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Research Article

Continuum Approximation Model for Transit Service Design with Stochastic Demand

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Travel demand is commonly predefined as a constant during the planning period in transit service design, but it varies daily with many factors, for example, weather, vacation, and social activity. Under the uncertain demand, the transit system operates in two states, that is, unsaturation and saturation, distinguished by whether or not the capacity of transit vehicle satisfies the possible demand. Thus, we propose a continuum approximation (CA) model for transit service design, including headway and station location, to account for the effects of the stochastic demand via a penalty cost, a service-reliability constraint, and equilibrium. The penalty cost is utilized to describe the saturation state. The service-reliability constraint is applied to ensure the robustness of the transit system. The equilibrium is introduced to allocate the household location where trip demand is generated in a corridor. Furthermore, we build a bilevel framework to find the solutions to the proposed model. In the numerical experiment, the proposed model is applied in the impact analyses of the service-reliability constraint, as well as the sensitivity analyses of the household numbers and value of time. The impact analyses indicate that the transit service design integrated with the effect of housing location choice is necessary under the stochastic demand. The sensitivity analyses show that the number of households and the value of time play a significant role in the performance of transit systems accounting for service reliability. The proposed model and findings serve to improve the design of the transit system under stochastic demand.

1. Introduction

With the economy and technology development, people heavily rely on passenger car transport, especially in the developed countries, for example, 84.6% in the US [1] and 83.6% in England in 2015 [2]. Correspondingly, the problem of congestion and pollution becomes more severe. To reduce the proportion of passenger cars in the daily commute, the government invests more and more in public transit service, which appears to be more economical and greenway as opposed to passenger cars.

Location choice and headway setting are two essential activities in transit service design. The service with the dense station location can reduce the access time of passengers to the station. Meanwhile, a low headway decreases the waiting time of passengers in the station. However, the above two measures will increase the cost of the transit agency, such as the building cost of the station and the operation cost of transit vehicles. Thus, there is a tradeoff between the passenger and transit agency with different objectives in transit service design, such as the system cost minimization and profit maximization.

Under the transit service design with stochastic demand, an additional type of waiting time should be considered [3]. Actually, there is a capacity constraint on transit vehicles; thus, not all of the passengers can board the first vehicle as they desire. In this case, some passengers have to wait until the next vehicle arrives, which induces an additional cost that must be accounted for. Different from the conventional waiting time, which is smaller than departure headway, the additional waiting time is longer for passengers. A transit service with small departure headway can efficiently reduce the probability of passengers who are unable to board, but it will increase the cost of the transit agency. Therefore, the tradeoff between oversaturated service and the cost to improve transit service also should be accounted for when designing transit service with stochastic demand. Furthermore, the phenomena with additional waiting time are hardly acceptable for passengers; thus, restricting the probability of the phenomena is necessary for providing a high-reliability level of transit service.

Another important observation is that the discrete models are adopted in most of the literature to optimize the transit stations from a series of candidate locations. Thus, the solution quality from the discrete model heavily depends on the set of candidate locations. Besides, the transit corridor design with the discrete model is a Nondeterministic Polynomial-time hardness (shortened as NP-hardness) problem, and its solution relies on the heuristic algorithm, such as Genetic Algorithm, Swarm Intelligence, and Artificial Neural Network. The processing time of these algorithms is exponentially related to the size of the candidate sets and the number of decision variables. Therefore, a continuum approximation (CA) method is introduced and widely utilized in transit location choice [4–8].

The main purpose of this study is to develop a framework to simultaneously determine the optimal station location and departure headway of the transit system with stochastic demand. Consequently, we develop a CA-based optimization model to incorporate the abovementioned waiting time when the transit service is oversaturated. The main contribution of this paper is as follows: (1) simultaneous determination of the station location and headway setting of transit vehicle departure, (2) constraint of the service reliability, which is introduced to ensure the service robustness and represents the probability that the transit service meets the stochastic demand [9], and (3) endogenous travel demand derived from the household distributed along the corridor according to the relationship of house location choice and property development.

The outline of the paper is as follows. The next section summarizes the related literature on transit service design. Section 3 introduces the service-reliability-constrained CA model to determine the transit service under the stochastic demand and its solution procedure. Section 4 demonstrates the usefulness of the proposed model by analyzing the impact of the penalty cost and service-reliability constraint and conducting the sensitivity analysis of the number of households and value of time. Finally, the conclusion and future research are presented.

2. Literature Review

Transit service design includes the network and the corridor design. The transit network design aims to find the optimal route alignment, station location, and frequency (i.e., headway of vehicles) to serve the travel demands [10]. Comparatively, in the transit corridor design, the route alignment is typically predetermined, and thus, the station location and the frequency need to be determined. There are plenty of studies related to the transit corridor design with different travel modes, for example, regular bus, bus rapid transit, rail, and mixed [4, 5, 11, 12], compositions of travel demand, for example, many-to-one, one-to-many, and many-to-many [13–15], and objectives, for example, coverage maximization, passenger cost minimization, and system cost minimization [15–18]. Typically, the discrete models are adopted in most of the above literature in transit service design.

To overcome the shortcoming of the NP-hardness problem brought from discrete models, Daganzo and Newell [19] developed a continuum approximation (CA) model to find a near-optimal solution. Then, the CA model is utilized to obtain the station location [6, 20] and routing [4, 5, 7] in the transit design with different operating schemes, for example, all-stop and skip-stop [21, 22], demand distributions, for example, uniform and heterogeneous [7, 8, 22, 23], and network structures, for example, grid network, ring-and-radial system, and hybrid network [4, 5, 24, 25]. However, the above researches predefined that the travel demand was fixed during the operation period, which was actually uncertain under the effects of various factors, such as socioeconomic characteristics, population development, land use property, and emergency traffic incident [26]. Particularly, under COVID-19, the transit demand is substantially reduced [27]. Therefore, the transit service design is expected to become more useful and robust with consideration of the stochastic nature of the travel demand.

Generally, there are two approaches to deal with the stochastic demand in the transit service design: the stochastic optimization to obtain the minimum expected cost of the transit system or the patron [3, 26, 28–30] and the robustness optimization to minimize the cost related to the worst-case scenario, for example, empty load and overloading [10, 31–33]. In contrast, a majority of the studies focus on stochastic optimization. For example, Hadas and Shnaiderman [28] minimized the expected cost to obtain the frequency and size of transit vehicles. Huang et al. [26] determined the frequency by minimizing the expected transit network cost, consisting of the passengers and the operation costs, under the effect of the variance in passenger travel time. Hassannayebi et al. [30] proposed a rail timetabling optimization model by minimizing the average passenger waiting time. Hassannayebi et al. [29] also proposed a multiobjective stochastic optimization model considering the expected overloading. Høyem and Odeck [3] optimized the transit frequency under the stochastic demand while incorporating a penalty cost for passengers who cannot board the first vehicles after arriving.

In summary, there are some limitations in the past literature on transit service design with stochastic demand. First, most researches emphasize the determination of the frequency and timetable for the transit operation, but not the station location. Second, researches on stochastic optimization typically ignore the service robustness. Third, the studies predefine the distribution of travel demand but

Table 1: Key notations mentioned in the manuscript.

Variables	Units	Descriptions
$B(x)$	Trips/h	Accumulative demand from the corridor boundary to location x
C_V	Trips/vehicle	The capacity of transit vehicles
$d(x)$	km	Access distance
$f(\theta)$	—	Probability density function of trips θ
h	H	Headway of vehicles departure
N_p	Households	The number of households in corridor
P^c	—	A probability level of service reliability
$p(x)$	$/m^2/year	Rent price per year at location x
p_0	$/m^2/year	Rent price per year at the center
r_a	$/m^2	Land value at the boundary of the corridor
t_d	h/station	Delay per station
v_c	km/h	Cruising speed of transit vehicle
v_w	km/h	Walking speed
X	km	The boundary of the corridor/transit line
Z	h/peak period	Generalized system cost with a certain value of trips θ
θ	Trips/household/peak period	Total number of trips per household during peak period
$\underline{\theta}$	Trips/household/peak period	Set of all possible θ
$\overline{\theta}$	Trips/household/peak period	Mean value of θ
θ_σ	Trips/household/peak period	Standard various of θ
$\theta_c(h)$	Trips/household/peak period	Threshold value of trips between the saturated and unsaturated conditions
$\lambda(x)$	Trips/km	Trip demand at location x
$\rho(x)$	Stations/km	Station density
$\widehat{\Gamma}$	h/peak period	Patrons' commuting cost with the penalty cost
Γ	h/peak period	Patrons' commuting cost without the penalty cost
Γ^A	h/peak period	Penalty cost at saturation state
τ	h	Duration of the peak period
μ	$/h	Value of time
v	Vehicles	Additional number of waiting vehicles
$\psi(x)$	$/household/year	Commute cost per household per year
$\varphi(x)$	h/trip	Commuting cost per commuter
$\varphi^A(x)$	h/trip	Access time to the closest transit station
$\varphi^W(x)$	h/trip	Waiting time at the transit station
$\varphi^I(x)$	h/trip	In-vehicle travel time from the station to the destination
Λ	$/peak period	Transit agency's cost
Λ^L	km	Length of the transit line
Λ^S	Stations	The number of transit stations
Λ^K	km	Transit vehicle kilometers traveled
Λ^M	h	Transit vehicle hours traveled
$\pi^L, \pi^S, \pi^K, \pi^M$	—	Unit costs of Λ^L, Λ^S, Λ^K, and Λ^M, respectively

ignore the relationship between transit service design and house location choice. Finally, the solution procedure in transit corridor design is complex due to the NP-hardness problem caused by the discrete model.

3. Models

The problem setting, including modeling assumptions, will be presented (Section 3.1), followed by the transit service design model (Section 3.2); and the solution procedure is introduced in Section 3.3. A table of notation is provided in the Appendix for ease of reading (Table 1).

3.1. Problem Setting. Consider a linear corridor where all employment, shopping, and other activities occur in a central business district (CBD) (located at ordinate 0 km). The population of households is continuously distributed along the corridor between the center and boundary of the corridor (at X km). Travel demand is derived from households that consist of multiple workers who commute from/to their residences to/from the predetermined locations of employment in the centers. To serve the daily commute trips, the transit vehicles run between the center and boundary, stop at each station along the commuting direction to pick up and deliver patrons, and return with no stopping. In the long run, considering the household location choices, the transit agency seeks to optimize the transit system design to meet the derived travel demand.

Without loss of generality, the following assumptions are made to facilitate the model development:

A1. The land within the corridor boundary is featureless, plain, identical, and ready for residential use. The value of the land at/beyond the boundary equals the agricultural rent or opportunity cost of the land [34].

A2. The total population of households in the corridor is exogenously given and fixed. All households are assumed to be homogeneous with respect to socioeconomic characteristics (e.g., the value of time). With the aim of maximizing the utilities, households make decisions on the choices of the residential locations and housing consumption (in terms of gross floor area per household) within their budget constraints [34–37]. The utility function follows the form of a Cobb–Douglas function [34], which is additive, separable, and logarithmic. Regarding their commute behaviors, the average number of trips (denoted by θ, trips/household/peak period) per household per period (such as morning/evening peak period) is assumed to be stochastic and obeys a distribution with mean $\overline{\theta}$ and variance (θ_σ^2). (It is a simplified assumption that the travel trips per household θ are the same in any location x along the corridor. The proposed model is easily expended to satisfy the heterogeneous trips $\theta(x)$ and their distributions via minor revision.)

A3. The property developers determine the intensity of capital investment in a perfectly competitive housing market to maximize their net profits (assumed as zero) from the supply of housing service [34, 38, 39]. The housing supply of property developers is assumed to follow a Cobb–Douglas function and the first-degree homogeneous function of the land and capital inputs [36, 39–41].

The next subsection presents the model formulation of the optimal design problem.

3.2. Optimization Model. We seek to minimize the expected generalized system cost $E(Z)$, with respect to station density $\rho(x)$ (stations/km, as a function of location x) and headway h (h). Z is the set of Z (h/peak period) which is the generalized system cost with a certain value of periodic commuting trips θ. The generalized system cost Z is the sum of patrons' commuting cost ($\widehat{\Gamma}$, h/peak period) and transit agency's cost (Λ, \$/peak period). Thus, the optimization problem is formulated as follows:

$$\min_{\rho(x),h} E(\mathbf{Z}(\rho(x),h|\boldsymbol{\theta})) = \frac{1}{N_p}\left(\frac{1}{\mu}E(\boldsymbol{\Lambda}(\rho(x),h|\boldsymbol{\theta})) + E(\widehat{\Gamma}(\rho(x),h|\boldsymbol{\theta}))\right), \tag{1a}$$

subject to

$$\text{Service reliability constraint: } P\left(B(x|\boldsymbol{\theta})h \le C_V\right) \ge P^c, \tag{1b}$$

$$\text{Non}-\text{negative constraint: } \rho(x) \ge 0,\ h \ge 0,\ \forall x \in [0, X], \tag{1c}$$

where $E(\cdot)$ is a function to obtain the expectation value regarding the random variable θ and the bold symbol is the set of the corresponding normal symbol; for example, $\boldsymbol{\theta}$ is a set of possible trips θ; μ is the value of time (\$/h); N_p is the number of households; P^c is the desired level of service reliability; $B(x)$ is the accumulative demand from the corridor boundary to location x (trips/h); C_V is the capacity of the transit vehicle (spaces/vehicle). The detailed formulations of Λ and $\widehat{\Gamma}$ are described in Sections 3.2.1 and 3.2.2. $B(x)$ is derived from the travel trips per household θ and housing density $n(x)$, which is introduced in Section 3.2.3.

3.2.1. Patrons' Commute Cost. Generally, when a commuter travels by transit from the residential location x, the travel time, denoted by $\varphi(x)$ (h/trip), is the sum of three components, that is, access time to the closest transit station, $\varphi^A(x)$ (h/trip), waiting time at the transit station, $\varphi^W(x)$ (h/trip), and in-vehicle travel time from station to destination, $\varphi^I(x)$ (h/trip), as given by

$$\varphi(x) = \varphi^A(x) + \varphi^W(x) + \varphi^I(x). \tag{2}$$

To formulate the parsimonious models, continuum approximation is applied to derive the above three components. First, it is assumed that the access time $\varphi^A(x)$ is represented by the access time to the station, that is, $d(x)/v_w$, which is the access distance $d(x)$ (km) divided by the walking speed v_w (km/h). The access distance is approximately defined as a quarter of the distance between two consecutive stations [4, 7], that is, $1/4\rho(x)$.

Second, when the commuter is assumed to board the desired first vehicle, the waiting time at the stations, $\varphi^W(x)$, is approximately half of the service headway encountered by patrons [42, 43].

Third, in-vehicle travel time, φ^I consists of two components: (i) the cruising time, that is, x/v_c, where v_c (km/h) is the cruising speed of transit vehicle; (ii) the delay at the station due to acceleration, deceleration, and dwelling of the transit vehicle for boarding passengers, which can be estimated by $\int_0^x t_d \rho(u)\mathrm{d}u$, where t_d (h/station) is the delay per station and assumed to be constant [44–47]. (Alternatively, some studies assumed that t_d was a linear function of the number of boarding patrons at stations. Modest changes can be made to our models if alternative assumptions were used instead.)

Thus, the total commute cost without penalty cost Γ (\$/peak hour) is obtained by integrating the product of their commuting cost per trip $\mu\varphi(x)$ (where μ is the value of time for the transit patrons (\$/h)) and the trip demand $\lambda(x)$ (trips/peak period). The trip demand is the product of the housing density $n(x)$ (households/km) and the trips per household per period θ (trips/household/peak period); that is, $\lambda(x) = \theta n(x)$. Then, Γ is given by

$$\Gamma = \mu\theta \int_0^X n(x)\varphi(x)\mathrm{d}x. \tag{3}$$

However, the transit service with certain headway could not satisfy all of the situations in the transit system with the stochastic demand. In this case, some passengers cannot board the first vehicle after they arrive at the station and have to wait for two or more vehicles. Research indicates that the extra waiting cost of those passengers leads to different optimal service levels and is highly relevant for decision-makers [3]. Therefore, the total additional waiting time Γ^A

(h/peak period) is considered as the penalty cost to avoid the occurrence of saturation circumstances [3] as follows:

$$\Gamma^A = \mu v h N_p \max\left(0, \theta - \theta_c(h)\right), \quad (4)$$

where v is the additional number of waiting vehicles and assumed to be one in this paper [3]. That means the passengers who do not get on the first vehicle could board the second vehicle. θ_c is the threshold value of trips between the saturated and unsaturated conditions of transit vehicle capacity (trips/household/peak period); that is, $\theta_c = \tau C_V / h N_p$.

Thus, the new patrons' commute cost $\widehat{\Gamma}$ in the stochastic optimization model is the sum of the general commuting cost Γ and the penalty cost Γ^A, as follows:

$$\widehat{\Gamma} = \Gamma + \Gamma^A. \quad (5)$$

3.2.2. Agency's Cost. The transit agency's cost depends upon four metrics [25]: the length of the transit line Λ^L (km), the number of transit stations Λ^S (stations), the transit vehicle kilometers traveled Λ^K (km/hour), and the transit vehicle hours traveled Λ^M (h/hour). So, the agency's cost is given by

$$\Lambda = \tau\left(\pi^L \Lambda^L + \pi^S \Lambda^S + \pi^K \Lambda^K + \pi^M \Lambda^M\right), \quad (6)$$

where τ is the duration of the peak period and π^L (\$/km/h), π^S (\$/station/h), π^K (\$/km), and π^M (\$/h) are the unit costs related to Λ^L, Λ^S, Λ^K, and Λ^M, respectively. The four cost metrics are formulated as follows:

$$\Lambda^L = 2X, \quad (7a)$$

$$\Lambda^S = \int_0^X \rho(x)\mathrm{d}x, \quad (7b)$$

$$\Lambda^K = \left(\frac{\Lambda^L}{h}\right), \quad (7c)$$

$$\Lambda^M = \left(\frac{\Lambda^K}{v_c} + \frac{t_d \Lambda^S}{h}\right), \quad (7d)$$

where "2" in equation (7a) indicates two operation directions. Equation (7b) implies that the lines of two operation directions share the stations (e.g., in rail and BRT systems, for the bus system, Λ^S accounts for the number of station pairs that are typically deployed symmetrically). In equation (7c), Λ^L/h is the vehicle kilometers traveled per operation hour. On the right-hand side of equation (7d), the first term is the vehicle hours traveled at the cruising speed, and the second term is vehicle hours delayed by stopping.

3.2.3. House Location Equilibrium. House location includes two types of decisions: house location choice of households and house supply of property developer. According to Assumption A2, all households choose the house location depending on their utility which consists of the income, cost of commuting, consumption of the house, and other non-housing goods. Meanwhile, the property developer will supply the house according to the demand and maximize the profit when deciding on the investment (Assumption A3). Finally, the housing market will reach an equilibrium state, where no household will benefit from unilaterally changing its residential location [14]. Furthermore, the household's utility function and property's input function are both assumed to obey Cobb–Douglas forms [34, 39, 41]; thus, annual land rental price $r(x, p_0)$ (\$/km/year) and household density $n(x, p_0)$ (households/km) can be deduced and expressed as follows:

$$r(x, p_0) = b^{(-\beta/1-\beta)}\left(\beta^{-1} - 1\right)\left(p_0\left(\frac{Y - \psi(x)}{Y - \psi(0)}\right)^{(1/(1-\alpha))}\gamma\beta\right)^{(1/1-\beta)}, \quad (8a)$$

$$n(x, p_0) = \frac{\gamma\left(p_0\left(Y - \psi(x)/Y - \psi(0)\right)^{(1/(1-\alpha))}\gamma\beta b^{-1}\right)^{(\beta/1-\beta)}}{q(x)}, \quad (8b)$$

where b is the unit cost of capital (i.e., the annual interest rate plus the annual cost of depreciation of a unit of capital); α is a positive constant parameter reflecting the "attraction" of each factor in the house location choice; γ, β are prespecified constants in house supply; Y (\$/year-household) is the household's annual income; that is, $Y = 2 \times 8w\mu$, where 2 indicates two workers per household, 8 indicates eight working hours per day, and w is the annual workdays, that is, 250; $\psi(x)$ is the annual travel cost per household in the corridors; p_0 is the rental price at the center, denoted by $p_0 \equiv p(0)$; $q(x)$ (m^2/household) is the quantity of housing service consumption (i.e., gross floor area). $\psi(x)$ and $q(x)$ can be expressed as

$$\psi(x) = w\left(\mu\theta\varphi(x) + \frac{\Lambda}{N_p}\right), \quad (9)$$

$$q(x, p_0) = \frac{(1-\alpha)(Y - \psi(0))^{1/1-\alpha}}{p_0}(Y - \psi(x))^{-\alpha/1-\alpha}. \quad (10)$$

For the detailed derivations of the formulas, readers can refer to studies [8, 14, 41, 48]. The methods for housing supply and house location choice are widely utilized in literature about housing location choice and property development [41, 48–51].

Moreover, $n(x|p_0, \psi(x))$ and $r(x|p_0, \psi(x))$ must satisfy the following two constraints:

$$\int_0^X n(x|p_0, \psi(x))\mathrm{d}x = N_p, \quad (11a)$$

$$r(X|p_0, \psi(x)) = r_a. \quad (11b)$$

Constraint (11a) ensures that the total number of households is fixed. Constraint (11b) indicates that the land value at city boundaries must equal the agricultural rent r_a.

To account for the stochastic demand, the household density $n(\cdot)$ is a random variable related to travel time $\psi(x|\theta)$. Correspondingly, the length of the transit corridor X

also varies with different time $\psi(x|\theta)$. Thus, two equilibrium solutions can be expressed as the expectation value, formulated as follows:

$$E\left(\int_0^{\mathbf{X}} \mathbf{n}(x, |\mathbf{p}_0, \boldsymbol{\psi}(x|\boldsymbol{\theta}))\mathrm{d}x\right) = N_p \forall\theta \in \boldsymbol{\theta}, \tag{12a}$$

$$E(\mathbf{r}(\mathbf{X}|\mathbf{p}_0, \boldsymbol{\psi}(x|\boldsymbol{\theta}))) = r_a \forall\theta \in \boldsymbol{\theta}. \tag{12b}$$

However, the household distribution is fixed for a long time once households choose their location; thus, the expected values of house density $\overline{n}(\cdot)$ (households/km), rent price $\overline{p}_0$, and corridor boundary $\overline{X}$ (km) are used to calculate the cost of the transit system, that is, the sum of agency's cost and patrons' commute cost. Meanwhile, the expected travel time $\overline{\psi}(x)$ can be calculated by $\int_0^{+\infty} \psi(x|\theta) f(\theta)\mathrm{d}\theta$, (where $f(\theta)$ is the probability density function of trips θ). Therefore, equation (9) could be rewritten as

$$\int_0^{X} \overline{n}(x|p_0, \overline{\psi}(x)) = N_p, \tag{13a}$$

$$r(\overline{X}|p_0, \overline{\psi}(x)) = r_a. \tag{13b}$$

Equations (13a) and (13b) have two independent variables p_0, which can be easily obtained via solving the equations when given (calculated using equation (9)). According to the probability theory, the expected household distribution can be derived subsequently using equation (10b).

Annual travel cost $\psi(x)$ is the key parameter connecting the models of the housing market and transit system (9). In terms of housing location choice, it is the primary parameter to decide when designing the transit service; it is the parameter that is used to evaluate the level of service. Moreover, daily transit demand θ is assumed to obey the same distribution among different days, which also holds among different years. Because the estimated period for the house location choice is at least one year (shown as annual travel cost), the estimated period for designing transit service and house location choice should be one or more years as well. Meanwhile, stochastic demand θ is a part of $\psi(x)$, which means that $\psi(x)$ plays a vital role in the estimated period in the models of the housing market and transit system. A similar approach has been adopted in some related literature such as [14, 41, 48].

3.3. Solution Procedure. Figure 1 shows a bilevel solution framework to seek the optimal transit design. To find the solution for the upper-level model (equations (1a)–(1c)), the solution for the lower-level model, that is, the house household density $\overline{n}(x)$ and the size of the corridor $\overline{X}$, is assumed as predetermined. Correspondingly, the solution for the lower-level model can be found by equations (13a)–(13b) with given parameters of the transit system (station density $\rho(x)$ and headway h).

The transit design model is solved using a two-stage iterative method similar to studies [7, 21, 22]. In stage 1, h is fixed, and the station density $\rho(x)$ is optimized for each point $x \in [0, \overline{X}]$. In stage 2, the variables h is optimized given $\rho(x)$. Closed-form solutions are derived in each stage for $\rho(x)$ and $\{H_i\}$, respectively.

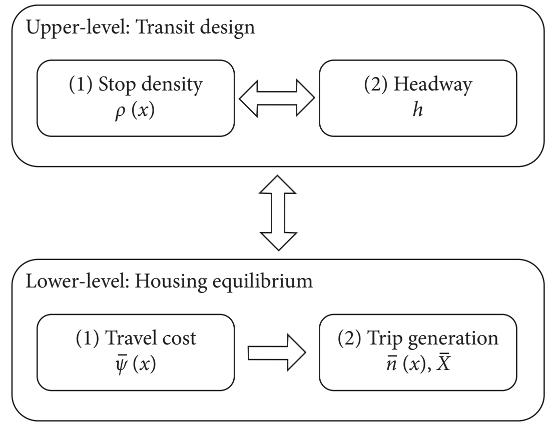

Figure 1: Flowchart of the bilevel solution method.

In stage 1, we could decompose the function variable $\rho(x)$ by location x because station densities $\rho(x)$ are independent between locations x. Then, we obtain the local extreme value at each location x (1a) by solving the first derivative condition of the objective function because the objective function is convex to the station density, which could be verified by using the second derivative test.

$$\rho^*(x) = \frac{1}{2}\sqrt{\frac{\mu\overline{\lambda}(x)h}{v_w\left(\mu t_d h\overline{B}(x) + \tau\pi^S h + t_d\tau\pi^M\right)}}, \tag{14}$$

where $\overline{\lambda}(x)$ and $\overline{B}(x)$ are the expected value of $\lambda(x)$ and $B(x)$, respectively.

In stage 2, the headway is obtained by comparing two results by (1a) without constraint (1b) and the critical value of constraint (1b) with the expected household distribution $\overline{n}(\cdot)$:

$$h^* = \underset{h\in\{h_c, \widehat{h}\}}{\arg\min}\; E(\mathbf{Z}(\rho^*, h)), \tag{15a}$$

where $\mathrm{argmin}_{z\in\{h_c, \widehat{h}\}} E(\cdot)$ returns the optimal headway between h_c and $\widehat{h}$; h_c is the critical headway determined by service-reliability constraint (1b); $\widehat{h}$ is the optimal solution of objective function (1a) without service-reliability constraint (1b), which could be obtained by finding the location where the slope is zero, given by

$$\frac{\mu\overline{\theta}N_P}{2} - \frac{\tau\pi^K\Lambda^L}{\widehat{h}^2} - \frac{\tau\pi^M}{\widehat{h}^2}\left(t_d\Lambda^S + \frac{\Lambda^L}{v_c}\right) + \mu v N\int_{\theta_c(\widehat{h})}^{+\infty}\left(\theta - \theta_c(\widehat{h})\right)f(\theta)\mathrm{d}\theta + \frac{\mu v\tau}{\widehat{h}^2}\int_{\theta_c(\widehat{h})}^{+\infty} f(\theta)\mathrm{d}\theta = 0. \tag{15b}$$

After deriving $\overline{\psi}(x)$ from the transit model solution, model (10) will be solved using off-the-shelf numerical solvers, for example, the "fsolve" function of MATLAB.

The main steps of the solution algorithm are summarized as follows:

Step 0. Initialization. Assign initial values to $\overline{X}^{(0)}$ and $\overline{n}^{(0)}(x) = N_p/\overline{X}^{(0)}$. Set the outer iteration index $n = 1$.

Step 1. Solve the upper-level transit design problem.

Step 1.0. Assign the initial headway $h^{(0)}$. Set the inner iteration index $n = 1$. Discretize the continuous range of $x \in [0, \overline{X}^{(n)}]$ to obtain a finite set of $x_n \in [0, \overline{X}^{(n)}]$ values.

Step 1.1. Compute station density $\rho(x_i)^{(m)}$ by equation (14) for each $x_i \in [0, \overline{X}^{(n)}]$, using the household distribution $\overline{n}^{(n-1)}(x_i)$ and headway $h^{(n-1)}$.

Step 1.2. Update $h^{(n)}$ by equation (15a) with $\overline{n}^{(n-1)}(x_i)$ and $\rho(x_i)^{(n)}$.

Step 1.3. Check the convergence. If the relative gap of $|h^{(m)} - h^{(m-1)}|/h^{(m-1)}$ and $|\rho(x_i)^{(m)} - \rho(x_i)^{(m-1)}|/\rho(x_i)^{(m-1)}$ in any location $x_�$ is smaller than a prespecified tolerance, for example, $\varepsilon = 0.001$, update the transit design, $\rho^{(n)}(x_i) = \rho^{(m)}(x_i)$, and $h^{(n)} = h^{(m)}$, and go to the next step; otherwise, set and go to Step 1.1.

Step 2. Solve the lower-level housing equilibrium problem.

Step 2.1. Calculate household's annual travel cost $\overline{\psi}^{(n)}(x_i)$ by equation (9) with the current transit design, $\rho^{(n)}(x_i)$ and $h^{(n)}$.

Step 2.2. Substitute $\overline{\psi}^{(n)}(x_i)$ into the nonlinear equations (11a) and (11b) and solve the housing market equilibrium to yield the results of $\overline{p}_0^{(n)}$ and $\overline{X}^{(n)}$ with all possible trips.

Step 2.3. Output the housing density $\overline{n}^{(n)}(x_i)$ by (8b).

Step 3. Check the convergence. If the relative gap $|E(Z^{(n)}) - E(Z^{(n-1)})|/E(Z^{(n-1)})$ is smaller than a prespecified tolerance, for example, $\varepsilon = 0.001$, stop and report the lowest-cost solution; otherwise, set and go to Step 1.

4. Numerical Experiment

We firstly present the parameter values and assumptions, followed by the numerical experiment with two steps: (1) uncovering the impact of the service reliability by analyzing the computational results from the proposed model and (2) analyzing the performance of the transit service from the proposed model with different numbers of household and value of time.

4.1. Parameter Values and Assumptions. The experiment assumes that the trip demand obeys a truncated normal distribution, that is, $\theta \sim \mathcal{N}(\overline{\theta}, \theta_\sigma^2, 0, +\infty)$. Table 2 summarizes the parameter values of the regular bus system and house equilibrium used in the numerical experiment. The parameters of the transit system are retrieved from references [14, 25, 43], and the value in house model is adopted/adapted from the previous studies [39, 52, 53].

Generally, the analysis includes the optimal preference (i.e., the boundary of corridor, household density, station density, and headway) and the corresponding cost related to the transit system (i.e., the costs of patrons, agency, and total system) [42, 43]. With the nature of the stochastic demand, standard deviation (SD) is used to estimate the undulation of the system cost. Due to the complex formulation of the objective function, the standard deviation of the transit system cost could not be calculated directly. Thus, the research estimates the standard deviation from the results of 100 thousand experiments with a specific trip (θ) randomly derived from a truncated normal distribution $\mathcal{N}(\overline{\theta}, \theta_\sigma^2,, 0, +\infty)$ each time.

4.2. Impact Analysis of Service Reliability. The section presents the reliability analysis of transit service in the regular bus with different desired levels of service reliability ($P^c = \{0, 0.5, 0.95\}$, where $P^c = 0$ means that the service-reliability constraint always holds). Table 3 summarizes the parameters of transit service design and housing market, as well as corresponding costs such as patrons, agency, and system costs. Figure 2 depicts the distributions of the station location. Figure 3 illustrates the distribution of expected empty seats, which is calculated by $(C_v - N\theta/\tau) f(\theta)$ with a special θ.

Table 3 shows that the service-reliability constraint (i.e., $P^c > 0$) leads to different transit service designs and household distributions. Compared to the result without service-reliability constraint (i.e., $P^c = 0$), the headway of vehicle departures decreases (26.2% when $P^c = 0.95$). Correspondingly, it leads to a higher expected agency's cost (24.7%) and a lower expected patron's commute cost (6.4%). With a lower commute cost, households can pay more in the housing market, thus resulting in a denser household distribution (2.3%), higher average house rental price (0.1%), and a smaller average floor area consumed per household (0.6%). Moreover, a high desired reliability level of transit service decreases the total additional waiting time Γ^A because the headway is small and the capacity of transit service is high, which decrease the probability of oversaturated service.

Due to the slight differences in parameters of transit system costs and housing market among different $P^c s$, we conduct the analysis of variance with t-tests (Table 4). The results show that p-values are less than 0.001, which verifies the statistical significance among different $P^c s$.

Interestingly, the study identifies less obvious difference in expected system costs with three desired levels after considering service-reliability constraint. It is due to the facts that (1) higher-frequency transit service decreases patron's commuting cost, (2) shorter transit corridor decreases both commute cost and agency's cost, and (3) agency scatters the station locations (3.4% in Figure 2) to reduce the agency's cost related to transit stations. Moreover, the standard deviation of system cost decreases. The result indicates that the service-reliability constraint not only improves the reliability of transit service but also decreases the variation of system cost with a small increase of expected system cost.

The above results also hold true in the transit system with a higher desired level of service reliability compared to the one with a lower level.

Table 2: Parameter values in the numerical experiment.

Parameters of transit system	Value	Parameters in house model	Values
v_w (km/h)	2	D (km)	5
v_c (km/h)	25	N_p (households)	10^3
t_d (min/station)	0.5	μ (\$/h)	20
C_V (spaces/vehicle)	80	$\overline{\theta}$ (trips/households)	2
π_L (\$/km/hour)	$6+0.2\mu$	θ_σ (trips/households)	0.25
π_S (\$/station/hour)	$0.42+0.014\mu$	γ	0.08
π_K (\$/vehicle/km)	0.59	α	0.75
π_M (\$/vehicle)	$2.66+3\mu$	β	0.7
τ (h)	2	b (year)	6%
w (days/year)	250	r_a (\$/km^2/year)	3,000

Table 3: Result of the proposed model under different desired levels of service reliability.

P^c	0	0.5	0.95
X (km)	22.98	22.79	22.45
Headway h (min)	5.39	4.80	3.98
Average station space (km)	0.714	0.723	0.738
Expected patrons' cost $\widehat{\Gamma}$ (h/household/peak period)	0.771	0.745	0.722
Expected agency's cost Λ (h/household/peak period)	0.227	0.247	0.283
Expected system cost Z (h/household/peak period)	0.998	0.992	1.004
Expected additional waiting time Γ^A (h/household/peak period)	0.022	0.008	0.0003
SD of system cost (h/household/peak period)	0.112	0.103	0.091
Average house rental price (\$/m^2/year)	141.75	141.81	141.93
Average floor area consumed (m^2/household)	130.88	130.63	130.09
Average household density (households/km)	43.52	43.87	44.55

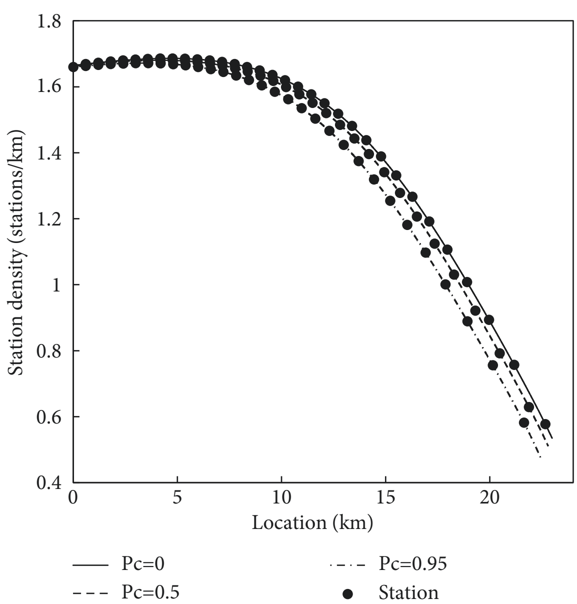

Figure 2: Station location with different desired levels of service reliability.

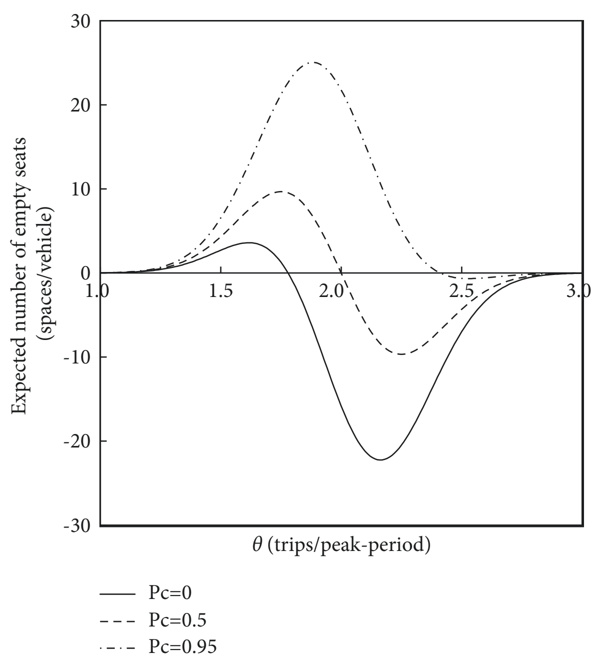

Figure 3: Distribution of the number of the expected empty seats.

In Figure 3, the negative value on the vertical axis means the expected number of passengers without boarding vehicles. Figure 3 shows that the seats of transit vehicle are effectively utilized in the transit service without service-reliability constraint, while numerous passengers cannot board the vehicle in most of the situations. Instead, there are more empty seats and fewer passengers without boarding vehicles under a high desired level (e.g., $P_c = 0.95$). The result indicates that a higher desired level brings about a serious waste of transit seat usage, but a low one generates more passengers without boarding vehicles and seriously oversaturated transit service. Therefore, it is important for the agency to choose an appropriate desired level to balance transit seat usage and service reliability.

Table 4: P-value of t-tests.

Method	$P^c = 0$ versus 0.5	$P^c = 0.5$ versus 0.95	$P^c = 0$ versus 0.95
Patrons' cost $\widehat{\Gamma}$	≤ 0.001	≤ 0.001	≤ 0.001
Agency's cost Λ	≤ 0.001	≤ 0.001	≤ 0.001
System cost Z	≤ 0.001	≤ 0.001	≤ 0.001
House rental price	≤ 0.001	≤ 0.001	≤ 0.001
Floor area consumed	≤ 0.001	≤ 0.001	≤ 0.001
Household density	≤ 0.001	≤ 0.001	≤ 0.001

4.3. Sensitivity Analysis. We further analyze the impact of service reliability under different numbers of households and values of the time.

4.3.1. Value of Time. The value of time (μ) related to the average income is the opportunity cost of the time that passengers spend on their trips. The variation of the time value in cities and countries leads to difference in costs of building the transit system and operating transit service, thus resulting in different transit designs. Hence, sensitivity analysis of time value is conducted in this section. Figure 4 shows the optimal headway of the transit service under different values of time.

According to Figure 4, the transit service is affected by the value of time when the desired level of service reliability is smaller than a special value, that is, the actual level without service-reliability constraint. When the desired level is higher than the special value, critical values of headway are adopted in the service-reliability constraint, which is unrelated to the value of time. Therefore, the headways remain the same among different values of time. The result indicates that the agency can directly determine the headway from the service-reliability constraint in this status. When the desired level is lower than the special value, the transit service has lower service reliability in the developing countries/cities (e.g., $\mu = 5$) in contrast to the status in the developed countries/cities (e.g., $\mu = 20$). The possible reason is that the passengers could endure higher waiting time compared with high agency's costs in the developing countries. The finding suggests that considering the service-reliability constraint is much more necessary in the developed countries.

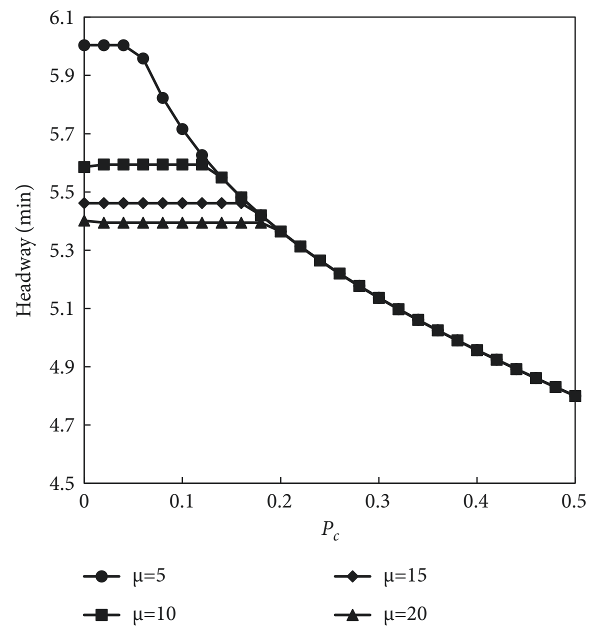

Figure 4: Optimal headway under different values of time.

4.3.2. The Number of Households. A sensitivity analysis is conducted to investigate the effect of service reliability under different travel demands. To quantify the relationship between the agency's cost and desired level of service reliability, the concept of investment-to-profit ratio (IPR) is introduced (h/household/10%). IPR is the increased agency's cost divided by the improved desired level. Figure 5 presents the IPR distribution in desired levels of service reliability under different numbers of households. Figure 6 shows the relationship between the optimal headway and desired level.

Figures 5 and 6 show that IPRs for the agency are zero and the headways are unchanged in many cases; for example, $P_c \leq 0.6$ when $N_p = 500$. The result means that the transit service without the service-reliability constraint also satisfies a desired level of service reliability ("actual level"). Meanwhile, the actual level is negatively correlated to the travel

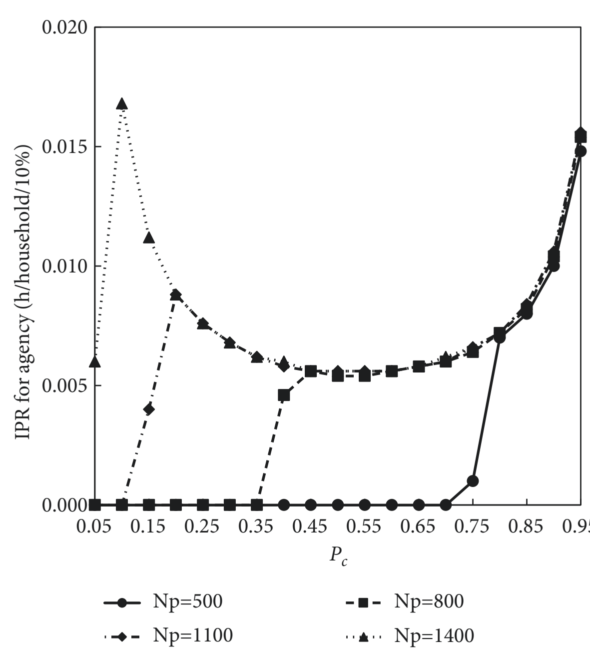

Figure 5: IPRs for agency under different numbers of households.

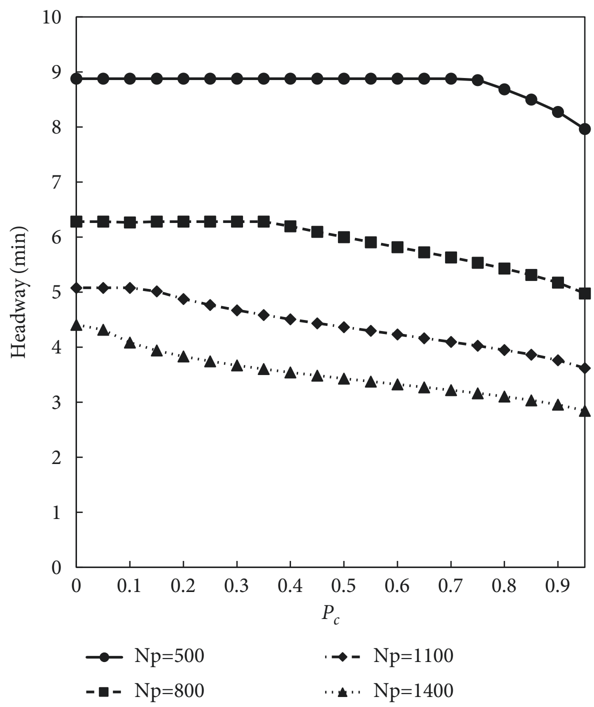

Figure 6: Optimal headway under different numbers of households.

demand. The phenomenon suggests that the agency should pay more attention to the selection of the desired level when the travel demand is high. Moreover, when the desired significance level is higher than the actual level, the total increased cost for the agency (to improve the service reliability) is linearly related to travel demand. It can be easily concluded from the same IPRs, which are averaged by the number of households, among different travel demands. Those findings suggest that the desired level of service reliability should be selected for the agency with the tradeoff between cost and service reliability according to the travel demand.

5. Conclusions

The paper proposes a continuum approximation model for transit service design under the stochastic demand. The model provides an optimal headway for departure vehicles and the location of stations based on the sum of the patrons' commute cost and the agency's cost. A penalty cost is introduced to account for the overcapacity scenario. Moreover, the model reflects the robustness of transit design through the constraint of service reliability. Through applying the proposed CA model, we conduct the impact analyses of the service reliability and a series of sensitivity analyses (e.g., the number of households and the value of time) in the numerical experiment. There are several interesting findings revealed from the numerical experiment:

(1) The service-reliability constraint leads to different parameters of transit designs (e.g., the headway of departure vehicles and location of stops) and housing markets (e.g., the density, floor area of the house, and rental price).

(2) The service-reliability constraint could not only improve the reliability of transit service but also decrease the variation of system cost with a small increase of expected system cost.

(3) A higher desired level of service reliability results in a serious waste of transit seat usage, while a low level of service generates a seriously oversaturated service.

(4) Transit service without the reliability constraint achieves the desired level of service, which is negatively related to the travel demand and positively related to the value of time.

(5) After the desired level of service reliability reaches the actual level without the service-reliability constraint, the increased cost for the agency is linearly related to travel demand to improve the service reliability.

(6) The transit service is affected by the value of time before the desired level of service reliability reaches the actual level without the service-reliability constraint.

(7) Agreeing with the real-world observations, the station is denser near the center than the boundary because the household and travel demand are denser near the center considering the behavior of house location choice. The phenomenon is somewhat different from previous studies that assume a uniform housing supply or transit travel demand [14, 17, 54].

(8) Consisting with the researches [3, 55], the optimal headway becomes smaller and thus decreases the total additional waiting time of passengers who can be able to board the first vehicle by means of including the overflow delay in the objective function.

(9) Due to the small headway, a high level of transit service decreases the total additional waiting time. The phenomenon is consistent with the result of study [56].

Based on the above findings, several suggestions are proposed as a reference for practical applications:

(1) It is necessary that the transit service design should be integrated with the effect of housing location choice and service-reliability constraint under the stochastic demand.

(2) The agency should consider seat usage and investment to balance the service reliability when choosing the desired level of service reliability.

(3) Considering the service-reliability constraint is more important in developed countries/cities than in developing countries/cities; it is also more important in the transit service with high demand.

(4) The agency could directly determine the headway from the aspect of service-reliability constraint when the desired level of service reliability is higher than the actual level without the service-reliability constraint.

Although we model the transit service design model under the stochastic demand, there are two main limitations that need to be addressed in future research. The first limitation is that the stochastic demand is a one-dimensional variable; that is, the travel trips at all locations are assumed to be the same, which may not be realistic. The biggest hindrance is to confirm the possible region of a multidimensional, even infinite-dimensional random variable when the worst scenario occurs, for example, overloading and empty loading. The second is the restrictive hypotheses, such as the homogeneity of the households, closed environment with only endogenous trip demand, and simple transit system with only one center, one transit line, and one transit mode.

Appendix

Notation

See Table 1.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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