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Research Article

A Benders Decomposition Algorithm for the Passenger Train Service Planning

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Railway transport becomes a more popular transportation in many countries due to its large transport capacity, low energy consumption, and benign environment. The passenger train service planning is the key of the rail operations system to balance the transport service and the passenger demand. In this paper, we propose a mixed binary linear programming formulation for the passenger train service planning to optimize the train route, frequency, stop schedule, and passenger assignment simultaneously. In addition, we analyze the computational complexities of the model and develop a Benders decomposition algorithm with valid inequalities to solve this problem. Finally, our model and algorithm are tested on a real-world instance of the Beijing-Shanghai high-speed railway line. The computational results show that our approach can solve these problems within reasonable solution time and small optimality gaps (less than 2.5%).

1. Introduction

Railway transport plays an important role for medium-long-distance transportation in countries with a vast territory. This is due to its large transport capacity, low energy consumption, and benign environment. For instance, China has built the largest high-speed rail (HSR) network in the world to solve many problems such as capacity restriction and congestion and promote its economic development [1].

Due to the complexity of the rail operations system, a hierarchically structured planning process is usually applied to generate and maintain passenger train schedules. It starts with demand analysis, which determines the passenger demand from origin station to destination station (passenger OD) for the railway transport system, usually followed by the passenger train service planning (PTSP), which determines the train route and corresponding frequency and stop schedule according to the passenger OD. Then operational planners allocate time slots to trains for each departure and arrival event at stations on their route, which is named as timetable. At last, planning of rolling stock and crew scheduling are proposed. In this paper, we focus on the PTSP, which is one of the most critical operational issues to balance the transport service and the passenger demand.

Railway transportation had been the primary choice of people due to its large transport capacity and cheap tickets. Figure 1 shows that, during a long time, more than half of the passenger volume had been completed by railroad in China [2]. However, the railway transportation had provided the low transport service level like low service frequency, low travel speed (53.3 km/h), serious overcrowding, and bad traveling environment for passengers on account of the limitation of the technical condition.

Nowadays, China has the longest high-speed rail (network) in the world, which is larger than the combined HSR networks of 13 European countries (7351 km). Compared with the existing lines, the HSR line is the only passenger-dedicated line that provides the high-quality transport service for passengers including high service frequency, faster travel speed (250 ~ 350 km/h), and comfortable traveling environment. Meanwhile, with the raise of economical level and the living level, Chinese people have the ability to select the multiple transportation modes, and the railway transport enterprises are facing fierce competition

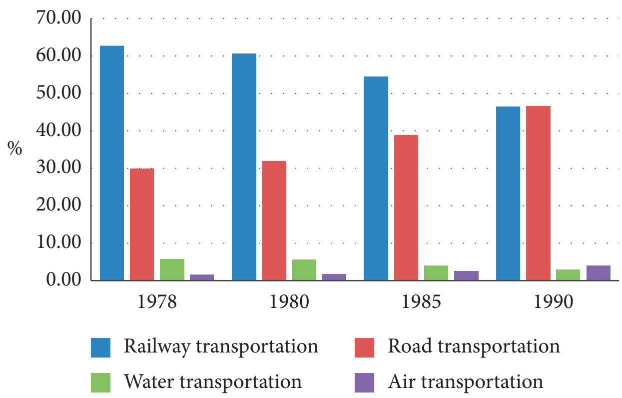

Figure 1: Passenger volume ratio for multiple transportation means during 1978–1990 in China

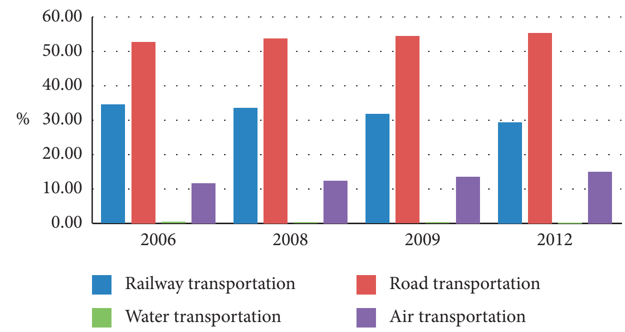

Figure 2: Passenger volume ratio for multiple transportation means during 2006–2012 in China

from other transportation modes. As shown in Figure 2, both road transportation and air transportation grow rapidly during the last ten years, and the road passenger volume has exceeded the railway passenger turnover [2]. Nevertheless, 30% of the passenger volume has still been completed by railroad.

As the marketization of railway transportation and the railway passenger turnover reduces year by year, the railway transport enterprises have to enhance their competitiveness by a good rail operations system. The PTSP is the primary operational issue in optimizing the rail operations system. However, the PTSP has always been determined manually based on experience by now, and it needs a systematic, analyzing approach instead of experience.

Compared with the medium-scale HSR lines in Japan and Europe, the HSR lines in China have the long distance (more than 1000 km) between the terminal stations operating a number of long-distance trains. The railway transport enterprises have to provide many intermediate train stops to meet the passengers' demand. However, the trains with more stops for a long-distance travel can reduce the travel speed and increase the travel time. As a result, the train stop-schedule plan, which is a key part of the PTSP for long-distance HSR lines, becomes too complicated to be pregiven like some medium-scale HSR lines do.

In this paper, we describe a mathematical programming for the PTSP and a solution approach based on the Benders decomposition. Given the daily passenger demand from origin station to destination station (passenger OD) as well as the railway resources, the objective function is to minimize the operating cost of the rail company as well as the total travel time of passengers, and constraints are designed to meet the daily passenger demand and railway resource constraints. This model determines the routes, frequencies, and stop schedule of the trains. We can summarize the main contributions of this paper as follows.

Firstly, we proposed a mixed-integer linear programming (MILP) model to optimize the train route, frequency, stop schedule, and passenger assignment simultaneously.

Secondly, we developed a Benders decomposition with valid inequalitiesto solve the PTSP. To the best of our knowledge, there are no other studies to solve the PTSP by the Benders decomposition.

Thirdly, the proposed approach is empirically applied to Beijing-Shanghai HSR line, which is one of the longest and busiest high-speed railways in China. Previous studies on this topic need to determine the alternative stopping stations for the train or the number of stop schedules in advance to limit the number of stop schedules [1, 3–5]. However, the long-distance HSR always operates the long-distance trains with low frequencies and sufficiently variable stopping patterns to serve as many passengers as possible; imposing restrictions on stop schedule may get a suboptimal solution [1].

The rest of this article is organized as follows. Section 2 gives the literature review related to the PTSP. In Section 3, the problem description is given. The mathematical formulation is given in Section 4 and the Benders decomposition is given in Section 5. Furthermore, Section 6 presents the empirical studies. The final section presents the conclusions and future research.

2. Literature Review

Over the last few decades, many mathematical models and solution algorithms studies have been proposed for the PTSP, which can be categorized as the following two types [6].

2.1. Optimizing the Train Routes and Service Frequencies with Pregiven Stops for the HSR Network. Earlier studies utilized a system split method to decompose both trains and stations into several levels. Consequently, the stop schedule of the train can be determined by assuming that classified trains stop at stations of the same and higher level [7–12]. Then the PTSP can be expressed as an integer programming to optimize the train frequencies with pregiven stops, and a common approach like branch-and-cut or branch-and-bound can be used to solve this problem. However, the system split method has two shortcomings: firstly, it gives passengers' traveling routes in advance instead of allowing the passenger OD to choose their routes freely; secondly, it does not optimize the stop schedule of the train . Thereafter, many studies extended the system split method by allowing passengers to select their routes freely. Guan et al. [13]

presented a linear binary integer program for simultaneous optimization of transit line configuration and passenger line assignment in a mass transit system with all-stop trains. Similarly, many studies proposed a MILP model based on multicommodity flow model and column generation approach for the PTSP in a city public transport network with all-stop buses [14–16].

2.2. Optimizing Train Routes, Service Frequencies, and Stops Simultaneously for the HSR Network. Given the number of stop schedules, Chang et al. [17] developed a multiobjective programming model involving large positive coefficients for the HSR line to determine the best-compromise train service planning including the train stop-schedule plan, service frequency, and fleet size without suggesting an algorithm. Consequently, some research extended the work of Chang et al. [17] and developed efficient solution algorithms for the PTSP. Reference [4] developed an implicit enumeration algorithm integrating the implicit enumeration, Lagrangian relaxation, and genetic algorithm, which can be viewed as a tool to accelerate the solution without splitting the original problem and improving its quality. Park et al. [5] presented the standard column generation algorithm to split the original problem. Nevertheless, the standard column generation cannot completely solve the PTSP, since its pricing subproblem is still open. Schmid and Walteros [18, 19] proposed the multicommodity flow model and metaheuristic methods for the bus rapid transit route design problem including the bus stop-schedule plan, service frequency, and fleet size. The metaheuristic methods based on genetic algorithm or hybrid large neighborhood search algorithm can solve some large-scale combinatorial optimization problems in reasonable run times. Nevertheless, the search strategy of the metaheuristic methods based on the probability might deteriorate the quality of the solution. Huiling et al. [1] described the PTSP as a four-stage hierarchical design problem with a series of heuristic algorithms to reduce its solving difficulty in large-scale HSR network. They also considered giving the optional train stop schedules. Moreover, the iterative computation algorithm based on enumeration for the stop schedule generation might not obtain the optimal solution in reasonable run times.

In the aforementioned literature, we find that most studies focus on the heuristic and metaheuristic algorithms due to the complexity of the PTSP. However, they may not guarantee the quality of the solution. The exact approaches may be an alternative. Besides, recently, some studies have obtained a good solution by using commercial software including CPLEX, GAMS, and Gurobi, which are widely used as a benchmark for both exact and heuristic approaches [3, 20, 21]. Therefore, we propose an exact approach based on Benders decomposition and compare it with the commercial software.

3. Problem Description and Model Formulation

3.1. Problem Description. Given the rail network consists of route sections, daily passenger demand from original stations to declinational stations (passenger OD), the operating cost including the fixed cost per train and unit cost per km, the velocity and dwell time for every type of trains as well as corresponding train capacities, the capacity of rail infrastructure, and the set of train ODs, the PTSP determines the routes, frequencies, and stop schedule of the trains to minimize the operating cost of the rail company as well as the total travel time of passengers. Generally speaking, the PTSP has two planning objectives: (a) minimizing the operating cost of the railway transport enterprise and (b) minimizing the passenger's total travel time. Moreover, the constraints for the PTSP mainly lie in two aspects: on the one hand, the railway transport enterprise must provide the trains as well as stop schedules for trains to meet the passenger OD's travel demand; on the other hand, all the trains and stop schedules for trains must meet the capacity restrictions of the rail infrastructure.

3.2. The Mathematical Formulation

3.2.1. Notations

Sets

$\mathbf{V}$: set of stations, indexed by s
$\mathbf{E}$: set of sections, indexed by e
$\mathbf{L}$: set of trains, indexed by l
$\mathbf{D}$: set of passenger ODs, indexed by $(i, j) \in \mathbf{D}$, $i, j \in \mathbf{V}$
TO D: set of the train ODs, indexed by $(i, j) \in D$, $i, j \in \mathbf{V}$
$r(f)$: set of nonstop running arcs for a train l, where $(i, j) \in r(l)$ represents a travel route for a train l between station i and station j without stopping at any station except station i and station j. It can be seen that the set $r(f)$ is equivalent to the stop schedule of a train l.

Parameters

fix_l: fix cost of train l
q^{od}: passenger demand from the original station o to the declinational station d
va_l: variable cost of train l per km
t_{ij}^l: dwell time at station i plus nonstop travel time between i and j of a train l
m_l: maximal stop times of a train l
Cap_l: capacity of a train l
(i, j): nonstop arcs
dis_{ij}: distance of the nonstop arc (i, j)
δ_e^{ij}: arc-section incident matrix, where $\delta_e^{ij} = 1$ if the nonstop arc (i, j) passes section e and $\delta_e^{ij} = 0$ otherwise.

Decision variables

w_l: $w_l = 1$ if train l is used; otherwise, $w_l = 0$
x_{ij}^l: $x_{ij}^l = 1$ if train l passes the nonstop arc (i, j); otherwise, $x_{ij}^l = 0$
v_{ij}^{odl}: number of the passenger ODs that travel nonstop arc (i, j) by train l

3.2.2. The Mathematical Formulation. In general, the HSR line has multiple train ODs, which is more difficult than one train OD. We can transform multiple train ODs into one train OD by constructing dummy source and sink nodes. As shown in Figure 3, train l_1 starts from v_1 to v_6, train l_2 starts from v_2 to v_5, and train l_3 starts from v_1 to v_3. By using dummy source node v_s and sink node v_t, all the trains start from v_s to v_t. Note that the dummy source node v_s and sink node v_t can only connect to the origin stations or destination stations. The number of passenger ODs and the cost between $v_s\,(v_t)$ and other nodes are zero. Based on the problem description, the PTSP can be stated as follows:

$$\min Z = \sum_{l\in L} \mathrm{fix}_l w_l + \sum_{l\in L}\sum_{(i,j)\in A} \mathrm{va}_l \mathrm{dis}_{ij} x_{ij}^l + \sum_{l\in L}\sum_{o,d\in V}\sum_{(i,j)\in A} t_{ij}^l v_{ij}^{\mathrm{odl}}. \tag{1}$$

It is subject to

$$\sum_{j\in \mathbf{TO\ D}} x_{v_s j}^l = w_l, \quad \forall l \in \mathbf{L}, \tag{2}$$

$$\sum_{i\in \mathrm{TOD}} x_{i v_t}^l = w_l, \quad \forall l \in \mathbf{L}, \tag{3}$$

$$\sum_{j:\ (i,j)\in \mathbf{A}} x_{ij}^l - \sum_{j:\ (j,i)\in A} x_{ji}^l = 0, \quad \forall l \in \mathbf{L}, i \in \mathbf{V}, \tag{4}$$

$$\sum_{l\in \mathbf{L}} \left(\sum_{j:\ (i,j)\in \mathbf{A}} v_{ij}^{\mathrm{odl}} - \sum_{j:\ (j,i)\in \mathbf{A}} v_{ji}^{\mathrm{odl}} \right) = \begin{cases} q^{\mathrm{od}}, & i = o, \\ -q^{\mathrm{od}}, & i = d, \ \forall l \in \mathbf{L}, i \in \mathbf{V}, \\ 0, & \text{else}, \end{cases} \tag{5}$$

$$\sum_{o,d\in \mathbf{V}} v_{ij}^{\mathrm{odl}} \le \mathrm{Cap}_l x_{ij}^l, \quad \forall l \in \mathbf{L}, (i,j) \in \mathbf{A}, \tag{6}$$

$$\sum_{o,d\in \mathbf{V}} x_{ij}^l \ge 1, \quad (i,j) \in \mathbf{A}, \tag{7}$$

$$\sum_{l\in \mathbf{L}} \sum_{(i,j)\in r(l)} x_{ij}^l \delta_e^{ij} \le n, \quad \forall e \in \mathbf{E}, \tag{8}$$

$$\sum_{(i,j)\in \mathbf{A}} x_{ij}^l \le m_l w_l, \quad \forall l \in \mathbf{L}, (i,j) \in \mathbf{A}, \tag{9}$$

$$w_l, x_{ij}^l \in \{0,1\}, \quad \forall l \in \mathbf{L}, (i,j) \in \mathbf{A}, \tag{10}$$

$$v_{ij}^{\mathrm{odl}} \in \{0,1\}, \quad \forall o,d \in \mathbf{V}, l \in \mathbf{L}, (i,j) \in \mathbf{A}, \tag{11}$$

where the objective function (1) minimizes the operating costs including costs of trains to be used as well as costs of train kilometers and the passenger travel time. Constraints (2)–(4) ensure that a train generates a route from the origin station v_s to the destination station v_t, if it is used. Constraints (5) and (6) are constraints to meet the passenger OD's travel demand, where constraint (5) indicates the flow conservation of the passenger, and constraint (6) guarantees passenger flow for every nonstop running arc of a train to be less than the capacity of the train. Constraint (7) ensures that there is at least one nonstop arc between any two stations; this is the strong connectivity of the transport network for trains which can guarantee that the model always has a solution. Although we can relax the constraint, our exact approach may not get a solution within reasonable solution time, since the solution space of the PTSP is enlarged. Constraints (8) and (9) are constraints to meet the capacity restrictions, where constraint (8) ensures that the total number of trains using the segment is less than a given number, and constraint (9) ensures that if a train is used, the number of stops of the train is less than a given number. Constraint (10) states the binary restriction for the decision variables x_{ij}^l and w_l. Constraint (11) states the nonnegative restriction for the decision variables v_{ij}^{odl} . In general, variables v_{ij}^{odl} should be integers, but a single passenger has less influence on the result of the PTSP. Therefore, we relaxed the integrality requirements on variables v_{ij}^{odl} to simplify the calculation as in previous research [14, 15, 18, 22].

3.2.3. Complexity of the PTSP. Even though the model defined by (1)–(11) fully describes the PTSP, it is difficult to solve. We now analyze the complexity of the PTSP as follows.

Theorem 1. *The model defined by (1)–(11) is NP-hard.*

Proof 1. Suppose that any two stations can be a train OD, then the set $\mathbf{L} = (i,j) | (i,j) \in \mathbf{A} = \mathbf{A}$, and variables w_l, x_{ij}^l, v_{ij}^{odl}, co_l, cv_l, and t_{ij}^l can be rewritten as w_{ij}, x_{ij}, v_{ij}^{od}, co_{ij}, cv_{ij}, and t_{ij} with the redundant variables w_{ij}. Let $n = m_l = \infty$ and eliminate constraints (8) and (9); the resulting model is a multicommodity capacitated network design problem, which is a NP-hard problem [23]. Therefore, the model defined by (1)–(11) is NP-hard.

Assume that railway network only consists of a high-speed rail line without branches. The number of binary decision variables is equal to $C_{\mathbf{TO\ D}}^2 (2^{|\mathbf{V}|-2} + |\mathbf{L}|)|$; besides $2^{|\mathbf{V}|-2}|\mathbf{L}||\mathbf{V}|^2 C_{\mathbf{TO\ D}}^2$ continuous decision variables are required as well as $2^{|\mathbf{V}|+1}|\mathbf{L}| + 2^{|\mathbf{V}|} + |\mathbf{V}|^3 + |\mathbf{L}||\mathbf{V}| + |\mathbf{E}| + 2|\mathbf{L}|$ constraints. Therefore, the number of variables and constraints grow exponentially with the number of stations. This number could be huge even for small HSP line. In general, the set of trains is tremendous; however, the upper bound $|\mathbf{L}|$ can be determined by maximal number of trains passing any given section or the capacity of the rail infrastructure [4]. Hence, we set the upper bound $|\mathbf{L}|$ for an input parameter in this paper. In addition, The PTSP is a two-way operation, where each one-way operation is assumed to be the same, and we can only consider a one-way operation to reduce the decision variables and constraints in half. □

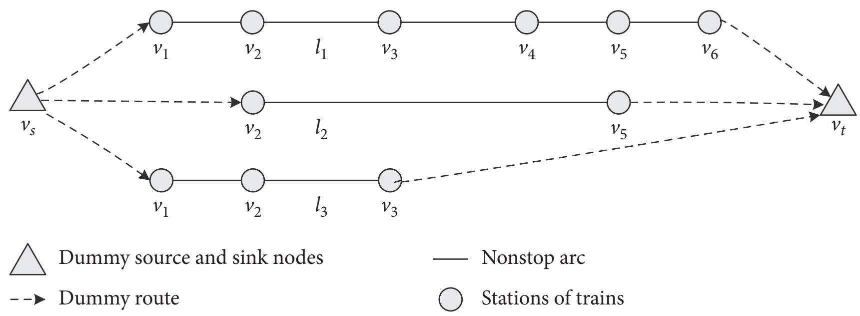

Figure 3: A small example for transforming multiple train ODs into one train OD.

4. Benders Decomposition

The model defined in (1)–(11) is a mixed-integer line programming (MILP) including binary variable and continuous variable. The Benders decomposition is an efficient method for solving MILP problems, which can decompose the original problem into a master problem and a subproblem and then solves them iteratively by utilizing the solution of one in the other. Besides, decision variables are divided into complicating variables consisting of binary variables and easier variables consisting of continuous variables.

4.1. Benders Subproblem. For given binary variables $\overline{w} = (\overline{w}_l)_{l\in\mathbf{L}}$ and $\overline{\mathbf{x}} = (\overline{x}^l_{ij})_{l\in\mathbf{L},(i,j)\in\mathbf{A}}$, we can state the subproblem by dual constraints (5) and (6) as follows:

$$\begin{aligned} \max Z_{sp} &= -\sum_{l\in\mathbf{L}}\sum_{(i,j)\in\mathbf{A}} \mathrm{Cap}_l \overline{x}^l_{ij}\beta^l_{ij} + \sum_{o,d\in\mathbf{V}} \left(\alpha^{\mathrm{od}}_o q^{\mathrm{od}} - \alpha^{\mathrm{od}}_d q^{\mathrm{od}}\right), \\ &\cdot \alpha^{\mathrm{od}}_i - \alpha^{\mathrm{od}}_j - \beta^l_{ij} \le t^l_{ij}, \\ &\cdot \alpha^{\mathrm{od}}_i, \quad o,d,i \in \mathbf{V}, \\ \beta^l_{ij} &\ge 0, \quad l \in \mathbf{L},\ (i,j) \in \mathbf{A}, \end{aligned} \tag{12}$$

where α^{od}_i and β^l_{ij} are the dual variables associated with constraints (5) and (6), respectively. If the subproblem is infeasible, the PTSP is also infeasible; if the subproblem is feasible and bound, the extreme points $\overline{\alpha} = (\overline{\alpha}^{\mathrm{od}}_o)_{o,d,i\in\mathbf{A}}$ and $\overline{\beta} = (\overline{\beta}^l_{ij})_{l\in\mathbf{L},(i,j)\in\mathbf{A}}$ can be obtained and the following constraint called optimality cut is added to the master problem:

$$\sum_{l\in\mathbf{L}}\sum_{o,d\in\mathbf{V}} \left(\overline{\alpha}^{\mathrm{od}}_o - \overline{\alpha}^{\mathrm{od}}_d\right) - \sum_{l\in\mathbf{L}}\sum_{(i,j)\in\mathbf{A}} \mathrm{Cap}_l \overline{x}^l_{ij}\overline{\beta}^l_{ij} \le \sigma, \tag{13}$$

where σ is an auxiliary continuous variable; if the subproblem is unbound, then the extreme rays $\widetilde{\alpha}$ and $\widetilde{\beta}$ can be obtained and the following constraint called feasibility cut is added to the master problem:

$$\sum_{l\in\mathbf{L}}\sum_{o,d\in\mathbf{V}} \left(\widetilde{\alpha}^{\mathrm{od}}_o - \widetilde{\alpha}^{\mathrm{od}}_d\right) - \sum_{l\in\mathbf{L}}\sum_{(i,j)\in\mathbf{A}} \mathrm{Cap}_l \overline{x}^l_{ij}\widetilde{\beta}^l_{ij} \le 0. \tag{14}$$

4.2. Benders Master Problem. Let $\mathbf{C}_o$ and $\mathbf{C}_F$ represent the sets of extreme points and extreme rays, respectively, where $(\overline{\alpha}, \overline{\beta}) \in \mathbf{C}_o$ and $(\widetilde{\alpha}, \widetilde{\beta}) \in \mathbf{C}_F$. Then the master problem can be expressed as

$$\min Z_{MA} = \sum_{l\in\mathbf{L}} \mathrm{fix}_l w_l + \sum_{l\in\mathbf{L}}\sum_{(i,j)\in\mathbf{A}} va_l \mathrm{dis}_{ij} x^l_{ij} + \sigma. \tag{15}$$

It is subject to

$$\sum_{l\in\mathbf{L}}\sum_{o,d\in\mathbf{V}} \left(\overline{\alpha}^{\mathrm{od}}_o - \overline{\alpha}^{\mathrm{od}}_d\right) - \sum_{l\in\mathbf{L}}\sum_{(i,j)\in\mathbf{A}} \mathrm{Cap}_l \overline{x}^l_{ij}\overline{\beta}^l_{ij} \le \sigma, \quad \forall (\overline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\beta}}) \in \mathbf{C}_O, \tag{16}$$

$$\sum_{l\in\mathbf{L}}\sum_{o,d\in\mathbf{V}} \left(\widetilde{\alpha}^{\mathrm{od}}_o - \widetilde{\alpha}^{\mathrm{od}}_d\right) - \sum_{l\in\mathbf{L}}\sum_{(i,j)\in\mathbf{A}} \mathrm{Cap}_l \overline{x}^l_{ij}\widetilde{\beta}^l_{ij} \le 0, \quad (\widetilde{\boldsymbol{\alpha}}, \widetilde{\boldsymbol{\beta}}) \in \mathbf{C}_F. \tag{17}$$

$$\text{Constraints } (2) - (4),\ (7) - (10). \tag{18}$$

4.3. Structure of the Benders Decomposition. Since the number of constraints (16) and (17) may be large and difficult to define in advance [24], an iterative approach of Benders decomposition that generates optimality cuts and feasibility cuts gradually is commonly used. The main structure of the Benders decomposition can be stated as follows:

Step 0: initialization: set upper bound UBD $= +\infty$ and lower bound LBD $= -\infty$, $\mathbf{C}_O = \varnothing$, $\mathbf{C}_F = \varnothing$.

Step 1: solve the master problem to obtain the binary decision variables $\overline{\mathbf{w}}, \overline{\mathbf{x}}$ as well as the current optimal value Z_{MA}; then update the lower bound LBD $= \min(Z_{MA}, \mathrm{LBD})$.

Step 2: solve the subproblem for $\overline{\mathbf{w}}, \overline{\mathbf{x}}$; then obtain the extreme points $(\overline{\alpha}, \overline{\beta})$ with the current optimal value Z_{SP} if it is feasible and bound or the extreme rays $(\widetilde{\alpha}, \widetilde{\beta})$ if it is unbound. Update the upper bound UBD $= \min(Z_{SP}, \mathrm{UBD})$.

Step 3: update the sets of extreme points and extreme rays: $\mathbf{C}_O = \mathbf{C}_O \cup \left\{(\overline{\alpha}, \overline{\beta})\right\}$, $\mathbf{C}_F = \mathbf{C}_F \cup \left\{(\widetilde{\alpha}, \widetilde{\beta})\right\}$.

Step 4: if $((\mathrm{UBD} - \mathrm{LBD})/\mathrm{UBD}) < \varepsilon$ (where ε is a tolerance parameter), then stop. Otherwise, continue with Step 1.

4.4. Accelerating Benders Decomposition by Valid Inequalities. There are various techniques that can be used to accelerate the algorithm potentially. Firstly, generate the initial cuts by

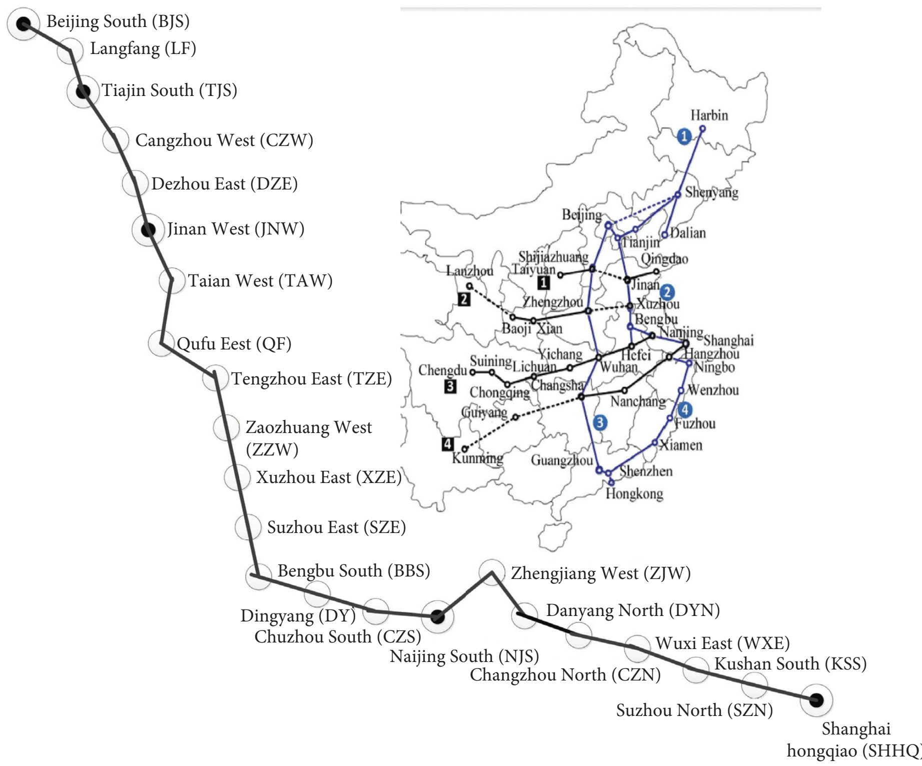

Figure 4: HSR network in China and Beijing-Shanghai HSR line.

defining x_{ij}^{l} for all $l \in \mathbf{L}$, $i, j \in \mathbf{V}$. This is a relaxation of the original problem and is proved to be quite effective to accelerate the algorithm [25]. Secondly, the following well-known cutest inequalities can be used to tighten the master problem [26]:

$$\sum_{l \in \mathbf{L}} \sum_{j:(i,j) \in r(l)} \mathrm{Cap}_l x_{ij}^{l} \geq \sum_{j:(i,j) \in \mathbf{A}} q^{ij}, \quad \forall i \in \mathbf{V}, \tag{19}$$

$$\sum_{l \in \mathbf{L}} \sum_{i:(i,j) \in r(l)} \mathrm{Cap}_l x_{ij}^{l} \geq \sum_{i:(i,j) \in \mathbf{A}} q^{ij}, \quad \forall j \in \mathbf{V}, \tag{20}$$

where inequalities (19)–(20) state that the total transport capacity offered from station i, which is a leaving or entering station, must be greater than or equal to the total number of passenger demands coming from or getting to station i. Finally, the upper bound $|\mathbf{L}|$ can be determined by the initial master problem, which is often much less than the number determined by the rail infrastructure. We can select a number from small to large to test the initial master problem until it is feasible.

5. The Empirical Study

In this section, we present the numerical experiment on the Beijing-Shanghai HSR line in China. The Benders decomposition is written in C# language with CPLEX 12.4 as the linear and integer programming solver. All experiments are run on an AMD A6-3420M 1.50 GHz PC with 4 GB RAM. In addition, we set the maximum CPU running time to be 1200s and the tolerance parameter $\varepsilon = 0.025$; other parameters in CPLEX are set to default values.

Table 1: Input parameters of the model.

Parameters	Value or descriptive calculation
Set of start or end stations	Beijing South, Tianjin South, Jinan West, Nanjing South, Shanghai Hongqiao
Fixed cost	10000 RMB/train
Variable operating cost	100 RMB/train.km
Dwell time	5 min
The velocity of train	300 km/h
The biggest number of available trains	228 trains/day
The biggest number of stopping stations	13 times/train

5.1. The PTSP of the Beijing-Shanghai HSR Line. The Beijing-Shanghai HSR line is one of the longest HSR lines in China. It is 1318 km long and goes through 23 stations along the Yangtze River Delta region, which is the most developed area in China. As shown in Figure 4, circles denote stations and black circles denote the origin or destination station for a train. The letters in parentheses beside each station name denote the abbreviations of the station names; for example, Beijing South is abbreviated to BJS and so on.

Input parameters of the model and the distance of each segment, respectively, are shown in Tables 1 and 2. The daily passenger OD is shown in Table 3. We test seventeen instances of the Beijing-Shanghai HSR line ranging from 15 to 23 stations, named as BJN-CZS to BJN-SHH. Moreover, we compare three different MIP solution methods: CPLEX,

Table 2: The distance in each segment.

Segment	Distance (km)	Segment	Distance (km)	Segment	Distance (km)
1	59.50	9	36.10	17	28.60
2	62.70	10	64.40	18	32.40
3	87.90	11	67.20	19	57.40
4	103.80	12	88.00	20	36.80
5	92.20	13	54.30	21	31.40
6	58.70	14	62.00	22	32.00
7	70.40	15	74.70		
8	56.00	16	61.70		

Benders decomposition (BD), and the Benders decomposition with valid inequalities (7) (BD & VI).

The computational results of the instances are shown in Table 4. The first and second columns denote the name of each instance and the number of stations, respectively. The third column denotes the upper bound **L** determined by the initial master problem. The fourth column denotes the gap, which is the percentage difference between the best LP bound and the best integer solutions. Other columns denote the optimal objective values (Opt) and the CPU computing times for three different MIP solution methods, respectively.

As seen from Table 4, the CPLEX cannot produce any feasible solution when the number of stations is more than 16. On all these instances, the BD can get the solution whose gap is less than 2.5% within 19 stations, while the gap increases from 10.75% to 49.80% as the number of the stations increases from 20 to 23. However, the BD & VI can get a solution whose gap is less than 2.5% for all instances with the least CPU computing times among the three MIP solution methods. Besides, the upper bound $|\mathbf{L}|$ determined by the initial master problem increases from 56 to 132, and all of them are less than the biggest number of available trains, 228, which is determined by the rail infrastructure. As can be observed in Figure 5, the BD & VI converges faster than the BD. From all experiments, the BD & VI outperforms BD and CPLEX.

Let us assume that trains with the same train OD and number of stops are classified as one type. The results of PTSP for the BJN-SHH are shown in Table 5. A total of 132 trains with 25 types are dispatched, most trains run in BJN-SHH section, about 39 trains with 6 types run in TJN-SHH section, and only one train runs from JNW to SHH. Most trains stop less than 5 times. In contrast to previous studies that provide the same stopping schedule for every type train, our results provide more stopping patterns for every type train (Figure 6), which can decrease the total cost by enlarging the solution space. This may be appropriate for long-distance HSR line in China. The reason is that a nonperiodic timetable was used in China with uneven passenger flow distribution, and the railway transport enterprise always provides sufficiently variable stopping patterns to serve as many passengers as possible.

5.2. The Sensitivity of the PTSP. The PTSP is needed to meet the passengers' demand as much as possible, and we can use the comparison between transport demand and transport capacity to evaluate the PTSP. The transport demand is the number of the passenger demands' volume getting in and out of every station and the transport capacity is the provided train capacity for a station according to the stop schedule. This approach is based on the same idea as that of Huiling et al. [1]. Figure 7 shows that the transport capacity curve does not accord with the transport demand well and a large capacity surplus is generated.

5.2.1. Train OD Changed. As seen from the transport demand curve in Figure 7, the largest capacity surplus is in TJS, while the transport demand of XZE is large. Hence, we use XZE instead of TJN in the set of train ODs. Figure 7 shows that the capacity surplus between transport demand curve and transport capacity curve is reduced. However, the number of stops, the operational cost, and the travel time increase (Table 6).

5.2.2. Train Capacity Changed. Since there is a large capacity surplus, we can reduce the capacity of a train in half through reducing cars of a train, which is very easy to operate in China. Figure 7 shows that the capacity surplus is reduced the most. However, the transport capacity of XZE and NJS cannot meet the transport demand, which may cause a crowded traveling environment for passengers who get in or out of XZE and NJS. Furthermore, the number of stops, the operational costs, and the travel time increase greatly (Table 6).

5.2.3. Analysis of the Objective Function. The PTSP is a bi-objective programming in essence, and the objective function can be modified to a combination as follows:

$$\min Z = \lambda\left(\sum_{l\in\mathbf{L}}\mathrm{fix}_l w_l + \sum_{l\in\mathbf{L}}\sum_{(i,j)\in\mathbf{A}}\mathrm{va}_l \mathrm{dis}_{ij} x_{ij}^l\right) + (1-\lambda)\left(\sum_{l\in\mathbf{L}}\sum_{o,d\in\mathbf{V}}\sum_{(i,j)\in\mathbf{A}} t_{ij}^l v_{ij}^{\mathrm{odl}}\right), \tag{21}$$

where $\lambda \in [0,1]$ is a weighting factor that can balance the operational cost for transportation enterprise and the travel time of passengers. Let λ vary from 0 to 1 in 0.1 interval, which results in 11 combinations in total and constitutes 11 Pareto optimal solutions, which are not dominated by each other. As shown in Figure 8, for $\lambda = 0$,

Table 3: Forecast of daily passenger OD for the Beijing-Shanghai high-speed railway.

Station	LF	TJS	CZW	DZE	JNW	TAW	QFE	TZE	ZZW	XZE	SZE	BBS	DY	CZS	NJS	ZJW	DYN	CZN	WXE	SZN	KSS	SHH
BJN	2634	1343	2248	1975	7790	1892	1724	487	774	2816	334	684	35	205	5289	686	100	1124	1508	1011	290	8311
LF		163	178	66	167	30	23	9	10	70	5	14	0	2	84	8	0	21	18	2	1	115
TJS			135	223	535	98	66	40	42	136	21	82	4	22	254	28	8	98	68	128	31	225
CZW				78	275	39	30	14	17	118	6	24	0	7	183	28	2	33	52	29	14	254
DZE					718	96	77	32	47	101	4	17	1	5	151	16	4	57	44	41	15	259
JNW						510	538	396	580	500	28	95	6	42	920	128	24	281	302	216	84	1777
TAW							95	43	74	171	6	43	4	13	219	23	0	32	68	48	22	399
QFE								17	43	133	8	27	3	9	233	19	4	69	80	19	19	614
TZE									16	57	3	24	4	6	87	7	0	17	23	12	12	156
ZZW										142	5	22	6	11	152	14	1	38	46	15	16	300
XZE											52	157	20	61	2186	127	36	280	434	192	189	1604
SZE												26	5	8	168	6	0	14	37	6	12	252
BBS													42	35	443	22	3	44	48	43	59	830
DY														35	77	3	0	10	21	9	20	88
CZS															262	4	1	18	39	15	28	286
NJS																265	37	333	422	332	361	4840
ZJW																	2	16	23	20	19	489
DYN																		2	0	1	1	
CZN																			54	0	54	536
WXE																				0	76	493
SZN																					45	499
KSS																						592

Table 4: Computational results for the instances.

Instance	V	L	BD & VI			BD			CPLEX		
			Gap (%)	Opt	Time (s)	Gap (%)	Opt	Time (s)	Gap (%)	Opt	Time (s)
BJN-CZS	15	56	1.42	1007030.22	60.87	0.34	1007025.22	84.99	0.26	1006415.22	1161.22
BJN-JNS	16	65	0.00	1320651.85	86.81	1.84	1321351.85	119.47	–	–	1200
BJN-CZN	19	90	2.03	1428609.89	135.07	2.14	1428094.89	176.08	–	–	1200
BJN-WXE	20	100	1.86	1527330.60	238.21	10.75	1527630.60	1200	–	–	1200
BJN-SZN	21	110	1.73	1675569.06	324.06	12.82	1700734.06	1200	–	–	1200
BJN-KSS	22	121	1.40	2148337.43	350.04	33.48	2151112.43	1200	–	–	1200
BJN-SHH	23	132	2.47	2773026.40	1111.57	49.80	2801321.40	1200	–	–	1200

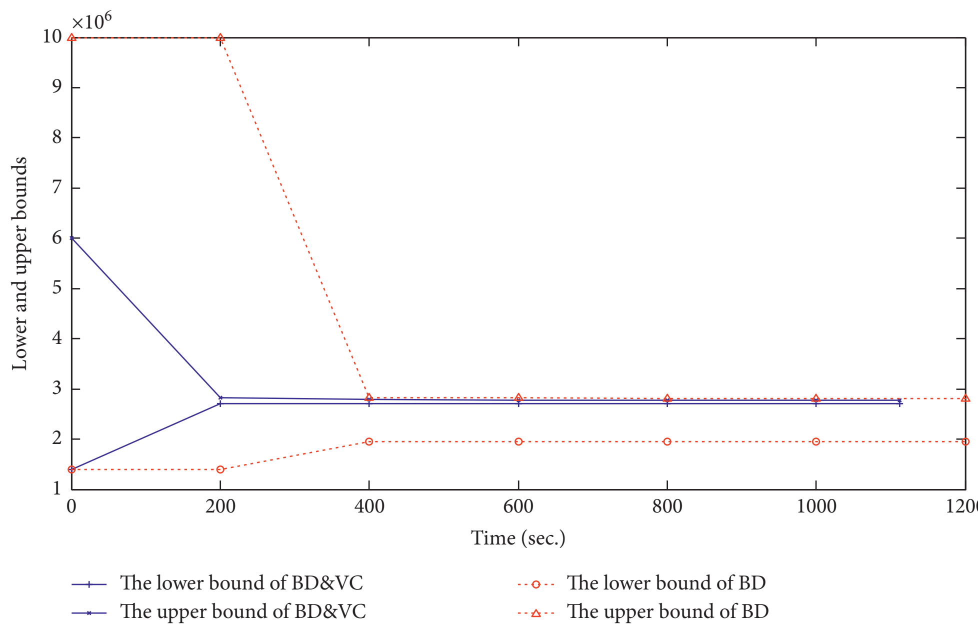

Figure 5: The convergence of the BD & VC and BD.

Table 5: The PTSP for the Beijing-Shanghai HSR line.

Train OD	Daily frequency	Stop planning
BJN-SHH	1	Nonstop
	10	Stop 1 time
	32	Stop 2 times
	17	Stop 3 times
	4	Stop 4 times
	3	Stop 5 times
	2	Stop 11 times
	1	Stop 12 times
TJN-SHN	1	Nonstop
	10	Stop 1 time
	16	Stop 2 times
	7	Stop 3 times
	4	Stop 4 times
	1	Stop 5 times
JNW-SHH	1	Nonstop
	5	Stop 1 time
	4	Stop 2 times
	1	Stop 3 times
	1	Stop 4 times
BJN-NJS	4	Stop 2 times
	2	Stop 3 times
TJS-NJS	1	Nonstop
	2	Stop 1 time
	1	Stop 2 times
JN-NJ	1	Stop 2 times

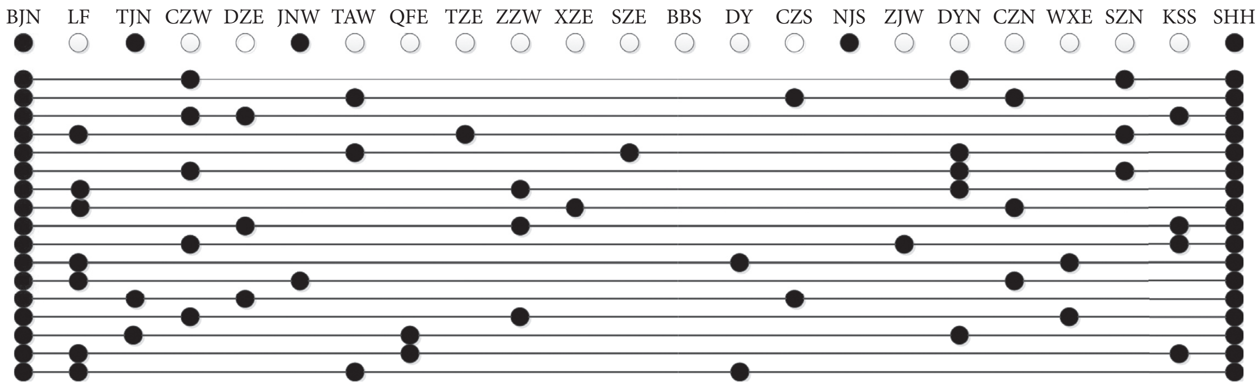

Figure 6: The stop schedule for BJN-SHH with 3 stops.

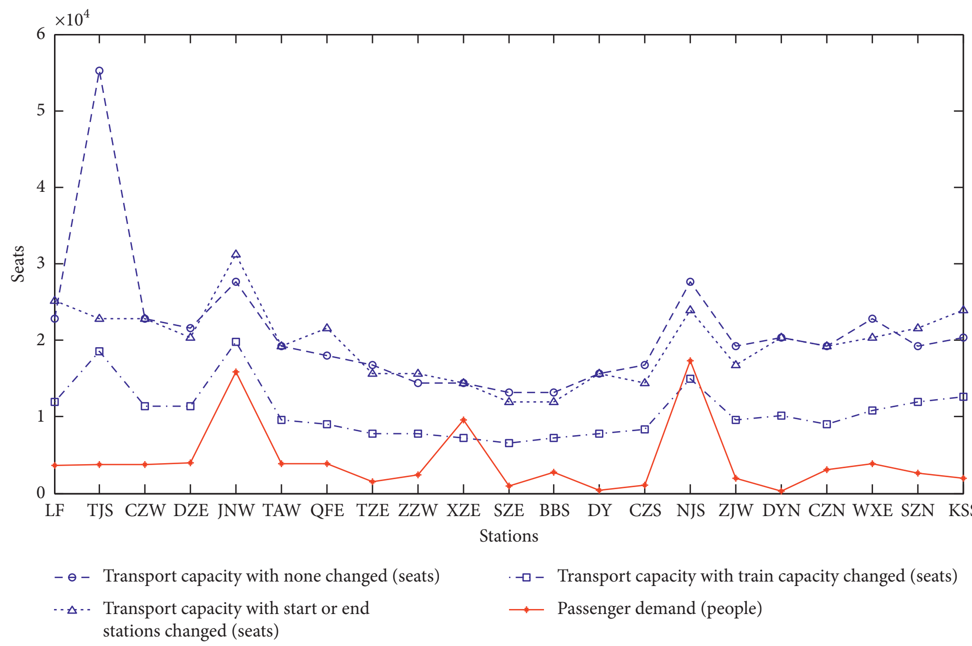

Figure 7: Comparison between transport demand and capacity of sensitivity analysis.

Table 6: Comparison for sensitivity analysis.

None changed				Start or end stations changed				Train capacity changed			
Trains	Stops	Cost	Travel times	Trains	Stops	Cost	Travel times	Trains	Stops	Cost	Travel times
132	341	1391000	1382026.40	132	367	1392900	1382226.40	132	373	1394600	1444046.40

only travel time contributes to the objective and is therefore low, while the operational cost is high. When $\lambda = 1$, the case is the opposite as well as the result. With increasing λ, the operational cost monotonically decreases, while the total traveling time increases. When $\lambda \geq 0.3$, the operational cost has no changes with increasing λ. When $\lambda \geq 0.6$, both the operational cost and the traveling time have no changes with increasing λ.

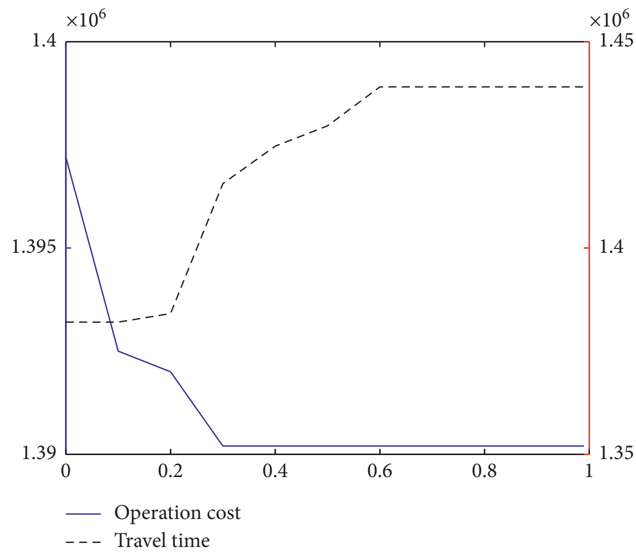

Figure 8: Influence of the weighting factor.

6. Conclusions and Future Research

In this paper, we proposed a mixed-binary linear programming model for the PTSP without large positive coefficients. By solving the model, we can determine the passenger train service planning including the train route, corresponding frequency, and stop schedule according to the passenger OD. We show that the PTSP is an NP-hard problem through the multi-commodity capacitated network design problem. We developed the Benders decomposition with valid inequalities. Our model and algorithm are used to test the Beijing-Shanghai HSR line in China without imposing restrictions on stop schedule to guarantee the quality of solution. Computational experiments show that the computational burden of solving the PTSP grows rapidly with the size of the HSR line. The standard commercial optimization packages CPLEX cannot find the feasible solution with 15 or more stations. However, the Benders decomposition with valid inequalities was able to find the solutions of all problems within 2.5% of optimality, which outperforms the standard Benders decomposition without valid inequalities on all of the considered instances. In addition, the results show that our approach can provide sufficiently variable stopping patterns. There are several directions for future research. Firstly, we will consider trains with different speed and capacity. Secondly, the uncertain passenger demand will be explored in the future. At last, the improved Benders decomposition should be designed for the PTSP with relaxed strong connectivity constraint (7).

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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