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Research Article

Parking Permit Scheme for Morning Commute considering Parking Search

Duo Xu and Huijun Sun

Key Laboratory of Transport Industry of Big Data Application Technologies for Comprehensive Transport, Beijing Jiaotong University, Beijing 100044, China

Correspondence should be addressed to Duo Xu; 516472407@qq.com

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Improving parking efficiency is essential to promoting the reform in urban transportation. But the large amount of deadweight costs caused by the parking is often underestimated because it is difficult to measure. Based on the existing investigations from the small fraction of cruising vehicles, this paper explores the influencing factors of the parking issue and describes it by the user equilibrium model. Then, two types of permit management schemes were proposed, lot-based and spot-based. By analyzing their performance in reducing system cost, three conclusions were drawn. Firstly, parking search leads to traveler's schedule and location adjustments, raises the trip cost, reduces the parking lot occupancy, and makes the parking issues "invisible." Secondly, permit scheme levels up managers' control, and it performs well in reducing deadweight loss, but only by eliminating the search cost, the deadweight loss can be fundamentally reduced. Thirdly, reducing parking search needs information guidance; with the rapid growth of urban parking demand, managers should make a transition to the permit scheme with parking information.

1. Introduction

Electrification and automated and shared vehicles will likely revolutionize urban transport and further promote travel efficiency, energy conservation, and emission reduction. Until then, parking issues will remain an inherent and rather unpleasant attribute of car travel in metropolitan regions [1].

Studies show that although green travel has been widely advocated, private cars are still the main way of urban travel. In the city of Beijing, private cars account for 30% of the travel demand, and each car travels more than 3 times with about 50 km distance a day on average [2]. However, as the endpoint of car travel, finding a parking spot often constitutes an appreciable fraction of the total travel time and contributes to traffic congestion and negative externalities [3–5]. According to the survey data, the time cost that a vehicle looks for a vacant spot accounts for 30–50% of the total travel time, nearly 70% of the direct travel cost, and 30% of traffic jams [6–9]. In Chicago, cruising for parking produces a total of 63 million miles of distance and 48,000 tons of carbon dioxide annually [10].

Over the years, charging (includes static, multi-phases, or dynamically varied with time and space) plays an important role in regulating demand, and it is widely adopted in practice [11–13]. However, it has limitations in dealing with parking searches. First, it cannot regulate demand accurately to meet the full utilization of parking lots with no vehicles cruising. Second, the charge itself does not contain information to help drivers avoid the search. In practice, managers often raise charges to keep a small number of parking spots vacant (5%–15%) and consider it an effective compromise [14, 15].

With the development of information and communication technology, quantity control approaches based on permits are introduced into the field of traffic demand management in recent decades. They originated from the regulation approach in environmental externalities [16, 17] and have developed to several forms, e.g., the license-plate rationing, the appointment-based schemes, the cap-and-auction scheme, the cap-and-trade scheme, etc. Permit scheme applied in the parking field is meaningful; first, it performs well in matching supply and demand, avoiding leap growth in costs when demand overflows; second, it is compatible with identity authentication, information release, and flexible price adjustment and is suitable for the integration of private and incomplete open parking resources, e.g., residential parking spots. They provide a potential opportunity to realize smart, shared parking.

In the empirical study of parking, the existing knowledge is strongly biased towards the reaction of drivers to parking prices [18–22], while the other critical factors, e.g., the occupancy rate, delay, and distance to destination, remain obscure [1]. In recent years, various methods were used to observe parking searching and cruising, e.g., follow vehicles, park-and-visit tests, in-car video, and GPS tracking [4, 23–31], but they did not reach completely consistent conclusions in studying travelers' search behavior and its impact on the system.

In the theoretical research field, the literature on parking permits is in two categories. One is the parking problem itself and to study the solution of reducing travelers' delay time or queuing time; the other is to make full use of multiple trip modes or road networks and to study how to reasonably allocate travel demand [32–39]. However, the parking search has not been fully considered. Search is more complicated than queuing. First, queuing cost is linear and definite, and travelers get parking services after waiting for a certain period. The parking lot supply is also fully utilized. But the search cost is nonlinear, and it increases with the reduction of vacant parking spots [6, 14, 40–42]. When it is higher than travelers' expectations (equilibrium point of the system), parking facilities may not be fully used, and the excess demand will expand the parking range. Second, queuing is affected by travelers' departure/arrival time, which can be reduced by dispersing travelers' departure/arrival. But searching is affected by parking lot utilization (essentially the arrival order). The time is longer for the late arrivals, which cannot be solved by decentralizing the departure time (as the current permit does).

Therefore, in order to explore solutions to the parking search problem, this paper summarizes the results of existing empirical research. First, parking search and cruise are common in the parking process, and in most cases, they take 3 to 5 minutes. Second, by comparing the data of city center and suburban, the scarcity of parking facilities affects the cruising time. Third, drivers who are more familiar with the road environment spent less time in cruising, indicating that the information is helpful for parking. Fourth, some recent studies have found that the search time and distance become shorter, and the proportion of travelers who stop before the destination increases. It is inferred that travelers can perceive the use of parking facilities at various locations more accurately and make trade-offs in advance. Accordingly, the parking problem is summarized into four aspects, the parking search cost, associated trip delay caused by departure time adjustment, the extra walking cost caused by parking location adjustment, and underutilization of the parking lot.

On this basis, this paper establishes a user equilibrium model to study the relationship between parking lot utilization and parking choices (parking position and departure time selection), then proposes permit schemes based on quantity control, and explores their feasibility in solving the parking search problem. Correspondingly, three schemes are introduced.

No Permit Scheme. Managers only charge and do not use permits (static charge is adopted). Travelers compete from the dimension of temporal and spatial. This scheme is set as a benchmark.

Parking Lot Permit Scheme. Manager issues parking permits based on independent parking lots, and travelers with permits can park anywhere in the corresponding parking lot. It is theoretically similar to the permit proposed by Zhang et al. [32] that does not distinguish between parking spaces. In practice, it is similar to the offline permit scheme (regularly in a parking facility), but the cycle is short (e.g., within a day).

Parking Spot Permit Scheme. Manager issues parking permits based on specific parking spots, and travelers with parking permits need to park in a designated spot. It is theoretically similar to the permit applied in the field of shared parking (the opening time of the supplier needs to match the parking time of the demander). In practice, it is applied in the management of private spots in a residential area or some spots reserved and bound to specific vehicles (parking spot marked with license-plate number in the office building) (Table 1).

The remainder of this paper is organized as follows. Section 2 presents the assumptions and analyzes the components of the trip costs. Section 3 deduces the user equilibrium in a no permit scheme. Two parking lot permit schemes are elaborated in Section 4. In Section 5, a numerical experiment is provided. Section 6 concludes the paper.

2. Assumptions

This paper concentrates on parking issues in a business district (not unique) during the morning peak, mainly considering the following.

Firstly, commuters can accumulate experience from daily trips, and they try to achieve user equilibrium. Secondly, commuters arrive at the parking lot in a concentrated time, which leads to a rapid and continuous change in parking lot occupancy rate from low to high in a short period. The impact of search problems is obvious. Thirdly, commuting trips have long parking times in general, and the problem can be simplified to a one-time parking problem (commuters do not leave away in the study period).

To facilitate the presentation of the essential ideas without loss of generality, we make the following basic assumptions.

A1: Network. As shown in Figure 1, in a linear network structure, the business district (office) is at O, and the residential area (home) is at the other end H of the city. A

Table 1: Description of three schemes.

Scheme name	Abbreviation	Description
No permit	Scheme n	The manager sets a single charge rate on the basis of first come first serve.
Parking lot permit	Scheme p	The manager issues permit for different parking lots. Commuters park in a lot by the permit of that lot.
Parking spot permit	Scheme r	The manager issues permit for every spot. Commuters park in a spot by the permit of that spot.

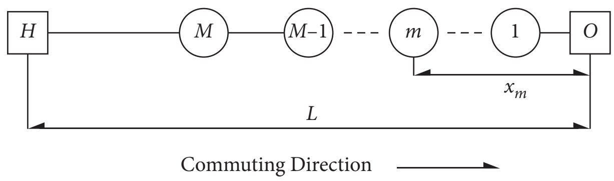

Figure 1: Linear urban structure and parking facilities.

highway with length L connects H and O (the capacity is large enough, and there is no congestion).

Assumption 1 is the simplification of a certain OD pair in a city. (1) Vehicle drivers prefer a short walking distance (generally no more than 0.3 km–0.6 km or within one block) after parking [43]. (2) The commuter number to a certain district is far less than the total commuter number in the city. Vehicle velocity is mainly affected by the external traffic; we set the capacity of the road network as large enough, and vehicle velocity is constant for simplicity.

A2: Commuters. There are N commuters who travel from H to O once a day, they choose the departure time and parking location, and their working start time is t^*. Their trip is composed of three parts: (1) drive to the parking lot; (2) search for a vacant spot in the lot; (3) walk to O. Their time value is α (during a trip) and β (arriving early or late), respectively.

Assumption 2 studies morning peak commuting. (1) Trips are concentrated, and parking competition is obvious. (2) The relatively long parking time can avoid the impact of leaving halfway.

A3: Parking Lot. There are M off-street parking lots distributed in a straight line within an acceptable walking distance approximately to O. The capacity and location of the parking lot m $(m = 1, 2, \ldots, M)$ are k_m and x_m, respectively. We set the walking distance inside the parking lot as 0. To satisfy both the scarcity of parking spots near O and the satisfiability of parking demand, we set $k_m < N < \sum_M k_m$.

Assumption 3 describes the current parking situation in urban areas. (1) Because the parking supply close to the destination is insufficient, some commuters may not be able to park at the destination but can park within a certain distance (e.g., in Beijing, there are more than ten parking lots in a square kilometer in the central area of the city). (2) Parking lot usually has more than one entrance and exit, and its internal spots are nonlinearly distributed. Therefore, we ignore the spots' differences for simplicity.

Further, the rest of the notations throughout the paper are listed in Table 2, and we will directly use them in the following sections.

Correspondingly, the trip cost includes travel cost (car driving and walking), search cost, delay cost, and parking charge.

2.1. Travel Cost. When commuter parks at x_m, his/her drive time and walk time are $(L - x_m)/v_c$ and x_m/v_w, respectively $(v_c > v_w)$. Then, the travel cost is

$$C^t(x_m) = \alpha\left[\frac{(L - x_m)}{v_c} + \frac{x_m}{v_w}\right]. \tag{1}$$

2.2. Search Cost. The number of occupied parking spots is no more than the capacity, and the formula is $n(x_m, t) \leq k_m$. If there is no parking guidance information, commuters will search for vacant parking spots one by one in the parking lot. The expected number of spots searched before finding a vacant one is $k_m/[k_m - n(x_m, t)]$ [40]. The expected search time satisfies $t_s(x_m, t) = \lambda k_m/[k_m - n(x_m, t)]$ $(\lambda \ll C_t(0))$. If commuters receive parking information in advance, they will go directly to the designated spot. Considering that the car speed will not be reduced with the spot information, but the search distance is much longer, we set $\pi > \lambda$. The expected search cost in the scheme j is

$$C_j^s(x_m, t) = \begin{cases} \dfrac{\alpha\lambda k_m}{k_m - n(x_m, t)}, & \text{scheme } j \text{ without information,} \\ \alpha\pi, & \text{scheme } j \text{ with information.} \end{cases} \tag{2}$$

2.3. Delay Cost. The delay cost is the product of the delay time and the time value β. The delay cost of commuters who park at lot m and time t is

$$C^d(x_m, t) = \beta\left|t^* - t - t_s(x_m, t)\frac{x_m}{v_w}\right|. \tag{3}$$

2.4. Parking Charge. We set all parking lots to have nonnegative parking charges (the charging price in the no permit scheme and the permit price in the parking lot and parking spot scheme), and there is $p_j(x_m, t) \geq 0$.

In conclusion, the total trip cost of the commuter park at lot m and time t in the scheme j is

$$C_j(x_m, t) = C^t(x_m) + C_j^s(x_m, t) + C^d(x_m, t) + p_j(x_m, t). \tag{4}$$

Table 2: Nomenclature.

Notation	Interpretation
x_m	Location of the parking lot m
x_j^*	Furthest parking location in scheme j
t_j^{s,x_m}	The first arrival time to the parking lot m in the scheme j
t_j^{e,x_m}	The last arrival time to the parking lot m in the scheme j
t_j^{i,x_m}	The $i-th$ arrival time to the parking lot m in the scheme j
$t_s(x_m,t)$	The expected search time at the parking lot m and time t
$n(x_m,t)$	Occupied parking spot number at parking lot m and time t
λ	Unit search time without information guidance
π	Average search time with spot information
v_c	Car velocity
v_w	Walking velocity
C	Trip cost
C^t	Travel cost
C_j^s	The expected search cost in the scheme j
C^d	Delay cost
$p_j(x_m,t)$	Parking charge at the parking lot m and time t in the scheme j
C^*	Trip cost under user equilibrium
τ	The minimum distinguishable time interval between two adjacent arrivals

Among the costs in (4), the travel cost is related to the parking location x_m, the delay cost is related to the parking time t and the parking location, and the search cost and charging price are also related to the scheme j.

3. User Equilibrium in the No Permit Scheme

At present, the most common form of urban parking management is to set a certain amount of charging fees (without spot information) in advance. They are generally applicable to a wide temporal and spatial range (e.g., day or night, suburban, or city center). When we study a small range (e.g., a traffic zone during morning peak), charges can be regarded as constant.

Since the total parking spot number is greater than the total commuter number, the trip time (cost) has a boundary. Rational commuters will weigh various factors and try to minimize trip costs by individually adjusting their strategy (e.g., schedule and parking location). If no one can reduce his/her trip cost by changing strategy individually, the system reaches user equilibrium.

3.1. Parking Lot Choice of the Commuter. In the scheme n, parking charges do not vary with time and location. We set parking charges that are all equal to zero for simplicity. Commuters' choices involve two dimensions, spatial (where to park) and temporal (when to park). In this section, we focus on the spatial dimension: (1) derive the user equilibrium between parking lots; (2) calculate the parking number in each parking lot and get the spatial parking distribution.

According to (2), when the parking number is close to the lot capacity, the commuter's search cost becomes so large that it will exceed the equilibrium cost. From this point, we get Remark 1.

Remark 1. If commuters are rational and punctual, there will be no traveler to arrive at the office after the time t^* under user equilibrium.

Proof 1. If a commuter arrives at O later than t^*, the positive delay cost can be replaced by a higher search cost. It means that the parking lot can attract more commuters who planned to park elsewhere and reduce their travel cost, which contradicts the assumption.

This means that the parking lot can attract more commuters who originally planned to park in other locations.

According to Remark 1, the final parking number of the parking lot m can be represented by $n(x_m,t^*)$.

Given the time point when the system reaches the final state, we further verify the existence and uniqueness of the user equilibrium of the scheme n. We construct the following model based on the Beckman programming model (Beckman programming model (proposed in 1956) which satisfies Wardrop's user equilibrium criteria):

$$\begin{aligned} &\min Z(N) = \sum_M \int_0^{n(x_m,t^*)} C_n(x_m,t^*,w)\mathrm{d}w, \\ &s.t. \sum_M n(x_m,t^*) = N, \\ &\quad n(x_m,t^*) \geq 0. \end{aligned} \tag{5}$$

When commuters are traveling without congestion, the parking lots they choose can be seen as routes without overlapping arcs in the road network. The final number of parking lots $n(x_m,t^*)$ is equivalent to the route flow, which satisfies the conservation condition.

The Lagrangian function of (5) is

$$La = Z(N) + \mu\left[N - \sum_M n(x_m,t^*)\right], \tag{6}$$

where μ is the Lagrangian multiplier. Omitting proof and derivation, the Kuhn–Tucker condition of (5) is

$$\begin{aligned} n(x_m, t^*) \cdot [C_n(x_m, t^*) - \mu] &= 0, \\ C_n(x_m, t^*) - \mu &\geq 0. \end{aligned} \tag{7}$$

Therefore, the minimum trip cost C^* exists and equals the Lagrangian multiplier μ. From (11), if $n(x_m, t^*) > 0$, $C_n(x_m, t^*) = C_n^*$, and if $n(x_m, t^*) = 0$, $C_n(x_m, t^*) \geq C_n^*$.

According to the cost settings in (2) and (4), trip costs are related to the parking number in the objective lot, so there is $\partial C_n(x_m, t^*)/\partial n(x_\chi, t^*) = 0$ (χ is also the serial number of the parking lot, and it is independent of m), and

$$\frac{\partial^2 Z}{\partial n(x_m, t^*) \partial n(x_\chi, t^*)} = \begin{cases} \dfrac{dC_n(x_m, t^*)}{dn(x_\chi, t^*)}, & x_m = x_\chi, \\ 0, & x_m \neq x_\chi. \end{cases} \tag{8}$$

According to equation (2), $dC_n(x_m, t^*)/dn(x_m, t^*) > 0$. The Hessian matrix of the objective function (5) is a positive definite matrix. It has a unique solution C_n^*.

The UE programming represented by (5) has a convex objective function (the cost of each parking location is a strictly increasing function of parking number), but the parking number at each location is not necessarily convex. Therefore, the programming problem is nonlinear, and there may be multiple solutions. Fortunately, different from the general traffic allocation problems, parking spot users instinctively prefer parking close to the destination (the destination is attractive for traffic flow). Based on this, we continue to discuss whether there is a unique parking distribution.

Since the total parking spot number is greater than the commuter's number, the parking distribution has a boundary in the scheme n. We denote the farthest parking location as x_n^*; then, for any $x_m > x_n^*$, there is $n(x_m, t^*) = 0$.

From (4) and (7), we get Remark 2 and Remark 3. □

Remark 2. Commuter who parks at the farthest parking location x_n^* does not bear the delay cost, and his/her search cost is $\alpha\lambda$.

Proof 2. If the sum of search cost and delay cost is greater than $\alpha\lambda$ at the location x_n^*, a commuter parked in a further location $x_n^* + \Delta x$ ($\Delta x \longrightarrow 0$) has a lower trip cost, which contradicts the assumption. □

Remark 3. In the scheme n, when the system reaches user equilibrium, parking distribution (or the utilization of each parking lot) is unique.

Proof 3. From Remark 2 and equation (4), the equilibrium cost C_n^* can be expressed as

$$C_n^* = \alpha \left[\frac{(L - x_n^*)}{v_c} + \lambda + \frac{x_n^*}{v_w} \right]. \tag{9}$$

From Remark 2 and (9), for the parking lot m with positive $n(x_m, t^*)$, there is $C^t(x_m) + C_n^s(x_m, t^*) = C_n^*$.

The search cost satisfies $C_n^s(x_m, t^*) = C_n^s(x_m, t^* - x_m/v_w)$ because all the commuters finish their parking at the time $t^* - x_m/v_w$ and there will be no commuters parked between $t^* - x_m/v_w$ and t^*. The final parking number $n(x_m, t^*)$ at each location satisfies $n(x_m, t^*) = n(x_m, t^* - x_m/v_w)$. Combined with (9), there is

$$n(x_m, t^*) = k_m - \frac{\lambda k_m}{(x_n^* - x_m)/v_w - (x_n^* - x_m)/v_c + \lambda}. \tag{10}$$

Combining conservation condition (5), there is

$$\sum_M \left[k_m - \frac{\lambda k_m}{(x_n^* - x_m)/v_w - (x_n^* - x_m)/v_c + \lambda} \right] = N. \tag{11}$$

Theoretically, we assume that the number of parking lots M is large enough and the distance between them is close enough; then, we can rewrite (11) as the integral form.

$$\int_{x_n^*} \left[k_x - \frac{\lambda k_x}{(x_n^* - x)/v_w - (x_n^* - x)/v_c + \lambda} \right] \mathrm{d}x = N. \tag{12}$$

We denote k_x as a constant instead of $k(x)$ because parking capacity is independent of location x. Take the derivative of the LHS of (12) with x, and we get $\lambda k_x (1/v_c - 1/v_w)/[(x_n^* - x)/v_w - (x_n^* - x)/v_c + \lambda]^2 < 0$. The LHS of (12) is a monotonically decreasing function, so there is a unique solution. Similarly, the discrete function (11) has a unique solution. □

3.2. Parking Time Choice of the Commuter. In Section 3.1, we discussed the commuters' choice in spatial dimension as well as the final utilization of each parking lot at the moment t^*. In this section, we are concerned about the parking time choices within the parking lot (temporal dimension). We take the last parker in each parking lot as a benchmark and extend the research to the time dimension to study commuters' arriving time (to the parking lot).

Under user equilibrium, the sum of search and delay costs is equal for commuters arriving at different times in the same parking lot. Referring to the cost of the last arrival, there is $C_n^s(x_m, t) + C^d(x_m, t) = C_n^s(x_m, t^*)$ (excluding the same travel cost). Then, we get

$$\begin{aligned} &\frac{\alpha\lambda k_m}{k_m - n(x_m, t)} + \beta \cdot \left[t^* - t - \frac{x_m}{v_w} - \frac{\lambda k_m}{k_m - n(x_m, t)} \right] \\ &= \frac{\alpha\lambda k_m}{k_m - n(x_m, t^*)}. \end{aligned} \tag{13}$$

For the first arriver, his/her search time is λ, and we calculate his/her arrival time at the lot m:

$$t_n^{s,x_m} = t^* - \frac{\alpha}{\beta} \cdot \frac{\lambda [n(x_m, t^*) - 1]}{k_m - n(x_m, t^*)} - \frac{x_m}{v_w} - \lambda. \tag{14}$$

For the last arriver, his/her search time is $\lambda k_m/[k_m - n(x_m, t^*) + 1]$, and we calculate his/her arrival time to the parking lot m:

$$t_n^{e,x_m} = t^* - \frac{x_m}{v_w} - \frac{\lambda k_m}{k_m - n(x_m, t^*) + 1}. \quad (15)$$

All the other commuters' arrival time t_n^{i,x_m} ($i \in [1, n(x_m, t^*)]$) is between t_n^{s,x_m} and t_n^{e,x_m}.

3.3. Graphic Representation of Equilibrium. In Sections 3.1 and 3.2, we discussed the equilibrium of commuters' choices between parking lots and at different times, respectively. In this section, we draw figures to describe the equilibrium of multiple dimensions.

According to (2), we present the curve of search cost in Figure 2. The search cost keeps at a low level when the occupancy rate of parking spots is low (e.g., under 80%) and then increases significantly when the occupancy rate approaches 100% (it is assumed that commuter spends one unit time searching for one spot; when the occupancy rate reaches 0.8, the expected search time will increase to 5 times, and when it reaches 0.95, the cost of parking search will increase to 20 times).

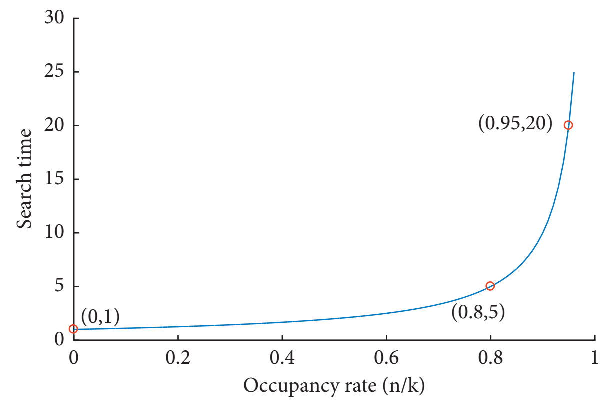

Figure 2: Search time varying with the occupancy rate.

3.3.1. Occupancy-Time Dimension. Figure 3 shows the relationship between a cumulative arrival rate and delay time. It takes four ultimate occupancy rates (0.96, 0.9, 0.8, and 0.5) as an example (blue, orange, yellow, and purple curves, respectively). It shows the parking lots with a high occupancy rate; most of the commuters will bear more delay time. When the system reaches equilibrium, the sum of search cost and delay cost of travelers in the same parking lot is equal; the high delay time compensates for the high search time of late arrivals. Parking lots with a higher occupancy rate (the curve deviating from the origin) are the locations close to the destination O.

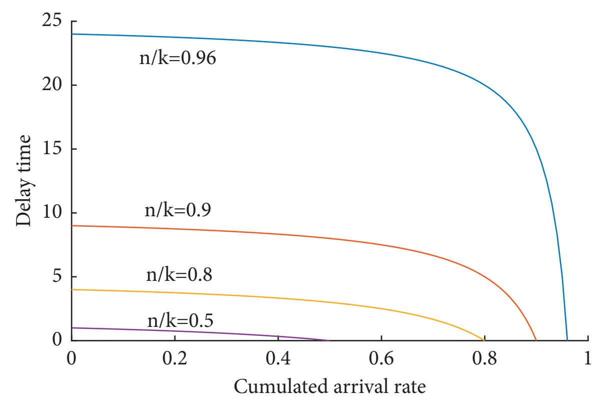

Figure 3: Delay time of locations with different n/k.

3.3.2. Time-Distance Dimension. To fully demonstrate the influence of distance factors on the occupancy rate of parking lots, we transform the discrete distribution model of the parking lot into the continuous distribution model and make a theoretical analysis. The distances between any two adjacent lots satisfy $x_{m+1} - x_m \longrightarrow 0$.

According to Remark 2, the trip costs of the last parkers only include travel cost and search cost, so they all depart from H at $t^* - C^*/\alpha$. We further derive the latest time of (1) departing from residence, (2) arriving at parking lots, and (3) leaving from parking lots, which are, respectively, plotted in Figure 4 by orange dot solid line (LT (depart)), orange solid line (LT (arrive lot)), and the red dotted line (LT (leave lot)). For the earliest arrivals at each location, the cost only includes the travel cost and delay cost, and the sum is fixed. According to equations (14) and (15), we derive the earliest time of (1) departing from residence H and (2) arriving at parking spots, which are, respectively, plotted by blue dot solid line (ET (depart)) and blue solid line (ET (arrive lot)).

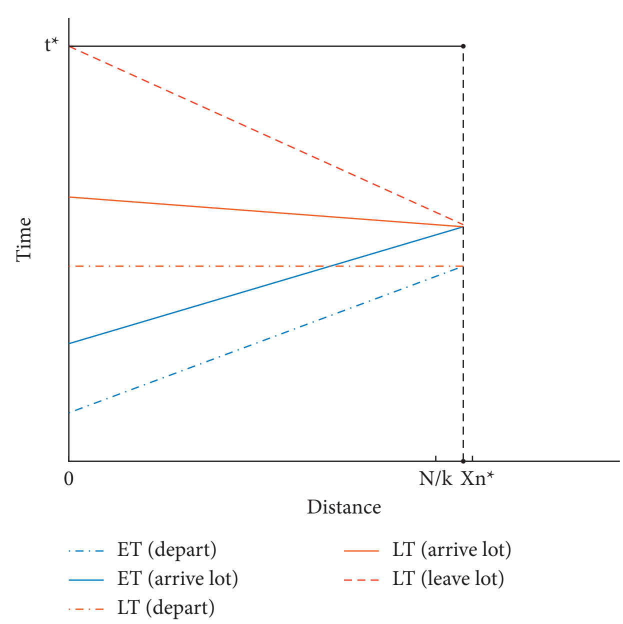

Figure 4: Trip schedule in scheme n.

As shown in Figure 4, the part between the red dotted line and the black horizontal solid line t^* is commuters' walk time, the part between the red dotted line and the solid orange line is the maximum search time, and the part between the red dotted line and the solid blue line is the sum of the maximum delay time (includes a unit search time). At the farthest location, the first arriver is also the last, so the blue and orange lines intersect x_n^*.

3.3.3. Occupancy-Distance Dimension. According to Remarks 1 and 2, the sum of travel cost and search cost for all the last commuters at each location is the same. We can calculate the final parking number at each location $n(x_m, t^*)$ from (10). Figure 5 shows the relationship between the occupancy rate of each parking lot with distance at the time t^*. The gentle change near the office shows that when the value of occupancy rate $n(x_m, t^*)/k_m$ is large, the linear variation in the travel cost can be offset by a small change in it (the small change produces a relatively large search cost variation). Similarly, the fast descending part near the farthest parking location x_n^* shows that when the value of occupancy rate is small, the linear variation in the driving cost needs to be offset by a large change in it.

Figures 2–5 indicate that the deadweight loss cost associated with parking for commuters in the scheme n is mainly reflected in search and delay. When the occupancy rate of the parking lot is high, the search time of commuters is longer. Under user equilibrium, commuters arriving earlier in the same parking lot also bear longer delays to avoid searching. In Figure 5, most parking lots within acceptable walking distance are with a high occupancy rate; only a small number of commuters choose farther parking lots to avoid the search cost and the associated delay cost. The parking problem is a combination of high search, delay, and travel costs, but its appearance may be as shown in Figure 5, where parking near O is not fully utilized. This can even give managers the illusion that parking spots are not scarce.

3.3.4. Time-Distance-Occupancy Dimension. By integrating time, distance, and occupancy, we draw a three-dimensional figure to depict temporal and spatial parking behavior (distribution), as shown in Figure 6. Its projection on the "time-distance" coordinate plane in Figure 4, that on the "occupancy-distance" coordinate plane in Figure 5, and that on the "occupancy-time" coordinate plane represent the cumulative arrival of each location over time (the upper yellow part of the occupancy rate no longer extends to time t^*, indicating that it has reached the final value at that time).

Figure 6 shows the following. (1) Search cost leads to the low efficiency of spot utilization while the occupancy rate of all locations is below 1, and it decreases as the distance increases. (2) The cumulative arrival surface reflects most commuters arriving early in nearly all parking locations, and the arrival rate tends to decrease with time.

Figure 6 is consistent with the parking survey results mentioned in Section 1. It provides a visual representation of parking utilization (individual parking utilization and parking range) and delays.

Therefore, in the most commonly adopted static charge scheme (single charge within a certain space), there are not only an explicit parking search but also some implicit issues,

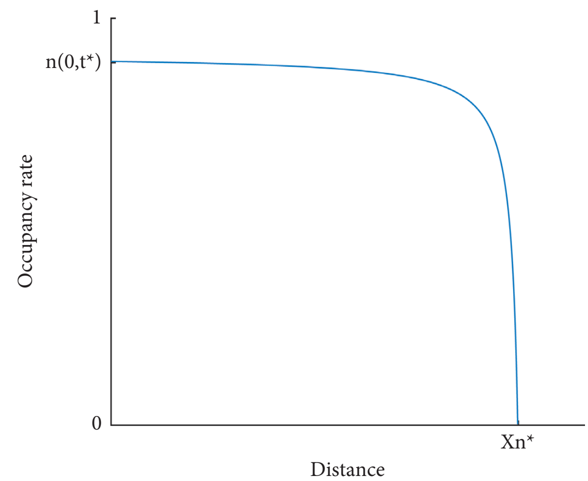

FIGURE 5: The final occupancy rate of all locations.

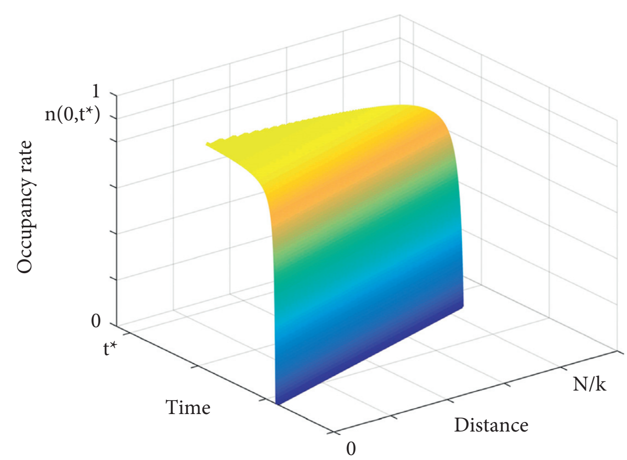

FIGURE 6: Arrivals at parking lots in scheme n.

e.g., a wide-range and large-scale delay cost occurs when parking spots are short in supply.

4. Improved Schemes of Parking Permit

Section 3 depicts the series of parking problems, including parking search, schedule delay, travel cost increase, and insufficient use of parking lot. In this section, two parking permit schemes are proposed, the parking lot permit scheme and the parking spot permit scheme. Both of them have basic practical application prototypes. The parking lot permit scheme is similar to a membership system, where travelers can purchase parking services from a parking lot for some time (weeks, months, and so on). It is usually for business uses, e.g., in institutions, companies, communities, etc. The parking spot permit scheme is similar to a reservation system, in which travelers reserve a designated parking spot where they can park directly. It is often used in a ticket booking system (containing the time and location information), e.g., in shared parking management or the airplane ticket or train ticket booking.

Aiming at reducing the total cost of the system and the equilibrium cost of commuters, this section explores the effect and limitations of permits in regulating each sort of cost. Combined with the actual parking process, we design a parking spot allocation mechanism (the number of open spots varies over time and space) and set prices to achieve the optimal parking distribution from the system optimal perspective.

4.1. Parking Lot Choice of the Commuter. To design the optimal pattern of permit issuance, an analysis of the components of travel costs is necessary so that we can find out which parts can be improved in the scheme p. According to Section 2, parking costs include four parts; among them, travel cost and search cost, respectively, depend on the location and occupancy rate (or arrival sequence) of parking spots; we define them as "invariable costs" (if the location and parked number are fixed, the costs are unique); delay cost and parking charge are "variable costs" which can be adjusted by the manager or commuters themselves.

Parking permits have two functions: one is to ensure parking for commuters and the other is to adjust commuters' parking behavior by setting the validity period and charging price. Referring to the method in Section 3, we still start by studying the final equilibrium state of the system and analyze the cost composition of the last arrival of each parking lot.

To begin with, we derive the equilibrium of the scheme p. As discussed in Section 3, we start with finding the farthest parking location and derive the equilibrium of the system.

According to Remark 2, all the $n(x_m, t^*) - th$ commuters at each lot in the scheme n only experience travel costs and search costs, both of which are invariable costs and cannot be internalized by charging. Therefore, we draw a further remark.

Remark 4. If the invariable costs of the system remain constant, the parking lot permit cannot reduce the equilibrium cost.

Proof 4. According to equations (4) and (9), the cost of the $n(x_m, t^*) - th$ commuter at each lot is

$$\begin{aligned} C_p\left(x_m, t^{e,x_m}\right) &= \alpha\left[\frac{\lambda k_m}{k_m - n\left(x_m, t^*\right)} + \frac{L - x_m}{v_c} + \frac{x_m}{v_w}\right] \\ &\quad + p_p\left(x_m, t^{e,x_m}\right). \end{aligned} \tag{16}$$

The equilibrium cost is equal to the sum of travel cost, search cost, and parking charge; when the charge $p_p(x_m, t^{e,x_m}) > 0$, the cost will exceed the equilibrium cost C_n^*. In addition, if the manager alleviates searching by reducing the permit issuance, then some commuters will move to other parking lots, $C_p^* > C_n^*$, which will appear in at least one parking lot, which contradicts the assumption.

Therefore, the final parking numbers of each parking lot $n(x_m, t^*)$ remain still, as shown in Figure 5. The equilibrium cost in the scheme p satisfies $C_p^* = C_n^*$.

By contrast, delay cost is variable. As shown in Figure 6, most commuters experience high delay costs in the spots with a high occupancy rate. If the manager raises the charge, commuters have less incentive to depart early and compete for parking spots under user equilibrium. Meanwhile, the deadweight loss of the system is also converted into internal revenue.

Theoretically, taking a parking charge which is close to the commuter's delay cost seems well in internalizing a great proportion of the system deadweight loss. However, there are two constraints.

First, if all commuters at the same parking lot do not delay or their arrival times are in a very small range, commuters arrive at nearly the same time (or in a very short time interval), but they strictly keep a specific arrival order. It is hard to implement.

Second, as the last commuters in each location have the longest search time, other commuters cannot arrive later than them. Therefore, commuters' delay time at the lot m is no less than the search time differences between them.

The search cost is determined by the order of arrival at the parking lot. To achieve the theoretical optimal charge, the manager needs to set different charges for every arrival time. We set the minimum distinguishable time interval between two adjacent arrivals in the scheme p as τ; then, the latest arrival time of the $i - th$ commuter at location x_m, t_p^{i,x_m} is

$$t_p^{i,x_m} = t_p^{e,x_m} - \left[n\left(x_m, t^*\right) - i\right] \cdot \tau. \tag{17}$$

As discussed in Section 3.1, when commuters leave the parking lot before $t^* - x_m/v_w$, they experience delay costs. The delay cost of the $i - th$ commuter satisfies

$$\begin{aligned} &C_p^d\left(t_p^{i,x_m} + \frac{\lambda k_m}{k_m - i + 1} + \frac{x_m}{v_w}\right) \\ &= \beta\left(t^* - t_p^{i,x_m} - \frac{\lambda k_m}{k_m - i + 1} - \frac{x_m}{v_w}\right). \end{aligned} \tag{18}$$

Since the last commuter at the lot m finishes parking at a time $t^* - x_m/v_w$ and arrives O on time, we calculate the delay cost of the $i - th$ commuter in the scheme p according to the difference between his/her leaving time (from the parking lot) and $t^* - x_m/v_w$.

Substituting (17) into (18), the delay cost satisfies

$$\begin{aligned} &C_p^d\left(t_p^{i,x_m} + \frac{\lambda k_m}{k_m - i + 1} + \frac{x_m}{v_w}\right) \\ &= \frac{\beta\lambda k_m}{k_m - n\left(x_m, t^*\right) + 1} - \frac{\beta\lambda k_m}{k_m - i + 1} + \left[n\left(x_m, t^*\right) - i\right] \cdot \beta\tau, \end{aligned} \tag{19}$$

where $t_p^{i,x_m} = t_p^{n(x_m,t^*),x_m} - [n(x_m, t^*) - i] \cdot \tau$. When $\tau \longrightarrow 0$, $t_p^{i,x_m} > t_n^{i,x_m}$, commuters in the scheme p arrive later than before. The delay cost is equal to the equilibrium cost minus the travel and search costs.

$$C_p^d\left(t_p^{i,x_m}+\frac{\lambda k_m}{k_m-i+1}+\frac{x_m}{v_w}\right)=C_p^*-\alpha\left(\frac{x_m}{v_w}+\frac{L-x_m}{v_c}+\frac{\lambda k_m}{k_m-i+1}\right). \tag{20}$$

$C_p^d(t_p^{i,x_m})\le C_n^d(t_n^{i,x_m})$ when $\tau\longrightarrow 0$.

If and only if $i=n(x_m,t^*)$, $C_p^d(t_p^{i,x_m}+\lambda k_m/k_m-i+1+x_m/\ v_w)=C_n^d(t_n^{i,x_m}+\lambda k_m/k_m-i+1+x_m/v_w)$.

Therefore, the charge fee (zero in the scheme n) equals the difference between the search costs of the scheme p and scheme n.

$$p_p\left(x_m,t_p^{i,x_m}\right)=C_p^d\left(t_p^{i,x_m}+\frac{\lambda k_m}{k_m-i+1}+\frac{x_m}{v_w}\right)-C_n^d\left(t_n^{i,x}+\frac{\lambda k_m}{k_m-i+1}+\frac{x_m}{v_w}\right). \tag{21}$$

The arrival time t_p^{i,x_m} of the $i-th$ commuter is stipulated to satisfy

$$t_p^{i,x_m}=t^*-\frac{x_m}{v_w}-\frac{\lambda k_m}{k_m-n(x_m,t^*)+1}-\left[n(x_m,t^*)-i\right]\cdot\tau. \tag{22}$$

The arrival time of the first and last arrivals t_p^{s,x_m} and t_p^{e,x_m}, respectively, satisfies

$$t_p^{s,x_m}=t^*-\frac{x_m}{v_w}-\frac{\lambda k_m}{k_m-n(x_m,t^*)+1}-\left[n(x_m,t^*)-1\right]\cdot\tau,$$
$$t_p^{e,x_m}=t^*-\frac{x_m}{v_w}-\frac{\lambda k_m}{k_m-n(x_m,t^*)+1.} \tag{23}$$

To conclude, the manager in the scheme p should issue $n(x_m,t^*)$ parking permits to the parking lot m between t_p^{s,x_m} and t_p^{e,x_m} for $x_m\le x_p^*$. The time interval of the two adjacent issuances is τ. No parking permits will be issued at other times and parking lots.

Combined with theoretical analysis, we further draw Figure 7 for the scheme p (the legend is identical to Figure 3).

Figure 7 shows the impact of charging on commuters' schedules in the scheme p. According to Remark 1, there is no delay cost for the last parker in each lot, and his/her search cost (time) is the largest among all parkers in the lot. On the other hand, the search cost (time) is determined by the arrival order. Therefore, if the search time of the last commuter is $t_s(x_m,t)$, then the sum of delay time and search time of other commuters are no less than $t_s(x_m,t)$ in the scheme p.

The inflection points of each curve in Figure 7 represent the delay time of the penultimate commuter. It is affected by the search time of the last commuter (the longer the search, the greater the search time difference between adjacent commuters and the longer the delay for the penultimate commuter). The curve part is consistent with Figure 3 described in the scheme n. Before the inflection point, the time

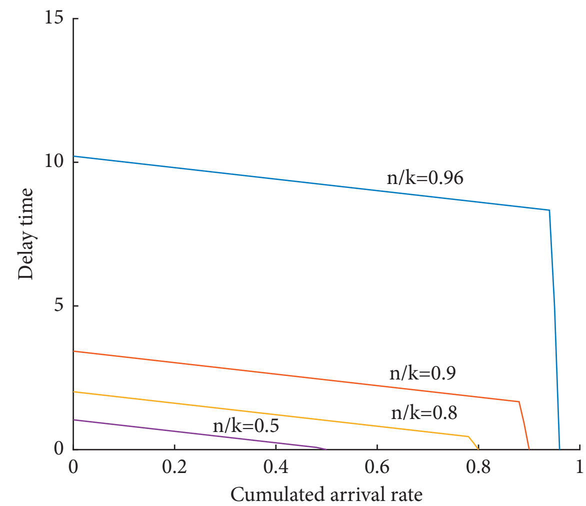

Figure 7: Delay time of different locations in scheme p.

interval for each commuter is τ, so it is a straight line with a slope of $-k_m\tau$. The multiplier $-k_m$ is caused by narrowing the definition field from $(0,\ k_m]$ to (0,1].

The first $n(x_m,t^*)-1$ commuters who arrive early will still experience nonnegative delay costs because the search time of the latest arrivals is not eliminated.

To better reflect the influence of the distance factor on the occupancy rate of parking lots, we make the discrete distribution of parking lots continuous.

Figure 8 has the same legend as Figure 4; it shows that for all parking locations, as τ is small enough, the blue dotted line (ET (depart)) is close to the orange dotted line (LT (depart)) and the blue solid line (ET (arrive lot)) is also close to the orange solid line (LT (arrive lot)), which depicts that the time composition of the first $n(x_m,t^*)-1$ commuters can approach the last commuter before they arrive. The delay time of the first $n(x_m,t^*)-1$ commuters has not been eliminated by permits; on the contrary, it is slightly higher than the search time of the last commuter.

From the dimensions of time, distance, and occupancy, the scheme p is depicted as three-dimensional in Figure 9. Compared with the scheme n described in Figure 6, the arrival time (to the office) of the majority of commuters is closer to t^*, and the change is more significant as the lot is closer to O.

We have analyzed the performance of the parking lot permit scheme. To approach the theoretical system optimally, it needs (1) the manager to distinguish the arrival time of each commuter and (2) all the commuters to arrive on time, which is difficult to implement in practice. As an alternative, the manager can set up a small number of groups (each group of commuters has the same charge price and expiration time). If there are η commuters in a group, the workload will be reduced to $1/\eta$ times. However, as a trade-off, the manager can only issue permits based on the last arrivals of each group and ignore the cost differences within the group. It is easy to prove that when $\eta=1$, the scheme is equal to the scheme p; when $\eta=n(x_m,t^*)$, the scheme is equal to the scheme n. We will examine this compromise scheme in Section 5. □

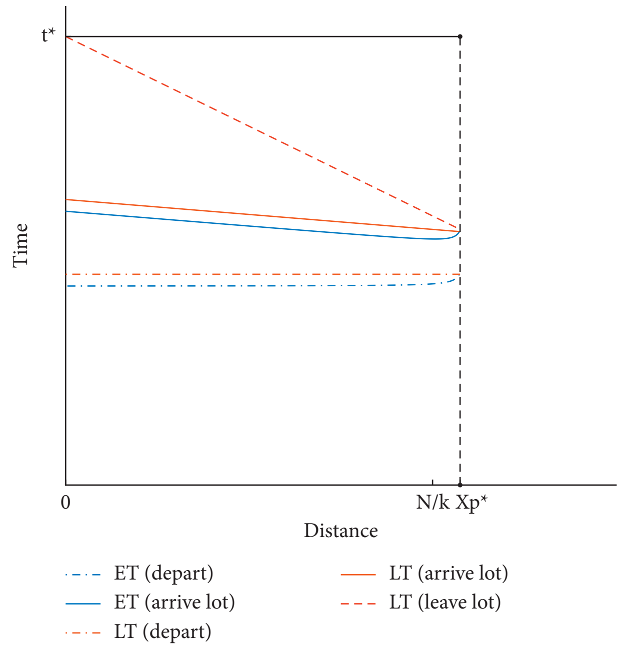

Figure 8: Trip schedule in scheme p.

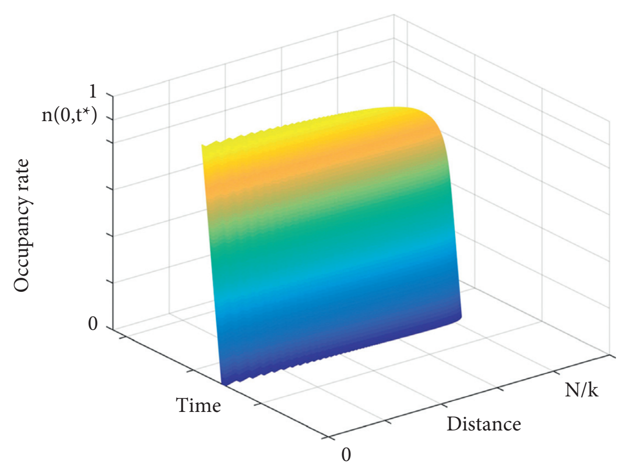

Figure 9: Arrivals at parking lots in scheme p.

4.2. Parking Spot Permit Scheme. According to Section 4.1, the contribution of the parking lot permit scheme is that a certain percentage of the delay cost can be internalized through the charging. However, it still has limitations. It cannot eliminate search cost and the efficiency loss directly related to it (e.g., early arrivals and insufficient utilization of parking lots).

Firstly, the scheme p does not change the equilibrium; the utilization of parking lots is the same as in the scheme n; despite the high demand for parking near O, the problem of parking lots not being fully utilized remains. Some commuters continue bearing more travel costs because they have to park in a farther location. The total travel cost in the scheme n is $\sum_{\{x_m|x_m \leq x_n^*\}} n(x_m, t^*) C^t(x_m)$, which is higher than the total travel cost when spots are fully utilized $(\sum_{\{x_m|\sum_{x_m} k_m \leq N\}} k_m C^t(x_m) + (N - \sum_{x_m} k_m) C^t(x_{m+1}))$.

Secondly, it is not able to effectively reduce the total delay cost of the system. The arrival of all commuters at one location is limited by both the arrival time and search time of the last (or last several) commuters, and the delay cost cannot be eliminated. Compared with the ideal case, it reduces the system's total delay cost by less than $1 - \beta/\alpha$.

The commuters resort to parking search when they cannot get information about the parking spots. They have to travel through the whole parking spot (or parking lot). To address this problem, we propose the parking spot permit scheme r. In this scheme, the manager issues parking permits with specific spot information. Permits are set and charged according to the location and serve time. Commuters get the spot information in advance (before departure) and go to the target spot directly. Their average search time is π.

Different from the schemes n and p, commuters' search cost no longer exceeds the equilibrium cost when $n(x_m, t) \longrightarrow k_m$ (they are not influenced by other parkers in the scheme r). Therefore, we infer that at the time t^*, all parking lots whose parking demand is greater than supply satisfy $n(x_m, t) = k_m$. The commuters park at these lots only have travel cost and fixed search cost pi. Denote the farthest parking location of the scheme r as x_r^*; according to (4), the trip cost is

$$C_r\left(x_r^*, t^* - \frac{x_r^*}{v_w} - \pi\right) = C^t(x_r^*) + \alpha\pi + p_r\left(x_r^*, t^* - \frac{x_r^*}{v_w} - \pi\right). \tag{24}$$

From (24), we can get the following remark.

Remark 5. When the scheme r achieves maximum efficiency, the charging price at x_r^* satisfies $p_r(x_r^*, t) = 0$. If the system achieves equilibrium, the minimum equilibrium cost is $C_r^* = C^t(x_r^*) + \alpha\pi$.

Proof 5. Excluding parking charges, the highest parking cost is at the farthest parking location x_r^*. If the charging price $p_r(x_r^*, t) > 0$, then commuters' cost at x_r^* and the equilibrium cost will increase, which contradicts the assumption.

We now study the utilization of each parking location. As we have discussed, the difference between the schemes r and p is that commuters in the scheme r do not need to search for parking spots when they are parking; their cost has no relationship with the current utilization of the parking lot or the order of arrival. For the lot in which m satisfies $x_m \leq x_r^*$, the last commuters do not experience delay cost, and their trip cost is

$$C_r\left(x_m, t^* - \frac{x_m}{v_w} - \pi\right) = C^t(x_m) + \alpha\pi + p_r\left(x_m, t^* - \frac{x_m}{v_w} - \pi\right). \tag{25}$$

In the scheme r, parking lot m (used) is still attractive to the $k_m - th$ commuter when the parking charge is appropriately set (e.g., no more than $C^t(x_r^*) - C^t(x_m)$). In addition, the costs excluding charging of the last commuters in each parking lot are no longer equal, so when the system is in equilibrium, the charges for parking lots $x_m \leq x_r^*$ are positive.

In the discrete model, if the location of the $m' - th$ parking lot $x_{m'} = x_r^*$ in the scheme r, then m' satisfies

$$\begin{cases} \sum_{m'-1} k_m x_m < N, \\ \sum_{m'+1} k_m x_m \geq N. \end{cases} \quad (26)$$

(26) indicates that when parking lots are discretely distributed, the $m' - th$ parking lot may not be fully used.

The equilibrium cost C_r^* of the scheme r satisfies

$$C_r^* = \alpha \left[\frac{(L - x_r^*)}{v_c} + \frac{x_r^*}{v_w} + \pi \right]. \quad (27)$$

According to (24), the charging fee for the last arrivers at x_m is

$$p_r\left(x_m, t^* - \frac{x_m}{v_w} - \pi\right) = \alpha\left(x_r^* - x_m\right)\left(\frac{1}{v_w} - \frac{1}{v_c}\right). \quad (28)$$

All commuters in the same parking lot arrive at the same time. Therefore, all commuters arrive at the same time (in theory), and the first commuter is also the $i - th$ or the last, and it satisfies $t_r^{s,x_m} = t_r^{e,x_m} = t_r^{i,x_m}$ and equation (30):

$$t_r^{i,x_m} = t^* - \frac{x_m}{v_w} - \pi. \quad (29)$$

To conclude, the manager in the scheme r should issue all k_m parking permits to the parking lot m at the time $t^* - x_m/v_w - \pi$ for $x_m \leq x_r^*$. No parking permits will be issued at other times and parking lots.

We continue to use the continuous distribution model of parking lots to ensure the accuracy of theoretical analysis. From the above derivation, we represent the trip schedule of commuters in Figure 10 (the legend is consistent with Figures 4 and8, and parking lots are continuously distributed). In the scheme r, the utilization rate of each parking lot is 100%, and the search time is π.

Different from Figures 4 and8, in Figure 10, (1) the farthest parking location x_r^* satisfies $\sum_{\{m|x_m \leq x_r^*\}} k_m x_m = N$, which is nearer than the previous two schemes; (2) two series of lines (blue and orange dotted lines and blue and orange solid lines), respectively, represent that the earliest and latest departing and arrival times overlap with each other, which shows that the time composition of commuters who park at the same location tends to be the same and the delay time is minimized; (3) the latest arrival (orange solid) line is close to the latest leave lot (red dot) line, which shows that the search time is minimized.

From the dimensions of time, distance, and occupancy, parking distribution in the scheme r further develops to Figure 11.

Compared with Figure 9, the scheme r continues to put off the departing and arrival times of commuters. The change nearly equals the change from the scheme p to the scheme n.

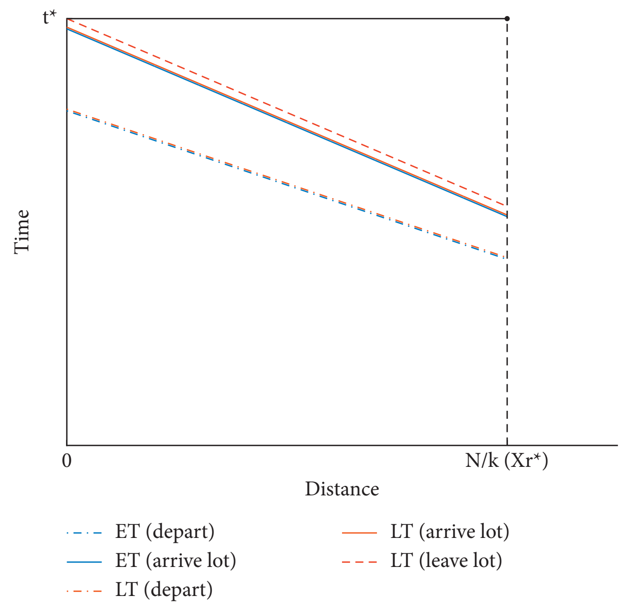

Figure 10: Trip schedule of different locations in scheme r.

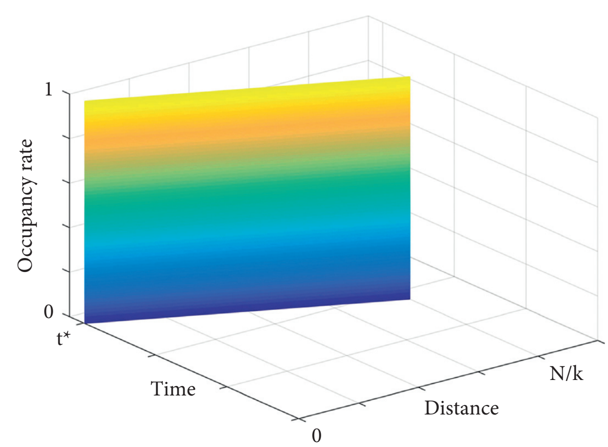

Figure 11: Arrivals at parking lots in the scheme r.

Compared with the scheme p, the delay cost in the scheme r is reduced by nearly two times, and the per-capita search cost changes to a constant value π. On the other hand, the surface changes into a plane; its projection on the "time-distance" coordinate plane is a straight line, depicting that the schedule of all commuters parked at the same location is consistent. Its projections on "occupancy-time" and "occupancy-distance" coordinate planes are rectangles, which show that all the parking lots are fully utilized in both temporal and spatial dimensions. □

5. Numerical Experiment

In this section, we provide a numerical experiment to analyze parking costs and user equilibrium of the scheme n and then quantitatively examine the effectiveness of the schemes p and r. We set the parameters as follows:

Parking Lots. Parking lot number is set as $M = 7$. The distance of the $m - th$ parking lot from the destination O is $x_m = 0.15 * (m - 1)$ kilometers.

Commuters. The commuter's number is set as $N = 1000$. The unit search time is set as $\lambda = 30$ seconds [14]. In the parking spot permit scheme r, commuters' search time under the guidance information π is set as 60 seconds according to the empirical research on factors of cruising speed and distance range [44–50].

The time value α is 1 cent per second, and that of β is 0.5 cents per second (see [50]). Commuters drive at 16 km/h and walk at 5 km/h, respectively.

Manager. The manager charges for parking spots. For simplicity, we set the parking charge as 0 yuan in the scheme n. In the schemes p and r, the manager charges variable fees according to the location and parking time.

Scenarios. The acceptable walking distance is generally within 600 meters. Therefore, we focused on the parking behavior within the scope of 1 km (L) from the destination O. From scarce to sufficient, we set four parking supply scenarios, corresponding to 7 parking lots with capacities of 200, 250, 400, and 800, respectively.

5.1. Equilibrium in the Scheme n. According to (7) and Remark 2, when the system gets to user equilibrium, the cost of all used parking lots will be equal, and when it is minimum, there is $n(x_m, t)[C_n(x_m, t) - C_n^*] \geq 0$. In theory, there is the farthest parking location x_n^*, the parking lots beyond which will not be used. Therefore, not all seven parking lots may be utilized by commuters.

From (10) and (11), the equilibrium and conservation conditions of the utilized parking lots are

$$C_n^* = \alpha\left[\frac{L - x_m}{v_c} + \frac{x_m}{v_w} + \frac{\lambda k_m}{k_m - n(x_m, t^*)}\right], \tag{30}$$

$$N = \sum_M n(x_m, t^*). \tag{31}$$

Substitute (29) into equation (30), and the equilibrium cost C_n^* can be calculated.

Based on the known conditions, we calculated the equilibrium occupancy rate $(n(x_m, t)/k_m)$ of each parking lot in four scenarios, as shown in Table 3.

The first two columns of Table 3 are the parking lot numbers and the distances from the destination O, and the last four columns represent the occupancy rate of four scenarios.

It shows that in four scenarios, the parking lot equilibrium occupancy rate decreases with the distance increase. For example, in scenario 1, the occupancy rate of parking one nearest to O is 0.932, and that of parking lot six farthest away is 0.589. Also, not all parking lots are used by commuters. With the increase in parking lot capacity, commuters tend to concentrate on parking lots closer to the destination. For example, only parking lot seven is not used in scenario 1, and the number of unused parking lots in scenarios 2–4 gradually increases.

We plot the information in Table 3 as Figure 12. It is consistent with Figure 5 (the four curves from far to near the origin, respectively, represent the occupancy rate of each parking lot when the capacity of a single parking lot is 200, 250, 400, and 800). Although the discretely distributed parking lots affect the smoothness of the curve, it still depicts the trend that the parking lot occupancy rate decreases with the increase of distance.

We further calculate the average occupancy rate of the utilized parking lots, equilibrium cost, per-capita travel cost, and per-capita deadweight loss (the sum of search and delay cost) of four scenarios in Table 4.

The first two columns of Table 4 are the scenario numbers and the single parking lot capacity; the third column is the average occupancy rate of the utilized parking lots, and the last three columns are commuters' equilibrium cost, per-capita travel cost, and per-capita deadweight loss.

In Table 4, with the increase of parking lot capacity, the equilibrium cost C_n^* decreases. The reasons are as follows. (1) There are more parking spots near the destination O that can accommodate commuters; thus, the travel costs are reduced. (2) More parking spots also reduce the commuters' search cost and the delay cost associated with it and then reduce the deadweight loss.

We can also see from Table 4 that the decline in the figures in column 5 is lower than the decline in the figures in column 6. It depicts that the increased parking capacity plays a more important role in the reduction in deadweight loss cost than the reduction in travel cost. From this point of view, it is reasonable for the manager to keep some parking spots close to the destination vacant and reduce the search cost (e.g., SFpark) and let some commuters park in distant locations.

Affected by the discrete distribution of parking lots, the average utilization rate does not decrease strictly, but we can infer from the decrease of deadweight loss that the parking supply is gradually getting ampler.

5.2. Improvement of the Parking Lot Permit Scheme. In the scheme p, because parking permits without information cannot eliminate the search cost, the trip schedule of the last commuter in each lot is the same as that in the scheme n. The utilization of each lot at the time t^* is consistent with the situation described in Table 3 and Figure 11.

Different from the scheme n, the manager in the scheme p can set time-varying charges to reduce the delay cost. Based on (2) and Section 3.2, commuters' search cost is determined by the occupied spot number, namely, the arrival order. The arrival time difference between two adjacent commuters is α/β times their search time difference. Then, we calculated the arrival time of each commuter from the search cost of the last commuters.

Table 3: The equilibrium occupancy rate of four scenarios in the scheme n.

Lot number	Distance	Scenario 1 occupancy	Scenario 2 occupancy	Scenario 3 occupancy	Scenario 4 occupancy
1	0	0.932	0.917	0.890	0.772
2	0.15	0.919	0.896	0.849	0.478
3	0.30	0.899	0.860	0.761	0
4	0.45	0.865	0.785	0	0
5	0.60	0.796	0.542	0	0
6	0.75	0.589	0	0	0
7	0.90	0	0	0	0

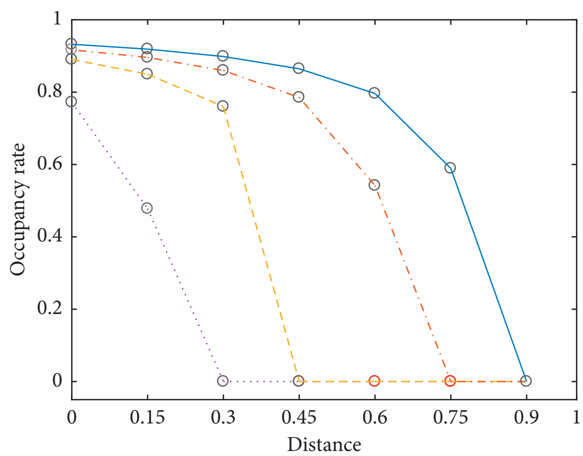

Figure 12: Occupancy rates of four scenarios (scheme n).

Table 4: The equilibrium occupancy and the costs of four scenarios in the scheme n.

Scenario number	k_m	Average occupancy	C^*	C^t	Deadweight loss
1	200	83.33%	6.69	3.95	2.74
2	250	80.00%	5.87	3.58	2.29
3	400	83.33%	4.99	2.95	2.04
4	800	62.50%	3.57	2.53	1.04

To compare with the scheme n, we also set the charge of the last commuter in the farthest lot as 0 and define two cases. Case 1 is theory-based, and τ_1 (from τ in (17)) satisfies $\tau_1 \longrightarrow 0^+$, which means the manager can accurately distinguish the arrival order of commuters and set a time-varying charge for each commuter. Case 1 can be regarded as the upper limit of system improvement. Case 2 is practice-based, and τ_2 satisfies $\tau_2 > 0$, which means it is difficult for the manager to accurately distinguish the arrival order. Instead, the manager set a small number of expirable times (set up groups include multiple commuters accordingly) and charged different parking fees. When the number of groups is 1, the scheme p is the same as the scheme n, which can be regarded as the lower limit of the system improvement (as shown in Table 4). τ_2 represents the cases between one group and $n(x_m, t^*)$ groups.

In Case 1, the manager reduces the delay time by charging the difference between the original arrival time of the commuter to the parking lot and the search time of the last commuter. Based on this, we calculate the revenue collected by charging under four scenarios.

Columns 3 and 4 in Table 5 are equal to columns 4 and 5 in Table 4. Column 6 is the parking per-capita charge in the scheme p; it is equal to the difference between the deadweight loss of scheme n (column 5 in Table 4) and scheme p (column 5). Column 7 is the percentage of parking charges to equilibrium costs, which represents the percentage of total costs that the system can reduce through charges (e.g., in scenario 1, the average parking charge is 1.01 yuan, accounting for 15.1% of the equilibrium cost of 6.69 yuan).

Table 5 depicts that although the permit scheme does not reduce the actual cost of commuters, it saves a certain percentage of the deadweight loss by charging from a system perspective. We can also see from Table 5 that with the increase of parking supply, the proportion of cost internalization by charging decreases, which is mainly due to the ease of parking search, and the delay cost associated with it also reduced.

In Case 2, we set the manager groups every 20 commuters in a parking lot and set charges based on the arrival time of the last commuter in each group. This setting greatly reduces the number of parking charges, and they are implemented simply. In this case, the deadweight losses internalized by charging of four scenarios are shown in Table 6.

The meanings of each column in Table 6 are the same as those in Table 5; since the 20 commuters are benchmarked against the last commuter in each group, their delay costs were not eliminated (only the delay cost of the last commuters in each group is eliminated); instead, the delay times are the arrival time differences between them and the last commuter.

By comparing columns 5–7 in Tables 5 and 6, the amount of deadweight loss that the manager internalizes through charges is reduced in Case 2. The effect is more obvious in scenarios 1 and 2, where parking spots are relatively scarce (with a decrease of 1.1%), while in scenarios 3 and 4, the decrease is 0.4% and 0.3%, respectively. The reason is that the increase in parking lot utilization leads to an increase in the search (delay) time for adjacent commuters.

Table 5: The equilibrium costs of four scenarios in the scheme p (Case 1).

Scenario number	km	C^*	Ct	Deadweight loss	Pp	Charge proportion
1	200	6.69	3.95	1.73	1.01	15.1%
2	250	5.87	3.58	1.48	0.81	13.8%
3	400	4.99	2.95	1.36	0.68	13.6%
4	800	3.57	2.53	0.78	0.26	7.3%

Table 6: The equilibrium costs of four scenarios in schemes s and p (Case 2).

Scenario number	k_m	C^*	C^t	Deadweight loss	p_p	Charge proportion (%)
1	200	6.69	3.95	1.80	0.94	14.1
2	250	5.87	3.58	1.54	0.75	12.8
3	400	4.99	2.95	1.38	0.66	13.2
4	800	3.57	2.53	0.79	0.25	7.0

Table 7: The equilibrium occupancy rate of four scenarios in the scheme r.

Lot number	Distance	Scenario 1 occupancy	Scenario 2 occupancy	Scenario 3 occupancy	Scenario 4 occupancy
1	0	1	1	1	1
2	0.15	1	1	1	0.25
3	0.30	1	1	0.5	0
4	0.45	1	1	0	0
5	0.60	1	0	0	0
6	0.75	0	0	0	0
7	0.90	0	0	0	0

For the same group of 20 commuters, their arrival time is relatively dense if they arrive earlier and relatively dispersed if they arrive later.

5.3. Improvement of Parking Spot Permit Scheme. In the scheme r, commuters are guided to parking spots by information other than searching for vacant spots. Therefore, the search time for each commuter is a fixed time. According to Section 4.3, the scheme r shortens commuters' parking search time and delay time and enhances the occupancy rate of parking spots.

To compare with the other two schemes, we set the charge of the commuters in the farthest lot as 0. In the scheme r, based on the known conditions, we calculated the equilibrium occupancy rate $(n(x_m, t)/k_m)$ of each parking lot in four scenarios, as shown in Table 7.

The meanings of each column in Table 7 are the same as those in Table 3; it shows that the parking distribution of commuters is more compact in the scheme r. Except for the farthest parking lots (the number of demands cannot be divided by the supply number), all the parking lots are fully used.

We plot the information in Table 7 as Figure 13. The four curves from far to near the origin, respectively, represent the occupancy rate of each parking lot when the capacity of a single parking lot is 200, 250, 400, and 800.

In Figure 13, the parking spot permit scheme changes the spatial parking distribution. Different from Figure 12, most parking lots have occupancy rates of 0 or 1. Commuters are closer to the destination O than scheme n and scheme p. More distant parking lots are left to be vacant.

According to Table 7, we further calculate the average occupancy rate of the utilized parking lots, equilibrium cost, per-capita travel cost, per-capita deadweight loss (search and delay cost), and per-capita parking charge in four scenarios in Table 8 (columns 3–7).

Compared with Table 4, a new column (column 7) (per-capita charge) is added to Table 8. In Table 8, later arrivals in each parking lot no more experience a relatively long search time, and the occupancy rate of the utilized parking lot increases. In scenarios 1 and 2, the occupancy rate of the parking lot reaches 100%; in scenarios 3 and 4, due to the discrete distribution of the parking lot, the occupancy rate remains unchanged. Compared with Table 4, the equilibrium cost and deadweight loss are all reduced in the scheme r (e.g., for scenario 1, the equilibrium cost in the schemes n and p is 6.69, while it is 5.52 in the scheme r; the deadweight loss is 2.74 in the scheme n, and 1.73 and 1.80 in two cases of the scheme p, while it is 0.60 in the scheme r). Additionally, with the increase in parking supply, the parking space span of commuters' parking decreases, so the commuters' travel cost and manager's charge also decrease with the increase of the case number.

Table 9 describes the comparison of various costs of commuters between scheme r and scheme n.

In Table 9, Columns 1–3 correspond to columns 1, 2, and 4 in Table 8, and columns 4 and 6, respectively, show the reduction of commuter's equilibrium cost and travel cost

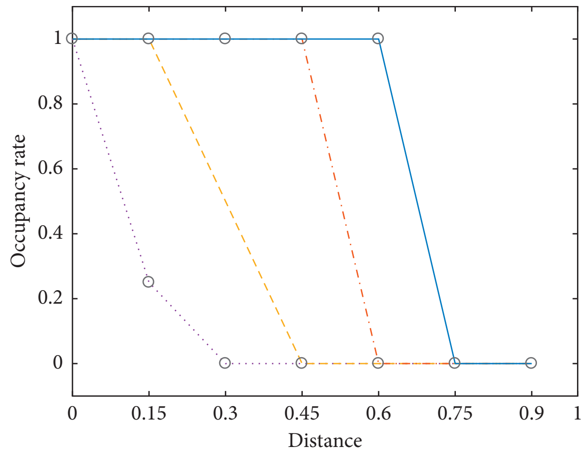

Figure 13: Occupancy rates of four scenarios (scheme r).

Table 8: The equilibrium occupancy and the costs of four scenarios in the scheme r.

Scenario number	k_m	Average occupancy	C_r^*	C^t	Deadweight loss	p_r
1	200	100%	5.52	3.74	0.60	1.18
2	250	100%	4.78	3.36	0.60	0.82
3	400	83.33%	4.04	2.84	0.60	0.60
4	800	62.50%	3.29	2.40	0.60	0.29

Table 9: Improvement of four scenarios in the scheme r.

Scenario number	k_m	C_r^*	C^* reduction	C^t	C^t reduction	p_r	System cost reduction
1	200	5.52	21.2%	3.74	5.3%	1.18	54.1%
2	250	4.78	22.8%	3.36	6.1%	0.82	48.2%
3	400	4.04	23.5%	2.84	3.7%	0.60	45.1%
4	800	3.29	8.5%	2.40	5.1%	0.29	19.0%

compared with the scheme n. Column 8 shows the reduction of system cost from the perspective of the manager (excluding commuters' parking charge). Different from schemes n and p, the scheme r brings benefits to both commuters and the system and reduces the equilibrium point.

Combined with Tables 9 and 5, in the scheme r, the equilibrium cost of commuters is significantly reduced (8.5%–23.5%) compared with the scheme n, which is about 1.2 to 1.7 times as effective as the scheme p (7.0%–14.1%). Compared with the schemes n and p, the travel cost of commuters is also reduced by 3.7%–6.1%. From the perspective of the system, the reduction of per-capita cost is more obvious (19.0%–54.1%), which is about 2.7 to 3.8 times the effect of the scheme p.

6. Conclusion and Discussion

At present, many reforms have been gradually implemented and are committed to building a smart, efficient, green, and shared urban transport system. As one of the most important trip modes in big cities, it is of great significance to improve the efficiency of parking management. However, due to the limited public parking spots in the downtown area, parking demand often exceeds parking supply. Searching for vacant spots not only raises travelers' trip costs but also intensifies the competition for parking spots, which makes travelers depart earlier or park further to guarantee a vacant spot, resulting in delay cost. In addition, the parking lot is surprisingly underutilized.

This paper studies the parking equilibrium and distribution of morning commuting and assumes all parking spots are utilized only once (in fact, the downtown parking spots often have high saturation and turnover rate, in which case the system deadweight loss is much higher).

This paper first analyzes the components of the trip cost that derives the user equilibrium and parking distribution in the no permit scheme. In that scheme, (1) parking spots cannot be fully utilized due to the high search cost at the last moment; (2) a large number of commuters arrive early, and commuters who arrive on time bear high search costs.

In response to this problem, this paper proposes two improved schemes based on the quantity control approach. The first is the parking lot permit scheme. By analyzing the causes of various costs, we classify them into two categories (invariable cost and variable cost) and find that (1) permit without parking information can only guarantee parking but eliminates the search cost depending on the utilization of the parking spots and (2) the delay cost can be reduced, but due to the arrival order, the early arrivals have to experience at least the same delay time as the search time spent by the last arrival. Therefore, the effect of the scheme p is restricted.

We further propose a parking spot permit scheme with precise spot information and find that information plays an important role in eliminating search costs as well as reducing system deadweight loss. Commuters in the scheme r can directly go to the target parking spot instead of going through all spots to find a vacant spot, they are no longer restricted by the current parking situation, and the interaction between commuters disappears as well. In the case of insufficient parking supply, it effectively reduces the high search and delay costs. By deriving the new equilibrium, we find that (1) as the search costs are controlled (they are no longer higher than equilibrium cost), the occupancy rate of the parking spots increases and the farthest parking location gets closer to the office, and the equilibrium cost reduces as well; (2) delay is no longer related to search cost, and it can be converted into the system revenue completely by charging fees; (3) for the system, the cost is only the travel cost C^t and the average search cost $\alpha\pi$.

A numerical example is introduced to study the costs within the last kilometer. It shows the following. (1) The parking lot permit scheme improves the system by obtaining parking revenue (its compromise also works well), but the user equilibrium cost does not change. (2) The parking spot permit scheme reduces the total cost of the system to a greater extent by both improving the occupancy rate of the parking lot and reducing the user equilibrium cost. This is consistent with the conclusion of the previous analysis.

With the development of the economy and society, urban parking resources are becoming increasingly scarce; fortunately, as the information collection and transmission become more convenient, the accuracy of obtaining parking information is greatly improved. The parking permit is not only a measure but also an information carrier. It (1) guarantees the parking right in advance and prevents travelers from going back and forth between parking lots (this does not occur in the theoretical equilibrium in this paper, but it may occur when there is demand disturbance in practice) and (2) adjusts parking effectively by setting different open times and charging prices. Compared with the no permit scheme, it changes the regulation mode of supply and demand based on "price" to that based on "quantity and price," which is more accurate. In addition, the parking permits establish a one-to-one relationship between users and parking lots/spots (which is suitable for authentication) and make the management scheme feasible in parking lots of all sorts (e.g., residential shared parking lots). It will promote the development of parking management towards intelligence, sharing, and automation.

We analyze the allocation and charging mechanism under the framework of two improvement schemes for system optimum purpose (minimizes the total system cost and user equilibrium cost at the same time). However, there are still some important issues unanswered. One of them is that parking management is managed not only by the government but also by the operators for the profitable purpose; there is a difference between the purpose of profit and the purpose of publicity (higher charge and lower occupancy in general). How to bridge the gap needs to be further studied. The other parking difficulty is not only reflected in a one-time parking process but also in finding vacant spots among many repeated used ones. When the parking lot keeps on both a high occupancy rate and turnover rate, the deadweight loss cost of the system will also be much higher, especially under schemes n and $p\pi$. Parking pricing and user equilibrium should be calculated based on both the situations of the previous period and the current time interval. It is worthy of further research.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] N. Fulman, I. Benenson, and E. Ben-Elia, "Modeling parking search behavior in the city center: a game-based approach," *Transportation Research Part C: Emerging Technologies*, vol. 120, Article ID 102800, 2020.

[2] Beijing Transport Institute, "2021 beijing transport development annual Report (in Chinese)," 2021, https://www.bjtrc.org.cn/List/index/cid/7.html.

[3] T. Litman, *Parking management comprehensive implementation guide*, Victoria Transport Policy Institute, Victoria, BC, 2011.

[4] R. C. Hampshire, D. Jordon, and O. Akinbola, "Analysis of Parking Search Behavior with Video from Naturalistic Driving," *Transportation. Research. Board*, vol. 2543, no. 1, pp. 152–158, 2016.

[5] E. Inci, J. N. Van Ommeren, and M. Kobus, "The external cruising costs of parking," *Journal of Economic Geography*, vol. 17, no. 6, pp. 1301–1323, 2017.

[6] K. Axhausen, J. Polak, M. Boltze, and J. Puzicha, "Effectiveness of the parking guidance information system in frankfurt am main," *Traffic Energy & Control*, vol. 35, no. 5, pp. 304–309, 1994.

[7] R. Arnott and J. Rowse, "Modeling parking," *Journal of Urban Economics*, vol. 45, pp. 97–124, 1999.

[8] V. R. Vuchic, *Transportation for Livable Cities*, Routledge, New York, NY, 1999.

[9] D. Shoup, "Cruising for parking," *Transport Policy*, vol. 13, pp. 479–486, 2006.

[10] D. Ayala, O. Wolfson, B. Xu, B. Dasgupta, and J. Lin, "Parking slot assignment games," in *Proceedings of the GIS '11 19th ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems*, pp. 299–308, Illinois, IL, USA, November 2011.

[11] R. Arnott, A. De Palma, and R. Lindsey, "A temporal and spatial equilibrium analysis of commuter parking," *Journal of Public Economics*, vol. 45, no. 3, pp. 301–335, 1991.
[12] W. H. Lam, Z. C. Li, H. J. Huang, and S. C. Wong, "Modeling time-dependent travel choice problems in road networks with multiple user classes and multiple parking facilities," *Transportation Research Part B: Methodological*, vol. 40, no. 5, pp. 368–395, 2006.
[13] X. Zhang, H. J. Huang, and H. M. Zhang, "Integrated daily commuting patterns and optimal road tolls and parking fees in a linear city," *Transportation Research Part B: Methodological*, vol. 42, no. 1, pp. 38–56, 2008.
[14] Z. Qian and R. Rajagopal, "Optimal dynamic parking pricing for morning commute considering expected cruising time," *Transportation Research Part C*, vol. 48, pp. 468–490, 2014.
[15] Z. Y. Gu, Najmi Ali, Saberi Meead, W. Liu, and H. Rashidi Taha, "Macroscopic parking dynamics modeling and optimal real-time pricing considering cruising-for-parking," *Transportation Research Part C*, vol. 118, Article ID 102714, 2020.
[16] T. D. Crocker, "The structure of atmospheric pollution control system," *The Economic of Air Pollution*, vol. 61, pp. 81–84, 1966.
[17] J. H. Dales, "Pollution, property, and prices: an essay in policy-making and economics," *Canadian Journal of Political Science*, vol. 2, no. 3, pp. 386-387, 1969.
[18] I. Benenson, K. Martens, and S. Birfir, "PARKAGENT: an agent-based model of parking in the city. Computers," *Environment and Urban Systems*, vol. 32, no. 6, pp. 431–439, 2008.
[19] K. Dieussaert, K. Aerts, T. Steenberghen, S. Maerivoet, and K. Spitaels, "SUSTAPARK: An agent-based model for simulating parking search," in *Proceedings of the AGILE International Conference on Geographic Information Science*, pp. 1–11, Hannover Germany, June 2009.
[20] N. Levy, K. Martens, and I. Benenson, "Exploring cruising using agent-based and analytical models of parking," *Transportmetrica*, vol. 9, no. 9, pp. 773–797, 2013.
[21] M. Karaliopoulos, K. Katsikopoulos, and L. Lambrinos, "Bounded rationality can make parking search more efficient: the power of lexicographic heuristics," *Transportation Research Part B*, vol. 101, pp. 28–50, 2017.
[22] S. Lehner and S. Peer, "The price elasticity of parking: a meta-analysis," *Transportation Research Part A*, vol. 121, pp. 177–191, 2019.
[23] J. N. Van Ommeren, D. Wentink, and P. Rietveld, "Empirical evidence on cruising for parking," *Transportation Research Part A*, vol. 46, no. 1, pp. 123–130, 2012.
[24] J. B. Lee, D. Agdas, and D. Baker, "Cruising for parking: new empirical evidence and influential factors on cruising time," *Journal of Transport Land Use*, vol. 10, no. 1, pp. 931–943, 2017.
[25] F. Alemi, C. Rodier, and C. Drake, "Cruising and on-street parking pricing: a difference-in-difference analysis of measured parking search time and distance in San Francisco," *Transportation Research Part A*, vol. 111, pp. 187–198, 2018.
[26] R. C. Hampshire and D. Shoup, "What share of traffic is cruising for parking?" *Journal of Transport Economics and Policy*, vol. 52, no. 3, pp. 184–201, 2018.
[27] A. Millard-Ball, R. C. Hampshire, and R. Weinberger, "The curious lack of cruising for parking," *Land Use Policy*, vol. 91, Article ID 103918, 2019.
[28] A. Behrang, B. Douglas, and A. Paz, "Searching for on-street parking: an empirical investigation of the factors influencing cruise time," *Transport Policy*, vol. 97, pp. 186–196, 2020.
[29] G. D. Chiara and A. Goodchild, "Do commercial vehicles cruise for parking?empirical evidence from Seattle," *Transport Policy*, vol. 97, pp. 26–36, 2020.
[30] R. R. Weinberger, A. Millard-Ball, and R. C. Hampshire, "Parking search caused congestion: where's all the fuss?" *Transportation Research Part C*, vol. 120, Article ID 102781, 2020.
[31] J. van Ommeren, M. McIvor, I. Mulalic, and E. Inci, "A novel methodology to estimate cruising for parking and related external costs," *Transportation Research Part B*, vol. 145, pp. 247–269, 2021.
[32] X. N. Zhang, H. Yang, and H. J. Huang, "Improving travel efficiency by parking permits distribution and trading," *Transportation Research Part B*, vol. 45, no. 7, pp. 1018–1034, 2011.
[33] H. Yang, W. Liu, X. L. Wang, and X. N. Zhang, "On the morning commute problem with bottleneck congestion and parking space constraints," *Transportation Research Part B*, vol. 58, pp. 106–118, 2013.
[34] W. Liu, H. Yang, Y. Yin, and F. Zhang, "A novel permit scheme for managing parking competition and bottleneck congestion," *Transportation Research Part C*, vol. 44, pp. 265–281, 2014.
[35] T. Akamatsu and K. Wada, "Tradable network permits: a new scheme for the most efficient use of network capacity," *Transportation Research Part C*, vol. 79, pp. 178–195, 2017.
[36] M. Xu and S. Muller, "Tradable credits scheme on urban travel demand: a linear expenditure system Approach and simulation in beijing," *Transportation Research Procedia*, vol. 25, pp. 2934–2948, 2017.
[37] J. Wang, X. N. Zhang, and H. M. Zhang, "Parking permits management and optimal parking supply considering traffic emission cost," *Transportation Research Part D*, vol. 60, pp. 92–103, 2018.
[38] J. Wang, X. N. Zhang, H. Wang, and M. Zhang, "Optimal parking supply in bi-modal transportation network considering transit scale economies," *Transportation Research Part E*, vol. 130, pp. 207–229, 2019.
[39] J. Wang, H. Wang, and X. N. Zhang, "A hybrid management scheme with parking pricing and parking permit for a many-to-one park and ride network," *Transportation Research Part C*, vol. 112, pp. 153–179, 2020.
[40] S. P. Anderson and A. De Palma, "The economics of pricing parking," *Journal of Urban Economics*, vol. 55, no. 1, pp. 1–20, 2004.
[41] Z. C. Li, H. K. William, S. C. Lam, Z. D. L. Wong, and H. J. Huang, "Modeling park-and-ride services in a multimodal transport network with elastic demand," *Transportation Research Record*, vol. 14, pp. 101–109, 2007.
[42] A. Horni, L. Montini, R. Waraich, and K. Axhausen, "An agent-based cellular automaton cruising-for-parking simulation," *The International Journal of Transportation Research*, vol. 5, no. 4, pp. 167–175, 2013.
[43] H. Ding, *Analysing and modeling on-street parking choice behavior. M.S. thesis*, Beijing Institute of Technology, Beijing, China, 2016.
[44] Y. Yang, Z. Z. Yuan, J. Y. Li, and Y. H. Wang, "Multimode public transit OD prediction and scheduling model," *Advances in Transportation Studies*, vol. 3, pp. 133–146, 2018.
[45] P. van der Waerden, H. Timmermans, and L. Van Hove, "GPS data and car drivers' parking search behavior in the city of

Turnhout, Belgium," *Geoinformatics for Intelligent Transportation*, pp. 247–256, Springer International Publishing, 2015.

[46] R. Liu, Y. Yang, D. Kwak, D. Hang, and B. Nath, "Your search path tells others where to park: towards fine-grained parking availability crowdsourcing using parking decision models," in *Proceedings of the ACM on Interactive, Mobile, Wearable Ubiquitous Technology*, vol. 1, no. 3, pp. 1–27, 2017.

[47] W. Li, L. Bao, and Y. Li, "Comparative evaluation of global low-carbon urban transport," *Technological Forecasting and Social Change*, vol. 143, pp. 14–26, 2019.

[48] W. Li, Z. Pu, and Y. Li, "How does ridesplitting reduce emissions from ridesourcing? a spatiotemporal analysis in chengdu, china," *Transportation Research Part D*, vol. 96, Article ID 102885, 2021.

[49] Y. Yang, Z. Z. Yuan, J. J. Chen, and M. Z. Guo, "Assessment of osculating value method based on entropy weight to transportation energy conservation and emission reduction," *Environmental Engineering & Management Journal*, vol. 16, no. 10, pp. 2413–2424, 2017.

[50] K. A. Small, "The scheduling of consumer activities: work trips," *The American Economic Review*, vol. 72, no. 3, pp. 123–132, 1982.