

Research Article

Vulnerability Analysis of Urban Rail Transit Network considering Cascading Failure Evolution

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Vulnerability analysis is the premise of operational risk management and control for the large-scale and complex urban rail transit network (URTN) under the operation interruption of important stations. The temporary operation interruption of one station in an emergency may lead to the cascading failure and the paralysis of the whole URTN due to the load of other stations exceeding the limited capacity. The priority of important stations is proposed by combining its location and function in URTN. In addition, focusing on the analysis of the travel behaviour of passengers and the synergy of public transport networks, a novel cascading failure evolution model is established to simulate the cascading failure process of URTN under different attack scenarios. The vulnerability indicators are constructed to dynamically evaluate the vulnerability of URTN considering cascading failure evolution, which are different from the traditional vulnerability indicators based on complex network theory. Taking the Beijing urban rail transit network as an example, the dynamic simulation results show that the cascading failure of URTN is closely related to the temporal-spatial distribution of passenger flows and malicious attacks are more destructive than random attacks. Compared with the important stations with the largest betweenness or degree, the interrupted stations with largest intensity have a greater impact on the operational stability of URTN. Moreover, increasing the capacity coefficient of the station can reduce the vulnerability of URTN.

1. Introduction

With the rapid development of URTN, the safety operation of URTN is facing new challenges and requirements. Once a station in the network fails, other stations may change from normal to failure when the load exceeded limited capacity due to the strong spatiotemporal correlation between stations, which may further cause cascading failure and seriously affect the travel quality of passengers. Therefore, the studies on the evolution mechanism of cascading failure in the URTN, identification of important stations, and dynamic assessment of vulnerability have become significant in optimizing the structure and improving the efficiency of the URTN.

Robustness indicates the ability of the system to operate under interference [1], and reliability indicates the dependability of the normal operation of the system under interference [2]. Vulnerability is widely used in measuring the performance of networks under external disturbances [3]. The concept of vulnerability is defined as the affection degree of external disturbance (e.g., unfavourable weather and equipment failure) on the performance of a system [4], which has been used by researches to measure the stability of various networks, such as the Internet [5], computer networks [6], social networks [7], and power networks [8]. In transportation networks, the vulnerability can be defined as the sensitivity to external disturbances that may decrease the

service performance [9]. Compared with the robustness and the reliability of the transportation network, vulnerability analysis pays more attention to the impact of external interference on the network.

In transportation networks, the studies on vulnerability assessment can be mainly divided into two categories: static and dynamic. Static vulnerability assessment refers to the analysis of the network topology, efficiency, and connectivity under a certain failure state, without considering the impact of external disturbances on the passenger travel behaviour and the propagation process of the passenger flows. On this topic, researchers often use the indices of the number of normal and failed components [10], network efficiency [11], and connectivity [12, 13] to measure the performance of the transportation network during the failure of edges, one or more stations. Many complex systems can be modeled as complex networks for analysis. At present, the research on complex networks has attracted extensive attention and research. When considering network characteristics, typical characteristics are usually used to measure the network. Liu and Song [14] introduced the concepts of degree, clustering coefficient, and average shortest path length in complex network theory into URNT to analyze the network accessibility and traffic efficiency under the failure of transfer stations. Zhang et al. [15] analysed the topological and functional characteristics of the Shanghai subway network and proposed to evaluate the function and connectivity of traffic network based on the function loss parameters and connectivity parameters. Combining the topology of URTN and the characteristics of passenger flow distribution, Lu [16] investigated the cumulative affected traffic flows caused by external disturbances and quantified the vulnerability of the network under different attacks. They also pointed out that the failure of the transfer station is less effective in the resilience of the network. Yang et al. [17] used the relative size of maximum connected subgraph (RSMCS) and Global Network Efficiency (GNE), two common indicators in the field of complex network analysis, to comprehensively evaluate the vulnerability of Beijing urban rail transit network (BRTN) under random and malicious attack strategies. Liu et al. [13] defined the fraction of acceptable trip and total generalized travel cost increase ratio to analyze the impact of link capacity reduction on the performance of urban rail transit network. Cats and Krishnakumari [18] deleted the nodes or edges of URTN according to the order of degree or betweenness. The relative size of the largest connected subnetwork when removing nodes or edges and the largest subnetwork of the original network and the normalized average shortest path are used to describe the change of network performance. Chen et al. [19] used the demand-impedance index to describe the performance curve of URTN in the process of external attack and repair. The results show that BRTN is a scale-free network, and malicious attack is more destructive than random attack. However, when the URTN is disturbed by external disturbance, the analysis of vulnerability cannot be limited to the network topology [20]. Because the external disturbance will also have an impact on travel decision making of passengers. Therefore, it is difficult to reflect the strong

spatiotemporal correlation between nodes and the relationship between external interference and passenger travel behaviour only using evaluation method for static vulnerability.

Dynamic vulnerability assessment refers to the evaluation of network stability, which considers the spatiotemporal dynamic changes of the network structure and the passengers within a certain failure duration [21]. The dynamic change of the passenger distribution and the topology of URTN can be described by cascading failure process during interruption time [22]. Sun et al. [9] evaluated the cascading failure process of URTN based on the coupled map lattice model and found that the failure was difficult to control when the loop was attacked. Liu et al. [23] built a CASCADE model with designing a traffic redistribution method based on the edge weight function, to analyze the survivability of URTN from the perspective of the scale of failure and degree of damage. Xing et al. [24] introduced the ORNL-PSERC-Alaska (OPA) model to describe the cascading failure phenomenon of urban regional transportation network under random attack. The essence of cascading failure is that when one or more nodes fail, the load will be redistributed, which may cause other nodes to fail because the load exceeds their limited capacity. Therefore, the proposed capacity-load model [25] is often used to describe cascading failures. The crux of cascading failure process analysis is load redistribution [26]. In the past, the load of the network was redistributed by constructing a load function of betweenness or degree [27, 28]. With the increasing interest, researchers gradually introduced the distribution characteristics of the passengers into the redistribution during the evolution process of cascading failure. Wu et al. [29] evaluated the survivability of weighted road traffic network considering cascading failure based on the user equilibrium model. Zhang and Wang [30] proposed a method of nonuniform load distribution to the neighborhood and evaluated the survivability of Nanjing Subway Network cascading failure based on the capacity-load model. In addition, Szymula and Bešinović [31] proposed a new network vulnerability model to evaluate the vulnerability of the railway system by finding out the combination of key links that cause the most adverse consequences to passengers and trains. Vulnerability is affected by the total travel cost of passengers, the number of passengers unable to reach the destination, and the cost of train service adjustment.

The researches of the above scholars provide extremely valuable results for the modeling of cascading failure and vulnerability analysis of URTN and provide a theoretical framework for subsequent research. Considering these findings, we found that the following points need attention:

- (i) The existing vulnerability research methods of URTN are usually limited to URTN. In the real world, URTN is coupled with other networks such as bus network, which does not exist in isolation. If the coordination between urban public transport networks is not considered, the impact of emergencies on the operation of URTN is difficult to be accurately evaluated.

- (ii) The key to judge whether the cascading failure of URNT occurs is to estimate whether the load of the station exceeds its limited capacity. In the past, most studies used departure frequency and topology metrics for complex networks to define the load of nodes and edges. Although the existing capacity-load model considers the passenger flow of URTN as the load of nodes or edges, the complex travel behavior of passengers under emergencies is not considered in the cascade failure process. Therefore, it is still difficult to assess the dynamic vulnerability of the URTN under emergencies.
- (iii) The problem of passenger redistribution is a research difficulty when a station is interrupted due to the disturbance. URTN serves the heterogeneous passengers, and their travel behaviour is complex and changeable when emergencies occur. Simply loading the passengers of adjacent stations according to the proportion of topology metrics for complex networks is difficult to meet the needs of the actual network.

The sequence data of passenger flow distribution of urban rail transit and other public transport are correlated in time and space dimensions, and the affected passengers due to the failure station may change their travel modes and travel paths, which causes the passengers in URTN transferred to other public transportation networks. Therefore, different from previous studies, the main contribution of this study is considered the coordination between the URTN and other public transport networks and the complex travel behaviour of affected passengers. During the analysis of cascade failure evolution, the edge weight function considering the capacity coefficient of passenger flow is used to define the states of stations. In the process of cascading failure, the reparability of stations is considered, that is, with the real-time change of network load, the failure state of each station of urban rail transit network changes, which is different from the previous situation that once the station is interrupted; it will be in failure state for a long time. In addition, the vulnerability indicators are constructed to dynamically evaluate the vulnerability of URTN considering cascading failure evolution when the station is attacked randomly or maliciously, which are different from the traditional vulnerability indicators (such as degree, clustering coefficient and average shortest path length) based on complex network theory.

This article is mainly divided into the following parts: Section 2 defines various attacks and analyses passenger travel behaviour under emergencies. Section 3 constructs the vulnerability evaluation model of URTN, including the cascading failure model and the establishment of the indicators for dynamic vulnerability assessment. Section 4 takes the BRTN as an example to analyze the dynamic vulnerability under the different attack scenarios. Finally, Section 5 summarizes our conclusions.

2. The Definition of Various Attacks and Travel Behaviour Analysis of Affected Passengers

Once a station of urban rail transit system is attacked, it is obviously characterized by high uncertainty, easy to spread, and trigger a chain reaction. At the same time, when the station is interrupted due to the attack, the travel choice behavior of passengers affected by the operation interruption will change. The behavior analysis of passengers when the station is attacked is the need of vulnerability analysis of the URTN.

2.1. The Definition of Various Attacks. Urban rail transit system plays a crucial role in urban public transport system, providing fast and convenient transportation services for the public. It has the characteristics of high construction requirements, high technical complexity, closed passenger transport environment, high operation intensity, and network operation. URTN usually encounters two kinds of attack events, namely, random attack and malicious attack [32], as shown in Figure 1.

In real life, it is difficult to quantitatively analyze the breaking force of the attack on URTN. Therefore, in this article, we define random attack as a random failure of a station in URTN, and the probability of random attack at all stations is equal. Malicious attacks are defined as targeted and destructive attacks, which often occur in stations with large traffic and vital stations in the network.

Under the action of attack events, the service of the station is often interrupted. The interrupted station will lead to the failure of other stations through the connection relationship between stations, resulting in cascading failure. It should be noted that the attack may take place at the stations or within the train operating line of the URTN. In addition, the failure of one position may also lead to the shutdown of the entire line [18]. Therefore, when studying the vulnerability of URTN under different attack scenarios, it is necessary to analyze specific scenarios.

Malicious attacks often occur at pivotal stations of URTN. Timely and efficient management and control of key stations under emergencies will help to improve the vulnerability of URTN. In this article, the important station of the URTN refers to the station that can cause the cascading failure and may have a large disruptive impact on the network. In order to identify the damage strength of the failed station to the network in a specific emergency scenario, it is necessary to traverse each node in the network for cascading failure analysis. By analyzing the damage degree of urban rail transit system, we can identify the pivotal stations in the network. For multiple stations, we also need to study each combination to judge the relative importance of the station [33]. However, this method is inefficient, and it is widely known that the important stations often play a key role in the structure and function of the actual URTN. In graph theory and network analysis, centrality is an index to

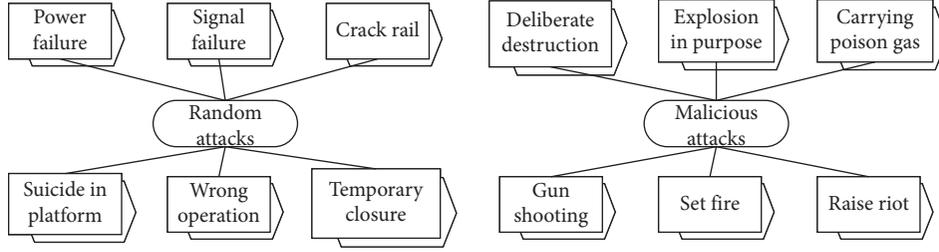


FIGURE 1: Examples of attack types.

judge the importance/influence of nodes in a network, mainly including degree and betweenness. In addition, the main object of urban rail transit service is passengers, that is, the number of transported passengers also determines whether the station plays a central role. Therefore, this article proposes to evaluate the importance of stations by degree, betweenness, and the intensity of the node.

Before calculating the indicators to evaluate the importance of the station, we first model the URTN based on L-space [34]. L-space method refers to a network modeling method in which stations are regarded as nodes and the connections between adjacent stations are regarded as edges. P-space [35] method regards the station as a node and the line relationship to which the station belongs as an edge. When multiple stations belong to the same operation line, any two stations are connected by one edge. R-space [36] method regards the connecting edges between stations as nodes and stations as edges. Through comparative analysis, it can be concluded that the physical significance of the characteristic parameters of P-space and R-space spatial models is relatively specific, which mainly reflects the transfer characteristics of URTN. However, L-space method can graphically reflect the corresponding relationship between nodes and edges, which is helpful to analyze the structural and functional characteristics of URTN. It is also convenient to integrate passenger travel information into the network and analyze the dynamic evolution characteristics of URTN under emergencies.

According to the following three evaluation indexes, we can rank the importance of the stations in the URTN.

(1) *Node Degree*. Node degree reflects the association of each node with other nodes in URTN, which is an important attribute of the node and reflects the embodiment of connectivity of the node.

$$k_i = \sum_{j \in N, j \neq i} \sigma_{ij}, \quad (1)$$

where k_i is the degree of node i . σ_{ij} represents the adjacency relationship between nodes i, j . N is the total number of nodes in the network. If there are directly connected edges between nodes i and j , $\sigma_{ij} = 1$, otherwise, $\sigma_{ij} = 0$.

(2) *Node Betweenness*. Node betweenness reflects the number of shortest paths through a node in a network and reflects the importance of node as a “bridge,” which is a measure of graph centrality based on shortest path.

$$B_i = \sum_{o, d \in N} \frac{g_i(L_{\text{short}}^{o,d})}{g(L_{\text{short}}^{o,d})}, \quad (2)$$

where B_i is the betweenness of node i . $g_i(L_{\text{short}}^{o,d})$ is the number of shortest paths through node i between o and d . $g(L_{\text{short}}^{o,d})$ is the number of shortest paths between o and d .

(3) *Intensity of Node*. The service subject of URTN is the passengers, so the intensity of node is introduced to reflect the passenger transport capacity of each station. The intensity of node is the sum of weights of edge directly connected to the node.

$$\omega_i = \sum_{j=1, j \neq i}^N \sigma_{ij} \lambda_{ij}, \quad (3)$$

where ω_i is the intensity of node i . λ_{ij} is the weight that is defined as the passenger flow on e_{ij} of the direct connective edge of node i and j .

2.2. *Travel Behaviour Analysis of Affected Passengers*. In the normal operation of the URTN, passengers may choose a relatively familiar route to travel, but when the network is interrupted due to external interference, the travel choice behaviour of passengers affected by the operation interruption will change. Part of passengers will be transferred to other modes of transportation, such as bus, which requires flexible service modes to adapt to the changing passenger demand [37]. If the interrupted station is the origin or the destination, passengers may cancel the trip or change the origin or the destination of the trip. When the fault occurs at the intermediate station of a trip, passengers can change the travel mode or path.

In this article, passengers whose travel path contains invalid stations are called affected passengers and the passenger travel path includes urban rail transit path, bus path, and the combined path. By analyzing the travel behaviour of passengers under emergencies, this article divides the travel schemes of passengers under emergencies into four categories: waiting for the station to recover (P_1), reselecting the shortest path of rail transit (P_2), choosing the shortest bus path (P_3), and choosing the combined mode of bus and rail transit (P_4), as shown in Figure 2.

In addition, before modeling passenger travel behaviour, we first put forward the following assumptions:

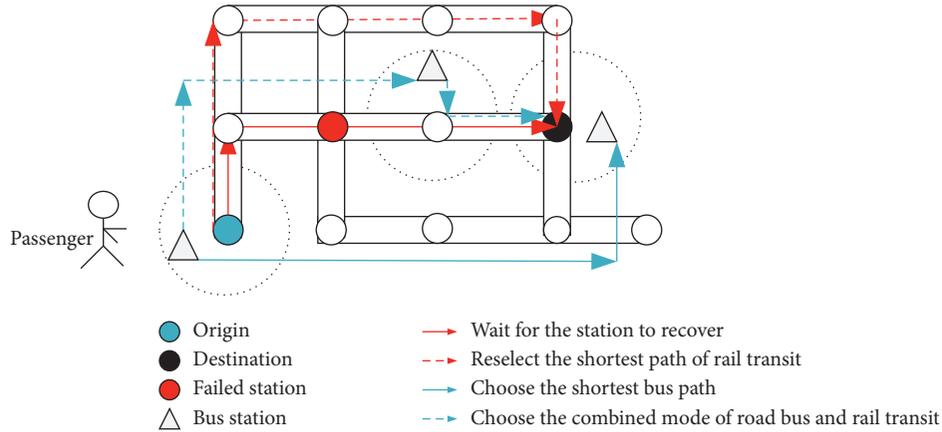


FIGURE 2: Travel schemes of passengers.

- (i) The travel demands of passengers during the study period are all rigid, i.e., passengers will not cancel their trips even if the operation is interrupted. In addition, the manager will not change the vehicle operation plan of the stations not affected by the interruption.
- (ii) During the entire period from interruption to recovery, passengers of all stations can complete their trips through the corresponding routes except the failed station.
- (iii) Passengers rely on public transport, namely, only considering strong dependence and correlation between the URTN and the bus network.
- (iv) It is assumed that passengers will select the shortest path in each scheme and then determine the probability of finally selecting a travel scheme according to the utility function.

When studying the vulnerability of URTN, the traditional method only redistributes the affected passengers by invalid stations to other rail transit paths, ignoring the diversity of passenger travel choice behaviour, which is inconsistent with the actual situation [38]. After the station operation is interrupted, passengers may choose other travel paths and modes, resulting in the loss of some passengers. However, at the same time, the lost passenger flow will not be redistributed to other rail transit feasible paths, thus reducing the failure risk of other stations. In order to simplify the model and reduce the complexity of calculation, this article only considers two travel modes: rail transit and bus travel.

The research on passenger travel choice behaviour is mainly based on utility function. The discrete choice model based on utility theory can make a scientific and reasonable explanation for specific behaviour decisions, such as the choice between multipath, waiting, or changing travel mode under emergencies. The multinomial logit model (MNL) [39] is actually a model describing probability selection, which can obtain the travel probability of different modes of transportation through the utility function.

MNL model is the basic form of the logit model. Based on the theory of probability, the MNL model with j options can be expressed as follows: $\partial_j = \exp(bV_j) / \sum_j \exp(bV_j)$, ∂_j represents the probability of choosing the j th option and b is the parameter.

The judgment of travel utility is mainly based on experience. Considering the impact of travel time, travel cost, transfer times, and station failure time on the travel scheme, the utility function of each travel scheme for each passenger is constructed as follows:

$$V(P_j^{o,d}) = \alpha_1 T_j^{o,d} + \alpha_2 C_j^{o,d} + \alpha_3 H_j^{o,d} + \alpha_4 T_e, \quad j = 1, 2, 3, 4, \quad (4)$$

where $V(P_j^{o,d})$ is the utility function of the passengers choose the j th travel scheme between o and d . α_i is the model coefficient, $i = 1, 2, \dots, 4$, which can be calibrated by the data of resident travel survey. $T_j^{o,d}$ and $C_j^{o,d}$ represent the travel time and cost of selecting the j th travel scheme, respectively. $H_j^{o,d}$ and T_e represent the transfer times of the j th travel scheme and the estimated duration of operational disruption of a station in the route, respectively. It should be noted that when passengers do not choose to wait for the station to recover, $\alpha_4 = 0$.

$$\partial(V(P_j^{o,d})) = \frac{\exp(V(P_j^{o,d}))}{\sum_j \exp(V(P_j^{o,d}))}, \quad (5)$$

where $\partial(V(P_j^{o,d}))$ represents the probability of choosing the j th travel scheme to travel for each passenger between o and d .

When the station fails, we need to obtain the range of affected passengers and further analyze the changes of their travel behaviour. Due to the different location of the failed station in the rail transit path, the scope of searching the feasible passenger travel path is different. Combined with the actual situation, this article sets the scope of searching the feasible passenger travel path that different affected passengers can choose to reduce the complexity of searching travel path in the process of allocating passenger flow:

- (1) When the failure station is the origin of the travel path, affected passengers who start from this station cannot directly enter the URTN system. These passengers can travel through three schemes: choosing the initial shortest rail transit path $M_{\text{shortest}}^{o,d}$ and waiting for the station to recover, taking the shortest bus path directly and choosing the combined mode of bus and rail transit. These travel schemes of affected passengers can be expressed as follows:

$$P_j^{o,d} = \begin{cases} M_{\text{shortest}}^{o,d}, \\ B_{\text{shortest}}^{o',d'}, \\ \min\{B^{o',i'} + M^{i,d}\}, \end{cases} \quad (6)$$

where o and d represent the origin and destination of one trip, respectively. o' and d' represent the nearest bus stop to o and d , respectively. i represents a new origin rail station for passengers to reenter the rail transit system. Similarly, i' stands for the nearest bus stop to i . $P_j^{o,d}$ represents the travel scheme for the affected passengers. $B^{o',i'}$ is the bus path from o' to i' . $M^{i,d}$ is the rail transit path from the new origin station to the initial destination.

- (2) When the failure station is in the middle of the rail transit path, affected passengers can complete their trips by the following ways. If there are other rail transit paths between origin and destination, passengers may choose another subshortest rail transit path or wait for the station to recover. If there is already no rail transit path between the origin and the destination, passengers may choose bus alone or the combined mode of bus and rail transit. In this scenario, the travel schemes for these affected passengers to complete this trip can be expressed as follows:

$$P_j^{o,d} = \begin{cases} M_{\text{shortest}}^{o,d}, \\ M_{\text{sub-shortest}}^{o,d}, \\ B_{\text{shortest}}^{o',d'}, \\ \min\{B^{o',i'} + M^{i,d}, M^{o,i} + B^{i',d'}\}, \end{cases} \quad (7)$$

where $M_{\text{sub-shortest}}^{o,d}$ represents the shortest path of rail transit reselected bypassing the failure station. Station i is a reselected transfer station in the path between the origin station and the destination. $M^{o,i}$ and $B^{i',d'}$ are the rail transit path from the origin o to the station i and bus path from the bus stop i' nearest i to bus stop d' nearest the destination d .

- (3) When the failure station is the destination of the trip, affected passengers cannot directly arrive at the destination by urban rail transit. In this scenario, passengers may wait for the station to recover. In addition, passengers may change travel mode and

choose the shortest bus path or choose the shortest combined path of urban rail transit and bus. The travel schemes for these passengers to complete their trip can be expressed as follows:

$$P_j^{o,d} = \begin{cases} M_{\text{shortest}}^{o,d}, \\ B_{\text{shortest}}^{o',d'}, \\ \min\{M^{o,i} + B^{i',d'}\}. \end{cases} \quad (8)$$

K-shortest path algorithm (KSP) [40] is mainly used to search the shortest path of the above passenger travel schemes. In the process of travel, when an emergency occurs at a station, in addition to waiting for operation recovery, passengers also hope to get decision-making references, such as suboptimal and suboptimal routes. Therefore, it is necessary to extend the shortest path problem, which is called K shortest paths. Assuming that $k=3$, take Figure 3 as an example to explain the behaviour of passenger selecting the travel path under emergencies. The “K” here is used to determine the number of paths searched by the algorithm. Under the normal operation of urban rail transit, 1-2-5 is the shortest path of rail transit, 1-3-5 is the second short path, and 1-2-4-5 is the third shortest path, according to the data of urban rail transit network. If the station 2 is suddenly interrupted, passengers can still choose 1-2-5 after the station is restored and choose the subshortest path bypassing the interrupted station. In addition, passengers can consider bus travel. Passengers can choose a bus station nearest to the rail transit station to assist in completing the trip. According to the data of bus network and URTN, the same method is adopted to obtain the shortest bus path of passengers and the shortest path combining bus and rail transit. Therefore, the optional path schemes for passengers can be determined under emergencies.

According to the shortest path of each scheme searched, the travel time, travel fare, and the transfer times of the shortest path can be calculated. In this article, the calculation of travel time ignores the walking time of passengers and only considers the operation time of vehicles. There is a fixed timetable for urban rail transit operation, which can easily obtain the operation time between stations. The travel time of bus is determined by the average link travel time by estimating approach with the bus operation data. In addition, the travel cost is determined by the ticket price.

3. Vulnerability Assessment of the URTN Based on the Cascading Failure Evolution Model

In this article, the vulnerability assessment of URTN is mainly divided into two parts: The cascading failure evolution rules are defined based on the constructed capacity-load model of URTN, and the redistribution algorithm of passenger flow in the cascade failure process is designed. Then, the vulnerability assessment index is constructed, and the vulnerability level is calculated by weighted analysis.

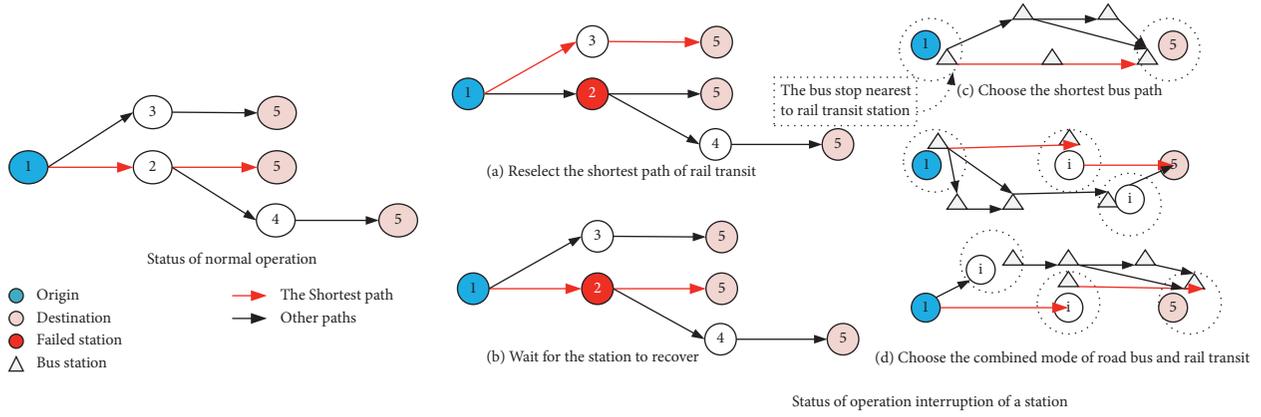


FIGURE 3: Passenger travel paths.

3.1. Cascading Failure Evolution Model

3.1.1. The Capacity-Load Model. Whether a station in the URTN is in the state of normal operation, by judging whether its load exceeds its capacity, if the load of the station is less than its capacity, the station is in the normal state; otherwise, it is in the failure state. In this article, the station capacity is defined as the maximum number of passengers per unit time. The capacity-load model suitable for URTN is proposed based on the research of Corman and D'ariano [41].

$$C_i = (1 + \varepsilon) \sum_{j=1, j \neq i}^N \sigma_{ij} \lambda_{ij}, \quad (9)$$

where C_i is the capacity of station i , ε is the capacity coefficient, which is used to adjust the station capacity. Because in the actual operation of urban rail transit, a certain amount of passenger flow overload is allowed. δ_{ij} represents the relationship of connection between station i and station j . If there is a direct connection edge between station i and j , $\sigma_{ij} = 1$; otherwise $\sigma_{ij} = 0$. λ_{ij} represents the initial flow of edge e_{ij} .

For example, the intensity of station i in Figure 4 is $\omega_i = \lambda_{ai} + \lambda_{ci} + \lambda_{di} + \lambda_{ei}$. The capacity of station i is: $C_i = (1 + \varepsilon)\omega_i$. If the intensity is less than the capacity of the station, the station is in the normal operation state. On the contrary, it is defined that the station changes from the normal state to the failure state, and the affected passenger flow generated by the failure station is redistributed according to the travel decision-making behaviour. It should be noted in this article that although the station whose capacity exceeds its load is regarded as a failure state, it still has the ability to transport passengers.

3.1.2. Algorithm of Passenger Flow Redistribution. In case of an emergency at a station, if the operation is interrupted, the station is in the failure state. The affected passenger flow selects different travel schemes according to the travel utility, that is, passenger flow redistribution. In the process of passenger redistribution, when the intensity of other stations is greater than its maximum carrying capacity, the

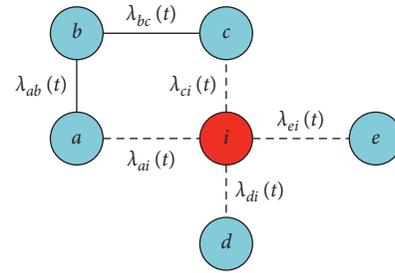


FIGURE 4: The analysis of traffic redistribution algorithm.

transportation capacity of the station is also regarded as invalid. The passengers affected by the failure station shall participate in the next redistribution.

Therefore, passenger flow redistribution is the key to the cascading failure of URTN, and the main algorithms are user equilibrium assignment, system optimal assignment model, and all-or-nothing allocation algorithm. User equilibrium assignment is based on passengers knowing exactly the traffic state of the network and choosing the shortest path. The optimal allocation of the system requires the passengers on the road network to be allocated according to the minimum average cost or total cost of the system. The all-or-nothing allocation algorithm does not consider the congestion of the road network. It directly allocates the passengers in each OD pair to the shortest path, which is the most basic allocation algorithm. The calculation is quite simple and only needs to be completed at one time. In the emergency scenario, the affected passengers need to make travel decisions again according to their travel utility and tend to choose an optimal travel path of each travel scheme. Due to the heavy calculation in the process of passenger assignment in large URTN, this article proposes a passenger redistribution algorithm based on all-or-nothing allocation algorithm. The specific steps are as follows:

- (1) **Basic Preparation.** Firstly, determine the station set $N = \{1, 2, \dots, n\}$ and edge set $E = \{1, 2, \dots, e\}$. Then, according to the AFC data (data collected by automatic fare collection system), the passengers of each OD pair in the initial network is obtained. By searching the shortest rail transit path of each OD

pair and distributing the initial passengers of each OD pair to each edge of the URTN, the passengers of each edge λ_{ij} can be obtained. According to the passengers of each edge, the initial intensity $\bar{\omega}_k$ and maximum capacity of each station C_k can be obtained, $k \in N$. According to the location of the emergency, determine the initial operation interruption station i and estimate the operation interruption duration of the station T_e .

- (2) Extract the passengers whose shortest rail transit path contains the station in abnormal operation state and determine the affected passengers $q_i^{o,d}(t)$. According to Section 2.2, search the travel options $P_j^{o,d}(t)$ for passengers. Then, according to the utility function $V(P_j^{o,d}, t)$, calculate the probability $\sigma_{P_j}^{o,d}(t)$

and affected passengers of each OD pair $q_i^{o,d}(t)\sigma_{P_j}^{o,d}(t)$, choosing the travel scheme $P_j^{o,d}(t)$. In addition, update the travel path of passengers.

- (3) Determine the time step Δt . Extract the affected passengers who still choose urban rail transit (including the combined travel mode of bus and rail transit) and redistribute the affected passengers according to the time $T_{P_j}^{o,b}(\cdot)$ of passing through the station of URTN.

Using Figure 4 as an example to illustrate the redistribution algorithm of affected passengers. When station i is the failure station, the weight of the edge connecting the station a and the station b at time $t + \Delta t$ is given as

$$\lambda_{ab}(t + \Delta t) = \begin{cases} \lambda_{ab}(t), & t + \Delta t < T_{P_j}^{o,b}(a), \\ \lambda_{ab}(t) + \sum_o \sum_d \sum_j \mu_{P_j}^{o,d}(t) \delta_{P_j}^{o,d}(e_{ab}, t) q_i^{o,d}(t) \sigma_{P_j}^{o,d}(t), & t + \Delta t \geq T_{P_j}^{o,b}(a), \end{cases} \quad (10)$$

where $\lambda_{ab}(t)$ and $\lambda_{ab}(t + \Delta t)$ are the weight of directly connected edges between the station a and the station b at time t and after passenger flow redistribution loading at time $t + \Delta t$, respectively. $\mu_{P_j}^{o,d}(t)$ indicates whether the travel path P_j includes the rail transit route. If P_j contains rail transit route, then $\mu_{P_j}^{o,d}(t) = 1$; otherwise, $\mu_{P_j}^{o,d}(t) = 0$. $\delta_{P_j}^{o,d}(e_{ab}, t)$ indicates whether edge e_{ab} will be passed. If the P_j will pass through edge e_{ab} , $\delta_{P_j}^{o,d}(e_{ab}, t) = 1$; otherwise $\delta_{P_j}^{o,d}(e_{ab}, t) = 0$. $q_i^{o,d}(t)$ represents the affected passenger flow between station o and d when the station i failed. $\sigma_{P_j}^{o,d}(t)$ is the probability that passengers between station o and station d choose the path P_j . $T_{P_j}^{o,b}(a)$ is the time of passengers choosing the path P_j leave the station a .

- (4) Compare the sizes of $\bar{\omega}_k(t + \Delta t)$ and $C_k(t + \Delta t)$. If $\exists \bar{\omega}_k(t + \Delta t) > C_k(t + \Delta t)$, then consider k as a failure station and record the set of failure stations $V(t + \Delta t)$. The reaffected passengers will be calculated and redistributed at the next time step. Let $t = t + \Delta t$, repeat steps (2–4). Otherwise, the process of passenger redistribution ends. In addition, it should be noted that when $t + \Delta t < T_{P_j}^{o,b}(a)$, the affected but not reaffected passengers passing the station a is not redistributed at time $t + \Delta t$ and will be judged whether to complete the redistribution at the next time step according to the time $T_{P_j}^{o,b}(\cdot)$.

According to the above analysis, during the cascade failure evolution of URTN, the passenger flow redistribution algorithm is as shown in Figure 5:

3.2. Construction of Vulnerability Assessment Indicators. Previous studies mainly used the metrics of complex network topology to evaluate the vulnerability of URTN under

different attack strategies, such as the size of the largest connected subnetwork when removing the failure stations. However, the dynamic vulnerability is not only related to the size of the affected area of URTN but also related to its functional characteristics. After the station is interrupted, the travel choice behaviour of passengers is affected, which is easy to cause local or large-scale traffic congestion and seriously affect the operation efficiency of the network. Therefore, this article establishes the vulnerability evaluation index of URTN by analyzing the travel behaviour and congestion propagation process of passengers under emergencies.

3.2.1. Loss Flow Ratio. The affected passengers due to the failure station may change their travel modes and travel paths, which causes the passenger flow in URTN transferred to other public transportation networks. This article defines the passenger flow transferred from the URTN to bus networks, which is the loss flow, and the loss flow ratio is defined as the loss flow to the total passenger flow of the URTN.

$$\eta_1(t) = \frac{R_0 - R_1(t)}{R_0}, \quad (11)$$

where $\eta_1(t)$ is the loss flow ratio of URTN. $R_1(t)$ and R_0 are the passengers flow to be loaded on the URTN at time t and the total passengers of the URTN, respectively.

3.2.2. Ratio of Node Failure. The ratio of node failure refers to the ratio of the number of failure stations to the total number of network stations in the process of cascading failures in URTN at time t .

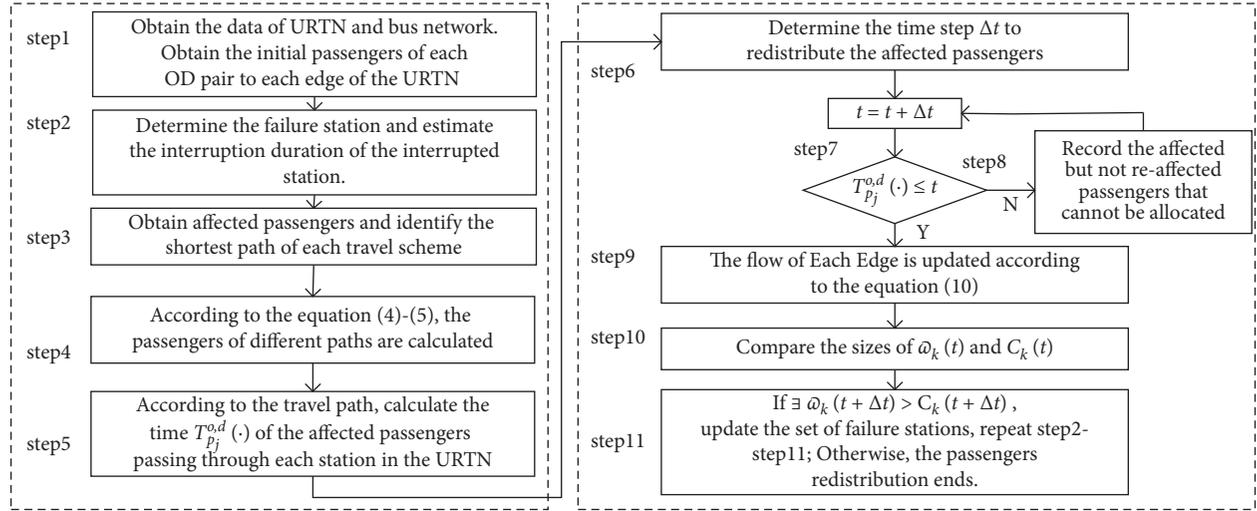


FIGURE 5: Passenger flow redistribution algorithm.

$$\eta_2(t) = \frac{N_1(t)}{N_0}, \quad (12)$$

where $\eta_2(t)$ is the ratio of node failure. N_0 and $N_1(t)$ are the total number of stations and failure stations at time t , respectively.

The ratio of node failure reflects the change of the affected area of URTN, and the ratio of passengers lost reflects the change of the transportation function of the URTN. Therefore, in order to comprehensively assess the vulnerability and identify the vulnerable stations in the URTN, this article constructs a weighted comprehensive vulnerability assessment index.

$$\eta_3 = \omega_1 \eta_1 + \omega_2 \eta_2, \quad (13)$$

where ω_1 and ω_2 are the weight of η_1 and η_2 , respectively. The larger the value of η_3 , it indicates that the attacked station is easy to lead to the failure of other stations, that is, the vulnerability of the station is high, and the vulnerability of urban rail transit network is also high.

Combined with the characteristics of the structure and function of the URTN, the assessment index of vulnerability is constructed. The algorithm for the vulnerability assessment is shown in Figure 6. The load of each station is calculated based on the redistribution of passengers. The presence of a failed station is determined by whether the load of any station exceeds its capacity.

4. Case Study

This method of vulnerability assessment is applied to the Beijing Rail Transit Network (BRTN) in January 2016. The BRTN model is composed of 16 operation lines and 262 stations, which is built based on L-space, as shown in Figure 7(a).

4.1. Preparatory Work. At first, the OD distribution flows of the entire network during the period of 7:00–9:00 is calculated based on the operation data of the BRTN on January

21, 2016. According to the KSP search algorithm, we get the k-shortest path for passengers of each OD pair, $K=3$. Selecting the shortest path and assigning the initial passengers of each OD pair to each edge of the BRTN according to the all-or-nothing allocation algorithm as show in Figure 7(b), the initial intensity of each station can be obtained as shown in Figure 7(c).

4.2. Set the Scene of Various Attacks on the Station. In the actual scene, emergencies often occur in a specific station, resulting in the operation interruption of the station. According to the calculation method of the importance of the station designed in Section 2.1, the degree, betweenness, and intensity of the stations are calculated by formulation (1)–(3) firstly and the ranking results corresponding to the top 5 are given, as shown in Table 1. It can be seen that Xizhimen has the largest degree and betweenness. The betweenness of Dongdan is the largest except Xizhimen Station. Therefore, Xizhimen and Dongdan are the structural important station in the BRTN. The intensity of Guomao accounts for about 3% of the intensity of all stations in the network, which indicates that Guomao is the station with the largest intensity.

This article selects Xizhimen with the largest degree, Dongdan with the largest betweenness, and Guomao with the largest intensity as three different types of attack objects to study cascading failure mechanism and network vulnerability dynamic evaluation. Combined with the survey of Beijing residents, the MNL model is calibrated using the least square method. The parameter calibration results are as follows: $\alpha_1 = -0.115$, $\alpha_2 = -0.145$, $\alpha_3 = -0.313$, $\alpha_4 = -0.086$, respectively. Assuming that the interruption time of the attacked station is $T_d = 30$ min, the vulnerability of BRTN under malicious attack and random attack of three types of important stations is simulated and analysed.

After determining the attacked station, extract the range of affected passengers, that is, extracting the passengers whose shortest rail transit path contains the attacked station.

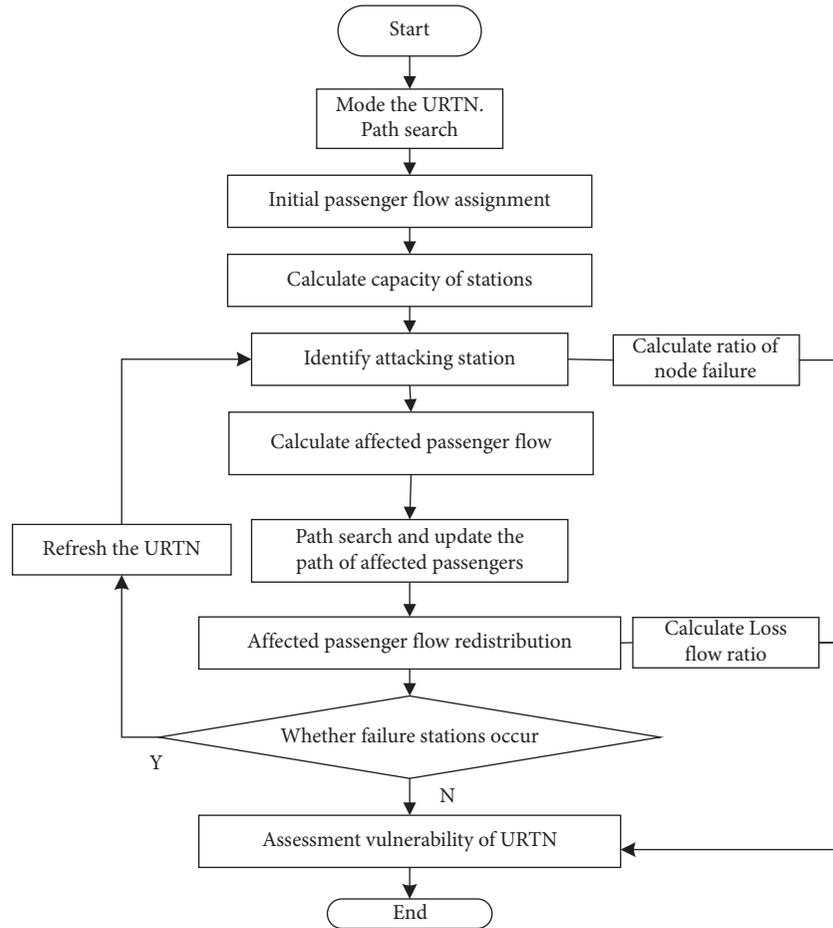


FIGURE 6: Vulnerability assessment algorithm.

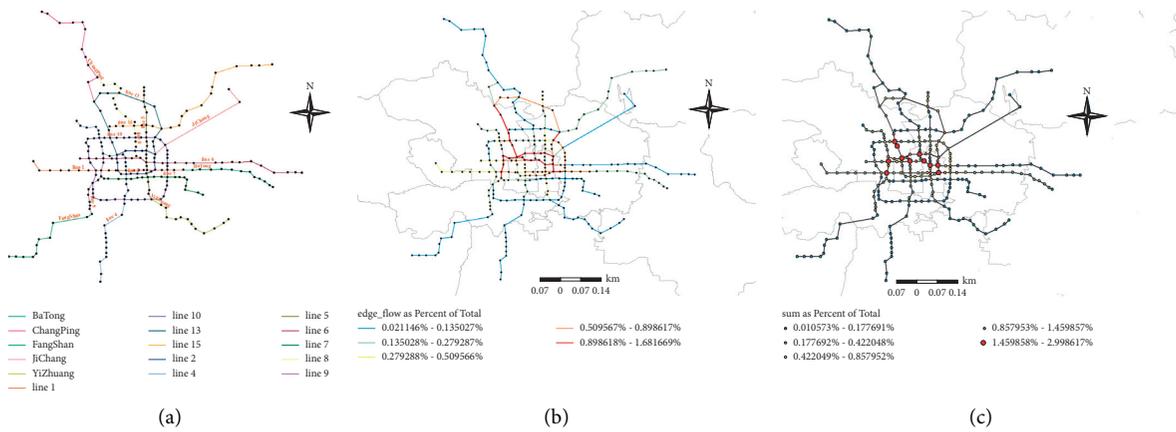


FIGURE 7: Topology and initial passengers of Beijing subway network (a-c).

TABLE 1: Results of top 5 importance nodes.

No.	Station	Degree	No.	Station	Betweenness	No.	Station	Intensity (%)
1	Xizhimen	5	1	Xizhimen	0.3169	1	Guomao	2.999
2	Dongsi	4	2	Dongdan	0.3112	2	Xizhimen	1.945
3	Xuanwumen	4	3	Baishiqaonan	0.2104	3	Caoyangmen	1.918
4	Guloudajie	4	4	Pinganli	0.2049	4	Junshibowuguan	1.917
5	Yonghegong	4	5	Dongsi	0.1973	5	Zhicunlu	1.911

According to Section 2.2, search the travel options for passengers. According to the utility function, the probability and affected passengers of each OD pair choosing the travel scheme are calculated. In addition, update the travel path of passengers. According to the passenger redistribution algorithm provided in Section 3.1, the affected passengers are redistributed to the BRTN.

4.3. Assessment Vulnerability of BRTN

4.3.1. Vulnerability Assessment of Different Types of Attacking Stations. Taking the capacity coefficient $\varepsilon = 0.25$ as an example, this article compares and analyses the cascading failures of three different types of important stations under malicious attack, as shown in Figure 8. It can be seen that attacking the stations with the largest betweenness, the largest degree, and the largest intensity, the duration of cascading failure is 18, 23, and 26. The peak failure ratios of stations are 0.42, 0.39, and 0.32, and the time steps to reach the peak are 3, 3, and 11, respectively. It can be seen from this set of data that the stations with the largest intensity have the longest duration of cascading failure process and the lowest ratio of node failure. In contrast, the station with the largest betweenness have the shortest duration, but its ratio of node failure is the highest and the time response to the highest ratio of node failure is the fastest. In the cascading failure process, when the ratio of node failure reaches the highest, the time of attacking the stations with the largest degree is also extremely short. In addition, when the capacity coefficient is constant, in the early stage of cascading failure (when the interruption duration of the station is 1–7 in Figure 8), attacking the station with the largest betweenness may cause a larger failure ratio of stations in the network. Furthermore, attacking the station with the largest betweenness can quickly recover, and attacking the station with the largest intensity requires more recovery time from Figure 8.

This is mainly because the station with the largest betweenness and the station with the largest degree play an important role in the topology of URTN. When these two types of stations are maliciously attacked, the connectivity of the network decreases, resulting in affected passengers being over reassigned to paths with limited capacity. It involves a wide range of passengers, so it is easy to cause cascade failure in a large area and the speed is fast. With the decline of network connectivity, a large number of passengers tend to choose bus in the later stage. With the loss of passenger flow, the status of urban rail transit stations quickly returns to normal. When the station with the largest intensity is attacked maliciously, the range of affected passengers is that the travel path includes the station. When the station with the largest degree or the largest betweenness is maliciously attacked, although the number of passengers affected is not as large as when the attack intensity is the largest, the range of passengers involved is relatively wide due to the station with the largest degree or the largest betweenness. Therefore, in the early stage, the proportion of failure stations is lower than the other two scenarios. Thus, with the redistribution of

passengers, the load of each station is balanced and the network gradually recovers.

Considering the loss of passenger flow, in the early stage of cascading failure, attacking the station with the largest intensity may cause a larger loss of passenger flow. It is mainly because the station serves the most passengers and the alternative routes of urban rail network are limited, resulting in some passengers changing their travel mode. In the middle stage of cascading failure, the loss flow ratio of the attacked station with the largest betweenness and the attacked station with the degree increases faster. This is mainly due to the increase in the affected area of the network, which leads to the change of travel mode and the loss of a large number of passengers. However, in the whole cascade failure process, the station with the largest intensity loses many passengers. To sum up, we can know that the cascading failure caused by different attacking will have different destructive effects on the network in different periods. Attacking the station with the largest betweenness and the largest degree may cause serious damage to the network rapidly. Attacking the station with the largest intensity will cause greater damage to the transportation function of the network because the loss flow ratio is high and the cascade failure process is long.

Therefore, in the actual operation process, when the network is under malicious attacks, the managers should increase the control strength of passenger flow in stations with the largest intensity. For example, managers should combine timetable and passenger flow control measures to relieve the pressure of massive stations and reduce the risk of failure of the largest intensity stations by increasing temporary buses. In addition, the risk management and control of the stations with the large betweenness and large degree should be strengthened to reduce the accidents as much as possible, such as increasing the inspection frequency of stations or tracks. Last but not least, when the travel operation of important stations is interrupted, it should be controlled in time to prevent large-scale paralysis of urban rail transit and massive loss of passenger flow.

Besides, Figure 9 shows the results of vulnerability assessment under both malicious and random attacks. We find that malicious attacks are more likely to cause a larger failure ratio and a loss flow ratio than random attacks. Further speaking, malicious attacks will accelerate the deterioration of the network and aggravate the degree of deterioration. Therefore, during the operation of urban rail transit system, we should strengthen the safety protection of the station to avoid malicious attacks, such as arson. In addition, although the damage degree of random attack is less than that of malicious attack, when some stations are attacked randomly, it will have an impact on the connectivity and function of the network that cannot be ignored.

4.3.2. Vulnerability Assessment under Different Capacity Coefficients. Figure 10 shows the process of cascading failure after the station with the largest betweenness interrupted in the BRTN. As can be seen from Figure 10(a), when the capacity coefficient $\varepsilon < 0.35$, the failure rate of the station

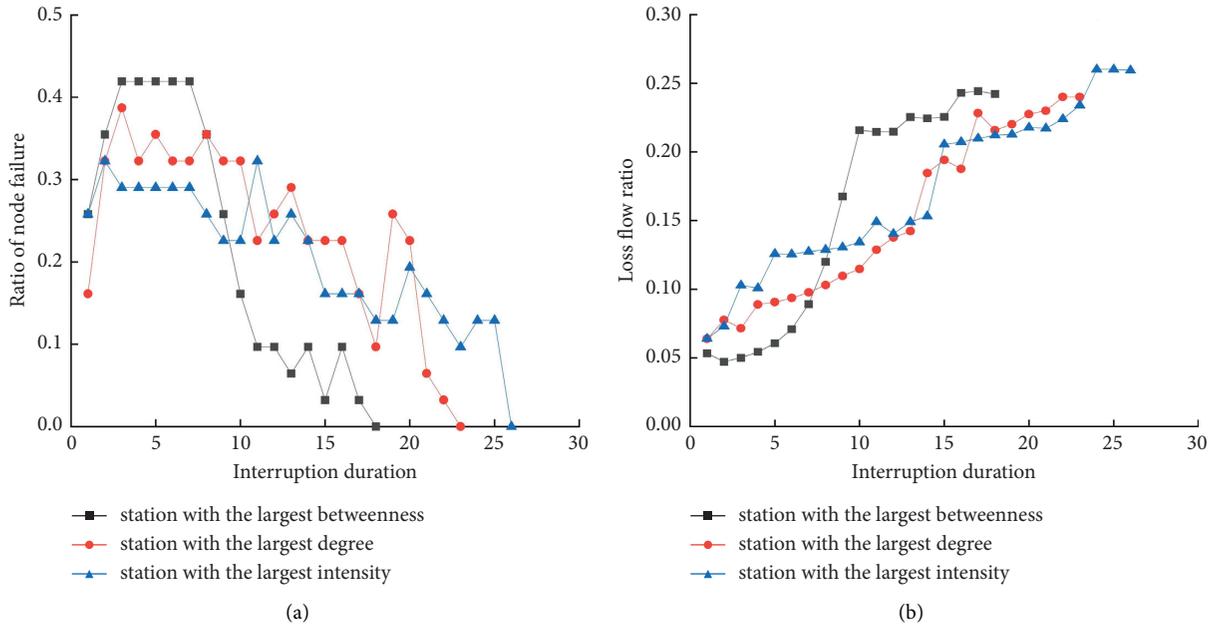


FIGURE 8: The impact of maliciousness on network vulnerability ($\epsilon = 0.25$). (a) Ratio of node failure. (b) Loss flow ratio.

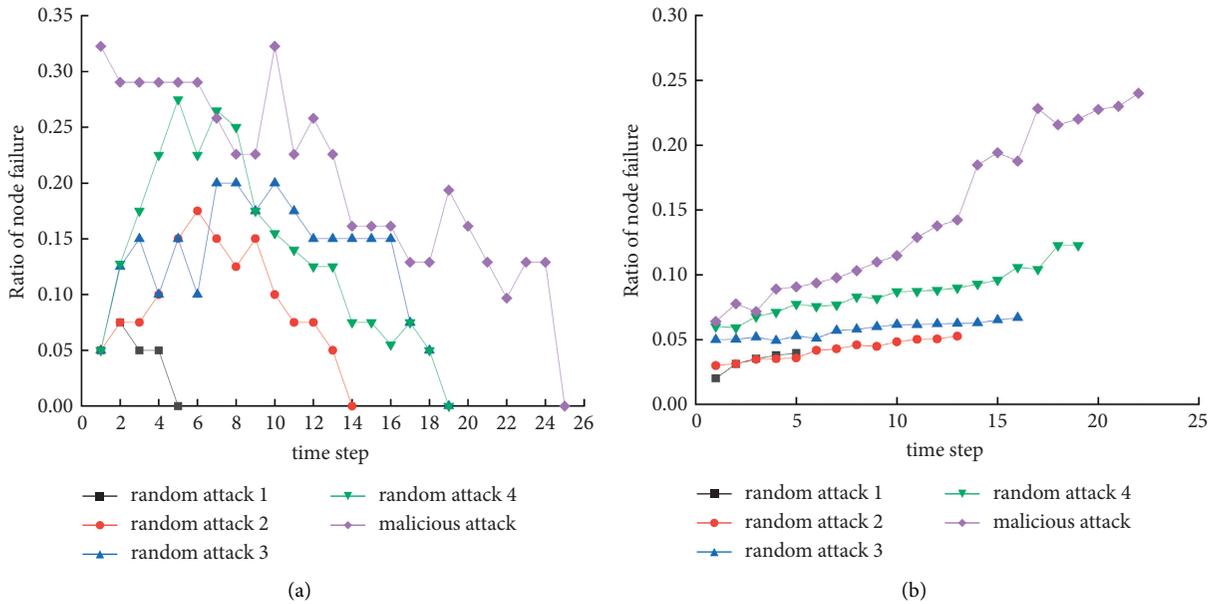


FIGURE 9: The vulnerability of the BRTN under different attacks: (a) ratio of node failure and (b) loss flow ratio.

reaches the peak after approximately 4-5 minutes of operation interruption, and the peak range is $\eta_2 \in [0.412, 0.508]$. When $\epsilon \geq 0.35$, the failure rate of the network station reaches the peak after approximately 7 minutes of operation interruption, and the range is $\eta_2 \in [0.288, 0.397]$. In addition, it can be seen from Figure 10(b) that when the capacity coefficient $\epsilon \geq 0.35$, the loss flow of the BRTN does not exceed 15.4%. Therefore, increasing the capacity coefficient of stations in BRTN can improve the robustness of the network and reduce the impact of operation interruption on network traffic efficiency.

Figures 11 and 12 show the BRTN cascading failure process after the station with the largest degree and the largest intensity of station interruption, respectively. It can be seen that with the increase of the capacity coefficient, the vulnerability of the BRTN under cascading failure has decreased. Therefore, reasonably determining the capacity coefficient can increase the robustness of the BRTN. There are many ways to improve the capacity coefficient, such as adjusting the train operation plan and improving the vehicle carrying capacity to increase the supply capacity of urban rail transit system.

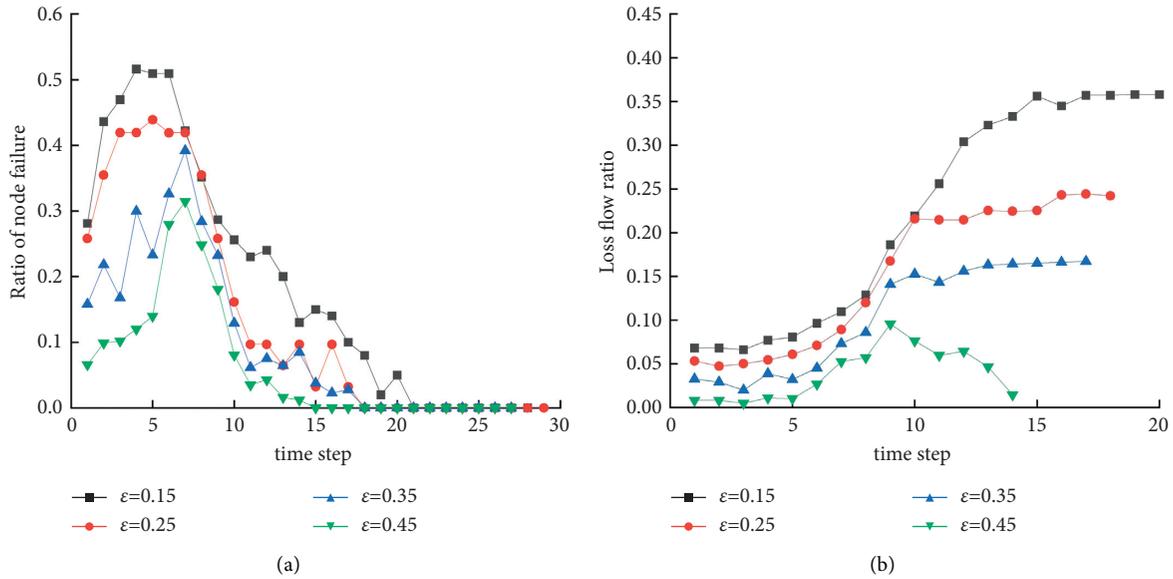


FIGURE 10: The vulnerability under the station with largest betweenness interruption. (a) Ratio of node failure. (b) Loss flow ratio.

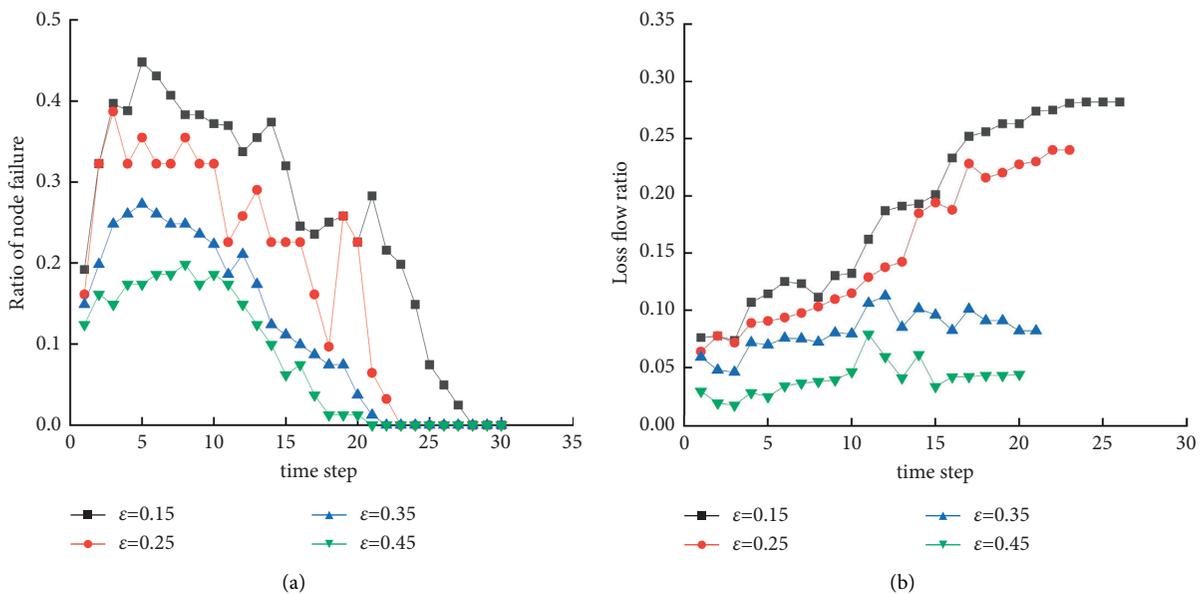


FIGURE 11: The vulnerability under the station with the largest degree interruption. (a) Ratio of node failure. (b) Loss flow ratio.

4.3.3. *The Vulnerability Level of the BRTN.* Finally, this study attacks each station in the network and obtains the weighted vulnerability index of the network to more clearly get the impact of station interruption on the vulnerability of BRTN. Taking the capacity coefficient $\epsilon = 0.25$, the weight $\omega_1 = \omega_2 = 0.5$ and the estimation interruption time of the attacking station $T_d = 30$ min as an example. In the risk control system of the station, it is necessary to pay different attention to the stations with different vulnerabilities. Different vulnerability stations are divided into several levels for differentiated control to facilitate the management of the station. In order to more clearly reflect the vulnerability differences of different stations according to the calculation

results of BRTN, the levels of vulnerability are graded as shown in Table 2. The results are shown in Figure 13.

It can be seen that most of the vulnerability stations in BRTN are located on the loop line or at the important transfer stations. The distribution of vulnerability stations is relatively concentrated. The station with the largest degree, the station with the largest intensity, and the station with the largest betweenness are at an extremely high level of vulnerability.

At the same time, this article has made statistics on the number of stations under different vulnerability levels, as shown in Figure 14. It can be seen that the number of stations in each vulnerability level is 3, 3, 34, 49, and 173,

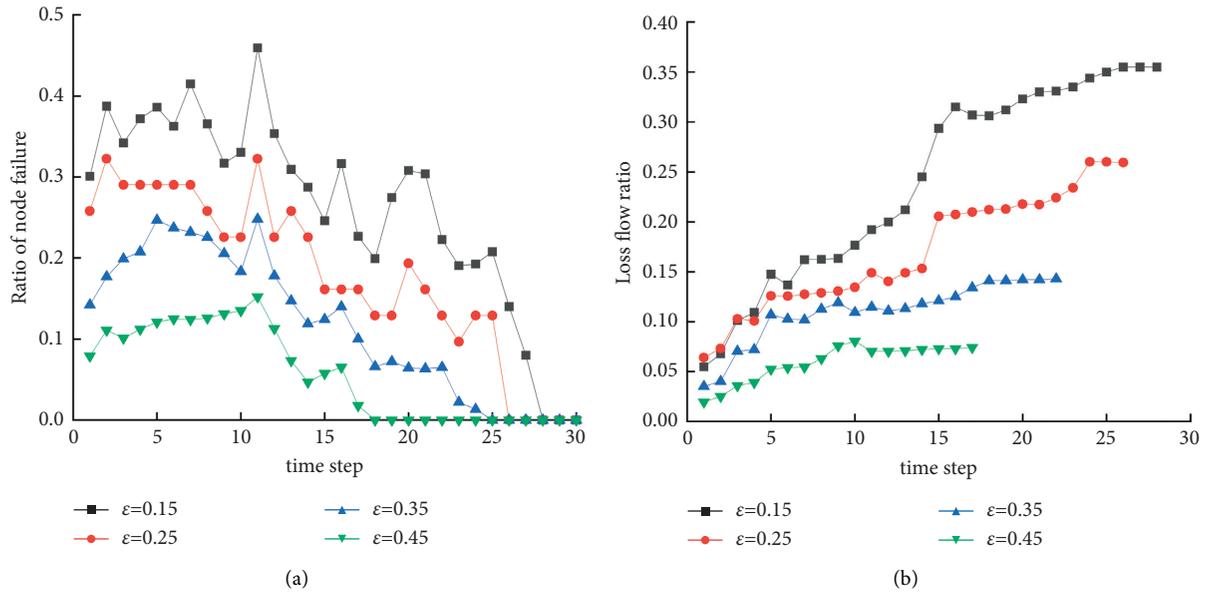


FIGURE 12: The vulnerability under the station with the largest intensity interruption. (a) Ratio of node failure. (b) Loss flow ratio.

TABLE 2: Vulnerability index range.

Vulnerability level	Vulnerability index range	Meaning
Extremely high level	[0.3, 1]	Station too prone to failure
High level	[0.25, 0.3]	Stations with a high probability of failure
Medium level	[0.15, 0.25]	Stations with a medium probability of failure
Low level	[0.1, 0.15]	Stations with a low probability of failure
Extremely low level	[0, 0.1]	Stations in robust state

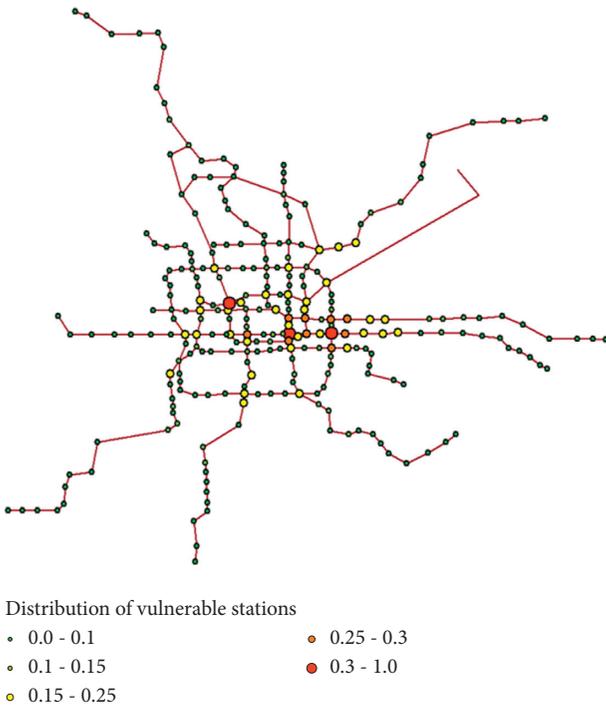


FIGURE 13: The distribution of vulnerability under cascading failure.

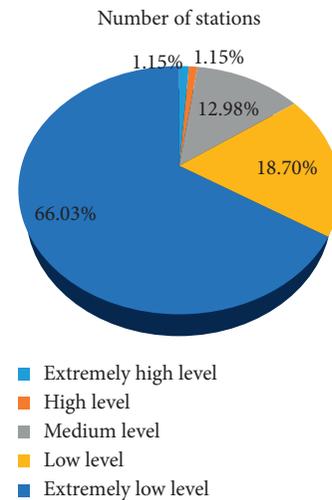


FIGURE 14: The levels of vulnerability under cascading failure.

accounted for 1.15%, 1.15%, 12.98%, 18.70%, and 66.03% of the total, respectively. Thus, we can draw that most stations in the operation failure cannot cause large-scale paralysis. The interruption of only a few stations with critical structures and functions may seriously reduce the efficiency of BRTN.

5. Conclusion

When a station in the urban rail transit network breaks down due to an emergency, the strong temporal and spatial correlation between stations is easy to cause the propagation of passenger congestion, further leading to station chain failure. In this scenario, the travel efficiency (such as travel time, and travel cost) of passengers is seriously affected. Hence, this article considers the changes in passenger travel modes in the case of abnormal disruption of the rail transit stations and designs travel plans for multiple modes of transportation (including road bus and rail transit). An algorithm for delay redistribution of passenger flows is proposed, and the dynamic vulnerability of the URTN under the phenomenon of cascading failure is finally evaluated.

By quantitatively analyzing the statistical characteristics of the structure and function of BRTN, this article finds that Xizhimen and Dongdan are important stations of the network structure, and GuoMao is an important station of the network function. The results of the dynamic simulation showed that the cascading failure of the URTN is closely related to the spatial-temporal distribution of passenger flow. Besides, compared with other important stations of the structure, the operational interruption of the station that has functional importance has a greater impact on the stability of the URTN operation. We also realize that decreasing the capacity coefficient of the stations can intensify the vulnerability of the URTN, especially in situations of interruption the largest intensity station. Therefore, increasing the capacity coefficient of stations in BRTN can improve the robustness of the network and reduce the impact of operation interruption on network traffic efficiency.

These conclusions provided guidance to operation managers. However, in order to simplify the model and reduce the complexity of calculation, this article only considers two travel modes: rail transit and bus travel. Reasonably speaking, the vulnerability of urban rail transit network must also be affected by other travel mode choices of passengers, such as private cars, taxis, and shared bicycles. When the station operation is interrupted, passengers may choose other travel modes, resulting in the loss of some passengers. However, at the same time, the lost passenger flow will not be redistributed to other rail transit feasible paths, thus reducing the failure risk of other stations. In addition, some scholars, such as Liu and Wang [39], have studied how dispatchers can quickly find a feasible and efficient operation table through the advanced dispatching decision support system to control passenger flow in case of traffic interruption. Therefore, in the future, we plan to further study the complex and diverse passenger travel choice behavior under emergencies and relevant passenger flow control measures, so as to more accurately describe the vulnerability change law of urban rail transit network under emergencies.

Data Availability

The Beijing Metro Corporation provided the operation data of Beijing Metro network from January 2016 to January

2017. However, because the data involves public security and personal privacy, they are not released to the public. If necessary, some sample data can be provided.

Conflicts of Interest

The authors declare no conflicts of interest.

Acknowledgments

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