occupant-free AV. Naturally, $t > \bar{t}$ because the time value of occupants is included in t but not in $\bar{t}$. As an example, at an average fuel consumption of 10 L/100 km in urban areas and average fuel cost of $\$1/L$, we have $\bar{t} = \$0.1$/km (excluding maintenance costs). To find t, at an average speed of 40 km/hr and a time value of \$20/hour for one driver, we have $t = \bar{t} + (20/40) = \$0.6$/km. Hence, t is six times larger than $\bar{t}$ in this example.

Under steady-state conditions, for every vehicle that leaves the home zone, another vehicle returns to the home zone. Similarly, for every vehicle that enters a parking lot, another vehicle leaves the same parking lot. In this way, we model the circular travel pattern of Figure 1.

We make the following assumption throughout the rest of this paper to ensure that the results of the model are practical. The assumption is

$$w - t > \bar{t}, \tag{3}$$

which is a realistic assumption because w (i.e., walking cost per km) is generally large. As an example, $w = \$4$/km (as a lower-bound on the true value of w) when the average walking speed is 5 km/hr, and the value of time is \$20/hour. Hence, equation (3) holds with $w = \$4$/km, $\bar{t} = \$0.1$/km, and $t = \$0.6$/km. Equation (3) implies that $C_{\mathrm{av}}(x) < C_{\mathrm{cv}}(x)$ which means that an AV that parks at X always experiences a lower travel cost than a CV that parks as X, for all $x \in [0, D]$.

Parking capacity at point X is denoted by $k(x)$ and measured in vehicles per km. CVs take one unit of parking capacity, but AVs take α units of capacity, where $\alpha < 1$ because of their higher maneuverability and because more AVs can be packed in each parking facility. It is estimated that AVs will take 60% of a CV parking space [12]. Hence, $\alpha \approx 0.6$.

On the road, CVs take one unit of road capacity and AVs take β units of road capacity where either $\beta \leq 1$ due to the V2X (i.e., vehicle to everything) connectivity of AVs or $\beta > 1$ due to the higher intervehicle gap [51] for safety assurance [25].

We now summarize the main assumptions of the model as the following:

(1) We consider both the inbound trip (home to downtown) and the outbound trip (downtown to home) in a static equilibrium setting.
(2) All vehicles have a fixed parking dwell time (i.e., stoppage time of a vehicle at a parking space).
(3) The drivers are familiar with parking search times on the corridor according to past experience. This is equivalent to deterministic user equilibrium conditions.
(4) The presented model is congestion-free; the cost of travel on the corridor does not depend on the traffic flow. We later discuss the implications of this assumption and highlight the importance of relaxing it for future research.

3.2. User Equilibrium Conditions. Each vehicle that parks at X incurs a parking search time cost $s(x)$, which is a decision variable of the model, and a travel time cost which is $C_{\mathrm{cv}}(x)$ for CVs and $C_{\mathrm{av}}(x)$ for AVs. The generalized cost of a CV that parks at X is

$$g_{\mathrm{cv}}(x) = C_{\mathrm{cv}}(x) + s(x), \tag{4}$$

and the generalized cost of an AV that parks at X is

$$g_{\mathrm{av}}(x) = C_{\mathrm{av}}(x) + s(x). \tag{5}$$

Under user equilibrium conditions, all CVs have the same generalized cost denoted by u_{cv} such that

$$g_{\mathrm{cv}}(x) = u_{\mathrm{cv}}, \tag{6}$$

for all $x \in [0, D]$ as long as at least one CV parks at x. Similarly, all AVs have the same generalized travel cost denoted by u_{av} such that

$$g_{\mathrm{av}}(x) = u_{\mathrm{av}}, \tag{7}$$

for all $x \in [0, D]$ as long as at least one AV parks at x.

3.3. Mathematical Model. Let $v_{\mathrm{cv}}(x)$ and $v_{\mathrm{av}}(x)$ be the flow of CVs and AVs per km that park at X. That is, $v_{\mathrm{cv}}(x)$ and $v_{\mathrm{av}}(x)$ are the probability-density-functions of a continuous distribution that represents parking occupancy on the corridor for CVs and AVs, respectively, when Vr and $V(1-r)$ are normalized to one. Under steady-state conditions, $v_{\mathrm{cv}}(x)$ and $v_{\mathrm{av}}(x)$ are also the flow of CVs and AVs per km that return home from point X.

We now formulate traffic flow on the corridor according to parking patterns of the vehicles. Let $f(x)$ be the stationary-person inbound flow of vehicles at X (i.e., the leftward flow in Figure 2(a)), i.e., for a person standing at point x, the road would observe $f(x)$ vehicles per hour. Under steady-state equilibrium conditions,

$$f(x) = \int_0^x v_{\text{cv}}(x) y \mathrm{d}y + \beta \int_0^x v_{\text{av}}(x) y \mathrm{d}y + 2\beta \int_x^D v_{\text{av}}(x) y \mathrm{d}y, \quad \forall x \in [0, D], \tag{8}$$

where the first term is the flow of CVs that park downstream of X, the second term is the flow of AVs that park downstream of X discounted by β to account for AV impact on road capacity, and the third term is the flow of AVs that park upstream of X discounted by factor β and multiplied by 2 because these AVs pass point X twice: once to drop off their occupants and the second time to pick them up. An alternative and equivalent way of defining flow at X is

$$f(x) = \int_0^x v_{\text{cv}}(x) y \mathrm{d}y + \beta V r + \beta \int_x^D v_{\text{av}}(x) y \mathrm{d}y, \quad \forall x \in [0, D], \tag{9}$$

where the first term is the flow of CVs that park downstream of X, the second term indicates that all AVs pass point X because they need to drop off their occupants downtown, and the third term indicates that only the AVs that park upstream of X have to pass X for a second time to pick up their occupants from the downtown zone.

We now present the following lemma which is applied in the mathematical model that follows.

Lemma 1. *Under user equilibrium conditions, the CVs and AVs do not share any parking facility such that $v_{av}(x)$ and $v_{cv}(x)$ cannot both be positive at any $x \in [0, D]$.*

Proof. Consider two points on the corridor located at x and $x + \epsilon$, where ϵ is an infinitesimal distance. Assume that CVs and AVs are present on both points (i.e., CVs and AVs share parking facilities) such that $v_{\text{av}}(x) > 0, v_{\text{av}}(x+\epsilon) > 0$, $v_{\text{cv}}(x) > 0$, and $v_{\text{cv}}(x+\epsilon) > 0$. Let $\gamma = \partial s(x)/\partial x$ be the marginal deviation in the parking search cost as travelers park ϵ km farther away from the downtown zone. Finally, assume $\alpha = 1$, although this assumption is not restrictive. We show that the equilibrium conditions (6) and (7) are not satisfied as long as CVs and AVs share parking facilities.

A CV that moves from x to $x + \epsilon$ incurs a marginal cost of

$$g_{\text{cv}}(x) n - g_{\text{cv}}(x) = \gamma \epsilon + 2\epsilon (w - t), \tag{10}$$

where the first term on the RHS is the extra search cost (note that $\gamma < 0$ is the marginal deviation in search cost per km) and the second term is the additional driving and walking cost. Similarly, an AV that moves from x to $x + \epsilon$ incurs the marginal cost of

$$g_{\text{av}}(x) n - g_{\text{av}}(x) = \gamma \epsilon + 2\epsilon \bar{t}, \tag{11}$$

where the first term on the RHS is the savings in parking search cost and the second term is the marginal occupant-free driving cost because the AV is now parking ϵ farther from downtown.

Under the equilibrium conditions, both (10) and (11) have to be zero; otherwise, (6) and (7) are not satisfied. As an example, if (10) is positive, then the CV users at $x + \epsilon$ have incentive to park at x to lower their generalized cost from $g_{\text{cv}}(x) n$ to $g_{\text{cv}}(x)$.

Consider the following two scenarios. First, assume that the CV user equilibrium conditions are satisfied. From $g_{\text{cv}}(x) = g_{\text{cv}}(x) n$ in (10), we have $\gamma = -2(w - t)$. According to (3) and (11), the AV equilibrium conditions are not satisfied because $g_{\text{av}}(x) n < g_{\text{av}}(x)$. Thus, the AVs move farther from downtown to the point where they no longer share parking spaces with CVs. In the second scenario, we assume that the AV user equilibrium conditions are satisfied. From $g_{\text{av}}(x) = g_{\text{av}}(x) n$ in (11), we have $\gamma = -2\bar{t}$. According to (3) and (10), the CV equilibrium conditions are not satisfied because $g_{\text{cv}}(x) n > g_{\text{cv}}(x)$. Thus, the CVs move closer to downtown to the point where they no longer share parking spaces with AVs. □

The following mathematical model, denoted as $\text{UE}(k)$, stipulates the user-equilibrium conditions for a given parking density $k(x)$:

$$\text{Minimize}\,[\text{UE}(k)]: \int_0^D C_{\text{cv}}(x) v_{\text{cv}}(x) \mathrm{d}x + \int_0^D C_{\text{av}}(x) v_{\text{av}}(x) \mathrm{d}x, \tag{12}$$

$$\text{subject to} \int_0^D v_{\text{av}}(x) \mathrm{d}x = V r, \tag{13}$$

$$\int_0^D v_{\text{cv}}(x) \mathrm{d}x = V(1 - r), \tag{14}$$

$$v_{\text{cv}}(x) + \alpha v_{\text{av}}(x) \le k(x), \quad \forall x \in [0, D], \tag{15}$$

$$v_{\text{av}}(x), v_{\text{cv}}(x) \ge 0 \forall, \quad x \in [0, D], \tag{16}$$

where equations (13) and (14) ensure that a total of Vr AVs and $V(1 - r)$ CVs are assigned to all the parking facilities. Constraints (15) ensure that occupancy in all parking facilities is within the available capacity. Constraints (16) ensure non-negativity. Note that $f(x)$ is an output of the model.

The mathematical model has a feasible solution if there is enough parking supply on the corridor to station all the vehicles. Precisely, the following should be satisfied for feasibility:

$$\int_0^D k(x) \ge V r \alpha + V(1 - r), \tag{17}$$

where the left-term is the total parking supply and the right-term is the total parking demand.

We now proceed to show that the mathematical model (equations (12)–(16)) is equivalent to the user equilibrium conditions of Section 3.2. Let λ_{av}, λ_{cv}, and $\lambda_p(x)$ be the Lagrange multipliers of constraints (13) to (15), respectively.

The Lagrange multipliers have the following interpretation. λ_{av} and λ_{cv} are the generalized cost of one additional AV and one additional CV, respectively, and $\lambda_p(x)$ is the parking search cost of a vehicle that parks at X.

According to Lemma 1, CVs and AVs do not share any parking facility. Hence, the parking search cost of a CV is $s(x) = \lambda_p(x)$ and the parking search cost of an AV is $s(x) = \alpha\lambda_p(x)$. If Lemma 1 did not hold (i.e., if CVs and AVs shared parking facilities), then constraint (15) would be incorrect because AVs would experience a lower search cost (of $\alpha\lambda_p(x)$) compared to CVs (who experience $\lambda_p(x)$) for parking at the same location which would be unrealistic. With Lemma 1, however, the mathematical model is valid.

We now present the Karush–Kuhn–Tucker (KKT) conditions which are first derivative tests and necessary conditions for a solution in nonlinear and linear programming to be optimal:

$$\left[C_{cv}(x) + \lambda_p(x) - \lambda_{cv}\right] v_{cv}(x) = 0, \quad \forall x \in [0, D], \tag{18}$$

$$C_{cv}(x) + \lambda_p(x) - \lambda_{cv} \geq 0, \quad \forall x \in [0, D], \tag{19}$$

$$\left[C_{av}(x) + \alpha\lambda_p(x) - \lambda_{av}\right] v_{av}(x) = 0, \quad \forall x \in [0, D], \tag{20}$$

$$C_{av}(x) + \alpha\lambda_p(x) - \lambda_{av} \geq 0, \quad \forall x \in [0, D], \tag{21}$$

$$\left[k(x) - v_{cv}(x) - \alpha v_{av}(x)\right]\lambda_p(x) = 0, \quad \forall x \in [0, D], \tag{22}$$

$$(13) - (16), \tag{23}$$

where equations (18) and (21) are equivalent to the user-equilibrium conditions in equations (4) and (5), respectively. For CVs, $g_{cv}(x) = C_{cv}(x) + \lambda_p(x)$ and $u_{cv} = \lambda_{cv}$. Equations (18) and (19) indicate that $g_{cv}(x) = u_{cv}$ when $v_{cv}(x) > 0$ and $g_{cv}(x) > u_{cv}$ when $v_{cv}(x) = 0$. Hence, all CVs experience the same generalized cost of $u_{cv} = \lambda_{cv}$. Similarly, for AVs, $g_{av}(x) = C_{av}(x) + \alpha\lambda_p(x)$ and $u_{av} = \lambda_{av}$. Equations (20) and (21) indicate that $g_{av}(x) = u_{cv}$ when $v_{av}(x) > 0$ and $g_{av}(x) > u_{cv}$ when $v_{av}(x) = 0$. Hence, all AVs experience the same generalized cost of $u_{av} = \lambda_{av}$.

The mathematical model has a unique solution because it is linear program. Using the KKT conditions of the equilibrium model, the search times are obtained as the Lagrangian multipliers $s(x) = \lambda_p(x)$.

3.4. Solution Algorithm. We present a solution algorithm for complex cases that cannot be solved analytically. The mechanism behind the algorithm is to segment the corridor into finite elements whereby each element represents the location of one parking facility with a given capacity. This strategy allows us to transform mathematical problems (12)–(16) into a linear program.

The algorithm has the following steps. We divide the corridor into e elements of length Δ such that the elements collectively make up the length of the corridor: $D = e\Delta$. Let $E = \{1, \ldots, i, \ldots, e\}$ be the set of elements such that element $i = 1$ touches the downtown zone and element $i = e$ touches the home zone, as shown in Figure 1.

The parking capacity of element i is denoted by k^i and defined as

$$k^i = \int_{i\Delta}^{(i+1)\Delta} k(x)\mathrm{d}x, \tag{24}$$

and travel cost of CVs and AVs that park at element i is denoted as C^i_{cv} and C^i_{av}, respectively, and defined as

$$\begin{aligned} C^i_{cv} &= 2(D - i\Delta)t + 2wi\Delta, \\ C^i_{av} &= 2Dt + 2i\Delta\bar{t}, \end{aligned} \tag{25}$$

where the two above equations are derived by replacing x with $i\Delta$ in equations (1) and (2). Let v^i_{cv} and v^i_{av} be the flow of CVs and AVs at element i, and let f^i be the traffic flow at element i:

$$f^i = \sum_{j=1}^{j=i} v^i_{cv} + \beta r V + \beta \sum_{j=i+1}^{j=e} v^i_{av}, \tag{26}$$

where the first term is the flow of CVs, the second term is the flow of all AVs that drop off their passengers downtown, and the third term is the flow of AVs that drive downtown for a second time to pick up their passengers.

The following mathematical model is the linear transformation of the original model (equations (12)–(16)):

$$\text{Minimize} \sum_{i=1}^{i=e}\left[v^i_{cv}C^i_{cv} + v^i_{av}C^i_{av}\right], \tag{27}$$

$$\text{subject to} \sum_{i=1}^{i=e} v^i_{av} = Vr, \tag{28}$$

$$\sum_{i=1}^{i=e} v^i_{cv} = V(1 - r), \tag{29}$$

$$\sum_{i=1}^{i=e} v^i_{cv} + \alpha v^i_{av} \leq k(i), \quad \forall i \in E, \tag{30}$$

$$f^i, v^i_{cv}, v^i_{av} \geq 0, \quad \forall i \in E, \tag{31}$$

which can be solved using the simplex method.

4. Optimal Parking Land Use

The large-scale adoption of AVs can transform existing parking land use in major cities. Urban planners will have higher flexibility in choosing the location of parking facilities because AVs do not need to park near their destinations. In this section, we present a mathematical model to investigate changes in parking land use.

We consider a Stackelberg game where the lower-level problem is the user equilibrium model (12)–(16) for given parking density $k(x)$, and the upper-level problem finds the optimal parking density along the corridor.

Let $R(x)$ be the rent cost per parking space per km. The rent cost $R(x)$ is a nonincreasing function of x, which, according to bid rent theory, says that it is more expensive to build parking near the downtown zone. Here, we consider $k(x)$ as a decision of a central agent in the upper-level model that plans to invest B dollars to build parking. The Stackelberg game is defined as

$$\text{Minimize} \int_0^D g_{\text{cv}}(x)v_{\text{cv}}(x)\mathrm{d}x + \int_0^D g_{\text{av}}(x)v_{\text{av}}(x)\mathrm{d}x, \quad (32)$$

$$\text{subject to} \int_0^D k(x)R(x) \le B, \quad (33)$$

$$k(x) \ge 0, \quad \forall x \in [0, D], \quad (34)$$

where objective function (32) minimizes the total cost of the system, constraint (33) ensures that the total expenditure is lower than the budget, and constraint (34) ensures nonnegativity of the parking density. The lower-level problem for a given parking density $k(x)$ is obtained from $\mathrm{UE}(x)$.

We can apply the same method in Section 3.4 to find the optimal parking land use. This is done by adding the following two equations, which are discretized versions of equations (33) and (34), to the mathematical program (equations (27)–(31)) of the solution algorithm:

$$\sum_{i=1}^{i=e} k^i R^i \le B, \quad k^i \ge 0, \forall i \in E, \quad (35)$$

where R^i is the expected rent cost of a parking facility at element i on the corridor: $R^i = \int_{i\Delta}^{(i+1)\Delta} R(x)/\Delta$.

5. Analytical Insights from a City with a Uniform Parking Distribution

We consider first the case with a mixed fleet of CVs and AVs and second a case where the entire fleet is either exclusively made of CVs or AVs.

5.1. Mixed Fleet of CVs and AVs. As a simple case, we assume that parking is uniformly distributed throughout the corridor at a density of $\bar{k}$ parking spaces per km such that $k(x) = \bar{k}, \forall x \in [0, D]$. The uniform parking density assumption is prevalent in the literature (see Arnott et al. [52]), as it enhances tractability. For other parking distributions, we can use the solution algorithm of Section 3.4. We also established previously that, (i) for CVs, the walking (access) cost per km, w, is larger than the driving cost per km, t, such that $w > t$, (ii) for AVs, $t > \bar{t}$ such that the driving cost per km with occupants, t, is larger than the driving cost per km without occupants, $\bar{t}$, and (iii) AVs that park at X experience a lower total travel cost than CVs that park at X, i.e., $C_{\text{av}}(x) < C_{\text{cv}}(x)$, because $w > t - \bar{t}$ as discussed in equation (3).

The parking pattern that emerges from the above assumptions has two properties. First, CVs always park closer to the downtown zone than AVs because this pattern leads to a lower objective function (12) since $C_{\text{av}}(x) < C_{\text{cv}}(x)$. In other words, the objective function (12) would always be lower if we move CVs closer to the downtown zone. Second, CVs and AVs do not share any of the parking spaces as discussed in Lemma 1.

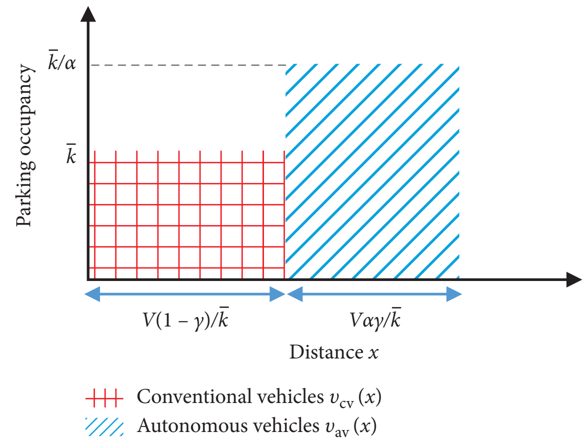


Figure 3: Parking occupancy of CVs and AVs.

Parking occupancy, $v_{\text{cv}}(x)$ and $v_{\text{av}}(x)$, is illustrated in Figure 3. It is evident that CVs park on a stretch of the corridor that is $V(1-r)/\bar{k}$ km long and AVs park on a stretch that is $V\alpha r/\bar{k}$ km long. As α gets smaller (i.e., each AV takes a smaller parking space), the AVs take a shorter stretch of the corridor and the height of $v_{\text{av}}(x)$ increases in Figure 3 because more AVs can be packed into each parking facility.

The parking search cost, $s(x)$, and the parking capacity Lagrange multiplier, $\lambda_p(x)$, are depicted in Figure 4, and their parameters are derived in Appendix. The relationship between the two is that the search time is discounted by a factor of α in the stretch of the corridor where AVs park. Figure 4 shows that vehicles experience a higher search cost as they park closer to the downtown zone because they have better accessibility. CVs that are closer to the downtown zone have to search for a longer time, but they walk a shorter distance, and AVs that park closer to the downtown zone experience a longer search time, but they travel a shorter distance when picking up and dropping off their occupants. Figure 4(b) shows that the maximum search cost of AVs is smaller than the minimum search cost of CVs by a factor of α (see point $x = V(1-r)/\bar{k}$ in Figure 4(b)). Hence, AVs experience a lower search cost than CVs as α increases.

Traffic flow along the corridor is depicted in Figure 5. Figure 5(a) shows that CVs approach the downtown zone at $V(1-r)$ vehicles per hour. As they get closer to downtown, some vehicles park on the corridor and the overall flow decays to the point where there is no CV flow at the downtown zone. AVs, on the other hand, show a completely different traffic flow pattern. AVs approach the downtown zone at a flow of $Vr\beta$ vehicles per hour. Every AV visits downtown twice: once to drop off a passenger and once to pick up a passenger. Hence, the AV flow at the downtown zone is $2Vr\beta$ vehicles per hour. The total flow of vehicles (i.e., sum of AV and CV traffic flow) is depicted in Figure 5(c), where it is evident that the peak flow occurs at $x = V(1-r)/\bar{k}$ which is the point where AV and CV parking is

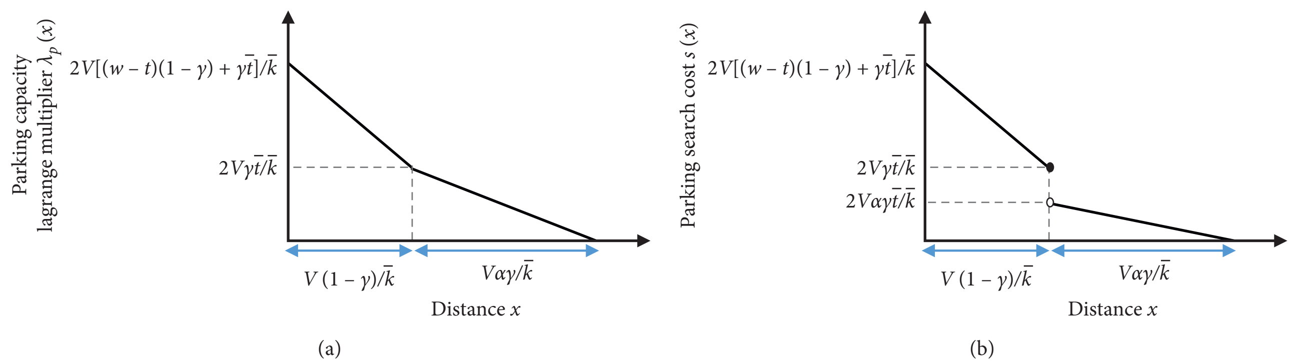


FIGURE 4: (a) Parking capacity Lagrange multiplier and (b) parking search cost.

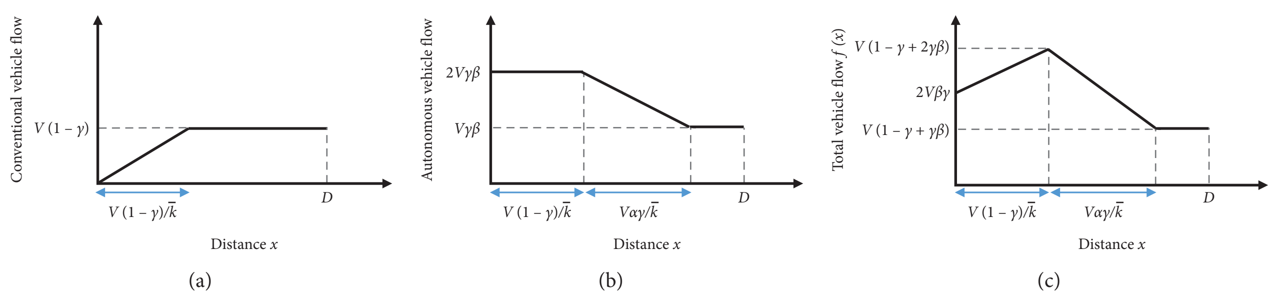


FIGURE 5: (a) Conventional vehicle flow, (b) autonomous vehicle flow, and (c) total vehicle flow.

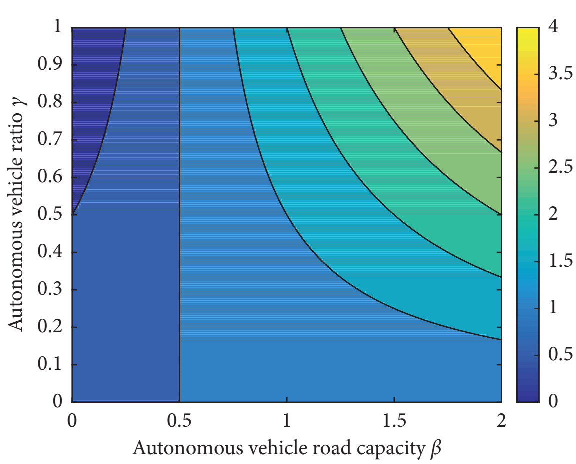


FIGURE 6: Peak traffic flow expansion factor: $\zeta = 1 - r + 2r\beta$.

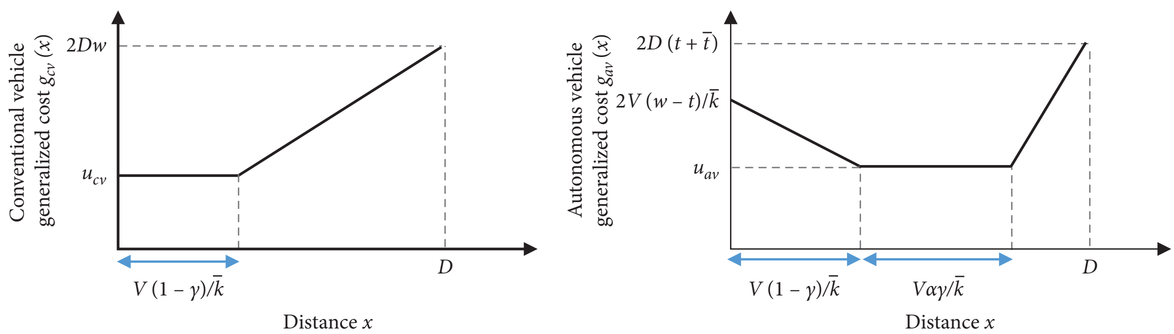
separated. From Figure 5(c), it is evident that increasing r (i.e., the ratio of AVs) pushes the peak flow location on the corridor closer to the downtown zone (because $V(1-r)/\bar{k}$ decreases) which may be problematic if one's objective is to reduce downtown traffic.

We investigate the changes in the height of the peak traffic flow which is $V(1-r+2r\beta)$, as shown in Figure 5(c). The peak flow has an expansion factor of $\zeta = 1 - r + 2r\beta$ compared to the no AV case with $r = 0$, where the peak flow is V. We illustrate the peak flow expansion factor $\zeta = 1 - r + 2r\beta$ in Figure 6 for different r and β. When $\beta > 0.5$ (with each AV taking more than half the road space allocated to each CV), ζ (the expansion factor) increases with r (AV ratio) because each AV traverses the peak location twice while taking more than half the space of a CV. Hence, increasing the AV ratio leads to higher peak traffic flow on the corridor. Moreover, when $\beta > 0.5$, then $\zeta > 1$, which indicates that the introduction of AVs, increases traffic.

Consider now the case where $\beta < 0.5$ with AVs each taking less than half the average road space allocated to each CV. The results here are the opposite of the previous case. When $\beta < 0.5$, then ζ (the expansion factor) decreases with r which shows that AVs can reduce traffic. Moreover, when $\beta < 0.5$, then $\zeta < 1$ which shows that the peak flow is lowered with the introduction of AVs.

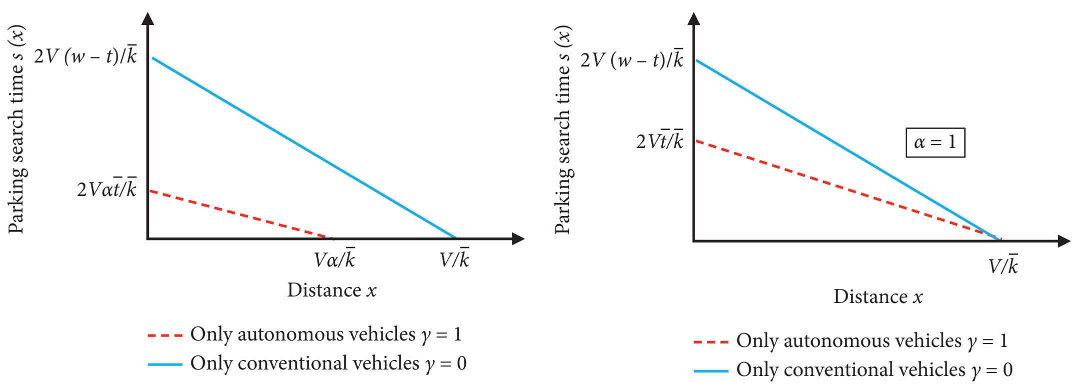
The generalized travel cost of CVs and AVs is depicted in Figure 7. CVs experience the lowest generalized cost in the domain $[0, V(1-\beta)/\bar{k}]$ where they park and the AVs experience their lowest cost in the domain $[V(1-r)/\bar{k}, V(1-r+\alpha r)/\bar{k}]$ where they park.

5.2. Fleet of Only CVs or Only AVs. We now consider a special case of the previous section where the entire fleet is either made of AVs (i.e., $r = 1$) or CVs (i.e., $r = 0$). The parking search cost is presented in Figures 8(a) and 8(b) for general α (AV parking capacity) and for $\alpha = 1$, respectively. As it is evident in Figure 8(a), the maximum search cost occurs at $x = 0$, the closes point to the downtown zone. The maximum parking search cost of AVs is lower than CVs by a factor of $(\alpha\bar{t})/(w-t)$ which shows that AVs experience a lower



(a) (b)

FIGURE 7: (a) User equilibrium for conventional vehicles, and (b) user equilibrium for autonomous vehicles.



(a) (b)

FIGURE 8: (a) Parking search cost with a general α and (b) parking search cost with $\alpha = 1$.

search cost when α is small so that more AVs are packed into each parking facility, or when $\bar{t}$ is small so that AVs can cruise around at a low cost of occupant-free driving per km. At $\alpha = 1$ (where CVs and AVs take the same amount of parking space), Figure 8(b) shows that AVs take the same stretch of the corridor to park, but they still experience a lower search cost compared to CVs.

The traffic flow is presented in Figure 9 for four separate ranges of β (AV road occupancy). When $\beta > 1$, as shown in Figure 9(a), the AV traffic flow is higher than the CV traffic flow all along the corridor. When $\beta = 1$, as shown in Figure 9(b), AVs and CVs have the same traffic flow at the home zone, but AVs have a higher traffic flow at the downtown zone which is larger than V. When $0.5 \le \beta < 1$, as shown in Figure 9(c), the AVs have a smaller traffic flow than CVs at the home zone but a larger flow at the downtown zone which is also larger than V. Finally, when $\beta < 0.5$, as shown in Figure 9(d), the AV traffic flow is smaller than V all along the corridor. Hence, the maximum traffic flow of AVs is smaller than CVs only in the fourth scenario, where $0.5 < \beta$. This is also shown in Figure 6 in the form of an expansion factor.

We investigate the impact of the AV ratio r on the equilibrium costs u_{cv} and u_{av}. For the uniform parking supply case, the equilibrium costs are

$$u_{cv} = 2DT + \frac{2V[(w-t)(1-r)+r\bar{t}]}{\bar{k}},$$
$$u_{av} = 2DT + \frac{2V[1-r+\alpha r]\bar{t}}{\bar{k}}. \tag{36}$$

It is evident that u_{cv} always decreases with r because $w - t > \bar{t}$. Given that CVs and AVs do not share parking, increasing the AV ratio r lowers the competition between CVs and helps them find parking easier. Moreover, the AV cost u_{av} also decreases with r because $\alpha < 1$. Hence, we conclude that increasing the AV ratio r lowers the equilibrium costs of both CVs and AVs.

We assess the impact of AVs on total system cost (including walking and driving) and the total driving cost (excluding walking). Figure 10 shows that increasing r (AV ratio) reduces the system cost because AVs do not incur the walking cost. Hence, the total experienced cost of drivers goes down as the AV penetration is increased. On the contrary, increasing r (AV ratio) increases the total driving cost because AVs travel more than CVs on the corridor as each AV needs to drive to the downtown zone twice. At $r = 1$ (full AV penetration), the total system cost is equal to the total driving cost because there is no walking in the system.

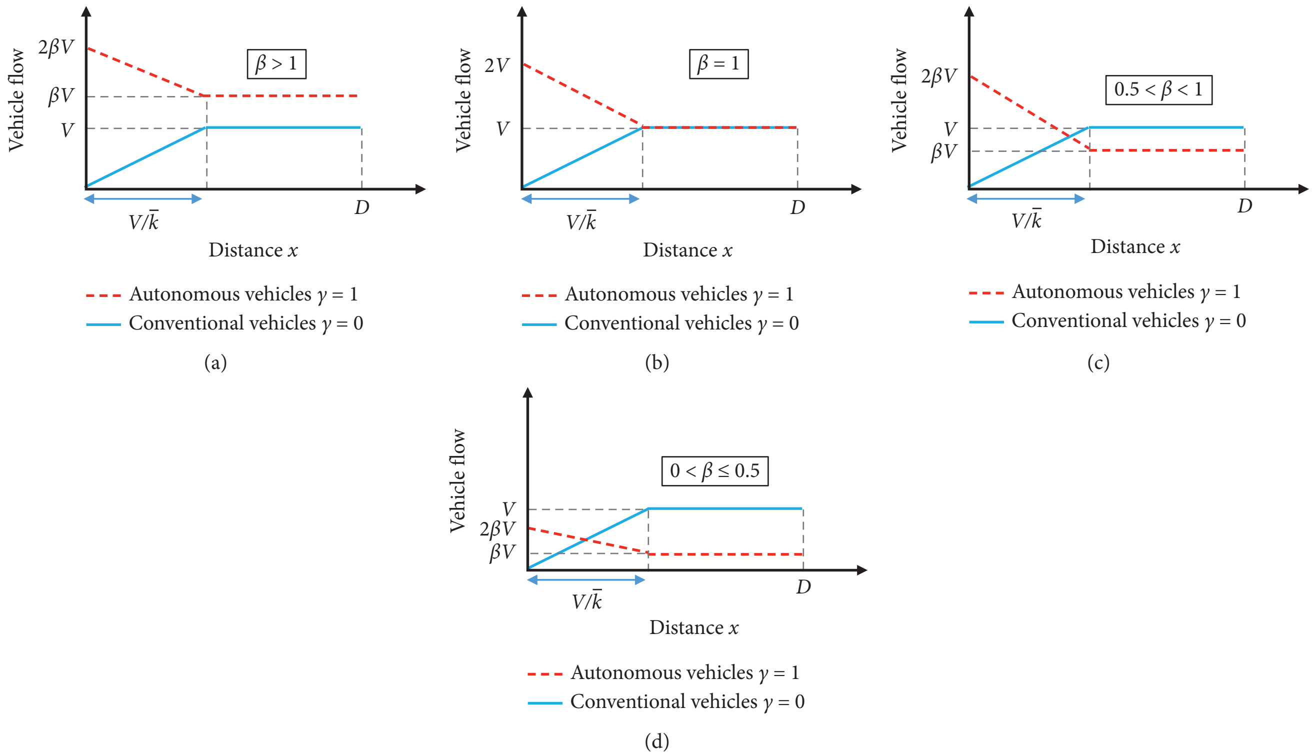


Figure 9: Traffic flow at (a) $\beta > 1$, (b) $\beta = 1$, (c) $0.5 < \beta < 1$, and (d) $\beta < 0.5$.

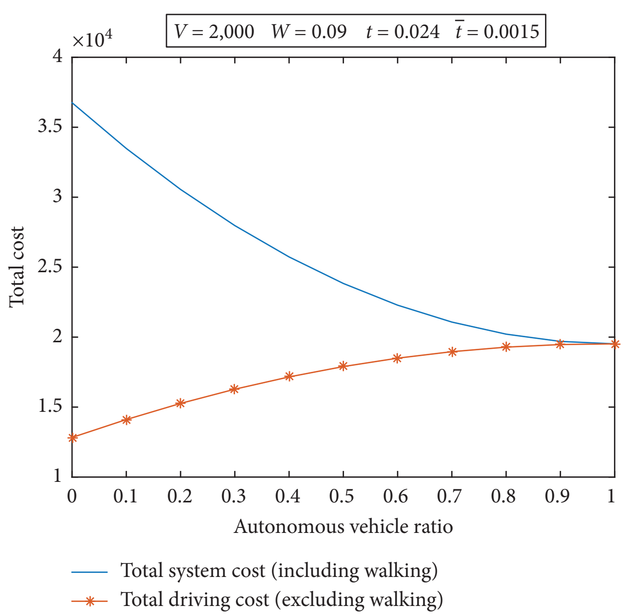


Figure 10: Total system cost and driving cost at different AV penetration rates.

6. Insights from Numerical Experiments

The analytical results of the previous section were derived for a case where parking was uniformly distributed throughout the corridor. In this section, we consider an alternative linear parking distribution in Section 6.1. We then find the optimal parking supply distribution in Section 6.2 for a given budget.

6.1. Linear Parking Distribution. Consider a linear parking supply of $k(x) = 90 + 20.5x$, where parking supply is 90 spaces/km at the downtown zone and 500 spaces/km at the home zone. Let $\alpha = 0.8$, $r = 0.7$, $W = 0.09$, $t = 0.25$, and $\bar{t} = 0.001$. The results of the linear parking supply case are presented in Figure 11. As it is evident from Figures 11(a) and 11(b), the CVs park closer to the downtown zone compared to the AVs. The generalized cost of CVs and AVs is presented in Figures 11(c) and 11(d), respectively, where it is shown that each vehicle type incurs its lowest generalized cost in the stretch of the corridor where it parks. The parking search cost is presented in Figure 11(e), where it is shown that vehicles incur a larger search cost closer to the downtown zone, and the traffic flow along the corridor is presented in Figure 11(f), where it is shown that flow increases and decreases nonlinearly compared to the case in the previous section with a uniform parking supply.

We present the user equilibrium costs u_{cv} and u_{av} with respect to r (AV ratio) and α (AV parking space) in Figure 12. Figure 12(a) shows that u_{av} decreases with r because the AVs are allowed to park close the downtown zones as r increases whereas u_{av} increases with α because each AV takes more parking space and AV has to search for a longer time to find an empty spot. The contours of Figure 12 show that AVs experience the same u_{av} for a range of r and α. That is, the negative impact of a large α is mitigated if r is large and there are enough AVs in the city.

For CVs, u_{cv} decreases with r because each CV competes with fewer CVs as r increases. On the other hand, u_{cv} is not sensitive to α because CVs do not compete with AVs for parking. That is, an increase in α benefits all AVs because

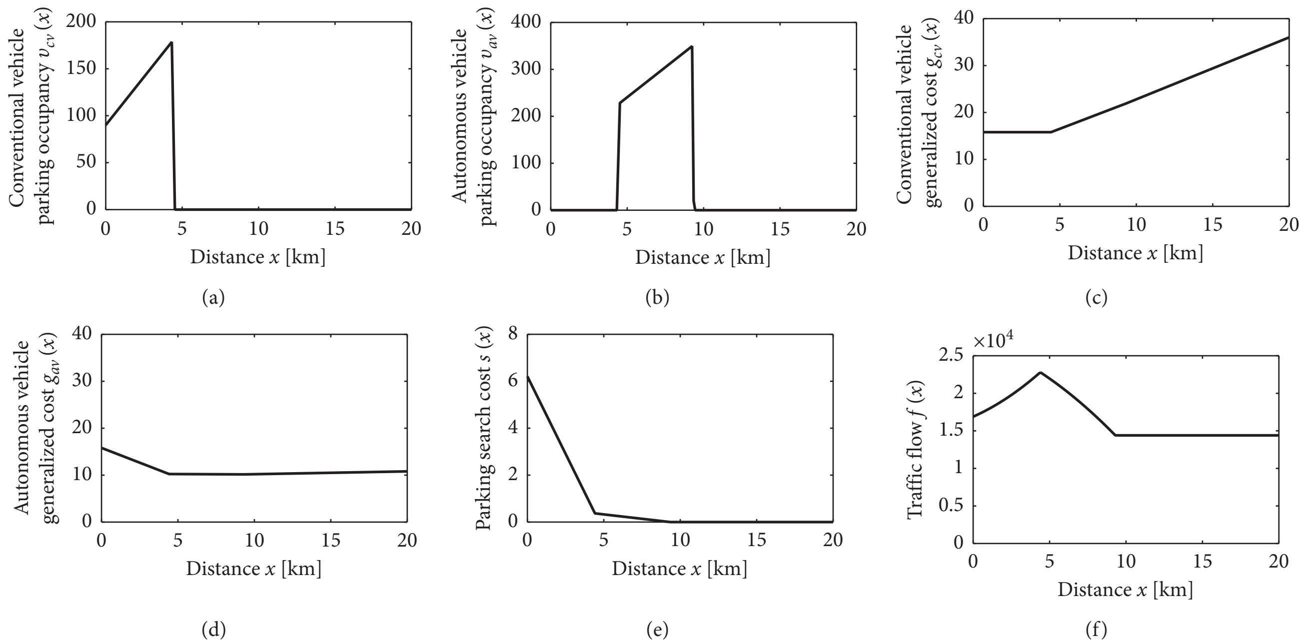


Figure 11: (a) Parking occupancy of CVs, (b) parking occupancy of AVs, (c) generalized cost of CVs, (d) generalized cost of AVs, (e) parking search cost, and (f) traffic flow.

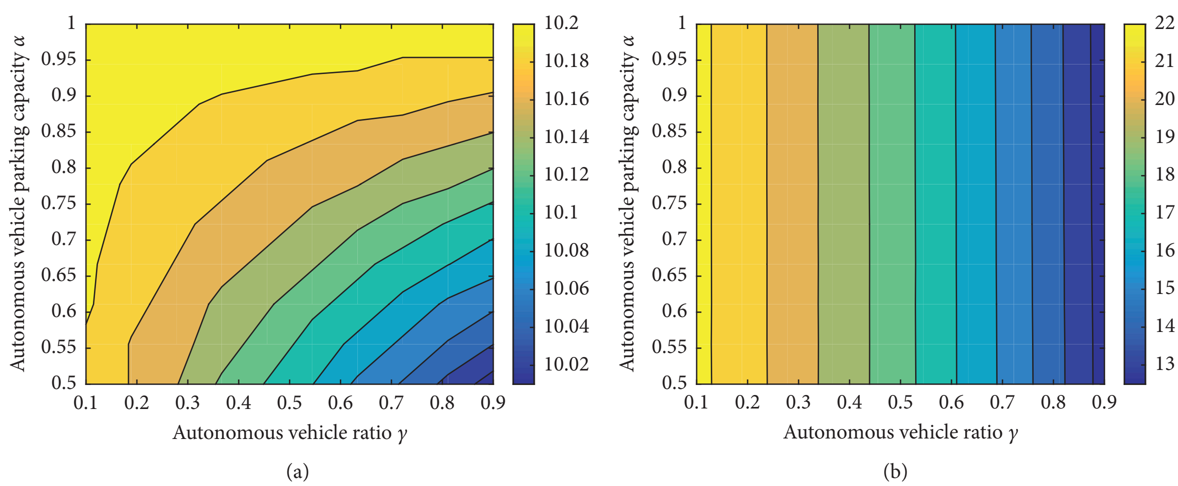


Figure 12: (a) Equilibrium cost of autonomous vehicles and (b) equilibrium cost of conventional vehicles.

they share the parking facilities together, but it does not influence the CVs. We conclude here that increasing r (AV ratio) generally helps both CVs and AVs and decreasing α helps AVs because they compete with each other but does not impact CVs because CVs do not compete with AVs for parking spaces.

6.2. Optimal Parking Supply. We now find the optimal parking supply distribution along the corridor. Let $R(x) = 5.2 \times 10^3 - 258.6x$ be the rent cost (and maintenance cost) of one parking space per km at x, and let the budget be $B = \$3 \times 10^7$. The rest of the parameters are the same as the previous section. The optimal parking supply is presented in Figure 13. The CV parking occupancy distribution $v_{cv}(x)$ is depicted in Figure 13(a), where it is shown that (i) CV drivers only have to walk a maximum of 2.7 km and (ii) there are more CVs parking closer to the downtown zone to minimize the walking distance. The AV parking occupancy distribution is depicted in Figure 13(b), where it is shown that AVs drive 11 km back to the home zone to park. Comparing Figures 13(a) and 13(b) shows that the priority is to develop parking near the downtown zone for CVs and have AVs drive back a longer distance to park at a facility with the lowest rent cost.

6.3. A Network Example. We now present a network example to validate the analytical findings in a more realistic setting. We use a static multiclass traffic assignment model with AVs

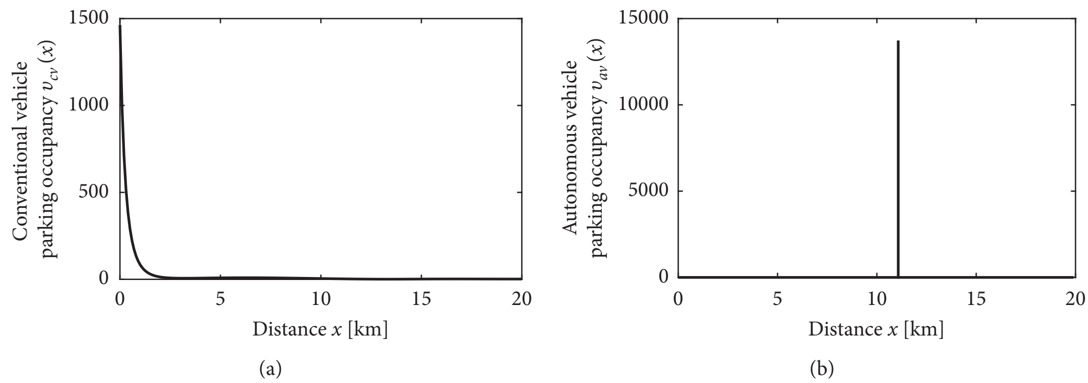


FIGURE 13: (a) Parking occupancy of conventional vehicles and (b) parking occupancy of autonomous vehicles.

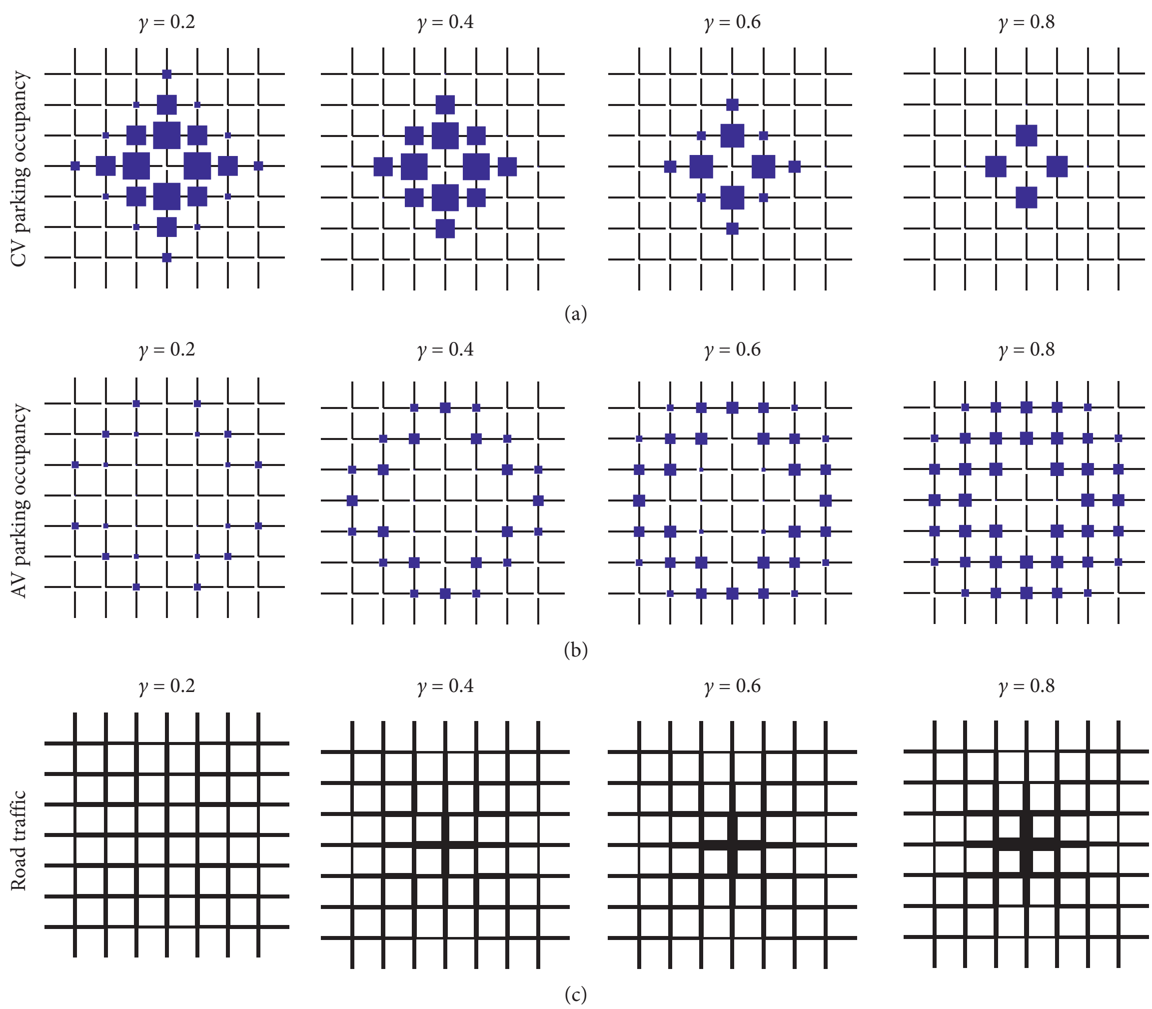


FIGURE 14: (a) Parking occupancy of conventional vehicles, (b) parking occupancy of autonomous vehicles, and (c) road traffic. The size of the squares in panels a-b shows the extent of parking occupancy. Similarly, the thickness of the lines in panel c represents the level of traffic on each link.

and CV. The network is grid with bidirectional links and one parking lot located at every crossing. A demand of 100 veh./hr is generated from each of the 28 bound boundary nodes; the entire demand has a single destination at the center node of the grid. The CV traveler walks from the parking lot to the destination, whereas the AVs send their car to self-park.

The travel time of link i is obtained from the BPR function $t_i(f) = T_i^0(1 + 0.15(f_i/c_i)^4)$, where f_i is the link flow, c_i is the capacity, and T_i^0 is the free flow travel time of link i. The cruising time at each parking lot i is also obtained from a BPR-type function $S_i(x) = \mu_i(1 + (x_i/k_i)^2)$, where μ_i is the cruising time at zero occupancy, x_i is the occupancy, and k_i is the capacity of parking lot i. This type of cruising function is described in studies, such as Axhausen et al. [53] and Nourinejad and Roorda [6].

The user costs are defined according to (4) and (5). The CV users experience a travel time in the network, a cruising time at the parking lots, and a walking time. The AV users are similar but do not experience walking times.

Figures 14(a) and 14(b) present the parking occupancy of CVs and AVs at varying AV penetrations rates, r. The CV parks closer to the final destination (i.e., center of the grid) because of their larger walking cost. In contrast, the AVs park farther away and are more spread out across the network. Moreover, CVs and AVs do not share parking lots and tend to exclusively occupy the lots. Thus, the analytical results of the corridor case are replicated in the network example as well.

Figure 14(c) presents the road traffic in the network. We show that the network is more congested at lager AV penetration rates as described in the corridor model. Moreover, there is more congestion in the center of the network because all AVs first drop off their passengers at the destination (i.e., center of the grid) before seeking an available parking spot.

7. Conclusions

It is anticipated that autonomous vehicles can provide a viable solution to the parking problem. In the next decade, AV drivers can use their vehicle as a personal valet where the AVs drop off their passengers and then head off occupant-free to find a parking space. This paper quantifies the impact of AVs on parking occupancy and traffic flow on a corridor that connects a home zone to a downtown zone. The model considers a heterogeneous group of AVs and CVs and captures their parking behavior as they minimize their generalized travel costs. The model is solved using a finite element method that transforms the problem into a linear program that can be solved using the simplex method.

Insights are obtained from two case studies with uniform and linear parking supply. The results of the model show that CVs park closer to the downtown zone because CV drivers cannot walk a long distance from where they park to their final destination. AVs, on the contrary, park far from the downtown zone and choose parking facilities where the search time is low. In light of this, we show that AVs experience a lower search time than CVs. The maximum AV search time is lower than the minimum CV search time. We investigate the impact of AV parking on traffic flow and show that AVs increase traffic flow because they make an additional trip to the downtown zone compared to CVs. In some instances, however, AVs can reduce the maximum traffic flow when they are highly connected and take less than half the road capacity compared to a CV.

While this study is the first to investigate the impact of AV parking, there are several remaining questions that need to be addressed in future research. The developed model in this paper can be extended in the following ways. First, to keep the model tractable, we did not account for the congestion effects on the corridor and assume a fixed cost per km traveled. Relaxing this assumption improves the accuracy of the congestion estimates of the model. Second, the model assumes that travelers originate from and terminate at one zone. A reasonable extension is to assume that vehicles originate from one zone but their destination location is randomly distributed along the corridor as it is done in Inci and Lindsey [40]. With random destination locations, new parking occupancy patterns may emerge. Third, to find the optimal parking land use, this paper assumes a fixed rent cost (per space) along the corridor. This cost structure in real life is comprised of a fixed construction cost and an amortized renting cost. We hypothesize that inclusion of the fixed parking cost leads to larger vertical economies of scale [39] for AVs than CVs so that AVs would be packed in a few high-rise parking facilities that are far from downtown. Finally, day-to-day dynamics of parking behavior should be considered if the users are not familiar with the parking conditions and instead learn from their daily experience. We believe that addressing these questions advances our understanding of the impact of AV parking on the future of cities. Finally, our analysis concerns a special case, where either the parking price is zero or the parking price is fixed and equal all along the corridor. For location-dependent pricing, we essentially have to add the price to the generalized cost of the users. While this is simple extension, the problem of finding the optimal prices along the corridor is more complex. To solve the pricing problem, there are several directions one can take by answering the following questions: (1) what is the optimal price for maximizing social-welfare, (2) what is the optimal price for maximizing profit, (3) what is the optimal set of prices when a set of agencies is individually responsible for maximizing their profit in a competitive market, and (4) should AVs and CVs be charged the same parking price or not.

Abbreviations

Parameters

α: Autonomous vehicle parking capacity
β: Autonomous vehicle road capacity
D: Length of the corridor (km)
V: Parking demand (vehicles per hour)
r: Ratio of autonomous vehicles
$k(x)$: Parking density at point X on the corridor (spaces per km)
t: Transportation cost of any vehicle with a passenger ($/km)
$\bar{t}$: Transportation cost of occupant-free autonomous vehicles ($/km)
w: Walking cost per km ($/km)
B: Parking construction budget
$R(x)$: Parking construction (rent) cost at point X ($ per space per km)

Decision variables

$v_{\text{av}}(x)$: Autonomous vehicle parking occupancy distribution
$v_{\text{cv}}(x)$: Conventional vehicle parking occupancy distribution
$f(x)$: Left-bound (west-bound) vehicle flow on the corridor at point X
$s(x)$: Parking search cost at point X on the corridor
u_{av}: User equilibrium cost of autonomous vehicles
u_{cv}: User equilibrium cost of conventional vehicles

Functions

$C_{\text{av}}(x)$: Transportation cost of autonomous vehicles that park at X
$C_{\text{cv}}(x)$: Transportation cost of conventional vehicles that park at X
$g_{\text{av}}(x)$: Generalized cost of autonomous vehicles that park at X
$g_{\text{cv}}(x)$: Generalized cost of conventional vehicles that park at X

Sets

E: Set of corridor elements.

Appendix

In this Appendix, we derive the search costs of Figure 4. Let $x_1 = V(1-r)/\bar{k}$ and $x_2 = V\alpha r/\bar{k}$ be the length of the corridor, where CVs and AVs park, respectively. The AV that parks at the point $x_1 + x_2$ does not incur any search cost, i.e., $s(x_1 + x_2) = 0$, because this AV can always move an infinitesimal distance to the right where parking occupancy and subsequently search time are both zero. Hence, the generalized AV travel cost at $x_1 + x_2$ is $g_{\text{av}}(x_1 + x_2) = 2Dt + 2(x_1 + x_2)\bar{t}$. We use this generalized cost shortly.

Consider now the AV that parks at point x_1 on the corridor. This AV experiences the same generalized cost, $g_{\text{av}}(x_1) = s(x_1) + 2Dt + 2x_1\bar{t}$, as the AV that parks at $x_1 + x_2$. By setting $g_{\text{av}}(x_1) = g_{\text{av}}(x_1 + x_2)$, which holds under user equilibrium conditions, the search cost of AVs at point x_1 is

$$s(x_1 + \epsilon) = \frac{2V\alpha r\bar{t}}{\bar{k}}, \tag{A.1}$$

where ϵ is an infinitesimal distance.

We now move on to find the search cost of CVs at points $x = x_1$ and $x = 0$. We start with the CV search cost $s(x_1)$. Recall that, according to Lemma 1, $s(x) = \lambda_p(x)$ for CVs and $s(x) = \alpha\lambda_p(x)$ for AVs. The following equality holds according to Lemma 1:

$$\lambda_p(x_1 + \epsilon) = \frac{s(x_1 + \epsilon)}{\alpha} \tag{A.2}$$

because point $x_1 + \epsilon$ is occupied by AVs. To obtain $s(x_1)$, we need to use the continuity condition of $\lambda_p(x)$ which states that

$$\lambda_p(x) = \lambda_p(x + \epsilon), \tag{A.3}$$

for small ϵ. Using equations (A.1)–(A.3), the search cost of CVs at point x_1 is

$$s(x_1) = \frac{2Vr\bar{t}}{\bar{k}}. \tag{A.4}$$

We now move on to derive the search cost of CVs at point $x = 0$. We do this using the user equilibrium condition which states that $g_{\text{cv}}(0) = g_{\text{cv}}(x_1)$. Given that $g_{\text{cv}}(0) = s(0) + 2t$, $g_{\text{cv}}(x_1) = s(x_1) + 2(D - x_1)$, and $g_{\text{cv}}(0) = g_{\text{cv}}(x_1)$, we have

$$s(0) = \frac{2V[(w-t)(1-r) + r\bar{t}]}{\bar{k}}. \tag{A.5}$$

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Mehdi Nourinejad carried out mathematical modeling, reviewed literature, and wrote the manuscript. Matthew Roorda reviewed literature, edited the draft, and wrote the manuscript.

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