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Research Article

Optimal Tradable Credit Scheme Design with Recommended Credit Price

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As an interesting research topic in transportation field, tradable credit scheme (TCS) has been extensively explored in the latest decade. Existing studies implicitly assumed that travelers are clear about the equilibrium credit price and make their trips accordingly. However, this may not be the case in reality, since the credit price is endogenously determined by the credit-trading behavior, especially in the early stages after the implementation of a TCS. Considering travelers' uncertainty on the equilibrium credit price, this paper aims to investigate the impacts of perception error on credit price and how to accommodate such errors by an appropriate scheme design. Transferring the perception error on credit price to a given and fixed value released by central authority, we first investigate the impacts of recommended credit price under a given TCS. The numerical results imply that it is necessary to simultaneously consider the choice of recommended credit price and charging scheme in TCS design. Regarding this, we combine the goals of social welfare and public acceptance of the scheme and propose a bilevel biobjective programming (BLBOP) model, by which the net economic benefit is maximized while the gap between the recommended and realized credit prices is minimized. Through two numerical examples, it is found that the rise in perception variance could intensify the contradiction effect between the two objectives. Additionally, a nonnegligible price gap must be allowed to occur to maintain the effectiveness of a TCS.

1. Introduction

Congestion, mainly attributed to the imbalance between supply of the transportation system and traffic demand, is becoming an increasingly disturbing problem worldwide. It is generally recognized that road congestion threatens urban prosperity due to its negative effect on the economy. In 2013, the US Department of Transportation stated that "Congestion in 498 metropolitan areas caused urban Americans to travel 5.5 billion hours more and to purchase an extra 2.9 billion gallons of fuel for a congestion cost of $121 billion"[1]. Therefore, how to relieve road congestion has become a highly attractive issue for policy-makers.

Among the variety of congestion-relief methods, a road pricing scheme has been studied for several decades and practiced around the world in various forms thanks to the marginal theory introduced by Pigou [2]. In Pigou's theory, the optimal toll charge is defined as the difference between the marginal social cost and the marginal private cost. However, despite its appeal, the Pigouvian toll appears politically and socially infeasible in terms of the equity problem. Specifically, a pricing scheme would only benefit those who value congestion relief more than the paid toll, and it could make a considerable portion of the travelers worse off if no compensation is provided [3]. Furthermore, such a scheme benefits the rich more than the poor, leading to political resistance of its implementation [4–6].

As an alternative to the pricing scheme, the cap-and-trade scheme, which has been successfully implemented to ensure internalization of environmental externalities [7–10], has been advocated by transportation scholars in the latest decade [11–13]. As such a scheme typically involves issuing mobility credits to travelers and allowing them to trade them in a market, it is also known as tradable credit scheme (TCS) [11].

In a TCS, the central authority initially issues credits to all eligible travelers, and the latter needs to consume them when using the road section with credit charge. Such a credit scheme has three major advantages over the monetary toll charge. First, it provides a framework for solving the congestion problem without increasing travelers' travel costs. In other words, it does not generate revenue from the travelers and could confront with less public controversy than congestion pricing. Second, travelers are motivated to limit their car use under such a scheme, because in this way they can sell their unused credits to gain benefit. Third, the burden of gathering information falls on the travelers since the prices of the tradable credits are endogenously determined by the market; hence, the cost of the political implementation can be greatly reduced.

Yang and Wang [11] first developed a mathematical model of TCS in a general network equilibrium context. Following this, a significant tranche of research has been conducted to seek the applicability of TCS in traffic management. The extensions include user heterogeneity [14–18], day-to-day dynamic [19–21], network design [22, 23], bottleneck management [12, 24–30], multimodal network [31–33], environmental issue [34–37], equity issue [38, 39], public-private partnership [40, 41], and autonomous vehicle management [42, 43]. This paper aims to investigate one aspect that has hitherto received little attention, namely, the optimal credit scheme design in the context of probit-based stochastic user equilibrium (SUE).

Following but different from the classical SUE principle in which travelers have a perception error on the travel time, we assume that the perception error lies in the equilibrium credit price recommended by the central authority. Applying a gradient projection method with a two-stage Monte Carlo simulation procedure embedded, we solve a linearly constrained minimization model in the context of probit-based SUE. Then, a bilevel biobjective programming model is proposed to design an optimal credit charging scheme with appropriate recommended credit price.

The main contributions of this paper are threefold. First, to the best of the authors' knowledge, it is the first time that application of TCS is investigated in the framework of probit-based SUE. We also combine the gradient projection and the two-stage Monte Carlo simulation methods proposed by Meng et al. [44] to solve the equilibrium problem. Second, we relax the implicit assumption in existing studies that travelers are clear about the equilibrium credit price. Instead, a perception error on the credit price is assumed to exist among travelers, which enhances realism in characterizing travel behaviors under TCS. Third, we propose a bilevel biobjective programming model for optimal design of TCS, in which minimizing the gap between the recommended price and the realized one is incorporated to enhancing the public acceptance of the proposed scheme. We believe that this work can provide useful managerial insights to the application of TCS.

The organization of this paper is listed as follows. In Section 2, we present the motivations and clarify the objective of this study. In Section 3, we list the assumptions adopted in this paper and propose a linearly constrained minimization model, based on which we demonstrate the impacts of recommended credit price under a given TCS through a small network. In Section 4, we propose a bilevel biobjective model for optimal TCS design and discuss about its solution procedure. In Section 5, two numerical examples are presented to investigate the features of the optimal TCS designed by the proposed model. In Section 6, two implementation issues are discussed to enhance the practicability of the proposed scheme. Finally, major conclusions and recommendations for future research are presented in Section 7.

2. Problem Statement

As stated in Zhu et al. [18], the essential difference between a congestion pricing scheme and a tradable credit scheme lies in the unit credit price in the latter, which is endogenously determined through market trading rather than a fixed value prescribed by the central authority. In other words, the equilibrium credit price is highly dependent on the network flow pattern that reveals the buyers and sellers in the credit-trading market. And intuitively, the credit price perceived by travelers in turn affects the route choice and thus the equilibrium flow pattern. Consequently, a user equilibrium (UE) as well as a market equilibrium (ME) can be achieved and the resultant flow pattern and credit price are obtained.

In the literature, existing studies implicitly assumed that travelers are clear about the equilibrium credit price and make their trips accordingly. This can surely direct the network flow towards the UE pattern solved by the mathematical programming model proposed by Yang and Wang [11]. However, unlike the congestion pricing scheme where the additional travel cost is clear and definite, we cannot expect that every traveler knows the operating mechanism of the market as well as the endogenously determined credit price in advance, especially when the scheme is initially implemented. Instead, what they know exactly before making the trips is the credit charge on each link, but they may have different attitudes toward the equilibrium credit price. For example, travelers who enjoy higher income levels or those who are satisfied with the mobility service provided by the authority may tolerate a higher credit price. In contrast, those who argue against the credit scheme tend to expect a lower price. Therefore, a more reasonable assumption is that perception error on the credit price exists. Along this line, this paper focuses on the initial stages after the implementation of a TCS and investigates how to accommodate such errors by an appropriately designed TCS.

Under the assumption that travelers have perception error on the credit price, however, the perception error itself will affect the equilibrium credit price by changing the network flow pattern, and such an intractable interaction between the perceived price and the realized one will significantly reduce the robustness of the scheme as well as the predictability of the flow distribution.

To overcome this difficulty, we can try to transfer such a perception error to a given and fixed value to maintain a controllable system. For central authorities, it is practically feasible to release a recommended credit price that travelers can refer to when making decisions. In this way, the perception error on realized credit price can be transferred to the recommended one. Since the recommended price is fixed and given by the central authority, it can make the network flow more controllable. Besides, the recommended price combined with the credit charging scheme can be a "guidance" for travelers, i.e., to coordinate their travel behaviors consistent with the control target.

With this in mind, it is then natural to wonder how to design a TCS with recommended credit price to maximize the network-level performance. Clearly, one of the most fundamental procedures for finding such a scheme is to capture user's stochasticity in making route choice. In the literature, it was frequently done by adding a random error term to the travel cost and presuming a specific distribution form for it. Typically, there are two kinds of problems, logit-based or probit-based SUE problem, assuming that the perception error follows multivariate Gumbel or normal distribution. Since the former possesses an elegant explicit expression [45], it has been extensively examined in the literature, for either model formulations or solution algorithms. However, despite its closed form, it is widely known that the logit-based approach suffers from overlapping issues and cannot accommodate the perception variance caused by independence of irrelevant alternatives (IIA), see Chapter 11.1 in [46]. In this regard, probit-based SUE problem is a better representative of SUE principle since it can avoid the above problems. Therefore, we adopt the probit-based SUE principle to characterize travelers' behavior in route choice in this paper.

In the next section, we shall formulate the equilibrium problem as an equivalent optimization program and present a solution algorithm for it.

3. Probit-Based Stochastic User Equilibrium under a Tradable Credit Scheme

3.1. Notations and Assumptions. Consider a general network $G = (N, A)$, with a set N of nodes, and a set A of directed links. Let W be the set of O-D pairs and R^w be the set of all paths connecting O-D pair w. Let q_w denote the given and fixed travel demand within O-D pair w and $f_{r,w}$ the flow on path r connecting O-D pair w. The relationship between the aggregate flow v_a and path flow can be expressed as

$$v_a = \sum_{w\in W}\sum_{r\in R^w} f_{r,w}\delta^w_{a,r}, \quad a \in A, \tag{1}$$

where the link-path incidence $\delta^w_{a,r} = 1$ if link a is on path r and 0 otherwise. And the path flow satisfies

$$\sum_{r\in R^w} f_{r,w} = q_w, \quad w \in W. \tag{2}$$

In this paper, the travel demand is assumed to be elastic. Let $\overline{q}_w$ denote the maximum travel demand over O-D pair $w \in W$ and $D_w(\bullet)$ be the nonnegative, nonincreasing, and continuously differentiable demand function. Then, the feasible set of flow patterns $(\mathbf{f}, \mathbf{v}, \mathbf{q})$, where $\mathbf{f} = \{f_{r,w}, r \in R^w, w \in W\}$, $\mathbf{v} = \{v_a,\ a \in A\}$ and $\mathbf{q} = \{q_w, w \in W\}$ can be defined by

$$\Omega = \left\{(\mathbf{f}, \mathbf{v}, \mathbf{q}) \mid \sum_{r\in R^w} f_{r,w} = q_w, v_a = \sum_{w\in W}\sum_{r\in R^w} f_{r,w}\delta^w_{a,r}, q_w \le \overline{q}_w, f_{r,w} \ge 0, \quad \forall a \in A, r \in R^w, w \in W\right\}. \tag{3}$$

Moreover, suppose that the link travel time function is separable and monotonically increasing with respect to the link flow, denoted by $t_a(v_a)$.

As presented in Yang and Wang [11], under a TCS, each traveler gets k credits from the central authority initially. Denote K as the total amount of distributed credits, which satisfies $K = \sum_{w\in W} k \cdot \overline{q}_w$. The link-specific credit charging scheme is denoted by $\kappa = \{\kappa_a,\ a \in A\}$, where κ_a is the credit charge for any traveler who uses link a. For the sake of presentation, we use (K, κ) to characterize a credit charging scheme κ under a total number of credits K issued in the market. Since not all TCSs can ensure the existence of feasible network flow patterns $(\mathbf{f}, \mathbf{v}, \mathbf{q})$, the corresponding feasible TCS set Ψ is defined as

$$\Psi = \left\{(K, \boldsymbol{\kappa}) \mid \exists (\mathbf{f}, \mathbf{v}, \mathbf{q}) \in \Omega, \sum_{a\in A} \kappa_a v_a \le K\right\}, \tag{4}$$

where Ω is defined by equation (3).

As stated in Yang and Wang [11], the credit market equilibrium (ME) conditions are given by

$$\sum_{a\in A} \kappa_a v_a = K, \quad \text{if } p > 0, \tag{5}$$

$$\sum_{a\in A} \kappa_a v_a \le K, \quad \text{if } p = 0, \tag{6}$$

where p denotes the credit price in time unit at ME. It implies that the credit price will be zero or equivalently, and the credit scheme will be nullified if there are remaining credits in the system.

Under a given TCS, the generalized link travel cost c_a on the link $a \in A$ is the sum of link travel time and credit cost measured in equivalent time unit. Equivalently,

$$c_a = t_a + p\kappa_a, \quad a \in A. \tag{7}$$

In this paper, we assume that the traveler's behavior in route choice follows the probit-based SUE principle. That is, each traveler has a perception error on the generalized travel cost and the perceived link travel cost C_a is equal to the actual generalized link travel cost c_a plus a normally distributed random variable ξ_a, i.e.,

$$C_a = c_a + \xi_a, \quad a \in A. \tag{8}$$

Then, the perceived path travel cost $C_{r,w}$ can be expressed by

$$C_{r,w} = c_{r,w} + \xi_{r,w} = \sum_{a \in A} c_a \delta^w_{a,r} + \xi_{r,w}, \quad r \in R^w, w \in W, \tag{9}$$

where $\xi_{r,w}$ is also a normal distributed random error term.

3.2. Perception Error on the Recommended Credit Price. In the published works related to probit-based SUE modeling, it is widely adopted that travelers have a perception error on the travel time. Specifically, it is usually assumed that the error term ξ_a regarding trip cost on link $a \in A$ has a zero mean and a variance linearly proportional to the free-flow travel time on that link, i.e., $\xi_a \sim N(0, \beta t^0_a)$, where β is the variance parameter and t^0_a is the free-flow travel time on link a. However, this may not be the case nowadays. On one hand, technological developments of prediction on network traffic flow has made the estimation on travel time in navigation services increasingly accurate and convincing in recent years. On the other hand, the advent and popular use of smartphones renders increasing drivers available to navigation services and naturally, more and more of them resort to navigation applications such as Google Map when making their route choices. As a result, travelers' perception error on travel time is in fact reduced significantly compared with a decade ago. By comparison, the perception error on credit price due to the uncertainty about the market operating mechanism under a TCS deserves more attention.

As mentioned in Section 1, to give a reference price of the credit and coordinate their travel behaviors accordance with the control target, a recommended credit price $\widetilde{p}$ released by the central authority is considered in this paper. Since $\widetilde{p}$ is merely a recommended price rather than a realized one, travelers may have different attitudes toward the relationship between $\widetilde{p}$ and the equilibrium credit price p^*, i.e., a perception error on the recommended credit price exists. Furthermore, we assume that the perception error on credit price is much more dominant than that on travel time so that the latter is negligible throughout the analysis.

Based on above, we have $\xi_a = \kappa_a \xi_p$ in this paper and then rewrite equation (8) as

$$C_a = c_a + \xi_p \kappa_a = t_a + p^* \kappa_a + \xi_p \kappa_a, \quad a \in A, \tag{10}$$

where the perception error ξ_p in terms of credit price is defined as a normally distributed random variable with a zero mean and a variance linearly proportional to $\widetilde{p}$, i.e., $\xi_p \sim N(\widetilde{p} - p^*, \beta\widetilde{p})$. Then, we naturally have $\xi_a \sim N((\widetilde{p} - p^*)\kappa_a, \beta\widetilde{p}\kappa_a^2)$, and the perceived path travel cost $C_{r,w}$ satisfies equation (9) with the variance of $\xi_{r,w}$ expressed as

$$\mathrm{Var}\left(\xi_{r,w}\right) = \beta\widetilde{p} \sum_{a \in A} {\kappa_a}^2 \delta^w_{a,r}, \quad r \in R^w, w \in W. \tag{11}$$

And the covariance between $\xi_{r,w}$ and $\xi_{l,w}$ can be expressed by

$$\mathrm{cov}\left(\xi_{r,w}, \xi_{l,w}\right) = \beta\widetilde{p} \sum_{a \in A} {\kappa_a}^2 \delta^w_{a,r} \delta^w_{a,l}, \quad r, l \in R^w, w \in W. \tag{12}$$

Based on above, the following proposition can be readily obtained.

Proposition 1. *The perceived link travel cost C_a follows statistically independent normal distributions $N(t_a + \widetilde{p}\kappa_a, \beta\widetilde{p}{\kappa_a}^2)$, $a \in A$, and the perceived path travel cost $C_{r,w}$ follows statistically normal distributions $N(\sum_{a \in A} (t_a + \widetilde{p}\kappa_a)\delta^w_{a,r}, \beta\widetilde{p}\sum_{a \in A}{\kappa_a}^2\delta^w_{a,r})$, $r \in R^w, w \in W$.*

Proposition 1 implies that with the given assumption, the travelers indeed make their route choice based on the recommended credit price rather than the equilibrium one.

As we can see from (11) and (12), there is a major difference between the proposed probit-based SUE model and the classic one. That is, the overall perception error on path travel cost in the proposed model actually relies on the link-based credit charge, which is a decision variable in designing the credit scheme. In other words, the higher credit charge on a path is, the higher the perception error on travel cost of that path becomes. However, the perception error in the classic model depends solely on the free-flow travel time, which is a fixed and given network attribute rather than a variable to be solved. This nature of the proposed model renders the solution procedure more complicated because it requires further effort to consider the change in perception error when determining the optimal credit charge scheme.

3.3. An Equivalent Minimization Model with Tradable Credit Scheme. By Sheffi [46], a path flow pattern $\mathbf{f}$ is a SUE one if and only if it satisfies the following SUE condition:

$$f_{r,w} = q_w P_{r,w}\left(\mathbf{c}_w(\mathbf{f})\right), \quad r \in R^w, w \in W, \tag{13}$$

where $P_{r,w}(\mathbf{c}_w(\mathbf{f}))$ represents the path choice probability, namely, the probability that path $r \in R^w$ is perceived by some drivers as the shortest one among all the feasible paths connecting O-D pair $w \in W$, given deterministic path travel time pattern $\mathbf{c}_w(\mathbf{f}) = \left\{c_{r,w}(\mathbf{f}), r \in R^w, w \in W\right\}$. The path choice probabilities can be expressed by

$$P_{r,w}\left(\mathbf{c}_w(\mathbf{f})\right) = \Pr\left(C_{r,w}(\mathbf{f}) \le C_{l,w}(\mathbf{f}), \forall l \in R^w \text{ and } l \ne r | \mathbf{c}_w(\mathbf{f})\right), \quad r \in R^w, w \in W. \tag{14}$$

To formulate the conventional SUE as an equivalent mathematical model, Daganzo [47] developed an unconstrained minimization model that can be applied to solve the conventional SUE problem in a straightforward way. However, as demonstrated by Meng et al. [48], when additional constraints are considered (such as link capacity constraints in their work or credit constraint in this paper), simply adding the constraints into Daganzo's model cannot yield the desirable optimality conditions. To solve the generalized SUE problem that contains link capacity constraints, they proposed a linearly constrained minimization model based on the work by Maher et al. [49] and proved that the model possesses desirable properties. In the same spirit, Han and Cheng [15] successfully applied Meng's model in the context of TCS, which is given by

$$\begin{aligned}\min z_1(\mathbf{f}) = &\sum_{w\in W} q_w S_w\left(\bar{c}_w(\mathbf{f}) + t\bar{d}_w n(\mathbf{f}_w)\right)\\ &- \sum_{w\in W}\sum_{r\in R^w} \bar{d}_{r,w}(\mathbf{f}_w) f_{r,w},\end{aligned} \tag{15a}$$

$$\text{s.t.} \sum_{a\in A} \kappa_a v_a \le K, \tag{15b}$$

$$\sum_{r\in R^w} f_{r,w} = q_w, \tag{15c}$$

where constraint (15b) indicates that the total credit charge cannot exceed the total distribution. $S_w(\cdot)$ is the satisfaction function [46] (page 269), which is defined as the expectation of the minimum disutility from a set of route choices $r \in R^w$, i.e.,

$$S_w\left(\mathbf{c}_w(\mathbf{f})\right) = E\left[\min_{r\in R^w}\left\{c_{r,w}(\mathbf{f}) + \xi_{r,w}\right\}\right], \quad w \in W. \tag{16}$$

By definition, the partial derivative of the satisfaction function in terms of path travel cost equals the choice probability of that path, i.e.,

$$\frac{\partial S_w\left(\mathbf{c}_w(\mathbf{f})\right)}{\partial c_{r,w}(\mathbf{f})} = P_{r,w}\left(\mathbf{c}_w(\mathbf{f})\right), \quad r \in R^w, w \in W. \tag{17}$$

The vector $\bar{c}_w(\mathbf{f})$ is given by

$$\bar{c}_w(\mathbf{f}) = \left\{\bar{c}_{r,w}(\mathbf{f}),\ r \in R^w, w \in W\right\} = \left\{\sum_{a\in A} \bar{t}_a(v_a)\delta^w_{a,r}, r \in R^w, w \in W\right\}, \tag{18}$$

where $\bar{t}_a(v_a)$ is defined by

$$\bar{t}_a(v_a) = \begin{cases} \dfrac{\int_0^{v_a} t_a(\omega)\mathrm{d}\omega}{v_a}, & v_a > 0,\\ t_a^0, & v_a = 0.\end{cases} \tag{19}$$

The vector $\bar{d}_w(\mathbf{f}_w) = \left\{\bar{d}_{r,w}(\mathbf{f}_w), r \in R^w, w \in W\right\}$ is a path-specific vector function satisfying the following condition [49]:

$$P_{r,w}\left(\bar{c}_w(\mathbf{f}) + \bar{d}_w(\mathbf{f}_w)\right) = \frac{f_{r,w}}{q_w} \quad r \in R^w, w \in W. \tag{20}$$

Then, we extend Han's model to an elastic-demand case. By Cantarella [50], the probit-based SUE conditions with respect to path flows considering demand elasticity can be formulated as

$$\begin{aligned} f_{r,w} &= q_w S_w\left(\mathbf{c}_w(\mathbf{f})\right), \quad r \in R^w, w \in W,\\ q_w &= D_w\left(S_w\left(\mathbf{c}_w(\mathbf{f})\right)\right), \quad w \in W.\end{aligned} \tag{21}$$

Based on Han's model, we can establish a minimization model incorporating demand elasticity:

$$\min z_2(\mathbf{f},\mathbf{q}) = \sum_{w\in W} q_w S_w\left(\bar{c}_w(\mathbf{f}) + t\bar{d}_w n(\mathbf{f}_w)\right) - \sum_{w\in W}\sum_{r\in R^w} \bar{d}_{r,w}(\mathbf{f}_w) f_{r,w} - \sum_{w\in W}\int_0^{q_w} D_w^{-1}(\omega)\mathrm{d}\omega, \tag{22a}$$

$$\text{s.t.} \sum_{a\in A}\kappa_a v_a \le K. \tag{22b}$$

The equivalence between minimization model (22a) and (22b) and the SUE conditions in the context of TCS is emphasized in the following proposition.

Proposition 2. *Any flow pattern solved by minimization model (22a) and (22b) fulfils the following conditions:*

$$f_{r,w} = q_w P_{r,w}(\mathbf{c}_w(\mathbf{f})), \quad r\in R^w, w\in W, \tag{23}$$

$$q_w = D_w(S_w(\mathbf{c}_w(\mathbf{f}))), \quad w\in W, \tag{24}$$

$$p^*\left(K - \sum_{a\in A}\kappa_a v_a\right) = 0, \tag{25}$$

$$p^* \ge 0, \tag{26}$$

where p^* is the optimal Lagrangian multiplier with respect to the credit amount constraint (22b).

Proof. The corresponding Lagrangian function of the minimization program can be obtained as follows:

$$L(\mathbf{f},\mathbf{q},\boldsymbol{\mu},p) = z_2(\mathbf{f},\mathbf{q}) + p\left(\sum_{a\in A}\kappa_a v_a - K\right). \tag{27}$$

Using equations (17) and (19), the Karush–Kuhn–Tucker (KKT) conditions equivalent to this model can be derived as follows:

$$\sum_{a\in A}\left(v_a^*\frac{\mathrm{d}\bar{t}_a(v_a^*)}{\mathrm{d}v_a^*} + p^*\kappa_a\right)\delta_{a,r}^w - \bar{d}_{r,w}(\mathbf{f}_w^*) \ge 0, \quad r\in R^w, w\in W, \tag{28}$$

$$f_{r,w}^*\left(\sum_{a\in A}\left(v_a^*\frac{\mathrm{d}\bar{t}_a(v_a^*)}{\mathrm{d}v_a^*} + p^*\kappa_a\right)\delta_{a,r}^w - \bar{d}_{r,w}(\mathbf{f}_w^*)\right) = 0, \quad r\in R^w, w\in W, \tag{29}$$

$$S_w\left(\bar{c}_w(\mathbf{f}^*) + \bar{d}_w(\mathbf{f}_w^*)\right) - D_w^{-1}(q_w^*) \ge 0, \quad w\in W, \tag{30}$$

$$q_w^*\left(S_w\left(\bar{c}_w(\mathbf{f}^*) + \bar{d}_w(\mathbf{f}_w^*)\right) - D_w^{-1}(q_w^*)\right) = 0, \quad w\in W, \tag{31}$$

$$p^*\left(\sum_{a\in A}\kappa_a v_a^* - K\right) = 0, \tag{32}$$

$$p \ge 0, \sum_{a\in A}\kappa_a v_a^* - K \le 0. \tag{33}$$

Since any feasible path flow under SUE conditions is strictly positive, from condition (29) we have

$$\bar{d}_{r,w}(\mathbf{f}_w^*) = \sum_{a\in A}\left(v_a^*\frac{\mathrm{d}\bar{t}_a(v_a^*)}{\mathrm{d}v_a^*} + p^*\kappa_a\right)\delta_{a,r}^w. \tag{34}$$

By the definition of $\bar{t}_a(v_a)$, we can obtain

$$t_a(v_a) = \bar{t}(v_a) + v_a\frac{\mathrm{d}\bar{t}_a(v_a)}{\mathrm{d}v_a}, \quad a\in A. \tag{35}$$

Using this relationship, it can be easily verified that

$$\bar{c}_{r,w}(\mathbf{f}^*) + \bar{d}_{r,w}(\mathbf{f}_w^*) = c_{r,w}(\mathbf{f}^*), \quad r\in R^w, w\in W. \tag{36}$$

Thus, equation (19) can be rewritten as

$$P_{r,w}(\mathbf{c}_w(\mathbf{f}^*)) = \frac{f_{r,w}}{q_w}, \quad r\in R^w, w\in W, \tag{37}$$

which implies that stated condition (23) holds. Then, by equations (31) and (36), we have

$$D_w^{-1}(q_w^*) = S_w(\mathbf{c}_w(\mathbf{f}^*)), \tag{38}$$

which is equivalent to condition (24). And the equivalence between the KKT conditions (32) and (33) and the stated conditions (25) and (26) is obvious. This completes the proof.

Based on Proposition 2, we can say that any flow pattern solved by model (22a) and (22b) satisfies the SUE conditions under a given TCS, and the optimal Lagrangian multiplier associated with credit conservation constraint (22b) is the equilibrium credit price.

Different from Han's work, in which the problem is discussed under the logit-based SUE conditions, we solve the model in the framework of probit-based SUE. In the next section, we move on to look at the solution method for minimization model (22a) and (22b). □

3.4. Solution Algorithm. In addition, the Lagrange multiplier of credit constraint (22b) is exactly the credit price p. Therefore, solving the equilibrium credit price of model (22a) and (22b) is consistent with the procedure of finding the optimal Lagrange multipliers of link capacity constraints in Meng et al. [48]. Thus, along the same line with Meng et al. [48], we turn to solve the Lagrangian dual problem of minimization program (22a) and (22b) by a gradient projection method with a predetermined step size sequence.

The Lagrangian dual formulation of (22a) and (22b) can be presented as [51]

$$\max_{p\geq 0} \varphi(p), \tag{39}$$

where the concave function $\varphi(p)$ is defined as follows:

$$\varphi(p) = \min_{\mathbf{f},\mathbf{q}}\left[z_2(\mathbf{f},\mathbf{q}) + p\left(\sum_{a\in A}\kappa_a v_a - K\right)\right]. \tag{40}$$

Since for any given credit price p, the link travel cost $c_a(v_a)$ is strictly monotone and continuously differentiable, the uniqueness of the equilibrium link flow pattern can be guaranteed [50]. The gradient of $\varphi(p)$ can be computed as

$$\nabla\varphi(p) = \sum_{a\in A}\kappa_a v_a - K. \tag{41}$$

To estimate the travel demand and SUE link flow in the solution procedure, a two-stage Monte Carlo simulation method proposed by Meng et al. [44] is embedded as a subroutine. The overall solution algorithm is stated as follows:

Step 0 (initialization): set an inaccuracy tolerance ε and two predetermined sample sizes, i_0 and j_0. Initialize the credit price with $p^{(0)} > 0$. Choose a step size $\{\alpha_n\}$ satisfying

$$0 < \alpha_n < 1, \sum_{n=1}^{\infty}\alpha_n = \infty, \sum_{n=1}^{\infty}{\alpha_n}^2 < \infty. \tag{42}$$

Set iteration counter $n = 1$.

Step 1 (SUE traffic assignment): generate an SUE demand pattern $\{q_w^{(n)}, w \in W\}$ and corresponding link flow pattern $\{v_a^{(n)}, a \in A\}$ by performing a two-stage Monte Carlo simulation procedure as follows:

(i) *Initialization of Satisfaction.* Set the initial satisfaction function $\overline{S}_w^0 = 0, w \in W$ and the sample counter $i = 1$.

(ii) *Sampling.* Sample the perception error $\overline{\xi}_p^i, a \in A$ from $N(\tilde{p} - p^{(n)}, \beta\tilde{p})$ for each link, and then, calculate the modified link travel cost by $C_a^i = t_a(v_a^i) + (p^{(n)} + \overline{\xi}_p^i)\kappa_a$, $a \in A$.

(iii) *Satisfaction Estimation.* Based on link cost pattern $\{C_a^i, a \in A\}$, find the shortest path between each OD pair and denote the path travel time as C_w^i. Calculate the estimated satisfaction function by

$$\overline{S}_w^i = \frac{(i-1)\overline{S}_w^{i-1} + C_w^i}{i}, \quad w \in W. \tag{43}$$

(iv) *Checking a Termination Criterion.* If the number of samples $i \geq i_0$, go to step (v); otherwise, set $i: = i + 1$ and go to step (ii).

(v) *Travel Demand Estimation.* Calculate the OD-based travel demand by

$$q_w = D_w\left(\overline{S}_w^i\right), \quad w \in W. \tag{44}$$

(vi) *Initialization of Flow Pattern.* Set the initial link flow $\mathbf{v}^0 = 0$ and the sample counter $j = 1$.

(vii) *Sampling.* Sample the perception error $\overline{\xi}_p^j$, $a \in A$ from $N(\tilde{p} - p^{(n)}, \beta\tilde{p})$ for each link, and then, calculate the modified link travel cost also by $C_a^j = t_a(v_a^j) + (p^{(n)} + \overline{\xi}_p^j)\kappa_a$, $a \in A$.

(viii) *All-or-Nothing Assignment.* Based on link cost pattern $\left\{C_a^j, a \in A\right\}$, assign q_w to the shortest path for OD pair $w \in W$. With an auxiliary link flow pattern $\left\{y_a^j, a \in A\right\}$ generated, calculate the link flow by

$$v_a^j = \frac{(j-1)v_a^{j-1} + y_a^j}{j}, \quad a \in A. \tag{45}$$

(ix) *Checking a Termination Criterion.* If the number of samples $j \geq j_0$, stop and obtain $\mathbf{q}^{(n)} = \mathbf{q}, \mathbf{v}^{(n)} = \mathbf{v}^j$; otherwise, set $j: = j + 1$ and go to step (vii).

Step 2 (checking a termination criterion): if the following inequality holds, stop. Otherwise, go to Step 3:

$$\left|p^{(n)} - \max\left\{0, p^{(n)} + \alpha^{(n)}\left(\sum_{a\in A}\kappa_a v_a^{(n)} - K\right)\right\}\right| \leq \varepsilon. \tag{46}$$

Step 3 (updating the credit price): update the unit credit price as follows:

$$p^{(n+1)} = \max\left\{0, p^{(n)} + \alpha^{(n)}\left(\sum_{a\in A}\kappa_a v_a^{(n)} - K\right)\right\}. \tag{47}$$

Let $n: = n + 1$ and go to Step 1.

Remark 1. As the name of the algorithm suggests, the left-hand-side in equation (46) is essentially the nonnegative projection of the gradient of the Lagrangian dual function with respect to p, $P_+[p^{(n)} + \alpha^{(n)}(\sum_{a\in A}\kappa_a v_a^{(n)} - K)]$. By Bertsekas [52], it can be readily obtained that p^* is the equilibrium credit price if and only if $p^* = P_+[p^* + \alpha\nabla\varphi(p^*)]$, which equivalently verifies the validity of the stopping criterion in equation (46) and the iterative update scheme in equation (47).

Remark 2. Note that the perception error in Step 1 (ii) and (vii) is sampled in a way such that travelers always choose their routes based on the recommended credit price rather than the equilibrium one. Toward this end, the mean of the random variable in the Monte Carlo simulation procedure changes correspondingly upon updating the credit price in each outer iteration.

Remark 3. The stop criterion in Step 2 (iv) and (ix) is based on two predetermined sample sizes. With a given accuracy level, the lower bounds for i_0 and j_0 can be determined by the study of Meng and Liu [53].

3.5. A Small Network Example. In this section, a toy network is adopted to demonstrate the impact of recommended credit price under a given TCS adopted in Bao et al. [14].

As shown in Figure 1, the example network consists of four paths connecting two O-D pairs: path 1 including link 1, path 2 including links 2-5-6, path 3 including link 3, and path 4 including links 4-5-7. The variance parameter is chosen as $\beta = 0.5$. The Bureau of Public Road (BPR) function is adopted to estimate the link travel time:

$$t_a(v_a) = t_a^0\left(1 + 0.15\left(\frac{v_a}{Q_a}\right)^4\right), \quad a \in A, \tag{48}$$

where Q_a is the capacity on link a. The travel demand function is given as

$$q_w = \overline{q}_w \exp(-0.01 * S_w), \quad w \in W, \tag{49}$$

with $\overline{q}_1 = 60$ for O-D pair 1 (node 1 $\longrightarrow$ node 2) and $\overline{q}_2 = 50$ for O-D pair 2 (node 3 $\longrightarrow$ node 4).

In the case of elastic demand, minimizing the system total travel time only is obviously inappropriate as an obviously unrealistic optimal solution would be the near zero traffic flow achieved by imposing unacceptably high charges. Instead, the SO problem in the elastic-demand case is defined in terms of maximization of the net economic benefit (EB) or social welfare (see Chapter 3.2.3 in [54]). By definition, the net economic benefit (EB) can be computed by subtracting the system travel time from the total user benefits, which is given by

$$\mathrm{EB} = \sum_{w\in W}\int_0^{q_w} D_w^{-1}(\omega)\mathrm{d}\omega - \sum_{a\in A} v_a t_a(v_a). \tag{50}$$

We assume that the central authority initially distributes 660 credits among travelers, and each traveler (including those who make their trips as well as those who give up their trips) gets 6 credits. The specific information of the link free-flow travel time, capacity, and credit charging scheme are given in Table 1. Based on the network topology and the given credit scheme, we know that travelers using paths 2 and 4 have remaining credits and those on paths 1 and 3 need to buy extra credits.

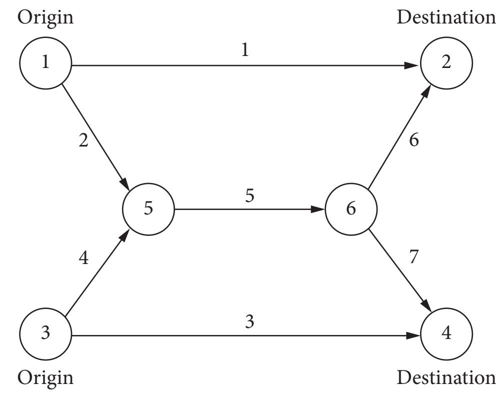

Figure 1: The toy network.

Table 1: Input data of the small network.

Link	Free-flow travel time	Capacity	Credit charge
1	10	35	9
2	3	30	2
3	12	35	8
4	4	35	1
5	5	35	3
6	3	35	2
7	4	25	1

First, we examine the convergent trend and stability of the gradient projection-based algorithm. The convergence results for the algorithm with different recommended credit prices are presented in Figure 2. It can be seen that it takes about 15 iterations to converge to the equilibrium credit price in all cases, and the figure depicts that the algorithm is insensitive to the initial credit price $p^{(0)}$. Taking $p^{(0)} = 5.0$, we further test the convergence performance and computational efficiency of the solution algorithm on three larger networks, as shown in Table 2 (the relevant network data were taken from the transportation network data sets maintained by Stabler et al. [55] (https://github.com/bstabler/TransportationNetworks)). From the table, it is clear that the algorithm does not scale very well as the size of the network becomes larger, since the computational time for the Winnipeg network is nearly 59 times more than that for Sioux Falls. This implies that the computational efficiency of the gradient method embedded with Monte Carlo simulation cannot be guaranteed for large-size problems.

Now, let us turn our attention back to the toy network, and we investigate the impact of recommended credit price on the actual one and system travel time. From Figure 3, it can be observed that the choice of recommended credit price affects both the resulting equilibrium credit price and economic benefit. Specifically, the equilibrium credit price declines as the recommended one increases. This can be explained by the change in demand and supply of credits. When the recommended credit price gets higher, travelers are expected to care more about the credit cost, and thus, less travelers will choose the paths with relatively higher credit charges. Then, the demand of credits in the trading market shrinks and the supply rises correspondingly since the total

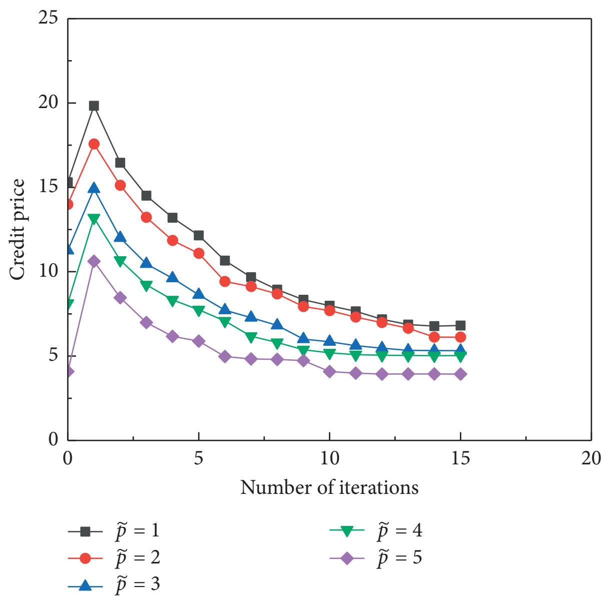

Figure 2: Convergence plots for equilibrium credit price with different recommended credit prices.

Table 2: Iterations and CPU times needed for convergence of the proposed algorithm.

Network	# Iterations	CPU time
Sioux Falls (SF)	24	31.36 s
Anaheim	46	263.24 s
Winnipeg	78	1876.31 s

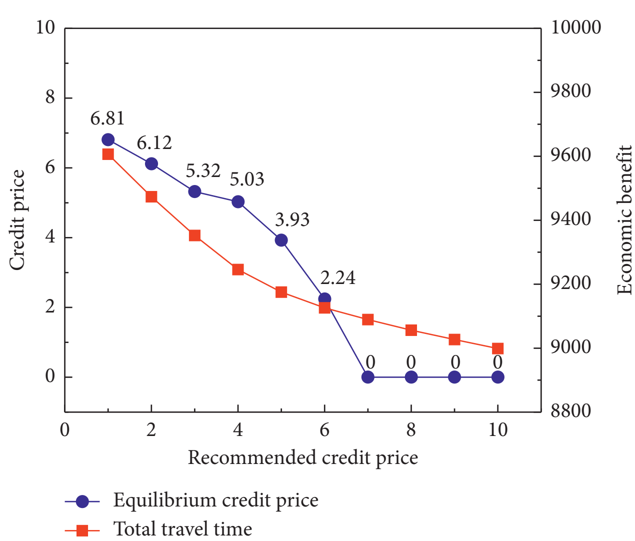

Figure 3: Change in equilibrium credit price and economic benefit with different recommended credit prices.

travel demand and issued credits remains the same. As a consequence, the equilibrium credit price will decrease. We can see that there is a decreasing trend in economic benefit with increasing recommended price. And it is interesting to note that when the recommended price is larger than (or equal to) 7.0, the actual credit price reduces to zero. This implies that the perceived credit price is too high to match sufficient buyers with the sellers in the market, leading to a nullified credit scheme. However, a nullified TCS does not necessarily mean a sharp loss in social welfare.

Furthermore, to see the differences between the proposed probit-based SUE condition (with perception error on credit price) and the conventional one (with perception error on free-flow travel time), we also solve model (22a) and (22b) with $\xi_a \sim N(0, \beta t_a^0)$ in equation (8) and compare the two resulting flow patterns in Figure 4. From the figure, we can see that the two flow patterns exhibit evident diversity. And it is observed that flows on path 1 and 3 (paths on which travelers need to buy extra credits) drop down as $\tilde{p}$ increases, while those on paths 2 and 4 (paths on which travelers have remaining credits to sell) show an increasing trend. This verifies the facts that more travelers will choose paths 2 and 4 when the recommended credit price gets higher.

From above, we learn that the choice of recommended credit price indeed affects the social welfare, and extremely high recommended price may nullify the credit scheme. Despite a socially optimal scheme, if the gap between the recommended price and realized one is too large, travelers will fail to have a reasonable expectation on their travel cost (e.g., underestimate their travel cost or overestimate their benefit from selling remaining credits) and thus lose their trust to the announced credit prices by the central authority. This means that over time, travelers will care less about the recommended credit price and the credit scheme will lose its impact on the system. Thus, to avoid this circumstance and enhance the public acceptability of the implemented TCS, it is critical and necessary for the central authority to minimize the gap between the recommended and the realized prices. Therefore, in addition to the network performance in terms of social welfare, the difference between the two credit prices also deserves particular attention.

4. Optimal Design of Credit Charging Scheme with Recommended Unit Credit Price

In the last section, we demonstrate that both the social welfare and the price gap should be considered in the design of a scheme. Along this way, in this section, we formulate the maximization of the economic benefit and the minimization of price gap as a bilevel biobjective programming (BLBOP) model, in which the credit charging scheme and recommended price are decision variables. Then, a weighted sum method is applied to transform the BLBOP model into a single-objective one, within which the values of objective functions are normalized to the interval [0, 1] to eliminate their dimension and order of magnitude. Furthermore, a genetic algorithm (GA) is applied to solve the problem.

4.1. A Bilevel Biobjective Programming Model. The optimal TCS design combines goals in terms of social welfare and public acceptance by simultaneously maximizing the economic benefit and minimizing the gap between the two prices. The biobjective programming model can be

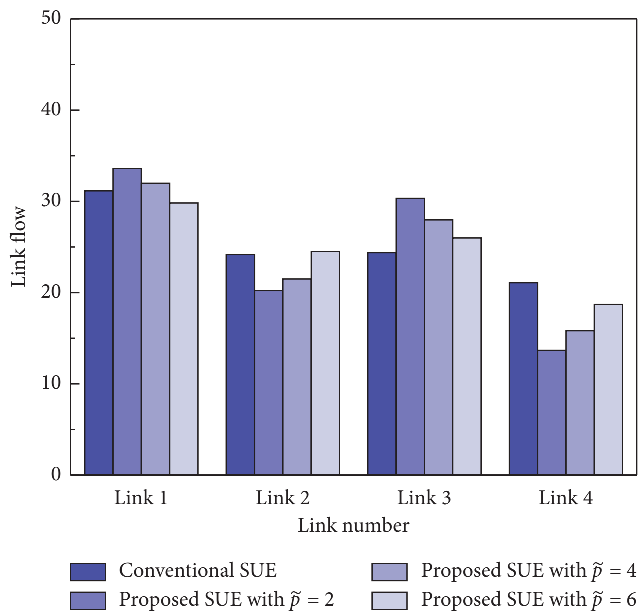

FIGURE 4: Comparison of equilibrium link flow patterns between the proposed and conventional probit-based SUE models.

presented as follows (in this paper, it is assumed that the initial issued credits K are given and fixed):

$$\left(\begin{array}{c} \max_{\boldsymbol{\kappa},\widetilde{p}} F_1(\boldsymbol{\kappa},\widetilde{p},\mathbf{v}) = \sum_{w\in W}\int_0^{q_w} D_w^{-1}(\omega)\mathrm{d}\omega - \sum_{a\in A} v_a t_a(v_a) \\ \min_{\boldsymbol{\kappa},\widetilde{p}} F_2(\boldsymbol{\kappa},\widetilde{p},\mathbf{v}) = \left|\widetilde{p} - p^*(\boldsymbol{\kappa},\widetilde{p},\mathbf{v})\right| \end{array}\right), \tag{51a}$$

$$\text{s.t. } 0 \le \kappa \le \kappa_{\max}, \tag{51b}$$

$$0 < \widetilde{p} \le p_{\max}, \ (28)-(33). \tag{51c}$$

The problem defined by (51a)–(51c) is essentially a Stackelberg network game which has a bilevel structure. In the upper level, the central authority takes into account the reaction of the travelers and attempts to improve the network performance as well as reducing the price gap between the recommended and the realized. The lower-level problem, which is characterized by constraints (28) through (33), is to find the equilibrium flow pattern $\mathbf{v}$ and credit price p^* from problem (22a) and (22b) with a given κ and $\widetilde{p}$. Moreover, constraints (51b) and (51c) aim to manage the link-based credit charge and announced credit price from being too high.

Since the link flows are positive and cannot be larger than the maximum potential travel demand, the travelers' reaction map is bounded. Given the continuous objective functions in (51a), in combination with the compact feasible sets of κ and $\widetilde{p}$, there always exists a solution to problem (51a)–(51c) as per Corollary 1 in Harker and Pang [56].

Although the existence of solution to problem (51a)–(51c) is guaranteed, the uniqueness condition cannot be established due to its nonconvexity incurred by constraints (28) through (33). In other words, there may be multiple local optima and it is prohibitively difficult to identify the global one in an analytical manner. In fact, however, even a locally optimal solution is acceptable for the central authority if it outperforms the alternative solutions, and hence, we try to find such a locally optimal solution using heuristic methods.

4.2. The Transformation of the BLBOP Problem. Among the existing methods to solve multiobjective optimization problems, the weighted sum method, by which a preferred or compromised solution obtained through a tradeoff of the multiple objectives, is widely used in research or engineering optimization due to its simple implementation [57]. Hence, we adopt the weighted sum method to solve BLBOP problem (51a)–(51c).

When applying the weighted sum method, the main technical difficulty lies in the different dimensions of the multiple objectives. That is, the weighting parameter fails to reflect the real emphasis on each objective if there are remarkable differences in their magnitudes. To eliminate such a negative effect, we normalize the objectives within the interval [0, 1]. Namely, problem (51a)–(51c) can be transformed into the following form:

$$\min_{\boldsymbol{\kappa},\widetilde{p}} F(\boldsymbol{\kappa},\widetilde{p},\mathbf{v}) = (1-\lambda)\frac{F_1(\boldsymbol{\kappa},\widetilde{p},\mathbf{v}) - F_1^{\max}}{F_1^{\min} - F_1^{\max}} + \lambda\frac{F_2(\boldsymbol{\kappa},\widetilde{p},\mathbf{v}) - F_2^{\min}}{F_2^{\max} - F_2^{\min}}, \tag{52}$$

where $\lambda \in [0,1]$ is the weighting parameter and $F_i^{\min}$ and $F_i^{\max}$ are the minimum and maximum of $F_i(\kappa,\widetilde{p},\mathbf{v})$, $i = 1, 2$.

Remark 4. As we can see, the values of $(F_1(\kappa,\widetilde{p},\mathbf{v}) - F_1^{\max})/(F_1^{\min} - F_1^{\max})$ and $(F_2(\kappa,\widetilde{p},\mathbf{v}) - F_2^{\max})/(F_2^{\max} - F_2^{\min})$ attain 0 and 1 when $F_1(\kappa,\widetilde{p},\mathbf{v})$ and $F_2(\kappa,\widetilde{p},\mathbf{v})$ reach their best and worst values, respectively. Hence both the two terms are decrease monotonically within the interval [0, 1] as the values of $F_1(\kappa,\widetilde{p},\mathbf{v})$ and $F_2(\kappa,\widetilde{p},\mathbf{v})$ move closer toward the optimal values. By doing this, the difficulty in quantitatively measuring the proportional importance of different objectives can be circumvented.

With the problem transformed, then an intriguing issue is how to choose the values of $F_1^{\max}$, $F_1^{\min}$, $F_2^{\max}$, and $F_2^{\min}$. Recall that F_1 is the economic benefit and F_2 is the positive gap between $\widetilde{p}$ and p^*. It is natural to obtain that

$$F_1^{\max} = \mathrm{EB}_{\mathrm{SO}}, \tag{53}$$

$$F_2^{\min} = 0, \tag{54}$$

where $\mathrm{EB}_{\mathrm{SO}}$ is the economic benefit at socially optimal (SO) status.

Then, let $\mathrm{EB}_{\min}$ denote the system travel time after an all-or-nothing assignment based on the free-flow travel time and $\widetilde{p}_{\max}$ the maximum recommended credit price (this term is also adopted in the initialization of the solution algorithm as seen later). Given that $\widetilde{p} \le \widetilde{p}_{\max}$ is always satisfied, $|\widetilde{p} - p^*| > \widetilde{p}_{\max}$ implies that $p^* > \widetilde{p} + \widetilde{p}_{\max}$ holds true, which means an unhealthy trading market with extremely high credit price and rarely occurs in practice (as seen later, the case with $|\tilde{p} - p^*| > \tilde{p}_{\max}$ could only occur when the minimization of price gap is completely ignored in optimal TCS design). Since there are few feasible solutions satisfying $F_1 < \mathrm{EB}_{\min}$ or $F_2 > \tilde{p}_{\max}$, it is safe to say that the two objectives are normalized to the interval [0, 1] by defining $F_1^{\max}$ and $F_2^{\max}$ as

$$F_1^{\min} = \mathrm{EB}_{\min}, \tag{55}$$

$$F_2^{\max} = \tilde{p}_{\max}. \tag{56}$$

With equations (53) through (56), problem (52) can be rewritten as

$$\min_{\boldsymbol{\kappa},\tilde{p}} F(\boldsymbol{\kappa}, \tilde{p}, \mathbf{v}) = (1-\lambda)\frac{F_1(\boldsymbol{\kappa}, \tilde{p}, \mathbf{v}) - \mathrm{EB}_{\mathrm{SO}}}{\mathrm{EB}_{\min} - \mathrm{EB}_{\mathrm{SO}}} + \lambda \frac{F_2(\boldsymbol{\kappa}, \tilde{p}, \mathbf{v})}{\tilde{p}_{\max}}. \tag{57}$$

Then, we move on to the solution algorithm to the proposed BLBOP model in the next section.

4.3. Genetic Algorithm to Solve the BLBOP Problem. Due to the complicated relationship between the recommended credit price and corresponding equilibrium credit price under a TCS and the approximation by the Monte Carlo simulation method, as well as the nonconvexity of the bilevel problem, it is extremely hard to develop an exact solution algorithm to solve the proposed problem analytically. Therefore, a heuristic method is preferred in designing the optimal credit charging scheme as well as determining the best choice of recommended credit price. In this paper, a genetic algorithm (GA) is applied due to its extensive generality, global perspective, and strong robustness.

The decision variables of the upper level, i.e., $\tilde{p}$ and $\kappa = \{\kappa_a, a \in A\}$ are coded as a single chromosome $\mathbf{x} = \{x_j | j = 1, 2, \ldots, |A| + 1\}$ (see Figure 5). A group of chromosomes is first generated randomly, in which $\tilde{p}$ and κ_a, $a \in A$ are continuous variables within $(0, \tilde{p}_{\max}]$ and $[0, \kappa_{\max}]$, respectively. Following the evaluation, selection, crossover, and mutation operations, a new population of chromosomes is generated at each iteration. After a given number of iterations, the genetic algorithm will terminate and return the best-found solution. The process is illustrated in Figure 6, where N_{pop}, N_{gen}, P_{cr}, and P_{mu} denote the population size, the maximum number of generations, crossover probability, and mutation probability, respectively.

5. Numerical Analysis

In this section, two numerical examples are presented to investigate the features of the optimal TCS with recommended credit price determined by model (57) with different values of the weight. The genetic algorithm-based procedure is performed with $N_{\mathrm{pop}} = 30$, $N_{\mathrm{gen}} = 100$, $P_{\mathrm{cr}} = 0.6$, $P_{\mathrm{mu}} = 0.15$, and $\kappa_{\max} = 10.0$. The algorithm was coded by Matlab 8.6, and the test was carried out on a laptop with an Intel(R) Core(TM) i7-6820HK CPU, 2.70 GHz × 8, RAM 16 G.

5.1. A Small Network. The first numerical analysis is conducted based on the same network given in Section 3.5. We solved the proposed BLBOP problem for the small network with different values of λ and β. The results are given in Table 3.

From the table, we can see that with higher weight on the price gap, lower price gaps and economic benefits are observed. With $\beta = 0.1$, it can be seen that the economic benefit is less sensitive to the changes in weight values (the economic benefit reduces about 1.2% while the price gap reduces about 79.5% as weight on the price gap changes from 0.2 to 0.8). Note that the maximum social welfare $\mathrm{EB}_{\mathrm{SO}}$ is 9727.1, which is very close to the values in the second column, while the social welfare at UE state is 9245.7, which is far smaller than the results in the table. It implies that we could minimize the price gap without too much sacrifice in social welfare in this case. However, this is not the case for larger networks, as shown later.

As β increases to higher values, the system travel time rises further away from $\mathrm{EB}_{\mathrm{SO}}$ and it becomes more sensitive to the changes in weight values. Similarly, the price gap obtained from the optimal solution also gets larger. Meanwhile, the economic benefits and the price gaps in the first and last rows do not change too much in either the case of $\beta = 0.5$ or $\beta = 1.0$. These demonstrate that the increase in perception variance could intensify the contradiction effect between the two objectives, making it harder to balance the goals.

We also investigate the total trading amounts and trading value in the credit-trading market (since the network is small and simple, it is not difficult to track down the path flow pattern through the link flows. Hence, the trading amounts of credits are ready to obtain). We find that emphasis on minimizing the price gap could evoke more trades in the market regardless of the value of β. However, no obvious tendency is observed in the variation of trading values.

5.2. The Sioux Falls Network. Then, we adopt a larger network, the Sioux Falls network (as shown in Figure 7) for the proposed BLBOP model. The Sioux Falls network consists of 24 nodes, 76 links, and 528 OD pairs. The demand function also adopts the form in equation (49). In regard to the credit scheme, we assume that the total amount of initially distributed credits is 361,100, and each traveler gets 10 credits. Due to larger size of network, we reduce the parameter $\kappa_{\max}$ to 5.0.

The BLBOP model is solved by varying the weight from 0 to 1 by 0.1 with $\beta = 0.1$ and $\beta = 1.0$. The Pareto frontiers are demonstrated in Figure 8. The two dashed lines refer to the economic benefit at SO and deterministic UE statuses (i.e., TCS is not implemented and naturally no perception error on the travel cost exists), respectively. The main findings are presented as follows.

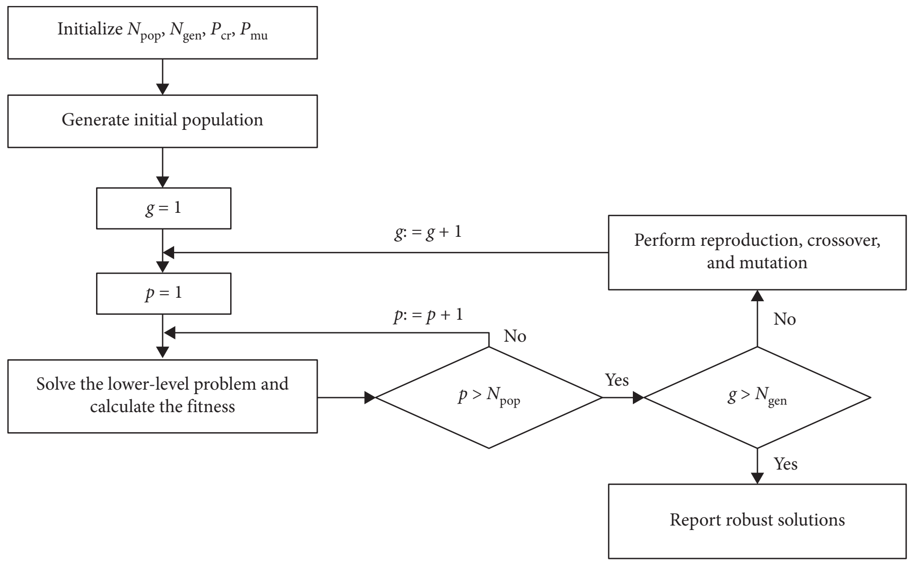

Figure 5: Structure of a chromosome.

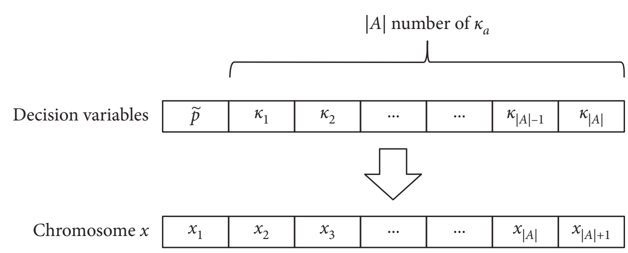

Figure 6: Flowchart of the genetic algorithm.

First, we focus our attention on the extreme cases. From the figure, it can be seen that when λ equals to 1.0, i.e., the goal in terms of social welfare is completely ignored in the TCS design, the economic benefit is even less than the UE level in either cases. Nevertheless, it can be remarkably reduced to around EB_{UE} once the social welfare is taken into consideration. However, as a cost, we must allow a nonnegligible price gap to occur, which is different from the toy-network case.

On the other hand, when the objective of minimizing the price gap is ignored in the TCS design, the maximized economic benefit is still lower than EB_{SO} regardless of the perception variance. This highlights the fact that the negative effect of perception error in terms of credit price cannot be entirely accommodated, even though we focus merely on the social welfare.

Second, when the weight on price gap changes from 0.1 to 0.9, we can see that price gaps in all the cases are significantly larger than those observed in the small network. It indicates that as the size of the network increases, it is much harder to achieve a zero price gap.

Furthermore, it is observed that the resulting price gaps with higher perception variance are always larger (i.e., for each value of λ, the blue point is always on the right of the green square). However, the relationship of the resulting system travel times with $\beta = 0.1$ and $\beta = 1.0$ is indefinite (i.e., for each value of λ, the blue point can be located either on the upper or on the lower of the green square, see the two sets of points regarding $\lambda = 0.5$ and $\lambda = 0.6$). In other words, a larger perception error always guarantees a larger resulting price gap, but it is not necessarily equivalent to a higher economic benefit.

6. Two Implementation Issues

The primary goal of this paper is to provide an efficient and engineering-oriented tradable credit scheme for mobility management considering perception error on credit price. Although the effect of the proposed credit scheme has been verified through numerical examples, there are still implementation issues that need to be dealt with in practice. In this section, we shall discuss about two implementation issues regarding model calibration and the selection of λ.

First, to characterize travelers' attitudes toward the recommended credit price and calibrate the distribution form of users' perception error, a sample survey or a virtual situational experiment can be conducted among potential travelers before implementation. For example, a similar methodology in Yu [58] can be adopted to portray the price perception for credit price, and user heterogeneities such as income level and resident area can be incorporated and further considered in the modeling framework. Moreover, it is worth noting that the framework proposed in this paper

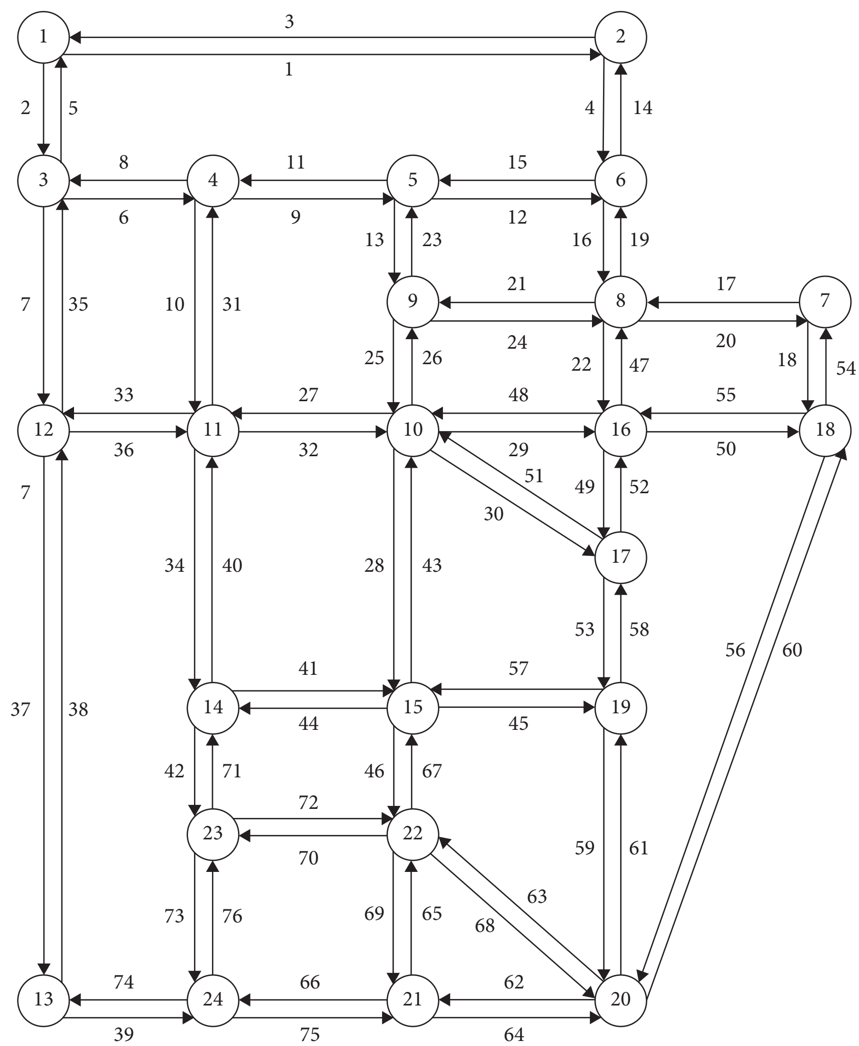

Figure 7: The Sioux Falls network.

Table 3: Numerical results for the small network.

λ	$\beta = 0.1$				$\beta = 0.5$				$\beta = 1.0$			
	EB	G	TA	TV	EB	G	TA	TV	EB	G	TA	TV
0.2	9704.9	1.17	159.3	100.2	9683.6	1.73	175.5	181.8	9672.7	2.61	132.1	166.3
0.4	9653.3	0.74	231.7	162.3	9613.8	1.52	191.1	289.1	9604.2	1.79	154.3	93.8
0.5	9620.6	0.66	252.5	152.1	9535.1	1.27	206.6	433.5	9519.9	1.32	176.5	339.8
0.6	9599.7	0.39	298.7	322.8	9502.5	0.85	207.2	373.6	9382.4	0.92	210.4	436.9
0.8	9584.0	0.24	345.2	251.8	9420.5	0.34	323.7	254.3	9242.3	0.51	227.8	417.8

*EB = economic benefit; G = price gap; TA = trading amounts; TV = trading value (=TA*unit credit price).

can be easily extended to other distribution forms of user's perception error by changing the sampling principle in the two-stage Monte Carlo simulation method.

Also worth noting is that the selection of the weighting parameter λ over time. Particularly, in the very initial stage after the implementation of the credit scheme, it is more necessary to gain public acceptance rather than achieving a target in terms of network efficiency. Therefore, in reality, more emphasis should be placed on reducing the price gap than the system travel time then. As the price gap is controlled within a reasonable range, travelers' perception error on the credit price may decrease over time, and the emphasis on price gap could be gradually transferred to travel time.

7. Concluding Remarks and Future Research

In this paper, we examine the practicability of tradable credit scheme (TCS) with recommended credit price in the initial stages after its implementation. Assuming that travelers have

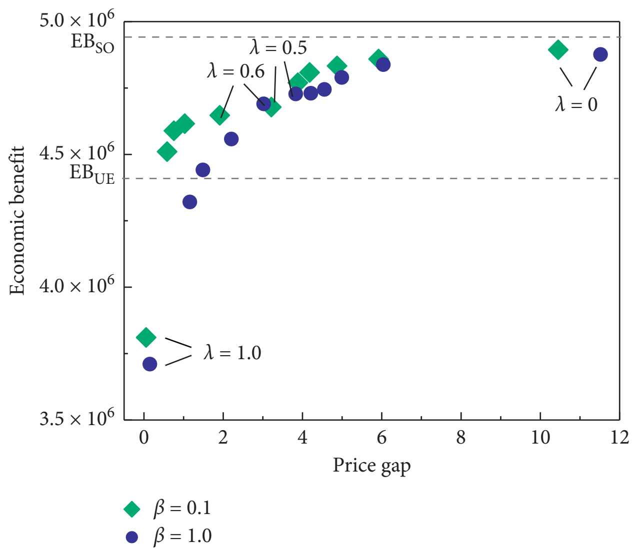

Figure 8: Pareto frontiers for the Sioux Falls network.

a perception error on the recommended unit credit price released by central authority, a bilevel biobjective programming model is established and solved by a genetic algorithm. By determining the optimal credit charging scheme with an appropriate recommended credit price, the social welfare is maximized and the gap between the recommended credit price is minimized. Based on the numerical results, the main findings are given as follows:

(i) With a given TCS, the realized credit price drops down as the recommended price increases due to the change in demand and supply of credits. Particularly, when the recommended price is sufficiently high, the scheme can be nullified with a zero credit price.

(ii) The rise in perception variance could intensify the contradiction effect between the two objectives, making it harder to balance the goals in terms of social welfare and public acceptability. Moreover, a larger perception error always guarantees a larger resulting price gap, but it is not necessarily equivalent to a higher economic benefit.

(iii) We could minimize the price gap without too much sacrifice in social welfare on small networks. However, to maintain the effectiveness of a TCS with recommended credit price for relatively larger networks, or networks with realistic size, we must allow a nonnegligible price gap to occur.

(iv) Emphasis on minimizing the price gap could evoke more trades in the market regardless of the perception variance. However, no regular trend is found in the variation of trading values.

These findings can provide managerial insights for the initial stages after implementation of a TCS. Future research studies can be carried out in the following three aspects. First, considering that restriction on car use or flow redistribution is only needed for a small area of an urban metropolis, it is burdensome to implement a charging scheme over the entire transportation network. However, the main difficulty of such a scheme is that the removal of credit charge in the uncongested area may compound the spatial inequity among travelers. One way to alleviate such equity concern is to adopt an O-D-specific initial distribution of credits. In such a case, the initial distribution acts as a benefit term in the travel cost and should be considered in solving the network equilibrium.

Second, as stated in Section 6, travelers' perception errors toward the credit prices are essentially time-dependent and region-specific. Specifically, a day-to-day evolution model, where a declining perception error and a time-varying weighting parameter are considered, can be applied for the former. While for the latter, we can categorize the origin nodes into various classes according to their economic and demographic features and adopt different perception errors for different O-D pairs. In this way, we can incorporate the heterogeneity in terms of regions where the trips occur.

Third, the modeling framework presented in this paper still leave out some real-world features, such as user heterogeneity and asymmetric link flow interactions. To conduct these extensions, the modeling framework adopted by Meng and Liu [53], Meng et al. [44], and Liu et al. [59] in the context of congestion pricing can afford useful insights.

Data Availability

The data about the Sioux Falls network were obtained from the GitHub repository "Transportation Networks for Research" (https://github.com/bstabler/TransportationNetworks).

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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