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Research Article

Pricing Method of the Flexible Bus Service Based on Cumulative Prospect Theory

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Pricing directly affects the sustainable development of the flexible bus service. This study proposes a profit maximization model and a social welfare maximization model for the flexible bus operator based on the cumulative prospect theory. Fares and uncertain travel time due to unforeseen detours in serving passengers jointly affect passengers' mode choice. On the other hand, fares and passengers' probabilistic choices over the flexible bus jointly determine the profits of the flexible bus company and social welfare. This study explores the relationship between the probability of passengers choosing the flexible bus, trip fares, and uncertain travel time. Serving more passengers indicates more profits, which also results in longer detour time thus decreasing the probability of passengers choosing the flexible bus. Considering the interactive influence among passengers, we further calculate the detour time distribution. Finally, a pricing model is established to compensate for the side effects of the detour. The results show that heterogeneous fares can help the flexible bus company to obtain higher profits but have negligible influence on social welfare. In addition, the development of long-distance services and regulations over the detour time can also help to obtain more profits.

1. Introduction

The flexible bus can be regarded as a bridging mode between the conventional fixed-route transit and dial-a-ride services. The flexible bus typically uses small or medium-sized vehicles to provide door-to-door travel services subject to passenger demand [1]. Since taxies are way more expensive despite their high level of service, the flexible bus service with its high flexibility and customization capability is an attractive alternative for high-to-medium income customers [2]. It is critical for operators to seek an appropriate pricing method for the flexible bus service. On the one hand, reasonable pricing can affect the behavior of travelers, reduce the demand for private cars/taxies, and transfer it to public transit, thereby alleviating urban traffic congestion. On the other hand, pricing also directly affects the financial sustainability of the flexible bus company.

In recent years, a growing number of studies investigated the pricing problem for many emerging travel modes such as flexible bus and customized bus. Li et al. [3] formulated a competitive game model in which the objective was to maximize the profits of customized bus services and ride-sharing based on passengers' transport mode choices. Sayarshad and Gao [4] developed a dynamic pricing scheme that utilized a balking rule considering socially efficient levels and revenue-maximizing price. An equilibrium-joining threshold was obtained by imposing a toll on the customers who joined the on-demand mobility system. Kaddoura et al. [5] proposed an agent-based transport simulation to investigate different design concepts for the demand-responsive transit. The simulation results show that a small service area and low prices may result in an unwanted mode shift effect from walk and bicycle to Demand Responsive Transit. Gong et al. [6] constructed a game theory model between the customized city bus service and the conventional urban bus transportation to maximize the profits of the two transportation modes. Wang et al. [2] integrated the disaggregated trip choice model with the vehicle routing model to determine incentive schemes. They concluded that passengers' sensitivity towards incentives is decisive to the result.

In addition, there are many studies on the pricing for other on-demand responsive travel modes, such as ride-hailing and ride-pooling. Ozkan [7] studied the interrelationship between pricing and matching decisions of a ride-sharing firm. He formulated a stylized ride-sharing model that captured customer and driver behaviors considering the geospatial nature of the system. The results showed that optimizing the pricing decisions alone with fixed matching rules did not increase the number of matchings in general. Yan et al. [8] designed and implemented a matching and pricing algorithm at scale to strike a balance between model complexity and accurate description of the marketplace dynamics. Wang et al. [9] found that if the platform offers the carpool service option, they can achieve a larger market coverage and the riders can enjoy more affordable rides without compromising on service quality. Bai et al. [10] considered an on-demand service platform using earning-sensitive independent providers with heterogeneous reservation prices (for work participation) to serve its time and price-sensitive customers with the heterogeneous valuation of the service. Zhong et al. [11] examined how an on-demand ride-hailing platform in competition with the traditional taxi industry designs its pricing strategies under unregulated and regulated pricing scenarios. They found that the monopolistic on-demand ride-hailing platform's price rate and profit under the unregulated pricing scenario are relatively higher than those under the regulated pricing scenario.

Detours are a problem faced by both the flexible bus and carpooling companies. However, few studies considered how the detour time would affect the pricing strategy of the flexible bus. Ke et al. [12] innovatively established a set of nonlinear equations to explore the relationships between the platform decision variables (i.e., trip fare) and endogenous variables (e.g., actual detour time) in ride-sourcing markets with and without on-demand ride-pooling services. By considering the extra detour time experienced by passengers and drivers, they found that decrease in trip fare not only directly increases passenger demand due to negative price elasticity, but also reduces actual detour time, which in turn increases passenger demand. Zhang and Nie [13] established a market equilibrium based on a spatial driver–passenger matching model that determines the passenger wait time for both solo and pooling rides. They found the system's benefit diminishes quickly as the average en-route detour time increases.

Both ride pooling and flexible bus are travel demand-responsive travel modes and how to pricing for both of them considering the uncertainty of the detour time is a challenging problem. However, a flexible bus can serve more passengers at the same time, making possible detours outcomes more complicated. The uncertainty in detour time affects passengers' travel decisions to a greater extent, thereby affecting demand. Only few pricing studies considered travel decisions under uncertain travel time conditions. Choi et al. [14] applied the mean-risk theory to analytically explore how the risk attitude of customers affects the optimal service pricing decision of the on-demand platform. Wu et al. [15] proposed a choice-based framework for modeling the supply/demand interaction in risky choice contexts. The model allowed the system operator to set an optimal pricing strategy regardless of whether user risk preferences are risk-seeking or risk-averse.

There are few studies on the relationship between pricing, detours, and passenger decision-making under uncertain conditions. Instead, most existing studies considered the relationship between price and demand from a macro perspective. Flat and homogenous fare is provided to all passengers [3, 6]. When passengers receive different service levels in terms of detour level, their fares should be different.

Cumulative prospect theory (CPT) is widely used to solve travel decision-making problems and transportation pricing problems. As a descriptive decision-making model under uncertainty, the prospect theory was proposed by Kahneman and Tversky [16] as a critique of the expected utility theory. In 1992, Kahneman and Tversky [17] extended their model to CPT. CPT is based on the assumption of bounded rationality of decision-makers and can describe decision-makers' behavior under uncertain conditions more accurately. Katsikopoulos et al. [18] found that drivers were risk-averse when choosing among routes in the gain domain and risk-seeking in the loss domain. It is consistent with the view of CPT. Sepehr et al. [19] introduced a CPT-based framework for mode choice modeling using observational data. The framework utilized CPT for modeling reliability in the trip-based models. CPT has also been widely applied to congestion pricing. Liu et al. [20] considered the psychological factors of passengers in the congestion pricing model and verified the feasibility of the model based on user equilibrium and CPT. Xu et al. [21] developed an optimal congestion pricing model in which user equilibrium was adopted to capture travelers' response to pricing signals under risk based on CPT. However, the application of CPT in the pricing of the flexible bus is limited.

We aim to study the pricing of the flexible bus service considering the interaction between passenger travel behavior and fares and thus propose a profit-maximization pricing strategy for the flexible bus system based on CPT. Considering the travel time of the flexible bus is uncertain due to its high route flexibility, we can apply CPT to our pricing model. The relationship of detour time distribution, fares, and acceptance of detours (choosing to take the flexible bus) are explored. On this basis, considering the detour problem of the flexible bus, we build a pricing model under static demand for the flexible bus company with the goal of expected profit maximization and social welfare maximization.

In summary, the aims of this research are as follows:

(1) Under uncertain travel time of trips by the flexible bus, we explore the influence of fares and uncertain travel time on passengers' travel mode choices based on CPT

(2) We establish two customized pricing models with the objective of profit maximization for the flexible

bus company and social welfare, considering the impacts of trip detours on-demand loss

The remainder of this paper is organized as follows. Section 2 establishes the mode choice model and the pricing model. The parameters are calibrated through a stated preference (SP) survey. Section 3 describes the design of two case studies and analyzes the results. The last section summarizes the innovations and conclusions.

2. Methodology

In this section, we first introduce the hypotheses used in the flexible bus pricing problem. Based on prospect theory, we calculate the perceived travel time utility of passengers. Taking the travel time utility and out-of-pocket costs as the components in the utility function, we can obtain the probability of passengers taking flexible buses for a given detour time. In addition, considering the mutual influence among passengers in each other's travel time, we develop a series of equations to obtain the detour time distribution. Finally, a flexible bus pricing model is constructed, which aims at maximizing the bus company's profit and maximizing the social welfare respectively. The model parameters are calibrated through the SP survey and Maximum Likelihood Estimation (MLE). The main ideas are shown in Figure 1.

2.1. Problem Description. We consider regional flexible bus service in this study [22], which allows vehicles to operate in a demand-responsive way through reservation within the service area. Passengers can only board or alight in the predefined service areas. The pick-up region and drop-off region are connected by a non-stop fast route. It can be regarded as a variant of customized buses. Figure 2 illustrates the operation of the regional flexible bus. In this paper, we aim to investigate the pricing problem for this regional flexible bus service.

The flexible bus adopts small and medium-sized vehicles and provides door-to-door services based on reservation. In the responsive area, the stops and bus routes depend on the demand of passengers, and the demand of passengers is elastic which is subject to the utility of the flexible bus. Compared with the taxi, the flexible bus will detour when delivering multiple passengers. More passengers to be served will probably lead to longer detour time. To passengers, a longer travel time would decrease the utility of the flexible bus thus affecting his/her mode choice probability. It is necessary to make reasonable pricing to compensate for the side effects of detours so as to make flexible buses competitive with taxies. In addition, as flexible buses may adjust routes dynamically while en-route, the travel time of passengers on-board may be prolonged and thus is uncertain. Passengers' risk attitude and personal preference also influence passengers' decision-making in the case of uncertain travel time. Therefore, it is necessary to accurately measure the impact of these factors on passenger decision-making to obtain the probability of passengers choosing the flexible bus. The mode choice probability and ticket price jointly affect the expected profit of the flexible bus company and the social welfare. This section solves the pricing problem considering the interactions among passengers in the system. The price for each passenger is optimized to maximize the expected profit and the social welfare respectively.

Due to the similarity of flexible bus and taxi services, we assume passengers can choose between the flexible bus and the taxi for their trip. They mainly consider the travel time and the fare assuming the other influencing factors are the same. The flexible bus knows the demand of passengers in advance through the reservation, but it is uncertain whether passengers will take it in the end. After knowing the passengers' demand, the flexible bus will plan the route. If the passenger eventually chooses to get on the bus, the bus will pick him up. According to the probability of passenger boarding, the flexible bus can estimate the travel time through the probability of passengers getting on the bus. Before boarding, the flexible bus displays passengers the fare, travel time, possible detour time, and corresponding probabilities (in this paper, they are captured by the detour time distribution). Detour time refers to the increased time for passengers due to picking up other passengers (the time loss for waiting for boarding, etc.) after boarding compared with the shortest travel time from the origin to the destination. The shortest travel time is equal to the travel time of a taxi. The distribution of detour time refers to the detour time and its corresponding probability.

2.2. Behavior of Passengers. When passengers make mode choice decisions, they will be affected by many factors such as travel time, cost, and uncertainty of travel time [5]. In addition, passengers will also be influenced by their risk attitudes and preferences [21]. Considering the characteristics of the flexible bus, serving other passengers leads to the increase of uncertain detour time for the loaded passengers. The uncertain travel time and fare jointly affect the passengers' mode choice decisions. This part explores the interrelationship between the distribution of detour time, the fare, and the acceptance of detours under the condition of uncertain travel time of the flexible bus.

2.2.1. Perceived Time Utility. Most of the traditional travel decision-making models are based on the expected utility theory, assuming that the decision-maker is entirely rational. In fact, people in the process of making travel decisions are often affected by the traveler's habits, attitude towards risk, preferences, and other factors. Decision-makers often can-no't be entirely rational.

CPT was developed by Kahneman and Tversky [17] based on prospect theory (PT), which is different from the traditional expected utility theory (EUT). CPT has three main observations about an individual's bounded rationality. (i) People usually consider possible outcomes relative to a certain reference point (x_0) rather than to the final state. The payoffs are defined as the gains or losses relative to x_0 before making choices. The payoffs that people perceive can be described by a concave function for gains and a convex function for losses which results in diminishing sensitivity.

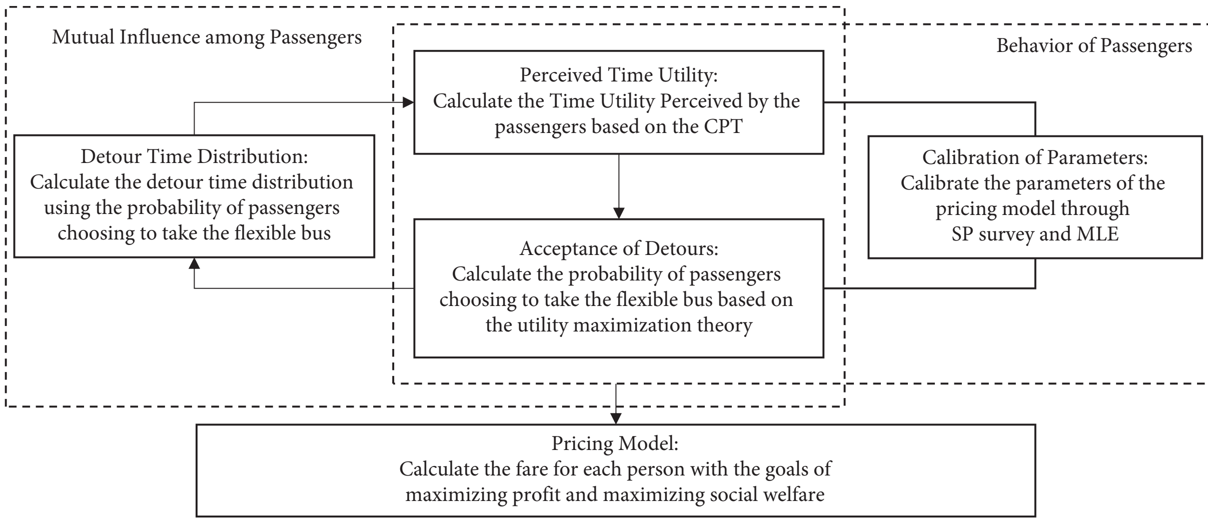

Figure 1: Main idea of the method.

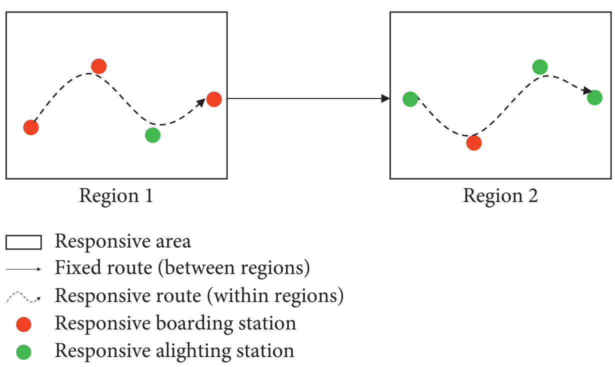

Figure 2: Operation of the regional flexible bus.

(ii) People tend to be loss aversion. People are more sensitive to losses than gains. (iii) People tend to overweight the probability of extreme, but rare events, and underweight more common events. People's decisions are affected by the decision weights, not actual probabilities. In conclusion, CPT argues that decision-makers perceive values $v(x_i)$ (travel time in the case of this paper) differently from the actual values x_i, and probabilities P_i (probability of experiencing a certain travel time for a given trip) are converted to decision weights $w(P_i)$.

Although some studies have considered the influence of detour time, they have not considered the uncertainty of detour [12]. Due to demand uncertainty, the travel time of the flexible bus is uncertain. Therefore, the choice between flexible public transportation and other modes of transportation is a typical travel decision-making problem under uncertain conditions. Given that CPT provides a well-supported descriptive paradigm for individuals' decision-making under risk or uncertainty, we can apply CPT to measure travelers' perceived utility of time.

We assume that a passenger intends to choose between a flexible bus and a taxi for a trip. Choosing each alternative can lead to k different outcomes, quantified by $x_1, x_2, \ldots, x_k$, The outcome i happens with probability P_i. Each alternative can be seen as a prospect (x_i, P_i). The following value function proposed by Kahneman and Tversky [17] is used in this paper:

$$v(x_i) = \begin{cases} (x_i - x_0)^{\alpha}, x_i \geq x_0, \\ -\lambda (x_0 - x_i)^{\beta}, x_i < x_0, \end{cases} \quad \text{where } 0 < \alpha,\ \beta < 1,\ \lambda \geq 1, \tag{1}$$

where x_0 is the reference point; α is the exponent of the value function over the gain region, and β is the exponent of the value function over the loss region. $v(x_i)$ is the value function, reflecting an individual's perceived value. The parameters α and β measure the degree of diminishing sensitivity of the value function. λ is the loss aversion coefficient, indicating that individuals are more sensitive to losses than gains.

The following probability weighting function by Kahneman and Tversky [17] is used:

$$w(P_i) = \frac{P_i^{\gamma}}{\left[P_i^{\gamma} + (1 - P_i)^{\gamma}\right]^{1/\gamma}} \quad \text{where } 0 < \gamma < 1, \tag{2}$$

where γ is the probability weighting parameter, representing the level of distortion in probability judgment in the decision-making process. $w(P_i)$ is the probability weighting function, reflecting an individual's perception of probability. Figures 3 and 4 show the value function and the weighting function. As shown in Figures 3 and 4, the value function $v(x_i)$ is S-shaped, concave in the gain region and convex in the loss region. Besides, individuals are more sensitive to losses than gains. Furthermore, the probability weighting function $w(P_i)$ expands the influence of rare events and shrinks the influence of common events.

The cumulative prospect value is defined as

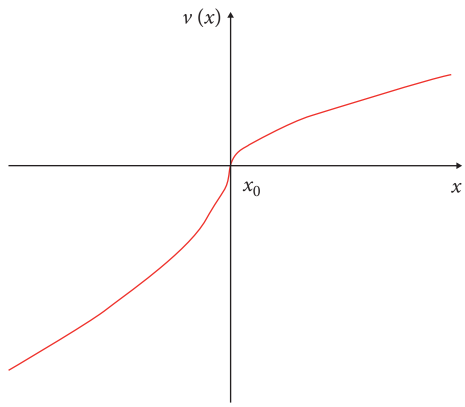

Figure 3: Value function.

$$CPV = \sum_{k} v(x_i)\pi(P_i), \quad (3)$$

$$\pi(P_i) = w(P_i + P_{i+1} \cdots + P_k) - w(P_{i+1} + P_{i+2} \cdots + P_k), \quad where\, i = 1, 2, \ldots, k-1, \quad (4)$$

$$\pi(P_k) = w(P_k), \quad (5)$$

The decision weight $\pi(P_i)$ is calculated based on the cumulative distribution function. All potential outcomes are ranked in increasing order in terms of preference. In these equations, k is the number of all potential outcomes, and i denotes a generic outcome.

For mode-alternative m, the travel time can be $t_{m,1}, t_{m,2}, \cdots, t_{m,k}$ with probabilities $P_{m,1}, P_{m,2}, \cdots, P_{m,k}$, respectively. To apply CPT, we need to define the reference point, the value function, and the weighting function first. In the case of the flexible bus detour problem, people will pay more attention to the increase in detour time compared with the shortest travel time when making travel decisions. Therefore, the shortest travel time t_0 (equal to taxi travel time) is selected as the reference point. (1) is assumed to be the value function. (2) is assumed to be the probability weighting function. Since $t_{m,i} \geq t_0, \forall i \in k$, we only model in the loss domain.

The calculation process of perceived time utility $CPV(t_m)$ is as follows:

$$v\left(t_{m,i}\right) = -\lambda\left(t_{m,i} - t_0\right)^{\beta}, t_{m,i} \geq t_0, \quad (6)$$

$$w\left(P_{m,i}\right) = \frac{P_{m,i}^{\gamma}}{\left[P_{m,i}^{\gamma} + \left(1 - P_{m,i}\right)^{\gamma}\right]^{1/\gamma}}, \quad (7)$$

$$\pi\left(P_{m,i}\right) = w\left(P_{m,i} + P_{m,i+1} \cdots + P_{m,k}\right) - w\left(P_{m,i+1} + P_{m,i+2} \cdots + P_{m,k}\right), \quad where\, i = 1, 2, \ldots, k-1 \quad (8)$$

$$\pi\left(P_{m,k}\right) = w\left(P_{m,k}\right), \quad (9)$$

$$CPV(t_m) = \sum_{i=1}^{k} v\left(t_{m,i}\right)\pi\left(P_{m,i}\right), \quad (10)$$

where $CPV(t_m)$ is the perceived time utility of mode m. (6) calculates the perceived utility $v(t_{m,i})$ corresponding to the travel time $t_{m,i}$. (7), (8), (9) calculate the decision weight $\pi(P_{m,i})$ corresponding to the travel time $t_{m,i}$. (10) calculates the total perceived time utility of mode m.

2.2.2. Acceptance of Detours. We assume that the speeds of the flexible bus and the taxi are equal. The difference in travel time between the flexible bus and the taxi is because that the flexible bus needs to detour to serve multiple passengers in one trip. Therefore, we define the probability of passengers choosing a flexible bus as the acceptance of detours. According to the assumption, the fare will also have an important impact on passengers' decision-making.

When travelers make travel decisions, they always choose the one with a larger utility, which is called the utility maximization theory. The formula of the utility maximization theory is as follows:

$$V_m = \sum_{A} \theta_m y_m, \quad (11)$$

where y_m is the characteristic variable and θ_m is the parameter reflecting travelers' perceived relative importance of different attributes. According to the discussion above, travel time and travel cost are selected as characteristic variables. Considering that travel time is uncertain, the utility of travel time is more suitable to be described by CPT. The travel cost is a deterministic factor. Therefore, the probability estimation and risk attitude of travelers will not affect the utility of the travel cost. The modified utility function is

$$MV_m = \beta_0\sigma_m + \beta_t CPV(t_m) + \beta_c C_m, \quad (12)$$

where C_m is the travel cost of mode m; $CPV(t_m)$ is the perceived travel time utility of mode m; σ_m is a dummy variable indicating whether a particular mode-alternative is the flexible bus or not; β_0 is the mode-specific constant added to capture the effect of unforeseen variables; β_c is the coefficient of cost, and β_t is the coefficient of time. CPT can be used to calculate the value of the modified utility (MV_m) of each mode (the flexible bus and the taxi) and the mode decision between the two modes. Acceptance of detours can be described by a Logit function:

$$p = \frac{\exp\left(MV_{fb}\right)}{\exp\left(MV_{fb}\right) + \exp\left(MV_{taxi}\right)}, \quad (13)$$

$$p = \frac{1}{1 + \exp\left(-\beta_0 + \beta_t\left(CPV(t_{taxi}) - CPV\left(t_{fb}\right)\right) + \beta_c \Delta C\right)}, \quad (14)$$

$$\Delta C = C_{taxi} - C_{fb}, \quad (15)$$

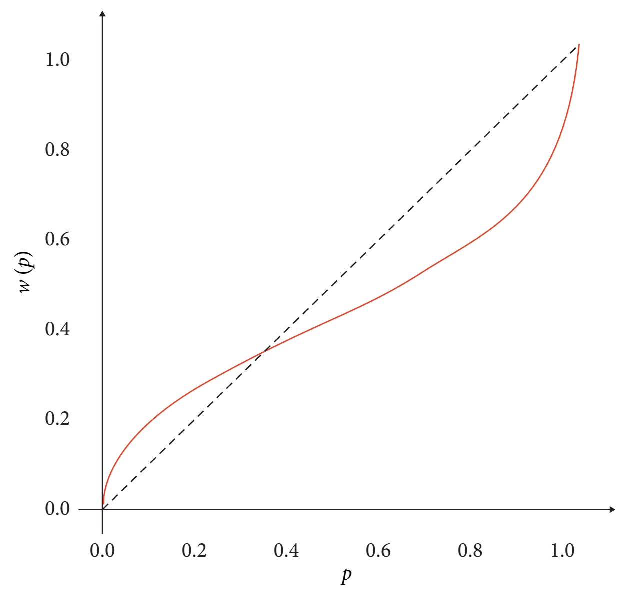

Figure 4: Weighting function.

where p is the acceptance of detours; fb refers to the flexible bus; MV_{fb} is the modified utility of the flexible bus; MV_{taxi} is the modified utility of the taxi, and ΔC is the difference between taxi cost and flexible bus cost. $CPV\,(t_{taxi})$ is equal to 0 and $CPV\,(t_{fb})$ can be calculated by (6), (7), (8), (9), and (10). Therefore, passengers' acceptance of detour can be simplified as

$$p=\frac{1}{1+\exp\left(-\beta_0+\beta_t\lambda\sum_{\mathrm{i}=1}^{\mathrm{k}}\left(\Delta t_i\right)^{\beta}\pi\left(P_i\right)+\beta_c\Delta C\right)}, \quad (16)$$

$$\Delta t_i=t_i-t_0, \quad (17)$$

where Δt_i is the detour time for taking the flexible bus.

2.3. Detour Time Distribution. In (16), the probability of taking the flexible bus is affected by the perceived time utility and fare. The perceived time utility is jointly affected by the detour time and its corresponding probability. Calculating the perceived time utility of a passenger requires the detour time distribution. By analyzing the reasons for the detour, we propose a method to calculate the time distribution of the detour. For a certain passenger, the detour he/she faced is caused by serving other passengers during his/her ride. Generally speaking, the more stations served, the longer the detour time the passenger suffers. However, the detour does no't necessarily happen. It is related to the probability of passengers choosing to take the flexible bus. Considering that the passenger demand is usually densely distributed in a certain area, it can be assumed that, for each passenger, the detour time increases by T for serving every extra station (including the boarding station and the alighting station).

To explain the method, we consider a case of five passengers, A-E, and they get on and off at given stations. Figure 5 shows everyone's boarding and alighting stations. Since the flexible bus provides door-to-door service, the

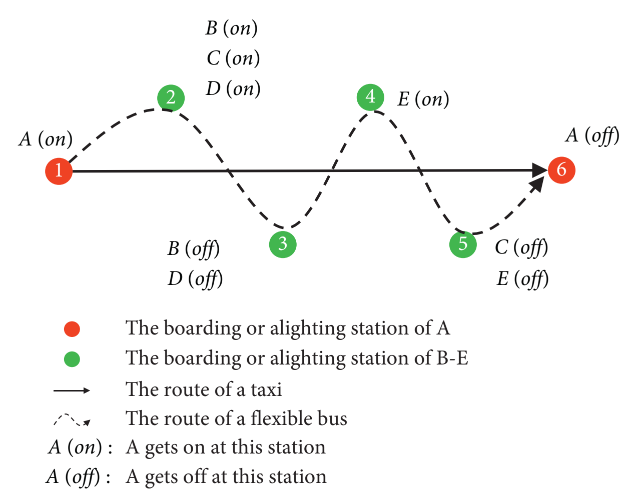

Figure 5: Boarding and alighting stations of A-E.

boarding and alighting stations are also the origin and the destination respectively. Take A as an example, if B-E choose to take the flexible bus, A will be affected by the detour caused by serving B-E. If B-E do no't take the flexible bus, A will not suffer a detour. This is related to the probability that B-E choose to take the flexible bus. The probabilities of B, C, D, and E taking the flexible bus are p_B, p_C, p_{D}, and p_E respectively. They further affect the probability of the flexible bus serving Stations 2–5. The probabilities of serving Stations 2–5 are sp_2, sp_3, sp_4, and sp_5 respectively, which can be calculated as follows:

$$sp_2=1-\left(1-p_B\right)\left(1-p_C\right)\left(1-p_{\mathrm{D}}\right), \quad (18)$$

$$sp_3=1-\left(1-p_B\right)\left(1-p_{\mathrm{D}}\right), \quad (19)$$

$$sp_4=p_E, \quad (20)$$

$$sp_5 = 1 - (1 - p_C)(1 - p_E), \tag{21}$$

According to the assumption, the detour time increases by T for serving every extra station. For A, the detour time can be 0, T, $2T$, $3T$, and $4T$ with probabilities P_1, P_2, P_3, P_4, and P_5, respectively. By combining Stations 2–5, the corresponding probability of the detour time can be obtained. Taking the detour time $\Delta t = 0$ as an example, the corresponding probability is calculated as follows:

$$P_1 = P(\Delta t = 0) = (1 - sp_2)(1 - sp_3)(1 - sp_4)(1 - sp_5), \tag{22}$$

where sp_2, sp_3 sp_4, and sp_5 can be calculated by (18), (19), (20), and (21). P_2, P_3, P_4, and P_5 can be calculated in the same way. The possible detour time for A is $(0, T, 2T, 3T, 4T, 5T)$. Then, we can get the detour time distribution of A. Similarly, we can use this method to calculate the detour time distribution for customers B-E. This idea of calculating detour time distribution can then be extended to a flexible public transport system with Q individuals. In general, for a passenger, his/her detour time is related to the number of other stations served by the flexible bus. The probability corresponding to the detour time is determined by the probability of passengers choosing to take the flexible bus in other stations.

2.4. Pricing Model

2.4.1. Service Provider Profit. The profit of flexible buses is the fare revenue minus the operating cost. The expected fare revenue of passenger is

$$r_j = x_j \cdot p_j, \tag{23}$$

where x_j is the fare of the flexible bus for passenger j and p_j is the probability of passenger j taking the flexible bus. According to (12), (13), (14), (15), (16), and (17), p_j is related to detour time and the difference between taxi fare c and flexible bus fare x_j:

$$p_j = \frac{1}{1 + \exp\left(-\beta_0 + \beta_t \lambda \sum_{i=1}^{k} \left(\Delta t_{i,j}\right)^{\beta} \pi\left(P_{i,j}\right) + \beta_c\left(c - x_j\right)\right)}, \tag{24}$$

Taxi fare c is related to the travel distance of the passenger. The travel distance is composed of the travel distance within the region and the travel distance between regions. The travel distance between regions is a constant, while travel distance within the region is related to the origin and the destination of the passenger. Considering that the travel distance between regions accounts for the main part, we assume that ticket fare and the distance between regions are positively correlated. The taxi fare c is:

$$c = \mu s, \tag{25}$$

where μ is the fare of taxi per kilometer and s is the travel distance between regions.

The operating cost is assumed to be L, which is the system parameter related to operating distance, the vehicle types, fuel consumption, and other factors of the flexible bus system. The calculation of operating cost is similar to the calculation method of taxi fare. The operating distance is composed of the operating distance within the region and the operating distance between regions. The operating distance between regions is a constant, while operating distance within the region is related to the spatial distribution of passengers. Besides, detours within the region will also affect the operation costs. The operating cost L is

$$L = \varphi(s + v\overline{\Delta t}), \tag{26}$$

where φ is the fare of taxi per kilometer; s is the travel distance between regions; v is the average speed of the flexible bus and $\overline{\Delta t}$ is the expected detour time. $\overline{\Delta t}$ can be calculated by the distribution of detour time mentioned in Section 2.4.

The profit of the service provider Z_1 can be expressed as

$$Z_1 = \sum_{j=1}^{Q} x_j \cdot p_j - L, \tag{27}$$

where L is the operating cost; p_j is the probability of passenger j taking the flexible bus, and c is the fare of the taxi.

2.4.2. Passenger Surpluses. The passenger surplus is the fare that a passenger is willing to pay for the flexible bus service minus the actual out of pocket costs. The fare that a passenger is willing to pay for the flexible bus service can be expressed as:

$$W = c - U_{\Delta t}, \tag{28}$$

$$U_{\Delta t} = \beta_t \lambda \sum_{i=1}^{k} (\Delta t_i)^{\beta} \pi(P_i), \tag{29}$$

where W is the fare that a passenger is willing to pay, and $U_{\Delta t}$ is the disutility caused by detours.

The passenger surpluses Z_2 can be expressed as:

$$Z_2 = \sum_{j=1}^{Q} \left(W_j - x_j\right), \tag{30}$$

2.4.3. Optimization Model of Fares. We consider two objectives (profit maximization and social welfare maximization) to optimize the fare of the flexible bus.

(i) Expected profit maximization: compared with the taxi, the flexible bus will detour, so the fare of the flexible bus will not exceed the fare of the taxi. The profit maximization model of the flexible bus can be expressed as

$$\max Z_1 \tag{31}$$

s.t. (12)-(27)

$$0 \le x_j \le c, \tag{32}$$

(ii) Social welfare maximization: social welfare includes the profit of the service provider and passenger surpluses, considering the interests of the flexible bus operator and the passengers simultaneously. The profit of the flexible bus operator should not be less than the threshold θ. The social welfare maximization model of the flexible bus can be expressed as

$$\max Z_1 + Z_2, \tag{33}$$

s.t. (12)-(30)

$$Z_1 \geq \theta, \tag{34}$$

(12), (13), (14), (15), (16), and (17) calculate the probability of passenger j taking the flexible bus. (18), (19), (20), (21), and (22) propose a method of calculating the time distribution of each passenger. (23) calculates the expected fare revenue of passenger j. (24) displays the probability of passenger j taking the flexible bus. (25) calculates the taxi fare. (26) calculates the operating cost. (27) calculates the service provider profit. (28) calculates the fare that a passenger is willing to pay for the flexible bus service. (29) calculates the disutility caused by detours. (30) calculates the passenger surpluses. The objective function aims at maximizing the expected profits of the flexible bus and maximizing the social welfare respectively. Constraint (32) ensures that the fare is within a reasonable range. Constraint (34) ensures that the profit of the flexible bus operator should not be less than a certain threshold.

Combined with the calculation of detour time distribution proposed in Section 2.3, we can find that the passengers in the flexible bus system affect each other. Take the service order as A-B-A-B as an example, the reduction in A's fare will increase A's probability of choosing the flexible bus. Because of the increase in the probability of A choosing the flexible bus, the probability of B's detour also increases, resulting in a decrease in the probability of B choosing to take the flexible bus. Therefore, the fare of each passenger directly affects his probability of taking the flexible bus, and indirectly affects the probability of other people's detours, thereby affecting the probability of other passengers' taking the flexible bus. Figure 6 shows the relationship between A and B. Therefore, a reasonable price should be set for each passenger in the flexible bus system to maximize the expected profit or the social welfare.

The final output of the pricing model is the optimal fare that is shown to each reserved passenger before the flexible bus departs. However, the model is under static demand. The actual operation of a flexible bus is a dynamic process. When a passenger refuses service or completes the service, new passengers will enter the system. This situation will lead to an increase in detour time. It will reduce the level of service for loaded passengers who have already received the initial fare on the bus. Correspondingly, the fare should be reduced. From the perspective of fairness to passengers, the result calculated by the profit maximization model should be the highest price given by the flexible bus in real-world operation.

2.5. Calibration of Parameters. To calibrate the parameters of our model, we conducted an online stated preference survey. The SP survey includes three parts: The first part is the individual socioeconomic attributes of the participants. The second part is regarding people's risk attitudes towards time. We design a set of binary travel plan choices. Each has a different travel time distribution. The average travel time of the choice is equal, but the variance is not. Participants need to choose their preferred travel plan. The third part is related to mode choice between the flexible bus and the taxi. Finally, 246 questionnaires were returned in this SP survey. After sorting the questionnaires, 205 valid questionnaires were obtained, with an effective rate of 83%. Overall, the collected data samples are representative. The model parameters were calibrated using the data of the SP survey.

In many existing studies on travel decision-making based on CPT, researchers use the parameters in the value and weighting functions which are directly estimated by Kahneman and Tversky. However, parameters in Kahneman and Tversky are estimated from the results of gambling in the economic field. In different scenarios, the parameters β of the value function in the loss domain of different travelers should be different [21].

The values of β in our model are estimated based on the experimental results of stated choice questions of the second part and the CPV parameter values (i.e., $\lambda = 2.25$ and $\gamma = 0.69$). A logit model and MLE are used to estimate the value function parameter β. On this basis, the values of β_0, β_t, β_c in our model are estimated by using logistic regression. Our estimation results are $\beta = 0.60$, $\beta_0 = -0.326$, $\beta_t = 0.252$, and $\beta_c = -0.285$. The rate of substitution between CPV(t) and travel cost in our study is $0.252/0.285 = 0.88$ RMB/min. For passengers, the detour is essentially a loss. Compared with taxi fares, the decrease of flexible bus fares is compensation for detours. Therefore, the results mean that travelers' willingness to accept every unit increase in CPV(t) is about 0.88 RMB per minute in our SP survey.

Passengers' willingness to accept CPT(t) includes the willingness to accept detour time and the willingness to accept the uncertainty of travel time. Most studies on the value of time or uncertainty addressed users of cars, buses, and taxis, while few considered ride-sharing or flexible bus passengers [23]. There are mainly three approaches considering reliability in the utility maximization theory. Small [24] proposed a scheduling model to analyze travelers' departure time choices to ensure on-time arrival. Small and Noland [25] improved Small's scheduling model to understand choices under uncertainty by adding the probability distribution of travel time. The mean lateness at departure and/or arrival [26] is another approach to measuring the value of uncertainty. The mean-variance (MV) model [25] estimates the values of travel time and uncertainty within the utility maximization framework.

We choose the mean-variance model to compare with our model based on CPT framework. In the mean-variance model, the utility of mode i is

$$U_i = \beta_t E(t_i) + \beta_{SD} SD(t_i) + \beta_c c_i + \beta_\sigma \sigma_i, \tag{35}$$

where $E(t_i)$ is the expected travel time of mode i, $SD(t_i)$ is the standard deviation of travel time, c_i is the ticket fare, σ_i is

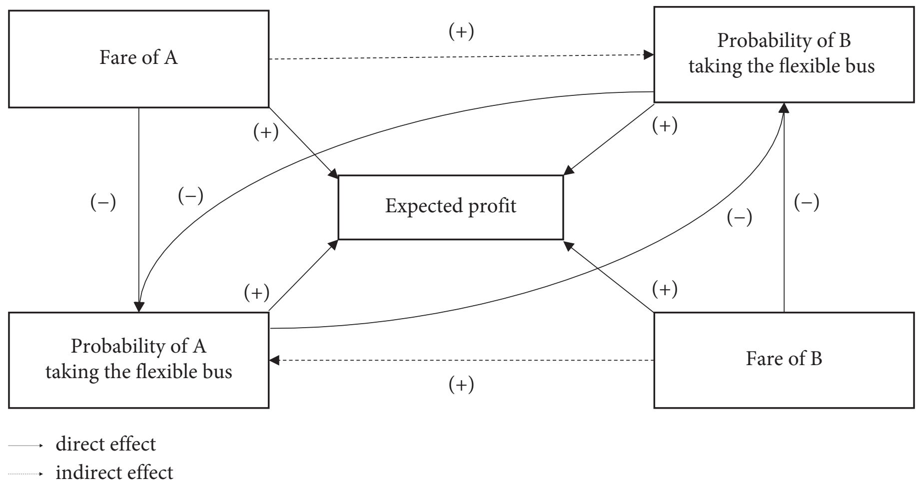

Figure 6: The relationship between A and B.

a dummy variable indicating whether a particular mode-alternative is the flexible bus or not, and β_t, β_{SD}, β_c and β_σ are the parameters. The value of travel time and value of uncertainty can be defined as β_t/β_c and β_{SD}/β_c. We used the data based on the experimental results of stated choice questions to calibrate the parameters in the mean-variance model. The estimation results are $\beta_\sigma = -0.054$, $\beta_t = 0.152$, $\beta_{SD} = 0.018$, $\beta_c = -0.311$. The value of travel time is 0.49 RMB/min and the value of time uncertainty is 0.06 RMB/min. The value of time uncertainty is too small, which is unreasonable. The rate of substitution between CPV(t) and travel cost in our model is 0.88 RMB/min, which is bigger than 0.49 RMB/min. Because our model considers the time uncertainty when calculating the value of travel time are shown in Table 1.

In addition, travel time variability generates utility or disutility depending on the relationship between the reference point. Therefore, the time value and uncertainty perceived by travelers cannot be estimated separately. And according to the above analysis, the subjective perception value of the traveler will affect the utility. Therefore, it is more reasonable to apply CPT to describe the passengers' decision-making behavior in this context.

Table 1: Parameter calibration results of the mean-variance model.

Parameters	β_σ	β_t	β_{SD}	β_c
Coef.	−0.054	0.152	0.018	−0.311
P-value	0.712	0.000	0.615	0.000

Table 2: Passengers' boarding and alighting stops in Case 1 and Case 2.

Stop	Passenger	Stop	Passenger
Case 1			
1	1 (on)	7	1 (off)
2	2 (on), 3 (on)	8	2 (off), 3 (off)
3	4 (on)	9	4 (off)
4	5 (on)	10	5 (off)
5	6 (on), 7 (on)	11	6 (off), 7 (off)
6	8 (on)	12	8 (off)
Case 2			
1	1 (on), 2 (on)	5	3 (off), 6 (off), 7 (off)
2	3 (on)	6	2 (off), 4 (off)
3	4 (on), 5 (on), 6 (on)	7	5 (off)
4	7 (on), 8 (on)	8	1 (off), 8 (off)

Note. 1 (on) means Passenger 1 gets on the flexible bus.

3. Case Study

3.1. Case Description and Parameter Settings. We suppose that there are two small areas, area R is a residential area, and area S is a commercial area. The distance between the two areas is 20 km. There are Q passengers going from R to S, and passengers can choose between flexible bus and taxi. The flexible bus can obtain the demand of all passengers in advance and determines the passengers' boarding and alighting order at the stops. Two cases are set up in this section. Table 2 describes the stops where passengers get on and off the bus. Case 1 adopts the first-on, first-off service principle. In Case 2, there is no specific service principle (random service principle).

The parameters of the two cases are set as follows: s, v, Q, T, μ, φ, and θ are system parameters. We set $s = 20$ km, $v = 30$ km/h, $Q = 8$, $T = 2$ min, $\mu = 3$ RMB/km, $\varphi = 4$ RMB/km, and $\theta = 100$ RMB. λ, γ, β are parameters in CPT. λ, γ are obtained by referring to the research of Tversky and Kahneman. β is calibrated through the SP survey. We set $\lambda = 2.25$, $\gamma = 0.69$, and $\beta = 0.60$. $\beta_0, \beta_t, \beta_c$ are the parameters in the pricing model, which are also calibrated through the SP survey. We set $\beta_0 = -0.326$, $\beta_t = 0.252$, and $\beta_c = -0.285$. In two cases, we omit the average detour time because the average detour time is much smaller than the travel time between regions.

3.2. Results

3.2.1. Case 1. Table 3 shows the fares for each passenger in profit maximization and social welfare maximization

TABLE 3: Results of case 1.

Passenger		1	2	3	4	5	6	7	8
Detour stops		5	5	5	5	5	5	5	5
Profit maximization model	Fares (RMB)	43.6	43.1	43.1	43.6	43.6	43.1	43.1	43.6
	Maximum profit (RMB)	235.9							
Social welfare maximization model	Fares (RMB)	23.7	21.0	21.3	24.0	23.3	21.3	21.9	23.4
	Maximum social welfare (RMB)	381.9							

models. In addition, the number of stops that affect each passenger's detour is also listed.

In the profit maximization model, the average fare is 43.4, and the std is 0.3. The expected maximum profit is 235.9. The fare of each passenger is almost the same. It is because that the number of detour stops (referring to the detour suffered by the passenger due to serving other stops) is the same. The increase in the fare of passenger 1 will result in a reduction in the probability of Passengers 2–8 taking the flexible bus. It is the same for every passenger. When the individual's fare is set as the average, the expected profit is reduced to 235.8 with negligible differences from the maximum profit.

In the social welfare maximization model, the average fare is 22.5, and the std is 1.2. The maximum social welfare is 381.9 with the constraint that the service provider profit is at least 100. The fare of each passenger is slightly different. When the individual's fare is set as the average, the social welfare is also with negligible differences from the maximum social welfare.

Figure 7 shows that the profit and social welfare (without the constraint that the service provider profit is at least 100 RMB) change with the fare. Here, we set the fare equal for each passenger. As the fare increases, the profit first increases and then decreases, reaching the maximum at around 43 RMB/trip, while the social welfare first remains almost unchanged and then gradually decreased after around 34 RMB/trip.

3.2.2. Case 2. Table 4 shows the fares for each passenger in profit maximization and social welfare maximization models. In addition, the number of stops that affect each passenger's detour is also listed.

In the profit maximization model, the average fare is 45.5, and the std is 2.2. The expected maximum profit is 254.8. In the profit maximization model, the average fare is 22.5, and the std = 2.64. The expected maximum profit is 254.8. Since the number of detour stops for each person is different, the individual fare is also different. Figure 8 shows the relationship between detour stops and fares in Case 2. When the travel distance is certain, the more stops you take, the lower the fare is. It is because the effective distance (the distance from the origin to the destination) is fixed for passengers. Detours caused by serving other stops increase the travel time of the passengers. Therefore, for the flexible bus system, the more detour stops a passenger travels, the lower the fare should be. This is consistent with the calculation results of our model. When an equal fare is used, the expected profit is 249.9, reduced by 1.9% compared to the maximum expected profit, and the social welfare is 388.7. It shows that it is better to take a different fare for each user when aimed at gaining more profits. However, taking a different fare has negligible influence on social welfare.

The travel distance, number of passengers, and other parameters of Case 1 and Case 2 are all the same except for the service order. By comparing Case 1 and Case 2, the maximum profit of Case 2 is 8.0% higher than Case 1, and the maximum social welfare of Case 2 is 1.8% higher than Case 1. This is due to the fact there are more detour stops in Case 1 that affect each passenger than in Case 2. It shows that the detour time has an important influence on the expected profit of the system.

3.3. Effects of the Travel Distance. We set the travel distance to vary from 10 to 40 km while maintaining the other parameters unchanged. Figure 9 shows that as the travel distance increases, the profit and the social welfare per unit distance increase, and the magnitude of the increase decreases. Take the maximum profit model as an example, when the travel distance is 10 km, the maximum profit per unit distance is 7.1 RMB; when the travel distance is 40 km, the profit per unit distance is 15.2 RMB. Compared with the travel distance of 10 km, the profit per unit distance increased by 114.1%, indicating that the flexible bus can obtain more profit in long-distance transportation. The trend of social welfare and profit is consistent. Therefore, the development of long-distance flexible bus services should be a priority strategy for the company and the whole society. Focusing on providing long-distance flexible public transportation services, such as between residential areas, transportation hubs, large shopping malls, and work areas can increase profit and social welfare. With the change of travel distance, the optimal fares per unit distance of the two objectives range from 1.8 to 2.5 and 1.1 to 1.2 respectively. The variation range is small, which indicates that the pricing model has good robustness when travel distance changes.

3.4. Effects of the Average Detour Time. The average detour time is caused by serving other stops for passenger boarding and alighting. We set the average detour time to vary from 1 to 9 minutes for serving one extra stop. Figure 10 shows that as the average detour time increases, the maximum expected profit and social welfare decrease, and the magnitude decreases. Take the maximum profit model as an example, when the average detour time is 1 minute, the maximum profit is 254.2 RMB; when the average detour time is

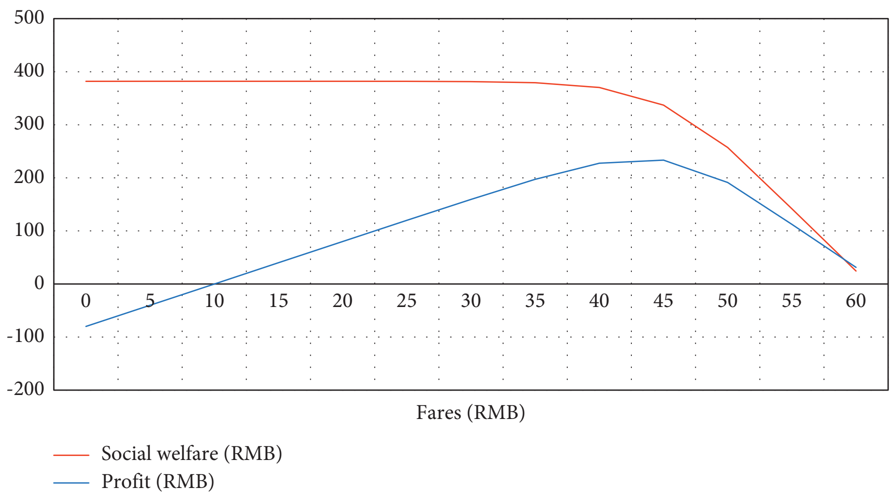

FIGURE 7: The relationship between fares, profit, and social welfare in Case 1.

TABLE 4: Results of case 2.

Passenger		1	2	3	4	5	6	7	8
Detour stops		6	4	2	2	3	1	0	3
Profit maximization model	Fares (RMB)	42.0	43.7	45.8	45.7	44.7	47.1	50.0	44.9
	Maximum profit (RMB)	254.8							
Social welfare maximization model	Fares (RMB)	16.6	21.2	24.5	22.4	24.3	23.2	25.9	22.1
	Maximum social welfare (RMB)	388.7							

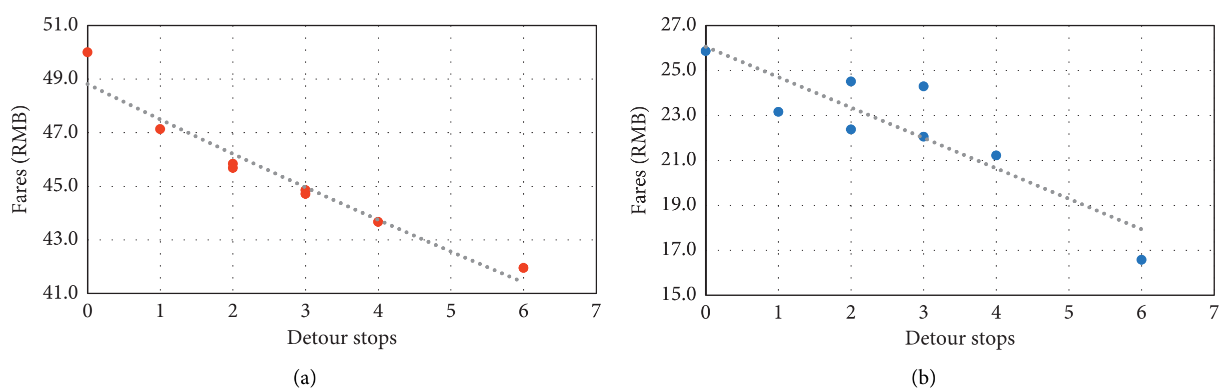

FIGURE 8: The relationship between detour stops and fares in Case 2. (a) The profit maximization model, (b) The social welfare maximization model.

9 minutes, the maximum profit is 161.1 RMB, substantially reduced by 36.6%. As the average detour time increases, a lower fare is needed to compensate for the delay suffered by passengers, resulting in a decrease in the maximum profit and social welfare. Therefore, the flexible bus company should pay attention to setting an upper limit for the detour time in operation. With the change of average detour time, the optimal fares per unit distance of the two objectives range from 1.7 to 2.3 and 1.1 to 1.2 respectively. This small range indicates that the pricing model has good robustness when average detour time changes.

3.5. Effects of the Value of Time. β_t/β_c measures the value of time, which is related to the individual's attributes. We set β_t/β_c to vary from 0.6 to 1.5 RMB/min while maintaining the other parameters unchanged. Figure 11 shows that as the value of time increases, the maximum expected profit and social welfare decreases, and the magnitude decreases. Take the maximum profit model as an example, when β_t/β_c is 0.4 RMB/minute, the maximum profit is 265.5 RMB; when β_t/β_c is 1.2 RMB/minute, the maximum profit is 217.0 RMB, substantially reduced by 18.3%. As the value of time increases, the operator needs to compensate more for the same

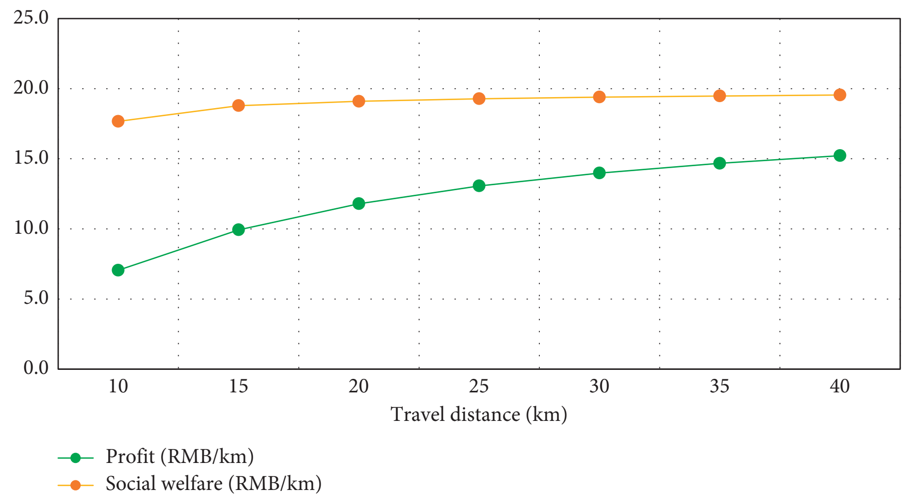

Figure 9: Effects of the travel distance (Case 1).

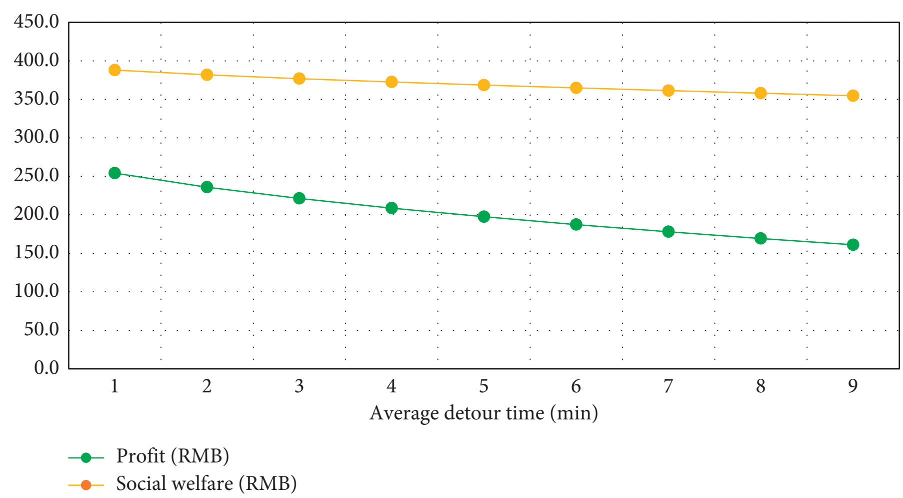

Figure 10: Effects of the average detour time (Case 1).

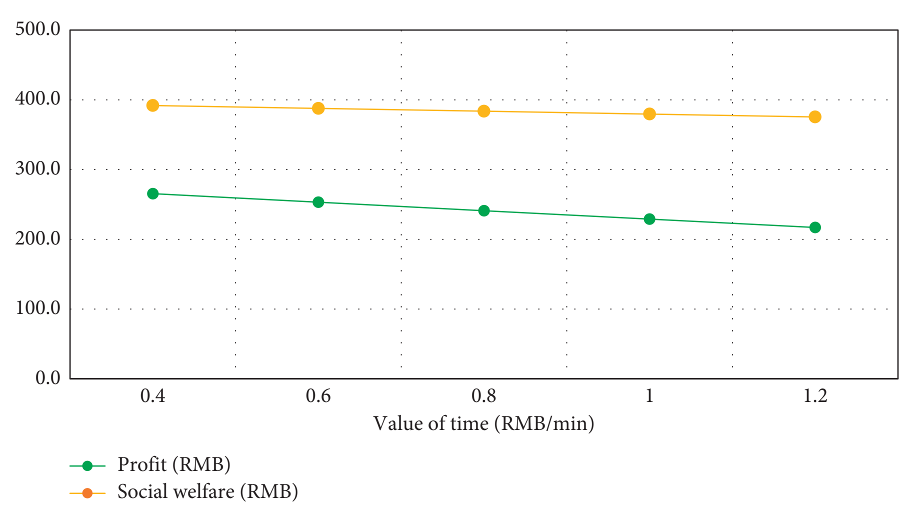

Figure 11: Effects of the value of time.

delay suffered by the passenger whose value of time is higher, resulting in a decrease in the maximum profit and social welfare. Therefore, the flexible bus company should determine the scope of target passengers in combination with the available service level in operation. With the change of the value of time, the optimal fares per unit distance of the two objectives range from 2.0 to 2.3 and 1.1 to 1.5, respectively, indicating good robustness when the value of time changes.

4. Conclusions

With the goals of maximizing the expected profit and maximizing social welfare, the pricing model is constructed based on the cumulative prospect theory for the regional flexible bus service. The parameters of the model were calibrated through a stated preference survey, and two flexible bus cases were designed to analyze the performance of the model. First, we considered the passenger detour in the flexible bus pricing problem and explored the influence of uncertain detour time and fare on passengers' mode choice probability. Second, we modeled the mutual influence among passengers and found that the certain passenger's mode choice probability would affect the detour probability of other passengers. Thus, a calculation method for detour time distribution was proposed.

The results showed that the detour time has a greater impact on the profit and social welfare of the system. More detour stops lead to a lower fare. As the travel distance increases, both the maximum expected profit and social welfare per unit distance increase. As the average detour time increases, the maximum expected profit and social welfare decrease. It is suggested that the flexible bus company may develop long-distance services and set a proper upper limit to the detour time to achieve higher profit. Besides, the passengers' value of time and uncertainty also has a vital influence on the profit and social welfare of the system. The flexible bus company should determine the scope of target passengers in combination with the available service level in operation. With the change of the parameters, the optimal fares per unit distance vary in a small range. The pricing model has good robustness.

There were some limitations to this study. When the area is small and the passengers are densely distributed, a detour time distribution can be calculated. But for few passengers in a large area, there may not be a specific detour time distribution. In addition, the current pricing model can only be used to handle static demand. How to dynamically price the flexible bus needs to be explored further.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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