

## Research Article

# A Data-Driven Urban Metro Management Approach for Crowd Density Control

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Large crowding events in big cities pose great challenges to local governments since crowd disasters may occur when crowd density exceeds the safety threshold. We develop an optimization model to generate the emergent train stop-skipping schemes during large crowding events, which can postpone the arrival of crowds. A two-layer transportation network, which includes a pedestrian network and the urban metro network, is proposed to better simulate the crowd gathering process. Urban smartcard data is used to obtain actual passenger travel demand. The objective function of the developed model minimizes the passengers' total waiting time cost and travel time cost under the pedestrian density constraint and the crowd density constraint. The developed model is tested in an actual case of large crowding events occurred in Shenzhen, a major southern city of China. The obtained train stop-skipping schemes can effectively maintain crowd density in its safety range.

## 1. Introduction

Large crowding events sometimes occur in massive commercial and recreational activities, posing great pressure to urban management agencies and urban transportation agencies [1]. Large crowding events are intrinsically a phenomenon of anomalous urban travel demand and manifested in extremely high crowd densities in some localized popular spots. In extreme case, the overly high density crowds can be drawn into a crowding accident that causes injuries or even deaths [2]. A number of crowd management strategies have been proposed to protect crowd safety. These strategies include arrangement of enough security guards, placing signs to indicate walking routes, opening additional exits [3], closing gates to the event place, maintaining long distances between festival attractions [4], and placing barriers at specific positions [5]. Well-designed crowd management strategies can, to some extent, avoid the occurrence of crowding accidents [1]. However, these strategies are mostly designed for already formed high-density crowds which are actually in a stressfully dangerous state and difficult to leave the

crowded area in a short time. Therefore, the more conservative and smart strategy is to avoid the situation of unsafely high-density crowds.

Large crowding events are intrinsically caused by anomalous urban travel demand as discussed. Consequently, a straightforward idea to avoid unsafe high-density crowds is to control the travel demand to the crowded spots. As many studies indicated, urban metro transports the majority of people to large crowding events which are usually held at the popular locations nearby urban metro stations. Urban metros become a major transport mode of crowds probably because they are characterized with the good features of large capacity and high reliability [6], and the severe traffic congestion in the vicinity of the crowd gathering place could be another reason. Hence, urban metro management could be a feasible approach to assist crowd management during large crowding events. More specifically, we can slow down the speed of crowds entering the overly crowded area through adjusting train operation schemes. On the other hand, developing specific train operation schemes for large crowding events is also beneficial to the safety of urban metro itself. Because, the anomalously high volume of passengers who

head to the crowd gathering spot also poses great pressure to the stations and the trains of urban metro.

In practice, a number of strategies were proposed to alleviate the passenger transport pressure of urban metro, such as closing urban metro stations or controlling passenger flow at some stations. Among these strategies, train stop-skipping strategy is probably mostly widely used because it is a low-cost approach to improve the efficiency of urban metro operation [7]. Train stop-skipping strategies were originally developed for reducing the travel time of passengers and saving the operation cost of urban metro agencies. Here, we revisit train stop-skipping strategy to investigate its possibility in controlling crowd density and develop an optimization model to generate the train stop-skipping scheme during large crowding events. The fundamental idea is that passengers may walk to the highly crowded area or wait for the following trains when the train stop-skipping scheme is implemented, which postpones the arrival of crowds.

The train stop-skipping strategy was first proposed for the Chicago metro in 1947 [7] and first implemented on a line of Santiago metro which suffered severe shortage of train capacity. The implementation of stop-skipping operation achieved good performance in Santiago metro, and the operational speeds of trains increased and the operation cost per km-train (per kilometer per train) was considerably reduced. The authorities of Santiago metro consequently implemented the stop-skipping strategies on other lines. Dong et al. [8] developed a Mass Passenger Flow Management System (MPFMS) for the "People Square" station of Shanghai metro. The developed system can be used to predict passenger flow and generate station stop-skipping schemes when mass passenger flow is going to occur. The system was proved to be able to prevent mass passenger flow backlog accident (accident caused by the number of passengers on the platform exceeding the safety threshold). Indeed, train stop-skipping strategies have been tested in many urban metros in the world, such as the SEPTA line of Philadelphia and the Helsinki commuter rail [9]. The train stop-skipping strategies have been demonstrated to be very effective in alleviating the pressure of urban metro during large passenger flow situations. In addition, the train stop-skipping strategy is easy to implement in practices. In what follows, we first make a review on previous works on train stop-skipping strategies.

The first research direction is on the development of stop-skipping strategies for different types of metro stations. Vuchic [10] proposed A/B stop-skipping strategy by dividing metro stations into three types: A, B, and AB, in which major stations are usually labeled with the type AB. Some trains stop at type A stations and type AB stations, whereas other trains stop at type B stations and type AB stations. Many researchers have investigated the A/B stop-skipping strategy. For example, Freyss et al. [7] used the A/B stop-skipping strategy to study the skip-stop operation of a one-way track. Economic parameters were employed in the model to generate the cost function. Abdelhafiez et al. [11], Cao et al. [12], and Salama et al. [13] compared the performances of different A/B stop-skipping strategies.

Different from the typical A/B stop-skipping strategy, He et al. [14] and Li et al. [15] classified the stations into local stations and express stations. He et al. [14] studied the station stop-skipping strategies for urban central lines and peripheral lines. Considering the overtaking behaviour between express trains and local trains, Li et al. [15] developed a mixed-integer nonlinear programming model and adjusted the train stopping patterns to minimize passenger travel time. Suh et al. [16] used a similar station classification method and investigated the stop-skipping strategy for the Korean express metro system. The authors showed that the peak-hour waiting time increased by 43.8% to 56.3% and the travel time decreased by 10.8% to 12.9% if their stop-skipping strategy is implemented.

The second research direction is on the development of stop-skipping strategies that do not classify metro stations. Elberlein [17] formulated the stop-skipping problem as the mixed-integer nonlinear programming (MINLP) problem. However, in this study, the stop-skipping schemes of consecutive stations were optimized as a set, and the stop-skipping decision was not generated for each station. To obtain the stop-skipping strategy for each individual station, Fu et al. [18] represented the stop-skipping decisions of trains at stations as binary variables and formulated the stop-skipping problem as the MINLP problem. The MINLP problem was solved using an exhaustive approach. However, the enumeration method cannot solve large-scale problems in real time and cannot make use of the informative real-time data such as travel time between each pair of stations and the number of passengers entering each station. The bilevel approaches [19, 20], the iterative convex programming approaches [21, 22], the two-step optimization strategies [23–25], and the sequential quadratic programming algorithms [26] were developed to reduce the computation cost of the optimization models. In addition, heuristic algorithms are also used to solve the optimization models. Jamili and Aghaee [27] put forward a robust mathematical model and solved it using two heuristic algorithms, a decomposition-based algorithm and a simulated annealing-based algorithm. Kang et al. [28] designed a heuristic evaluation-based optimization algorithm to solve a last train operational model, which aimed to save energy and decrease passenger transfer time.

The optimization models above were extended from different perspectives. For instance, Sun and Hickman [29] developed an alternative scheme to implement a stop-skipping policy in a real-time manner. The alternative policy is more convenient because not all stations on the skipping segment must be skipped. Altazin et al. [30] studied the problem of real-time rescheduling for rapid transit railway systems when the systems were disturbed by unexpected large number of passengers or technical problems. The goal of the stop-skipping strategy was to minimize the impact of accidental disturbances. In addition to the studies on real-time management, Jiang et al. [31] investigated the stop-skipping strategy under given total inbound demand. To modify the inbound passenger distribution among stations, the authors built a passenger original station choice model based on utility theory.

Previous studies have focused on reducing the travel time and waiting time of passengers or the operational cost of urban metro operators. Only a few studies [7, 8] have considered the effectiveness of the stop-skipping strategy on crowd management in large crowding events. Here, we propose a dynamic stop-skipping strategy in which the skipped stations change simultaneously according to the passenger travel demand obtained from large-scale urban metro smartcard data. By limiting the number of passengers arriving at the crowd gathering place, the crowd density during a crowding event can be well controlled. In the proposed model, we assume passengers can obtain the train stop-skipping information through personal digital devices or screens at stations and adjust their travel behaviours accordingly. The contributions of the present study are summarized as follows:

- (1) The proposed station stop-skipping strategy can guarantee the crowd density be well below the safety threshold at the crowd gathering place and protect the safety of crowds
- (2) A pedestrian network is generated, and metro stations are used as the connection points to connect the pedestrian network and the urban metro network to form a two-layer network model, which is used to simulate large-scale crowd gathering
- (3) The constraint of pedestrian density on the roads connecting to the crowding station and the constraint of crowd density at the crowd gathering place are simultaneously taken into account in the model.

The following sections are organized as follows. Section 2 presents the optimization model for generating the train stop-skipping schemes. Section 3 presents the genetic algorithm to solve the optimization model. In Section 4, the train stop-skipping schemes are solved and analyzed in the case study of Shenzhen metro. Section 5 draws a conclusion of the proposed model, the developed algorithm, and the findings in this study. Limitations of the research and future research directions are also discussed.

## 2. Model Formulation

In this paper, we propose a model for generating the stop-skipping schemes during large crowding events. The proposed model belongs to nonlinear integer programming models [32], which were often used in urban metro management [18, 19] and public transportation management [33, 34]. In Section 2.1, we make a general problem statement. In Section 2.2, the properties of train operation and the characteristics of passengers are analyzed, and the objective function and the constraints of the model are determined.

*2.1. Problem Statement.* In this study, the urban metro station that is closest to the place where a crowd gathering event takes place is referred as the crowding station, the urban metro stations that are within a particular walking distance to the crowd gathering place are referred as

alternative stations, and the remaining stations are referred as ordinary stations (see Figure 1).

During a large crowding event, a large number of passengers may use urban metro to arrive at the crowd gathering place. This may cause extremely high density of individuals at the crowd gathering place and the crowding metro station. In such a situation, the stop-skipping strategy can be used to control the number of passengers arriving at the crowding station by not stopping at the station. The generated stop-skipping scheme determines the value of the binary variable  $p(x, i)$  at each station  $i$ , which reflects the stop-skipping decision of train  $x$ . Passengers may have different choices according to their origin stations when a train skips their origin stations or the crowding station (see Figure 2). For passengers departing from the ordinary stations, they can (1) take the urban metro to an alternative station and then walk to the crowd gathering place or (2) wait for the next train which stops at the ordinary station and the crowding station. For passengers departing from alternative stations, they can (1) walk to the crowd gathering place or (2) wait for the next train which stops at the alternative station and the crowding station.

Two objectives are addressed in this study: (1) minimizing the total waiting time of passengers and (2) minimizing the total travel time of the passengers. The mathematical expressions of these two objectives as well as the relevant constraints are provided in Section 2.2.

*2.2. The Optimization Model.* We firstly introduce the assumptions used in the model.

*Assumption 1.* The distance between the crowd gathering place and the crowding metro station is ignored.

*Assumption 2.* Passengers whose departing stations are skipped will behave according to the distance from their departing stations to the crowding station. The passengers may choose to walk to the crowd gathering place if the distance is within 2250 meters [35]; otherwise, passengers will wait for later trains.

*Assumption 3.* Passengers can receive timely stop-skipping information.

*Assumption 4.* The number of passengers at the crowd gathering place and the number of passengers walking along the roads connecting to the crowding station are initially set to 0 [36].

*Assumption 5.* Anomalous large passenger flow states can be predicted with high accuracy. For example, the EAD-OF (elastic anomaly detection for out-flow) model can send an alarm 2 hours in advance [37].

*Assumption 6.* Passengers arrive at an urban metro station at a constant rate [38].

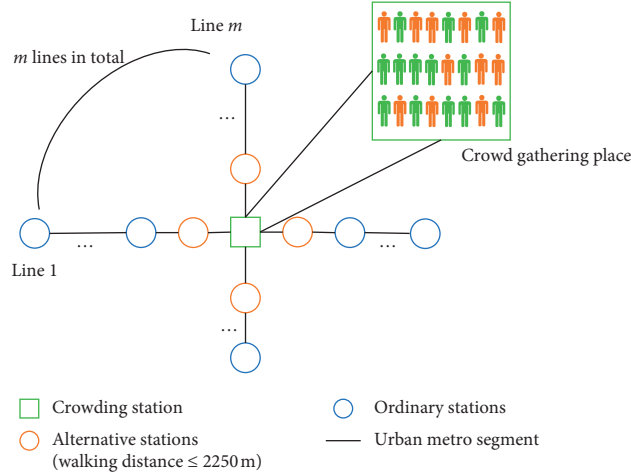


FIGURE 1: Illustration of urban metro lines and urban metro stations. The green square represents the crowding station, and the black lines represent the urban metro segments. The orange circles and the blue circles represent the alternative stations and the ordinary stations, respectively. The passengers colored in orange represent the passengers who walk to the crowd gathering place, whereas the passengers colored in green represent the passengers who take urban metro to the crowd gathering place.

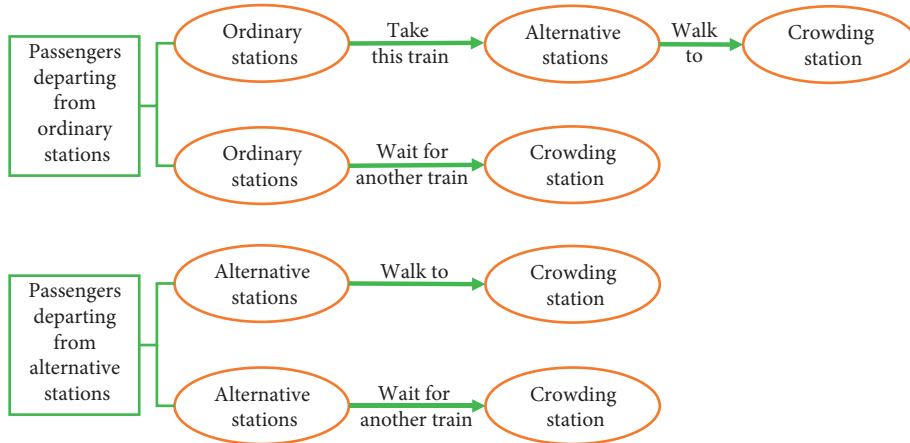


FIGURE 2: Different ways to arrive at the crowding station when train stop-skipping strategy is implemented.

*Assumption 7.* Train schedule will not change with the implementation of the stop-skipping strategy, which means the trains still stop at skipped stations, but passengers cannot get on or get off.

The parameter and variable notations are introduced in Table 1. In the following, we further explain some variables listed above.

The arrival time  $t_A(x, i)$  and the departure time  $t_D(x, i)$  of train  $x$  at station  $i$  of an urban metro line are calculated

using equations (1) and (2). The departure time of train  $x$ ,  $t_D(x, i)$ , equals to its arrival time  $t_A(x, i)$  plus the dwell time  $\tau$ , where  $N_x$  is the total number of trains running on the urban metro line and  $N_s$  is the total number of stations in the urban metro line. The arrival time of train  $x$ ,  $t_A(x, i)$ , equals to its departure time  $t_D(x, i - 1)$  at the station  $i - 1$  it just passes through plus the train travel time  $t_{\text{train}}(i - 1, i)$  between station  $i - 1$  and station  $i$ :

$$t_D(x, i) = t_A(x, i) + \tau, \quad x = 1, 2, \dots, N_x, i = 2, 3, \dots, N_s, \tag{1}$$

$$t_A(x, i) = t_D(x, i - 1) + t_{\text{train}}(i - 1, i), \quad x = 1, 2, \dots, N_x; i = 2, 3, \dots, N_s. \tag{2}$$

The number of passengers alighting train  $x$  at the crowding station  $C$  of an urban metro line,  $N_{\text{alight}}(x, C)$ , is calculated for the following two cases.

*Case 1.* If train  $x$  stops at the crowding station  $C$ , the number of passengers alighting train  $x$  at crowding station  $C$  equals to the total number of passengers that are skipped by

TABLE 1: Parameter and variable notations.

Parameters	Description
$i, j$	Index of urban metro stations
$x$	Index of trains
$l$	Index of urban metro lines
$t$	Index of time windows
$\Omega_a$	Set of alternative stations
$\Omega_o$	Set of ordinary stations
$C$	Crowding station
$N_{\text{train}}(x, i, j)$	The number of passengers boarding train $x$ at station $i$ and alighting the train at station $j$
$\lambda(i, j)$	The arriving rate of passengers at station $i$ , whose destination is station $j$
$t_{\text{train}}(i, j)$	Train travel time between station $i$ and station $j$
$t_{\text{walk}}(i, j)$	Walking time from station $i$ to station $j$
$\tau$	Dwell time of a train at a station
$h$	Headway between consecutive trains of a line
$S$	The area of the crowd gathering place
$S_{\text{road}}(i)$	Area of the roads connecting the alternative station $i$ and the crowding station $C$ , $i \in \Omega_a$
$\rho_{\text{max}}$	Safety threshold of crowd density and pedestrian density
Variables	Description
$p(x, i)$	The binary variable that determines whether train $x$ stops at station $i$ (i.e., if $p(x, i) = 1$ , train $x$ stops at station $i$ , otherwise if $p(x, i) = 0$ , train $x$ skips station $i$ )
$N_{\text{skipped}}(x, i, j)$	The number of passengers skipped by train $x$ but waiting for the next train that stops at both station $i$ and station $j$
$N_{\text{residual}}(x, i, j)$	The number of passengers skipped by previous trains who intend to board train $x$ at station $i$ and alight at station $j$
$N_{\text{board}}(x, i)$	The number of passengers boarding train $x$ at station $i$
$N_{\text{alight}}(x, i)$	The number of passengers alighting train $x$ at station $i$
$N_{\text{walk}}(x, i, j)$	The number of passengers skipped by train $x$ , who intend to walk from station $i$ to station $j$
$N_{\text{road}}(i, t)$	The number of passengers walking on the roads connecting alternative station $i$ and the crowding station $C$ during time window $t$ , $i \in \Omega_a$
$t_A(x, i)$	Arrival time of train $x$ at station $i$
$t_D(x, i)$	Departure time of train $x$ at station $i$
$N(t)$	Number of passengers at the crowd gathering place during time window $t$
$\Delta N(t)$	Increase of the number of passengers at the crowd gathering place during time window $t$
$\rho(t)$	Crowd density at the crowd gathering place during time window $t$
$\rho_{\text{pedestrian}}(i, t)$	Pedestrian density on the roads connecting alternative station $i$ and the crowding station $C$ during time window $t$ , $i \in \Omega_a$

previous trains at station  $i$   $N_{\text{residual}}(x, i, C)$  plus the passengers arriving on schedule  $N_{\text{train}}(x, i, C)$ :

$$N_{\text{alight}}(x, C) = \sum_{i=1}^{N_s} p(x, i) \cdot (N_{\text{train}}(x, i, C) + N_{\text{residual}}(x, i, C)), \quad x = 1, 2, \dots, N_x, \quad (3)$$

where  $N_{\text{residual}}(x, i, C)$  is the number of passengers who fail to board previous trains:

$$\% N_{\text{residual}}(x, i, C) = \sum_{x=lt}^{x-1} N_{\text{skipped}}(x, i, C), \quad x = 2, 3, \dots, N_x, \quad (4)$$

where  $lt$  is the index of the last train stopping at both station  $i$  and crowding station  $C$ .

Case 2. If train  $x$  skips the crowding station  $C$ , there is no passenger alighting train  $x$  at crowding station  $C$ :

$$N_{\text{alight}}(x, C) = 0, \quad x = 1, 2, \dots, N_x. \quad (5)$$

In the same way, the number of passengers departing from crowding station  $C$  by taking train  $x$ ,  $N_{\text{board}}(x, C)$ , can be calculated using

$$N_{\text{board}}(x, C) = \sum_{i=1}^{N_s} p(x, i) \cdot p(x, C) \cdot (N_{\text{train}}(x, C, i) + N_{\text{residual}}(x, C, i)), \quad x = 1, 2, \dots, N_x. \quad (6)$$

$\Gamma_A$  and  $\Gamma_D$ , respectively, denote the set of trains arriving at and departing from crowding station  $C$  during time window  $t$  and  $\Gamma_S$  denotes the set of trains skip crowding station  $C$  and passengers that are skipped by this set of trains can walk to crowding station  $C$  during time window  $t$ .

The increase of the number of passengers at the crowd gathering place equals to the number of passengers arriving at crowding station  $C$  by trains,  $\sum_{x \in \Gamma_A} N_{\text{alight}}(x, C)$ , plus the number of passengers arriving at crowding station  $C$  through walking,  $\sum_{i \in \Omega_a} \sum_{x \in \Gamma_S} N_{\text{walk}}(x, i, C)$ , and minus the number of passengers departing from the crowding station  $C$ ,  $\sum_{x \in \Gamma_B} N_{\text{board}}(x, C)$ :

$$\Delta N(t) = \sum_{x \in \Gamma_A} N_{\text{alight}}(x, C) + \sum_{i \in \Omega_a} \sum_{x \in \Gamma_S} N_{\text{walk}}(x, i, C) - \sum_{x \in \Gamma_B} N_{\text{board}}(x, C), \quad t = 1, 2, \dots, T, \quad (7)$$

$$N_{\text{walk}}(x, i, C) = (N_{\text{train}}(x, i, C) - N_{\text{skipped}}(x, i, C)) + \sum_{j \in \Omega_o} p(x, i) \cdot p(x, j) \cdot (N_{\text{train}}(x, j, C) - N_{\text{skipped}}(x, j, C)), \quad x \in \Gamma_S, i \in \Omega_a, \quad (9)$$

where  $N_{\text{skipped}}(x, i, C)$  is the number of passengers who are skipped by train  $x$  at an alternative station  $i$  and wait for later trains, and  $N_{\text{train}}(x, j, C) - N_{\text{skipped}}(x, j, C)$  is the number of passengers who are skipped by train  $x$  at an ordinary station  $j$  and do not wait for later trains.

The number of people at the crowd gathering place during time window  $t$  is calculated using the number of people in previous time window  $N(t-1)$  plus the change of number of people in the current time window  $\Delta N(t)$ :

$$N(t) = N(t-1) + \Delta N(t), \quad t = 2, 3, \dots, T. \quad (10)$$

The crowd density during time window  $t$  is calculated [36]:

$$\rho(t) = \frac{N(t)}{S}, \quad t = 1, 2, \dots, T, \quad (11)$$

where  $S$  is the area of the crowd gathering place.

In the same way, the pedestrian density of the road connecting alternative station  $i$  and the crowding station  $C$  can be calculated using

$$\rho_{\text{pedestrian}}(i, t) = \frac{N_{\text{road}}(i, t)}{S_{\text{road}}(i)}, \quad t = 1, 2, \dots, T; i \in \Omega_a. \quad (12)$$

where the  $T$  is the total number of time windows.

If train  $x$  stops at an alternative station  $i$  and the crowding station  $C$ , the number of passengers walk from alternative station  $i$  to crowding station  $C$ ,  $N_{\text{walk}}(x, i, C)$ , is zero:

$$N_{\text{walk}}(x, i, C) = 0, \quad x \in \Gamma_S, i \in \Omega_a, \quad (8)$$

where the  $\Omega_a$  represents the set of alternative stations.

If train  $x$  skips the alternative station  $i$  or the crowding station  $C$ , the number of passengers walk from alternative station  $i$  to crowding station  $C$  is

The area of the road connecting alternative station  $i$  and the crowding station  $C$ ,  $S_{\text{road}}(i)$ , is equal to the width of the road times the length of the road. The length of the road is measured using the online map software [39], and the width of the road is obtained according to road construction specifications [40].

The optimization model is formulated as follows.

Assuming passengers whose destination is station  $j$  arrive at station  $i$  at a constant rate of  $\lambda(i, j)$ , the average waiting time  $\bar{w}(x, i)$  of the passengers at station  $i$  waiting for train  $x$  is [38]

$$\bar{w}(x, i) = \frac{h}{2}, \quad (13)$$

where  $h$  is the headway of consecutive trains.

If train  $x$  skips station  $i$ , passengers have to wait for later trains. The train that stops at both station  $i$  and station  $j$  is denoted as train  $x+k$ . Passengers who board later trains have to wait for  $((2k+1)h)/2$  at station  $i$ .

The proposed model only considers the passengers who used the urban metro lines implementing stop-skipping strategy, and the total waiting time of passengers of an urban metro line is

$$Z_{\text{wait}} = \sum_{x=1}^{N_x} \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} \left( N_{\text{train}}(x, i, j) \cdot \frac{h}{2} + N_{\text{skipped}}(x, i, j) \cdot k(x, i, j) \cdot h \right) - \sum_{x=1}^{N_x} \sum_{i \in \Omega_a} (N_{\text{train}}(x, i, C) - N_{\text{skipped}}(x, i, C)) \cdot \frac{h}{2}, \quad (14)$$

where  $N_{\text{train}}(x, i, j) \cdot (h/2)$  is the total waiting time of passengers at station  $i$  without considering train skipping and  $N_{\text{skipped}}(x, i, j) \cdot k(x, i, j) \cdot h$  is the additional waiting time of passengers who intend to wait the following trains.  $\sum_{x=1}^{N_x} \sum_{i \in \Omega_a} (N_{\text{train}}(x, i, C) - N_{\text{skipped}}(x, i, C)) \cdot (h/2)$  is the total waiting time of passengers who directly walk from an alternative station to the crowding station.

$$Z_{\text{train}} = \sum_{x=1}^{N_x} \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} p(x, i) \cdot p(x, j) (N_{\text{train}}(x, i, j) + N_{\text{residual}}(x, i, j)) \cdot t_{\text{train}}(i, j). \quad (16)$$

Here,  $(N_{\text{train}}(x, i, j) + N_{\text{residual}}(x, i, j)) \cdot t_{\text{train}}(i, j)$  is the in-vehicle time (time spent in trains) of passengers who take train  $x$  from station  $i$  to station  $j$ .

The walking time of passengers is

$$Z_{\text{walk}} = \sum_{x=1}^{N_x} \sum_{i \in \Omega_a} N_{\text{walk}}(x, i, C) \cdot t_{\text{walk}}(i, C), \quad (17)$$

where  $N_{\text{walk}}(x, i, C)$  is the number of passengers walking from the alternative station  $i$  to crowding station  $C$  and  $t_{\text{walk}}(i, C)$  is the walking time from alternative station  $i$  to crowding station  $C$ .

Two objectives are addressed via the weighted sum method:

$$\min \sum_{l=1}^{N_l} Z = w_1 \sum_{l=1}^{N_l} Z_{\text{wait}} + w_2 \sum_{l=1}^{N_l} Z_{\text{travel}}, \quad (18)$$

where  $w_1$  and  $w_2$  are the common units of cost in dollars of the total waiting time  $Z_{\text{wait}}$  and the total travel time  $Z_{\text{travel}}$  of all passengers and  $N_l$  is the number of metro lines connecting the crowding station. The decision variables are  $p(x, i)$ ,  $x = 1, 2, \dots, N_x$  and  $i = 1, 2, \dots, N_s$ . If  $p(x, i) = 1$ , train  $x$  stops at station  $i$ , otherwise if  $p(x, i) = 0$ , train  $x$  skips station  $i$ .

The constraints of the objective function are as follows. Firstly, the crowd density should be lower than the safety threshold:

$$\rho(t) \leq \rho_{\text{max}}, \quad t = 1, 2, \dots, T, \quad (19)$$

where  $\rho_{\text{max}}$  is the safety threshold of crowd density. Secondly, the pedestrian density should be lower than the safety threshold of pedestrian density:

$$\rho_{\text{pedestrian}}(i, t) \leq \rho_{\text{max}}, \quad t = 1, 2, \dots, T. \quad (20)$$

Thirdly, the first station and the last station are not allowed to be skipped, which generates

$$p(x, 1) = p(x, N_s) = 1, \quad x = 1, 2, \dots, N_x. \quad (21)$$

Fourth, the initial number of passengers at the crowd gathering place is set to 0:

$$N(0) = 0. \quad (22)$$

The travel time of all passengers can be formulated as

$$Z_{\text{travel}} = Z_{\text{train}} + Z_{\text{walk}}, \quad (15)$$

where the time that passengers spend in the trains is

Fifth, the initial number of passengers on the roads connecting alternative station  $i$  and the crowding station  $C$  is set to 0:

$$N_{\text{road}}(i, 0) = 0, \quad i \in \Omega_a. \quad (23)$$

### 3. Problem Solving

The proposed optimization model is a nonlinear integer programming model with a nonconvex objective. The problem is a NP-hard problem and cannot be solved using a deterministic algorithm. We use the Genetic Algorithm incorporating Monte Carlo simulation [41] to solve this problem.

#### 3.1. Monte Carlo Simulation

Step 0 (initialization): set the iteration of simulations to  $m = 1$ , and let  $\bar{Z}^m$  denote the estimated value of the objective function  $Z$ .

Step 1 (sampling): the walking time from an alternative station to the crowding station is a random variable with a predetermined mean and variance. Sample the walking time for each alternative station using the distribution of walking time.

Step 2 (parameter calculation): determine the number of passengers walking to the crowding station in equation (9) according to the sampled walking time and update the value of walking time in equation (17).

Step 3 (objective value): calculate the value of the two objectives using equations (14)–(18) and calculate the objective function value  $\hat{Z}^m$ .

Step 4 (stop test): if  $m > m_{\text{max}}$  (the predetermined sample size), stop the algorithm and output the estimated objective function value  $\bar{Z} = \bar{Z}^m$ .

Step 5 (update): calculate the value of  $\bar{Z}^{m+1}$  using

$$\bar{Z}^{m+1} = \bar{Z}^m + \frac{1}{m} (\hat{Z}^m - \bar{Z}^m). \quad (24)$$

3.2. Genetic Algorithm Incorporating Monte Carlo Simulation. Based on the Monte Carlo simulation above, the performance of each stop-skipping scheme can be evaluated.

Step 0 (initial population): set the population size to  $n$ . Use pseudorandom numbers to generate the chromosomes. Set the number of generations  $k = 1$ , and set the value of crossover rate  $p$  and mutation rate  $x$ .

Step 1 (remove unreasonable solutions): each chromosome indicates one train stop-skipping scheme. According to the constraints of the optimization model, unreasonable solutions in the offspring are removed.

Step 2 (calculate the value of the fitness function): estimate the objective function value for each generated chromosome using equation (24) and find out the optimal solution of this generation.

Step 3 (reproduce): calculate the total fitness of this generation, the proportion of fitness of each solution, and use roulette method to select the offspring.

Step 4 (crossover): based on the existing chromosomes, set a uniformly distributed random number  $\gamma_k$  between  $[0, 1]$  for each chromosome. If  $\gamma_k < p$ , exchange the genes with the latter chromosomes.

Step 5 (mutation): set a uniformly distributed random number  $\bar{\gamma}_k$  between  $[0, 1]$  for each gene in all the existing chromosomes. If  $\bar{\gamma}_k < x$ , change the value of this gene (from 0 to 1, or from 1 to 0) and record a new chromosome.

Step 6 (stop test): if  $k > k_{\max}$  (the predetermined generation size), stop the algorithm and output the minimal  $Z$  of the survivors and record the corresponding chromosome; otherwise, set  $k = k + 1$  and go to Step 1.

## 4. Numerical Experiments

**4.1. Data.** An actual crowd gathering event is used as a case study to test the proposed optimization model for generating the train stop-skipping scheme. The crowd gathering event occurred at the plaza of Window of the World of Shenzhen on October 31, 2014. The plaza is located at the vicinity of the Window of the World station of Shenzhen metro. In this study, the geographic information data and the smartcard data of Shenzhen metro are used.

The geographic information systems (GIS) data of Shenzhen metro and the smartcard data were provided by Shenzhen Transportation Authority. The smartcard data of Shenzhen metro were collected during October 2014. During the data collection period, there are 5 lines and 118 stations in Shenzhen metro. The time, the card ID, and station ID are recorded each time a passenger enters or exits a station, which generates a total of 46 million passenger trip records during the data observation period.

We split one day into 36 thirty-minute time windows from 6:00 a.m. to midnight according to the service period of Shenzhen metro. We calculate the number of passengers entering a station  $s$  during each time window  $t$ , in-passenger-flow  $N_{\text{in}}(s, t)$ , and the number of passengers exiting a station  $s$  during each time window  $t$ , out-passenger-flow  $N_{\text{out}}(s, t)$ . Next, we estimate the number of passengers gathered at the

plaza of the Window of the World following the method proposed in [36]

$$N(s, t) = \sum_{t'=6 \text{ am}}^t \Delta N(s, t'), \quad (25)$$

where  $N(s, t)$  is the estimated urban metro passengers at the plaza and  $\Delta N(s, t)$  is the change in the number of passengers during time window  $t$ :

$$\Delta N(s, t) = N_{\text{out}}(s, t) - N_{\text{in}}(s, t). \quad (26)$$

The estimated number of passengers gathered at the plaza of the Window of the World is shown in Figure 3(b). The red line and the gray lines represent the numbers of passengers gathered at the Window of the World on October 31 and on other working days in October, respectively. And, the black line represents the average number of passengers gathered at the Window of the World on October working days. As shown in Figure 3(b), the number of passengers gathered during the crowding event is much higher than on other days.

In this study, we focus on the two lines intersecting at the "Window of World" station (see Figure 4). There are 57 stations in the two urban metro lines. According to the definition in the Model Formulation Section, there is one crowding station (Window of the World), where the crowd gathering event occurred, six alternative stations that are within the walking range to the crowding station, and 50 ordinary stations.

The two-layer network, which includes the urban metro network and the pedestrian network, is shown in Figure 5. The two layers of networks are connected through urban metro stations. The bottom diagram of Figure 5 shows the urban metro network of Line 1 and Line 2 of Shenzhen metro, and the top diagram shows the pedestrian network between six alternative stations and the crowding station.

The length of a time window  $t$  is 30 minutes. The area of the crowd gathering place in equation (11) is  $S = 17477 \text{ m}^2$  [36]. The maximum safe crowd density in equation (19) and the maximum safe pedestrian density in equation (20) are both set to  $1.08 \text{ persons/m}^2$  [42], and the dwell time  $\tau$  in equation (1) is set to 30 s [43]. According to the train operation schedule, the headway  $h$  in equation (13) is set to 5 minutes. According to the model data setting in [18], the two weights in equation (18) are set to  $w_1 = 20/h$  and  $w_2 = 10/h$ . According to the model data setting in [33], the values of the crossover rate and mutation rate are set to  $p = 0.25$  and  $x = 0.01$ , respectively.

**4.2. Results.** We calculated the minimum and average fitness values among all the 100 chromosomes in one generation. The minimum and the average fitness values for all the 60 generations are shown in Figure 6. In Figure 6, the minimum fitness value of each iteration is relatively stable after 40 generations, which means that the genetic algorithm incorporating Monte Carlo simulation can obtain a solution close to the optimal solution.



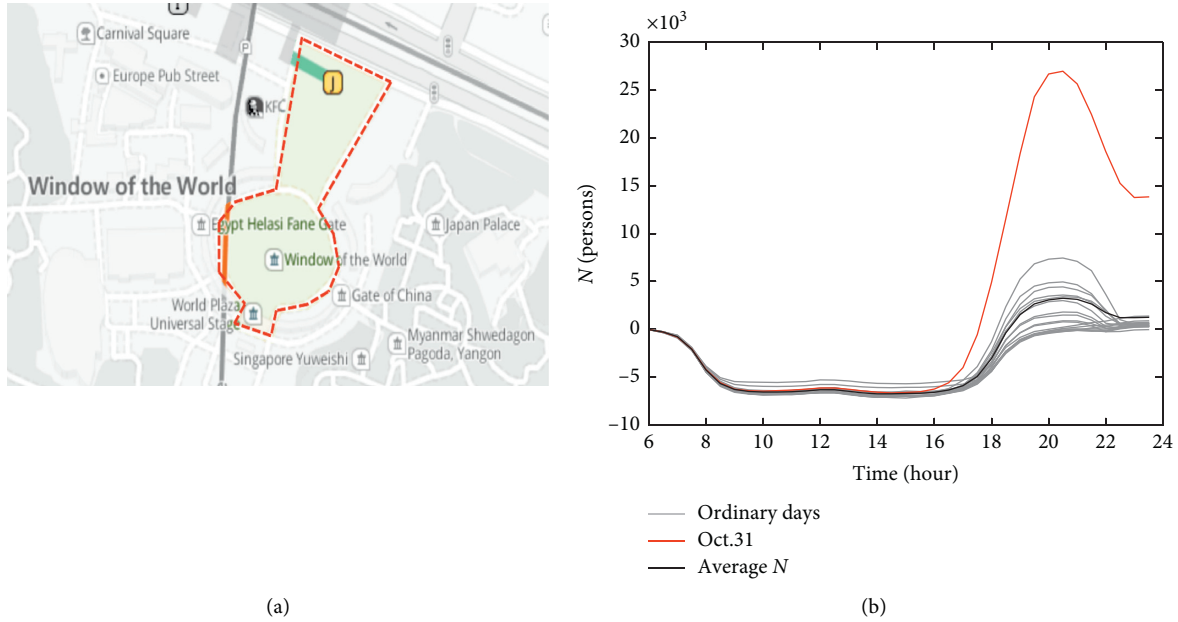


FIGURE 3: (a) The crowd gathering area of the Halloween event. (b) Estimated number of passengers gathered at the plaza of the Window of the World.



FIGURE 4: Line 1 and Line 2 of Shenzhen metro. The black circle represents the “Window of World” station, and the blue circles and the green circles represent the alternative stations and ordinary stations, respectively.

In full-stop strategy, trains stop at every station. Compared with the full-stop strategy, the implementation of the stop-skipping strategy increases passenger waiting cost by 346.6% and passenger travel cost by 0.24% (see Table 2). After the implementation of the stop-skipping strategy, the total cost of a passenger increases by 2.89 dollars. Although stop-skipping management increases both travel cost and

waiting cost of passengers, it effectively controls the crowd density (see Figure 7). The stop-skipping strategy can effectively maintain the crowd density under the safety threshold. Without urban metro management, crowd density can reach as high as 1.55persons/m<sup>2</sup>. However, when implementing the stop-skipping schemes, the maximum crowd density is 1.07persons/m<sup>2</sup>.

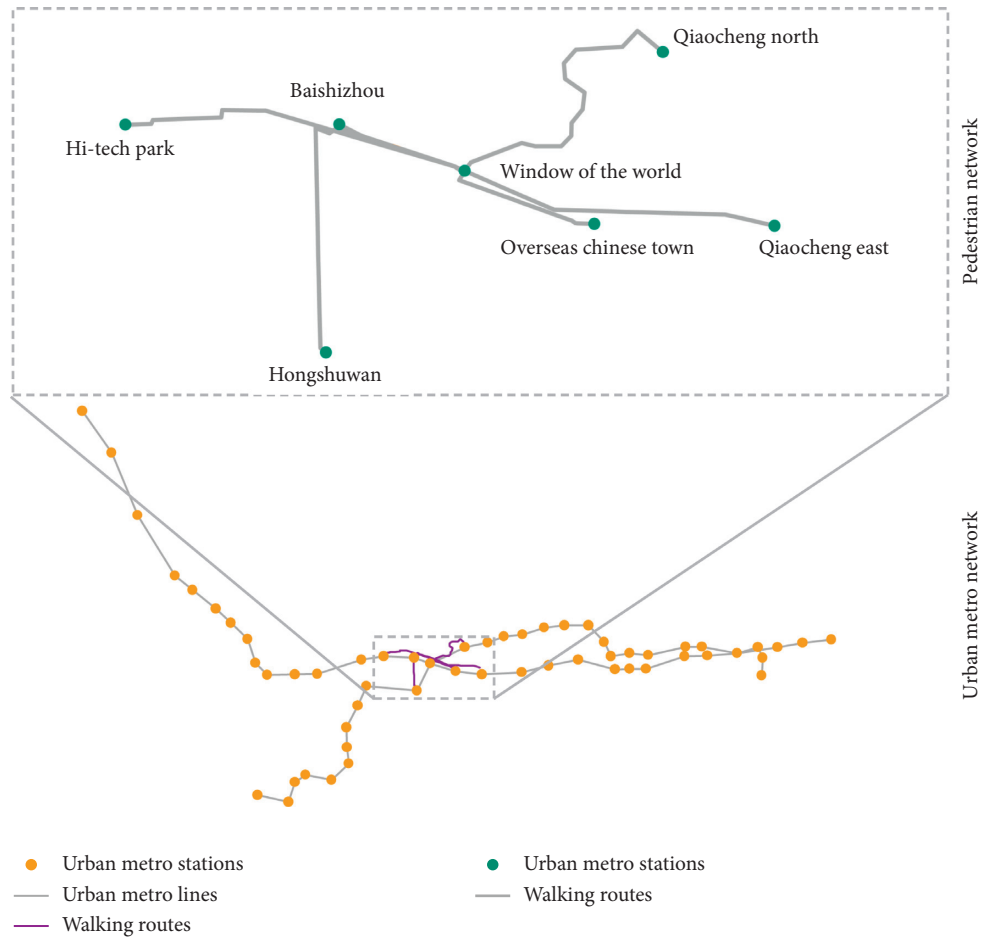


FIGURE 5: The two-layer network. The bottom diagram shows the urban metro network, and the top diagram shows the pedestrian network between six alternative stations and the crowding station.

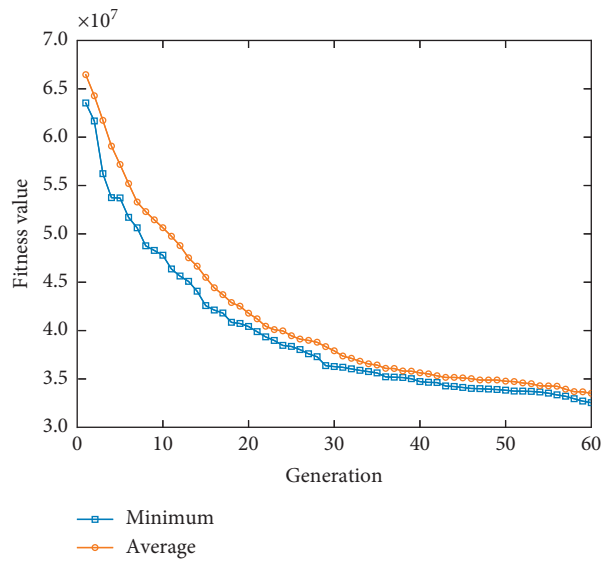


FIGURE 6: Convergence trend of the genetic algorithm.

TABLE 2: Objective function values of full-stop strategy and stop-skipping strategy (dollars).

Function	Z	$w_1 Z_{\text{wait}}$	$w_2 Z_{\text{travel}}$	Total cost per passenger
Full-stop strategy	$5.14 \times 10^6$	$1.03 \times 10^6$	$4.10 \times 10^6$	4.14
Stop-skipping strategy	$8.72 \times 10^6$	$4.60 \times 10^6$	$4.11 \times 10^6$	7.03

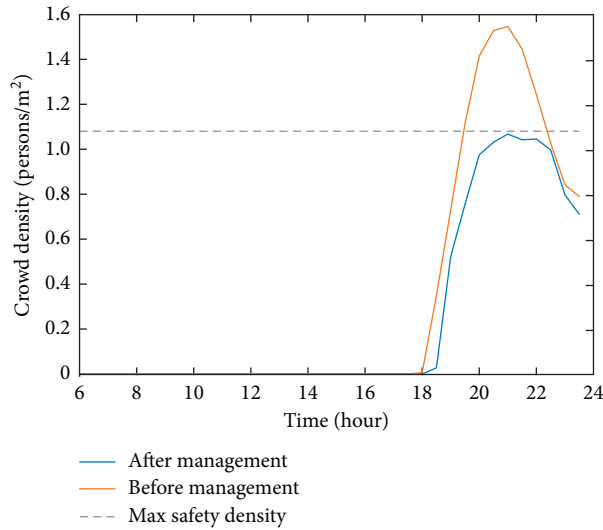


FIGURE 7: Crowd density is well controlled when implementing the stop-skipping management. The gray dotted line represents the maximum safe crowd density.

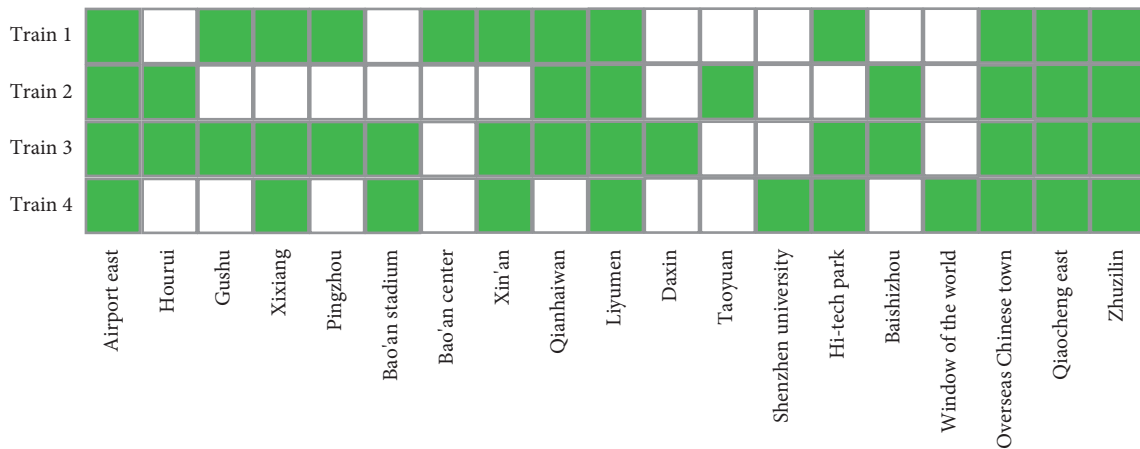


FIGURE 8: The solved train stop-skipping schemes for some urban metro stations on Shenzhen metro Line 1.

During the period from 20:10 p.m. to 20:30 p.m., the solved train stop-skipping schemes for Shenzhen metro Line 1 are shown in Figure 8. In Figure 8, a green grid represents the train stops at the station, while a white grid represents the train skips the station. As shown in Figure 8, train 3 skipped Bao'an Center station, Taoyuan station, Shenzhen University station, and Window of the World station.

According to the obtained train stop-skipping scheme, the average waiting time of passengers at each station of Line 1 and Line 2 is calculated. As shown in Figure 9, the average waiting time of passengers at 52 stations (57

stations in total) does not exceed 15 minutes, which means that these stations are seldomly kept being skipped for more than three times.

Some passengers walk to the plaza of Window of the World when implementing the stop-skipping strategy. Figure 10 shows the proportion of the number of pedestrians departing from each alternative station to the total number of pedestrians at different stages of the crowding event. In the early stage of the Halloween event, passengers mainly depart from the Baishizhou station and Overseas Chinese Town station and walk to the Window of the World. In the mid-late

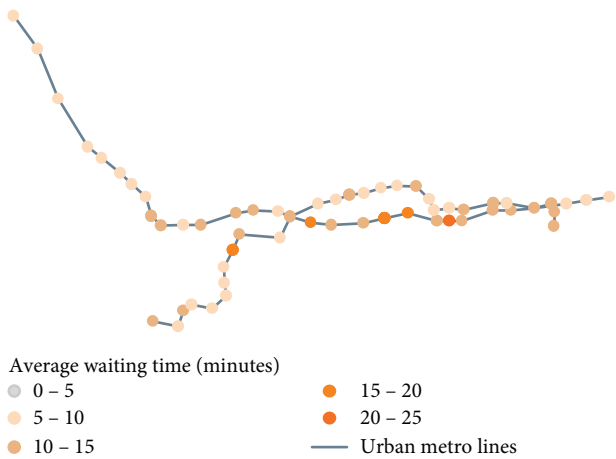


FIGURE 9: The average waiting time of passengers at each station of Shenzhen metro Line 1 and Line 2.

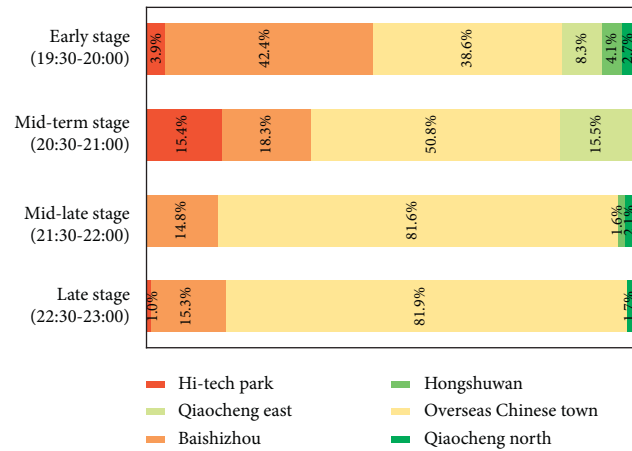
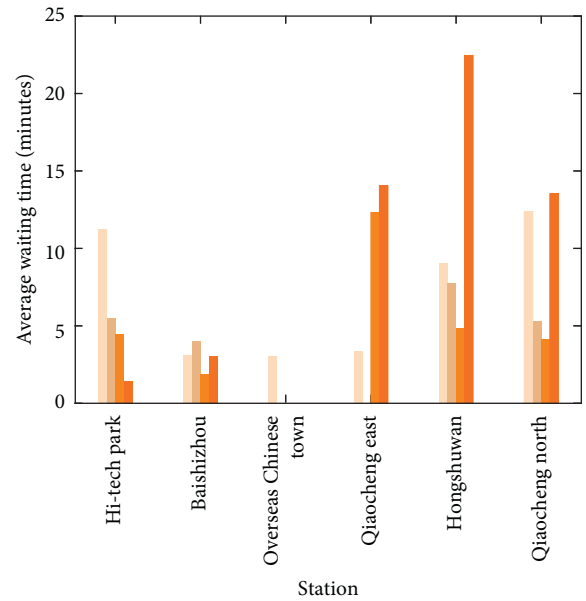


FIGURE 10: The proportion of the number of pedestrians departing from an alternative station to the total number of pedestrians at different stages of the crowding event.

and late stages, more than 80% of pedestrians walk from the Overseas Chinese Town station to the Window of the World. This may be because the walking distance from Overseas Chinese Town station to Window of the World is relatively short, and the passenger flow from the Overseas Chinese Town station to the Window of the World is relatively large.

The average waiting time of passengers at six alternative stations and the proportion of pedestrian passengers to the total passengers at each alternative station are shown in Figure 11. In Figure 11(b), passengers at Overseas Chinese Town station all choose to walk directly to Window of the World in the mid-term, mid-late, and late stages of Halloween event instead of waiting for later trains, so the waiting time of passengers during these three stages in Figure 11(a) is 0. And, as shown in Figure 11(b), no passenger at Qiaocheng North station chooses to walk, probably, because the walking distance from Qiaocheng North station to Window of the World is relatively long.

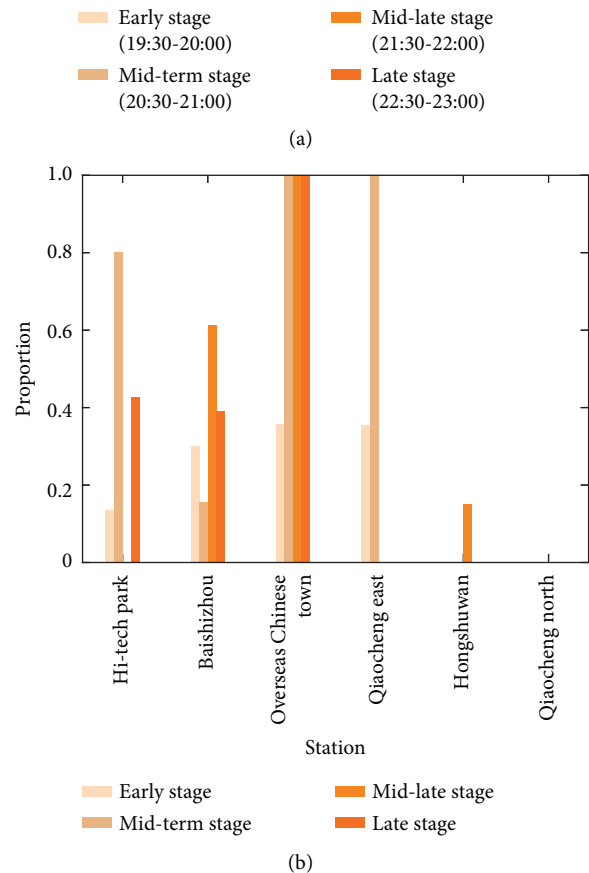


FIGURE 11: (a) The average waiting time of passengers at six alternative stations. (b) The proportion of pedestrian passengers to the total passengers at each alternative station.

The estimated maximum number of pedestrians is 1544, appearing on the road connecting Overseas Chinese Town station and the Window of the World station at 18:30 p.m. (see Figure 12). Each road connecting an alternative station and the crowding station is divided into 10 road segments

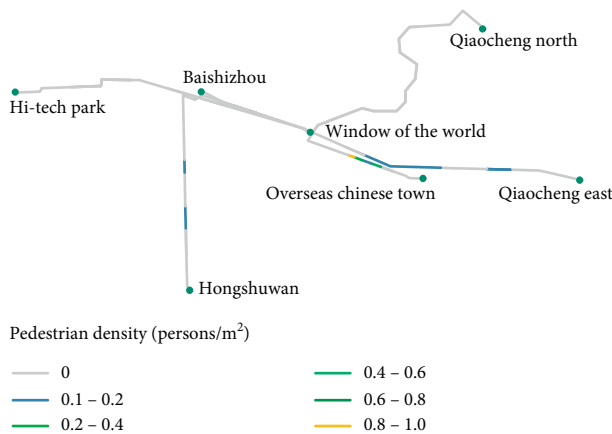


FIGURE 12: The pedestrian density on the roads at 18:30 p.m.

with the same length. In Figure 12, the pedestrian density of most road segments is between 0.1persons/m<sup>2</sup> and 0.2persons/m<sup>2</sup>, and only for some road segments connecting Overseas Chinese Town station and the Window of the World station, the pedestrian densities exceed 0.2persons/m<sup>2</sup>. Besides, the maximum pedestrian density of the road segments is 0.97persons/m<sup>2</sup>, which is well below the safety threshold (1.08persons/m<sup>2</sup>).

## 5. Conclusions

In summary, this paper studies the stop-skipping problem in urban metros. The stop-skipping problem is formulated as an optimization model to minimize a weighted sum of two objectives: the passengers' total waiting time cost at stations and the passengers' total travel time cost. To control the crowd density at the crowd gathering place, we take the crowd density limit as one constraint of the optimization model. In addition, the pedestrian density constraint is also considered in the optimization model. A two-layer network, which includes the pedestrian network and the urban metro network, is used to better simulate the crowd gathering process. Since the stop-skipping problem is a MINLP problem, a genetic algorithm is employed to solve the problem. Results show that the stop-skipping strategy can effectively limit the crowd density below the safety threshold, indicating that the stop-skipping strategy can be potentially used during large crowding events.

Given that urban metro transports the majority of crowds to the crowd gathering place, only urban metro passengers are considered in our model. And, only one strategy, the train stop-skipping strategy, is investigated. In the future work, some extensions might be considered. For example, taxi and bus passengers can be considered in future study, and taxi GPS records and bus smartcard data can be incorporated in the optimization model. In addition, the optimum results could be improved by combining a variety of crowd management strategies. For instance, the crowd density at the crowd gathering place may be further reduced by simultaneously limiting the number of passengers entering urban metro stations and implementing the train stop-skipping strategy.

## Data Availability

The metro smart card data used to support the findings of this study have not been made available because of the confidentiality agreement.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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