

Research Article

An Optimization Approach considering Passengers' Space-Time Requirements for Bus Bridging Service under URT Disruption

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Rapid urbanization and growth of population in megacities generate severe pressures on urban rail transit (URT) system. The quantity and frequency of disruptive events have increased significantly, which might have obvious adverse impacts. A large number of passengers are stranded at disrupted URT station when a disruptive event occurs. One essential solution for passenger evacuation is the bus bridging service. This paper is aimed at addressing the passenger evacuation problem caused by a disruptive event in the URT network, by proposing a bus bridging service model considering the passengers' space-time requirements. The model is proposed to minimize the waiting time of passengers and considers factors including bus service capacity limitations, bus stop parking capacity, and the maximum bridging time limit of a single bus. Buses are assumed to provide bridging service on either the local bus route or the direct bus route. The optimal routes and scheduling plans of bridging bus are designed. The model is applied to an example of a disruptive event in Shanghai URT line 9. The results of this example show that the proposed model is capable of reducing the waiting time of passengers and the number of buses used by 3.2% and 24.7%, compared with the traditional bus bridging service. Further analysis of the example shows that it is not a cost-effective solution to reserve a large number of buses for URT disruption. Decision-makers should comprehensively trade off between passengers' space-time demands and monetary costs of bus bridging service.

1. Introduction

The accelerated urbanization has led to gradual increases in the population of large cities, and the demand for urban public transport is growing. Moreover, population expansion has raised a series of concerns regarding environmental pollution and traffic congestion. As an efficient, green, and economical transport mode, urban rail transit (URT) could effectively meet the above challenges and thus has become the backbone of urban transport system.

The daily operation of URT is highly dependent on infrastructure such as metro lines, metro stations, and technologies like train operation control and signal communication, which makes it vulnerable to external uncertainties. Common factors of disruptive events include infrastructure damage (e.g., signal failure, faulty trains) and unexpected events (e.g., geological disasters and extreme weather). Sta-

tistics show that Beijing Subway has encountered 504 unexpected operational accidents in the last decade.

The inherent characteristics of the URT system make it difficult to directly evacuate passengers after a disruption. On the one hand, the operational plan of the URT system is made in advance, so it is difficult to immediately adjust the operation in case of an emergency. On the other hand, majority of the rail transit stations and lines are located underground or elevated, and the operating environment is relatively closed. Hence, it is urgent and difficult to rescue and evacuate passengers in case of a disruption.

Rail transit emergencies could cause train delays and stranded passengers in faulty lines or stations, while increasing travel delays for passengers. When an unexpected disruptive event occurs, two types of emergency measurements are normally taken by URT operators. One is the adjustment of the rail transit system itself to transfer passengers to adjacent

stations via other URT lines. Another is evacuation of the stranded passengers with the help of the surface bus system.

The study shows that passengers could be transferred in short-term rail transit emergencies by adjusting the rail transit system itself [1]. However, when the disruption lasts for more than 30 minutes, it is difficult to meet the passengers' travel demand by solely adjusting the rail transit. In this situation, it is necessary to provide the bridging bus service to transfer passengers.

This paper proposes a model that considers passengers' space-time demand to optimize the total passenger bridging time incurred in bus bridging service after URT disruption. The main contributions of this paper include

- (1) proposing an optimization model of bus bridging service considering passengers' space-time requirements; the example shows that the model could reduce the waiting time of passengers and the number of buses used
- (2) considering the maximum dispatch time constrain of bus, as well as the maximum waiting time constraint of passengers to improve the service efficiency of bridging bus, compared with the optimization approach in literature [2, 3]
- (3) taking the demands of passengers whose origin and destination are both located at the disrupted station into account, on the basis of considering the time demand of passengers whose destination is the turn-over station

The paper is organized as follows: Section 2 reviews the current research related to this paper. In Section 3, the model formulation is designed, which includes problem description and content of model construction. Section 4 analyzes the model in a case study and conducts sensitivity analysis among the major factors. The conclusion of this paper and future outlook are presented in Section 5.

2. Literature Review

Extensive research and exploration of URT disruption problems have been conducted by scholars from various perspectives. Some studies summarize disruption management concepts, sources, and their applications. On this basis, scholars have studied the rail transit disruption problem from operational adjustment and bus bridging. This paper is motivated by the real scenario and the gap of existing research.

The unexpected URT disruptions are a disruption management problem from a macro perspective. Disruption management means that the operator has to quickly adjust the original schedule in order to minimize the adverse effects when disruptive event makes the original plan infeasible or in a nonoptimal condition [4]. The concept of disruption management was first introduced in airline operations management to reduce losses under disruptions and enhance service for passengers [5]. Subsequently, disruption management is widely used in production management, inventory

control, supply chain management, and rail transit operation management. URT disruption management is aimed at quickly evacuating passengers and restoring the original level of rail transit operation as soon as possible when an unexpected service disruption occurs.

Rail disruption management has been widely studied and applied. Pender et al. summarized the reasons of rail disruptions and some common measures to deal with unexpected disruptive events [3]. Zhang et al. analyzed disruption management in the metro system from getting prepared for metro network disruption, managing disruption within the metro system, and managing metro disruption with SB service and finally proposed future research directions [6]. De-Los-Santos et al. concluded that bridging bus effectively transferred passengers affected by URT disruptions through comparative experiments [7]. Jin et al. pointed out that bus bridging service can effectively improve the reliability of the rail network [8]. In addition, the behaviour of passengers after URT disruptions has also been explored. Pnevmatikou et al. used SP and RP survey methods to analyze the degree of influence factors on passengers' choice of different transport modes after URT disruption [9]. Wang et al. modelled the queuing behaviour of passengers after disruptions through Poisson process, which is demonstrated using Monte Carlo method [10].

URT operation adjustment includes flexible adjustment of the frequency, train halting pattern, route planning, and train operation direction according to the operation situation. Gao et al. pointed out that the operation can be degraded by means of train skipping stations when passengers are stranded after URT disruption [11]. Leo et al. proposed an innovative model which dynamically adjusts the rolling stock based on dynamic passenger flow [12]. Xu et al. proposed a bridging bus model that reduces passenger waiting time when the last train delays as well as maximizes the transport of passengers [13]. Xu proposed a train schedule adjustment model that considers passenger transfer behaviour at the route and network level [14].

For the bus bridging service after disruptions, related studies mainly focus on the necessity of bridging buses, bridging bus modelling, bridging bus route design, bridging bus schedule planning, passenger choice behaviour while waiting for bridging buses, and research on bridging buses from the perspective of new technologies.

Several studies have analyzed the importance of bus bridging for URT. Pender et al. investigated the disruption of URT that occurred across 71 international transportation organizations; according to the investigation, bus bridging service is the most common method to tackle the problem [3]. Kepaptsoglou et al. studied the bus bridging problem from the perspectives of conceptual framework, model, and algorithm [15]. Yang et al. analyzed the importance of bus bridging service for the URT network connectivity [16].

Some studies explored the bus bridging service from the perspective of mathematical modelling. Brendan et al. put forward a method for assessing satellite bus reserve locations [17]; Deng et al. investigated the generation of alternate paths via the shortest route to select the optimal route and allocate bus resources [18]. Hu et al. designed a nonlinear

integer programming model in order to schedule bridging buses for the evacuation of passengers [19]. Wang et al. proposed a flexible dispatching strategy to solve the bus bridging problem [20]. Yang et al. analyzed the way to design bus bridging model [21], while Liang developed an effectual bridging network to handle URT disruption with regard to network robustness [22]. Itani and Shalaby analyzed the impact of bus bridging strategies on Toronto subway operation resilience by using machine learning methods such as *K*-mean clustering and CART analysis [23]. Tang et al. quantified the effectiveness of rail transit system resilience and provision of bus bridging services based on several disruption scenarios. The results show that the impact of bus-bridging services on improving the resilience of different types of rail transit systems is robust and dynamic, ranging from 14% to 30% [24]. Itani et al. analyzed the differences of bus bridging service from four perspectives: bridging direction, initial time of bus bridging, duration of bus bridging, and variables in relation to passenger requirements [25]. Zheng et al. proposed a comprehensive bridging strategy based on considerations of the experience of passengers, the reliability of conventional bus system, heterogeneity of passenger, underutilized capacity, and dynamic passenger demand changes, with the aim of balancing the benefit of stranded metro passengers and conventional bus passengers [26].

There are some literatures on the bus bridging route design: Codina et al. planned bridging routes in the event of congestion [2]; Wang et al. established bridging routes when passenger arrival is uniformly distributed [27]; Gu et al. allowed a single bus to serve different bridging routes and obtained the bridging bus routes by minimizing total evacuation and passenger delay time [28]. Wu et al. considered the passenger delay in the bus bridging process and the metro short turning process and developed a coordinated emergency response model dealing with urban metro disruptions [29]. Bojic et al. pointed that multiple types of bridging buses are allowed in the optimal bridging plan, in order to decrease the travel delay of passengers and increase the number of passengers who can be served [30]. Furthermore, Ding proposed bridging routes based on the robustness of URT [31]. In addition to studying the issue from single line, some scholars also analyze the bus bridging route design in a way of network. For instance, Jian proposed optimal routes for bridging buses with limited bus resources for URT network problem [32]. Wang studied bus bridging route design based on URT network [33]. Luo et al. determined the bus bridging routes and frequency based on an integrated network consisting of remaining available rail lines, existing operating bus lines, and newly introduced bridging bus lines, considering the uncertainties in remaining capacity of the existing rail and the passenger demand [34]. Further to research on the design of bus bridging routes, some studies focus on bus evacuation schedule. Chen applied an integrated optimization framework for the purpose to obtain bridging bus routes and timetables under time-varying demand [35]. Moreover, Wang studied bus evacuation timetables and scheduling and paid particular attention to the transfer passengers [36].

Moreover, some studies explored bus bridging service from the perspective of efficiency and passenger behaviour. Zhang et al. studied the best initial time of bridging bus [37]. Yin established a three-layer discrete selection model for dynamic passenger flow demand after disruption and set up a bridging bus plan to minimize the total passenger evacuation time and bus operation cost [38]. Wang studied the influencing factors of passengers' travel choices under URT network disruption [39], while Wei explored the impact of disruption on passenger travel behaviour based on multiagent [40]. Ehsan analyzed the waiting time tolerance of passengers in response to unexpected service disruptions [41]; and Zeng et al. investigated the possibility of collaborating with a taxi company to provide the recovery service for short-term disruptions in public tram systems [42].

In recent years, advancement of technology and rapid development of the Internet promote the shared autonomous vehicle industry to mature rapidly. With its unique combination of convenience and flexibility, shared autonomous vehicles can provide a better travel service for passengers. Moreno et al. discovered that four shared autonomous vehicles could provide the demand equivalence of ten conventional vehicles [43]. Wang et al. found that shared automated vehicle systems have the ability to reduce average waiting time, vehicle travel kilometers, and the number of empty shared automated vehicles trips [44]. In a word, shared autonomous vehicles have the capability to provide better bridging service in response to URT disruption.

Although the research related to bridging buses is relatively well established, there are still some issues that should be explored. It is worth noticing that the majority of studies do not consider the passenger travel demand whose origin and destination happen to be the disrupted URT stations, which motivates the current paper. Additionally, the bus parking capacity at URT stations should be considered, since the attachment area of a URT station is normally very limited. Lastly, passengers' choice behaviour under URT disruption needs to be intensively investigated, because they might shift to other transportation mode if their waiting time for bridging bus is intolerable.

In order to solve the above problems, the paper investigates the bus bridging problem considering passengers' space-time requirements. An innovative model is developed, which considers passenger demand between disrupted stations. In addition, the model limits the maximum waiting time of passengers at a bus bridging stop.

3. Mathematical Modelling

In this section, we firstly describe the scenario of the research problem, then outline the contents and drawbacks of the traditional methods for the problem, and finally propose the bus bridging model in URT disruption.

3.1. Problem Description. A disruptive event may occur at a URT station, section of a URT line, or multiple stations/sections simultaneously. In this paper, the study of bus bridging

is based on the scenario of disruption in the middle section of a single URT line.

As shown in Figure 1, the upward direction of the URT line is from station 1 to station S . When the accident occurs in the middle section $(1, S)$, stations affected by the disruption like $\{2, 3, 4, \dots, S-1\}$ are the disruption stations. The turnover stations as $\{1, S\}$ are used for short-turn. The bus depots such as $\{D_1, D_2, D_3\}$ are the bus dispatch stations. The faulty train located in the disruption section $(1, S)$ needs to be towed to the adjacent stations for passenger clearance which results in a large number of stranded passengers. In order to reduce the impact of disruptive events to the URT, short-turn strategy is provided in the downward direction of station 1 and the upward direction of station S . There are two stages of implementing bus bridging service including bus dispatch and bus evacuation. Bus dispatch refers to the process of deploying buses from depots $\{D_1, D_2, D_3\}$ to URT bridging stations $\{1, 2, 3, 4, \dots, S-1, S\}$. In the evacuation process, buses make roundtrips between stations of $\{1, 2, 3, 4, \dots, S-1, S\}$ to evacuate passengers.

Combined with the actual requirements and research needs, the model assumptions in this paper are as follows:

- (1) For this bridging model, the passengers' space-time requirements are given and static at the beginning
- (2) Bridging buses are assumed to be identical
- (3) Speed of bridging buses is assumed to be constant, and traffic congestion is not considered

3.2. Traditional Solutions and Problems. Conventionally, the design of bus bridging service is based upon artificial experience. As shown in Figure 2, firstly dispatch the bus from a bus depot in set of $\{D_1, D_2, D_3\}$ to the nearest turnover stations at any of $\{1, S\}$, and then, set routes along the URT disruptive section of $(1, S)$. Additionally, the bus visits the URT station stop by stop.

As indicated in Table 1, bus bridging service has been applied in response to URT disruptive events in some Chinese cities including Shanghai, Shenzhen, and Guangzhou.

It is observed that the traditional bus bridging mode might increase passengers' waiting time while decrease their satisfaction level. The reason is that the traditional method does not consider the time and space requirements of passengers. In reality, it is very difficult to evacuate all stranded passengers within the time period of URT stations during the course of the disruptive event. As shown in Figure 3, there are some passengers whose waiting time for bridging bus might be significant. For example, when 200 passengers are stranded at a station, as bus capacity of 70, 70 of the remaining passengers need to wait for the return of the bridging bus to be transferred, and the remaining 60 need to wait for the second return of the bridging bus before they could successfully depart. It is important to solve the problem mentioned by [9].

3.3. Model Framework. A large number of passengers will gather in a short period of time after a URT disruptive event. Passengers are more time sensitive compared with the regu-

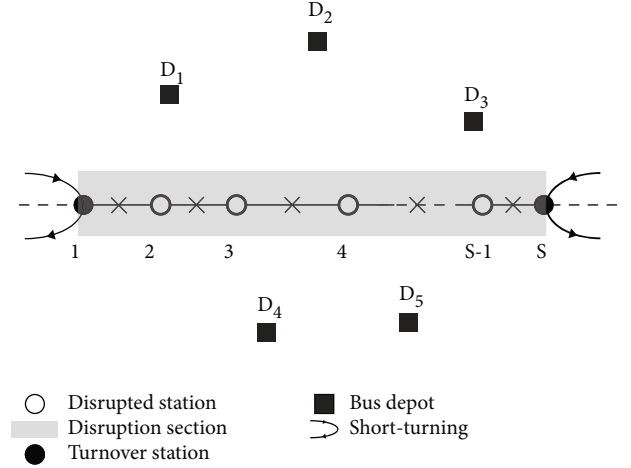


FIGURE 1: Disruptive section in the URT line.

lar bus bridging approach. This paper adopts bus bridging service to solve the problem of URT disruption management considering the passengers' space-time demands and proposes a bus bridging service model which could optimize the overall bridging time of stranded passengers. The cooperation with URT system should be considered in this bus bridging scenario.

The passenger demands whose origin and destination located at disrupted station such as stations of $\{2, 3, 4, \dots, S-1\}$ are far less than the demands in turnover station as $\{1, S\}$. The time requirements for the former passenger are less than the latter. In view of the differences between these two categories of passengers' demands, the bridging routes of the model are divided into two categories including the local bus route (LBR) and the direct bus route (DBR).

Local bus routes (LBR) are shown in Figure 4. Firstly, a bus is dispatched from depot D_i to the left-most disrupted station 2 or the right-most disrupted station $S-1$; then, the bus makes round trips following the URT line. In this process, the bridging bus visits every disrupted station, enabling passengers to embark and disembark. There are two local bus routes in Figure 4 as $D_1 - 2 - 3 - 4 \dots (S-1) - (S-2) \dots - 4 - 3 - 2$ and $D_2 - (S-1) - (S-2) \dots 4 - 3 - 2 - 3 - 4 \dots (S-2) - (S-1)$.

Direct bus routes (DBR) are indicated in Figure 5. There are three types of DBR in the model. When a bus operates in the first kind of route, it is dispatched from the depot D_i to a disrupted station in $\{2, 3, \dots, S-1\}$ and makes round trips with the destination of turnover stations as $\{1, S\}$. $D_2 - 4 - S - 4 - S$ is one of the first type as shown in the figure. When a bus transports in the second kind of route, it is dispatched from D_i to the turnover station of $\{1, S\}$ and makes round trips with the destination of disrupted station in $\{2, 3, \dots, S-1\}$. $D_1 - 1 - 3 - 1$ is one of the second type. When a bus runs in the third kind of route, it is dispatched from D_i to the turnover station of $\{1, S\}$ and makes round trips with the destination of the other turnover station of $\{1, S\}$. $D_3 - S - 1 - S$ is one of the third type.

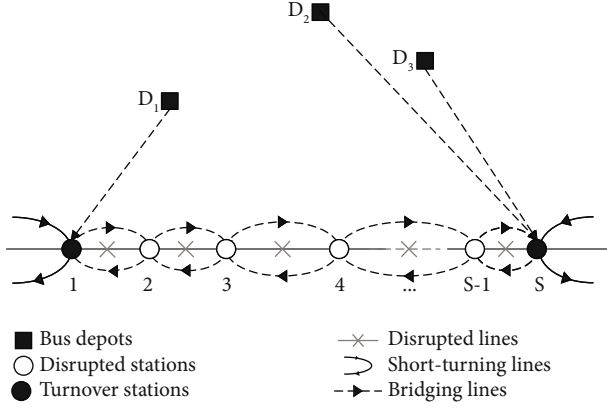


FIGURE 2: Traditional bus bridging service.

Apart from the designed routes, the model parameters and variables are introduced. The model parameters are shown in Table 2, and the variables are presented in Table 3.

This paper focuses on the bus bridging service after a URT disruptive event and proposes a model considering the space-time requirements of passengers. To minimize the waiting time of passengers, the model considers the maximum number of bridging buses in LBR or DBR, capacity limit of the bus, parking capacity of bus stations, and maximum bridging time limitation of a single bus.

The model objective is to minimize total bridging time for passengers. There are two parts in the model including time spent on LBR as T_1 and time taken on DBR as T_2 . The bridging time includes dispatch time and evacuation time. For example, the first term in Equation (2) represents the dispatch time spent on LBR, and the other term indicates the evacuation time of LBR.

$$\min T = \sum_{m \in \mathcal{M}} (T_1 + T_2), \quad (1)$$

$$T_1 = \sum_{i \in \mathcal{L}} (x_i^m \times t_i^m) + \sum_{i \in \mathcal{L}} (t_{i,i+1} + t_{i+1,i}) \times v^m, \quad (2)$$

$$T_2 = \sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{P}, i \neq j} [(y_{i,j}^m \times (t_i^m + t_{i,j}))] + \sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{P}, i \neq j} [(t_{i,j} + t_{j,i}) \times z_{i,j}^m]. \quad (3)$$

There are 16 model constraints as follows:

$$\sum_{i \in \mathcal{L}} x_i^m + \sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{P}, i \neq j} y_{i,j}^m \leq 1, \quad \forall m \in \mathcal{M}. \quad (4)$$

Equation (4) indicates that each bus can serve one bus route only.

$$v^m \leq O \times x_i^m, \quad \forall m \in \mathcal{M}, i \in \mathcal{L}, \quad (5)$$

$$z_{i,j}^m \leq O \times y_{i,j}^m, \quad \forall i \in \mathcal{S}, j \in \mathcal{P}, i \neq j, m \in \mathcal{M}. \quad (6)$$

Equation (5) explains that the number of round trips is allocated to bus m only after the bus is dispatched to

the LBR. Equation (6) explains that the number of round trips is allocated to bus m only after the bus is dispatched to the DBR.

$$v^m \leq \alpha, \quad \forall m \in \mathcal{M}, \quad (7)$$

$$z_{i,j}^m \leq \beta, \quad \forall i \in \mathcal{S}, j \in \mathcal{P}, i \neq j, m \in \mathcal{M}. \quad (8)$$

Equations (7) and (8) define the maximum number of round trips of bus m on the LBR and the DBR, respectively.

$$C \times \sum_{m \in \mathcal{M}} v^m \geq q_{i,j}, \quad \forall i \in \mathcal{L}, j \in \mathcal{L}, i \neq j, \quad (9)$$

$$C \times \sum_{m \in \mathcal{M}} (y_{i,j}^m + z_{i,j}^m) \geq q_{i,j}, \quad \forall i \in \mathcal{L}, j \in \mathcal{P}, \quad (10)$$

$$C \times \sum_{m \in \mathcal{M}} z_{j,i}^m \geq q_{j,i}, \quad \forall i \in \mathcal{L}, j \in \mathcal{P}, \quad (11)$$

$$C \times \sum_{m \in \mathcal{M}} (y_{i,j}^m + z_{j,i}^m + z_{i,j}^m) \geq q_{i,j}, \quad \forall i \in \mathcal{P}, j \in \mathcal{P}, i \neq j. \quad (12)$$

The above four equations are all about bridging requirements of passengers. Equation (9) determines that the transport capacity provided by the LBR between disrupted stations could satisfy the passengers' demand. Equation (10) refers to the fact that bus bridging could provide transport service for the passenger from disrupted stations to turnover stations. Equation (11) represents that the passengers' demand from turnover stations to disrupted stations could be satisfied. Equation (12) indicates any buses assigned to the turnover station can provide direct transport service to the passengers with demand from one turnover station to the other.

$$\sum_{m \in \mathcal{M}} x_i^m \leq \gamma, \quad \forall i \in \mathcal{L}, \quad (13)$$

$$\sum_{m \in \mathcal{M}} y_{i,j}^m \leq \gamma, \quad \forall i \in \mathcal{S}, j \in \mathcal{P}, i \neq j. \quad (14)$$

The number of buses parking at bus stops is limited. Equation (13) considers the maximum number of bridging bus dispatched to LBR. Equation (14) considers the upper limit of buses about to be accommodated by the URT station for the DBR.

$$x_i^m t_i^m + \left(\sum_{i \in \mathcal{L}} (t_{i,i+1} + t_{i+1,i}) \right) \times v^m \leq D, \quad \forall i \in \mathcal{L}, m \in \mathcal{M}, \quad (15)$$

$$y_{i,j}^m \times (t_i^m + t_{i,j}) + (t_{i,j} + t_{j,i}) \times z_{i,j}^m \leq D, \quad \forall i \in \mathcal{S}, j \in \mathcal{P}, i \neq j, m \in \mathcal{M}. \quad (16)$$

TABLE 1: Some applying cases of bus bridging service in China.

Disruptive events	Time point of occurrence	Duration	Dispatched buses	Affected passengers
Shanghai URT line 1	12/22/2009, 6:50	5 h 25 min	105	—
Shanghai URT line 1	7/28/2015, 6:10	3 h 15 min	110	10000
Shenzhen URT line 4	9/5/2012, 13:37	6 h	170	20700
Guangzhou URT line 1	06/02/2017, 06:31	1 h 45 min	17	2400
Guangzhou URT line 1	05/07/2019, 19:40	1 h 30 min	60	—

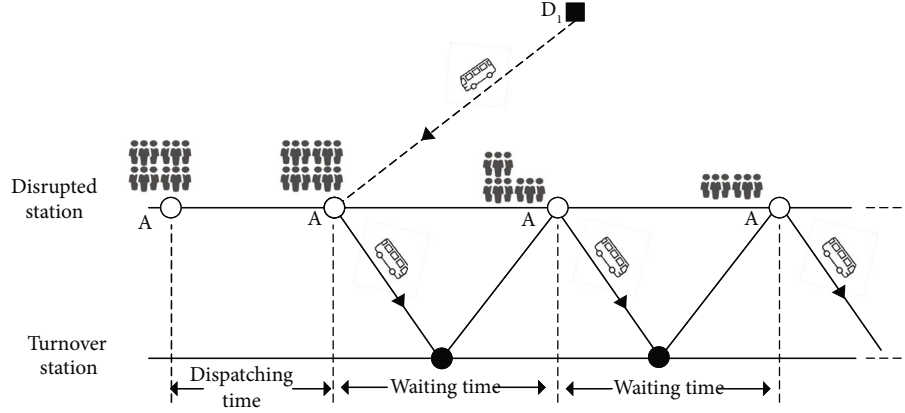


FIGURE 3: Passengers' waiting time in the bus bridging service.

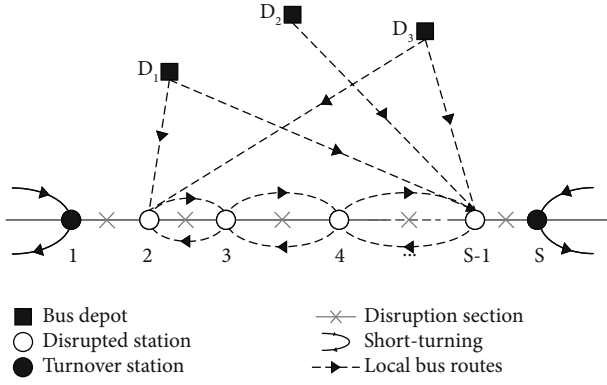


FIGURE 4: The local bus routes (LBR).

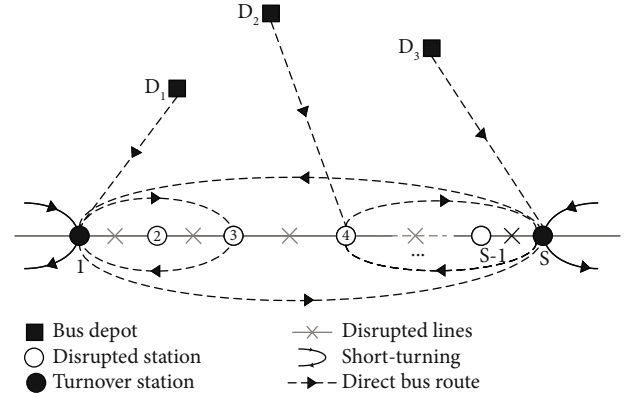


FIGURE 5: The direct bus routes (DBR).

Equations (15) and (16) limit the bridging time of every bus which is less than the duration of the disruptive event.

$$x_i^m t_i^m \leq T_d, \quad \forall i \in \mathcal{L}, m \in \mathcal{M}, \quad (17)$$

$$y_{i,j}^m t_{ij}^m \leq T_d, \quad \forall i \in \mathcal{S}, j \in \mathcal{P}, i \neq j, m \in \mathcal{M}. \quad (18)$$

The above two formulas limit the maximum deployment time of a bus. Equations (17) and (18) calculate the dispatch time from the depots to URT station, which should be less than passengers' maximum waiting time T_d in LBR and DBR, respectively.

$$y_{i,j}^m t_{ij}^m + z_{i,j}^m (t_{i,j} + t_{j,i}) \leq T_w + t_{i,j}, \quad \forall i \in \mathcal{H}, j \in \mathcal{P}, m \in \mathcal{M}. \quad (19)$$

Equation (19) illustrates that maximum bridging time T_w from the disrupted stations to turnover stations should be less than the sum of waiting and travel time for passengers.

4. Numerical Example

4.1. Numerical Example Setup. Parts of the data in numerical example draw from Gu [2]. The research takes Shanghai URT line 9 in China as an example: it is assumed that a

TABLE 2: Model key parameter definition.

Parameters	Parameter meaning
\mathcal{S}	Set of URT stations affected by the URT disruption, including disrupted stations and turnover stations. $\mathcal{S} = \{1, 2, 3, \dots, S\}$, indexed by i, j
\mathcal{P}	Set of URT turnover stations, $\mathcal{P} = \{1, S\}$, $\mathcal{P} \subset \mathcal{S}$
\mathcal{K}	Set of URT disrupted stations, $\mathcal{K} = \{2, 3, \dots, S-1\}$, $\mathcal{K} \subset \mathcal{S}$
\mathcal{L}	Set of disrupted URT stations nearest to the turnover stations, $\mathcal{L} = \{2, S-1\}$, $\mathcal{L} \subset \mathcal{K}$
\mathcal{M}	Set of available buses, $\mathcal{M} = \{1, 2, 3 \dots, M\}$, indexed by m
C	Bus capacity
D	Disruption duration
$q_{i,j}$	Travel demand between i and j , $i, j \in \mathcal{S}, i \neq j$
O	A large number
α, β	Maximum number of local bus route round trips and direct bus route round trips
γ	Bus stop parking capacity
t_i^m	Dispatch time from bus depot to URT station i for bus m
$t_{i,j}$	Bus travel time between i and j
T_d	Maximum dispatch time
T_w	Maximum waiting time for passengers from disrupted station to turnover station
T	Total bridging time for all buses
T_1	Total bridging time of the buses in LBR
T_2	Total bridging time of the buses in DBR

TABLE 3: Variables in the model.

Variables	Variable meaning
x_i^m	Binary variable, designated value for bus m dispatched to station i of set L with operation through LBR as 1
v^m	Integer variable, indicating the number of round trips of bus m at LBR
$y_{i,j}^m$	Binary variable, designated value for bus m dispatched to disrupted station i ($i \in \mathcal{S}$) with turnover station j ($j \in \mathcal{P}, i \neq j$) while operation through DBR as 1
$z_{i,j}^m$	Integer variable, indicating the number of round trips of bus m at DBR between station i ($i \in \mathcal{S}$) and station j ($j \in \mathcal{P}, i \neq j$)

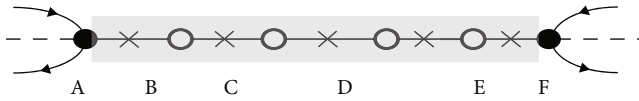


FIGURE 6: Topology of a disruptive event in Shanghai URT line 9.

TABLE 4: Passenger demand between two stations.

	A	B	C	D	E	F
A	—	574	626	604	590	1076
B	578	—	98	113	149	440
C	434	92	—	102	134	449
D	458	125	86	—	92	433
E	398	152	137	101	—	630
F	1147	485	469	472	484	—

TABLE 5: Bus travel time in stations (units: min).

	A	B	C	D	E	F
A	—	9	16	17	18	23
B	10	—	8	12	17	20
C	14	7	—	4	9	13
D	16	9	2	—	7	11
E	19	14	6	4	—	6
F	22	18	10	8	3	—

disruptive event occurs in the URT single line, and the disruption lasts more than 30 minutes. It is challenging for the operator to evacuate passengers by adjusting train timetable. Consequently, the bus bridging service has to be initiated. The URT is represented by the topology

TABLE 6: The depot capacity and travel time to URT stations.

Depots	Number of buses	Stations (units: min)					
		A	B	C	D	E	F
1	10	6	5	9	12	15	26
2	9	19	14	11	10	10	14
3	11	19	15	11	7	5	9
4	16	8	3	5	7	12	21
5	13	12	10	10	13	16	18
6	13	16	12	9	6	8	12
7	11	14	9	5	5	8	11

TABLE 7: Comparative analysis of the optimized bridging service and traditional bridging service.

Different bus bridging service	Traditional bridging service	Proposed bridging service	Degree of optimization
Bridging time (units: min)	3385	3278	LBR: 240 DBR: 3038 3.2%
Dispatched buses	73	55	LBR: 3 DBR: 52 24.7%

TABLE 8: Comparative results of the optimization bridging service and traditional bridging service.

Depots	Traditional bridging service						Optimization bridging service					
	A	B	C	D	E	F	A	B	C	D	E	F
D1	9	0	0	0	0	0	6	2	0	0	0	0
D2	0	0	0	0	0	5	0	0	0	0	0	0
D3	0	0	0	0	0	10	0	0	0	0	6	4
D4	15	0	0	0	0	0	0	11	4	0	0	0
D5	12	0	0	0	0	0	0	0	0	0	0	0
D6	0	0	0	0	0	12	0	0	0	7	3	2
D7	3	0	0	0	0	7	0	0	6	3	0	1
Buses used	39	0	0	0	0	34	6	13	10	10	9	7
Total buses used	73						55					

shown in Figure 6, where the disruptive section is from stations A to F . Ending points A and F are turnover stations, and the four stations in the center signify disrupted stations. The operator would take a short-turning strategy at nondisruptive section.

Passenger demand between two stations is indicated in Table 4.

Bus travel time in stations is expressed in Table 5.

The number of buses in each depot and the travel time from the depot to the URT stations are shown in Table 6.

The relevant parameters are introduced as follows:

Number of available buses in depots: $M = 76$

Capacity of a bus: $C = 90$

(1) Maximum number of round trips for LBR and DBR:
 $\alpha = \beta = 3$

(2) Bus stop parking capacity: $\gamma = 7$

(3) Disruption duration: $D = 90$ min

(4) Maximum dispatch time of a bus: $T_d = 15$ min

(5) Maximum waiting time for passengers from disrupted station to turnover station: $T_w = 50$ min

Additionally, each bus stops at the station in one minute to allow passengers to embark and disembark.

4.2. Solution Algorithm. CPLEX Component Libraries could solve linear programming (LP) and related problems. As a successful computational software, CPLEX has batteries included with branch-bound and cutting plane method. The solution algorithm is performed on a laptop which possesses an Intel Core i5-7500, 2.00 GHz CPU, and 16 GB memory. The ILOG CPLEX 12.6 is applied as the LP solver for the paper. It is a Python language that is used to realize

TABLE 9: Optimized bus bridging routes and bus bridging time.

Bus depots	Bridging routes	Bridging time (units: min)
D1	A-F	31
	B-F-B-F	67
	B-A-B-A	38
D3	E-A-E-A	65
	F-A-F-A	80
	E-F-E-F-E-F-E-F	46
D4	B-C-D-E-D-C-B-C-D-E-D-C-B	80
	C-F-C-F-C-F	70
	B-F-B-F	65
	B-A-B-A-B-A	57
D6	C-A-C-A	53
	D-F-D-F-D-F	61
	F-A-F-A	83
	D-A-D-A	59
	E-F-E-F-E-F-E-F	49
D7	E-A-E-A	68
	D-A-D-A	58
	C-F-C-F-C-F	70
	D-F-D-F-D-F	60
	C-A-C-A	53
	F-A	35

the model. The solving algorithms are all executed within one minute.

4.3. Results and Discussions. The calculations suggest that the model proposed in this paper is superior to the traditional bus bridging service in terms of total bridging time as well as the total number of buses used. It requires 73 buses and 3385 minutes to evacuate all passengers using the traditional bus bridging service. By contrast, only 55 buses and 3278 minutes are required for the model developed in this paper. Bridging time decreases by 3.2%, while the number of buses dispatched is reduced by 24.7% as Table 7 shows. It is worth noting that the bridging time and the number of buses used by LBR are 240 minutes and 3 buses, which are far fewer than the bridging time and number of buses used by DBR.

It is observed from Tables 6 and 8 that the bus is not dispatched to the nearest disruption station. For example, the D3 depot is closer to URT stations *D* and *E*; however, the buses at this depot are dispatched to stations *E* and *F* according to the optimization bridging model. The reason is that the model is to optimize the entire bridging time instead of dispatch time or evacuation time only.

The bus takes round trips on LBR and DBR to meet the demand of passengers. A statistical analysis of the bridging time of all buses is described in Table 9. There are only one LBR (D4-B-C-D-E-D-C-B-C-D-E-D-C-B) in the optimized bridging routes with bridging time 80 minutes. The

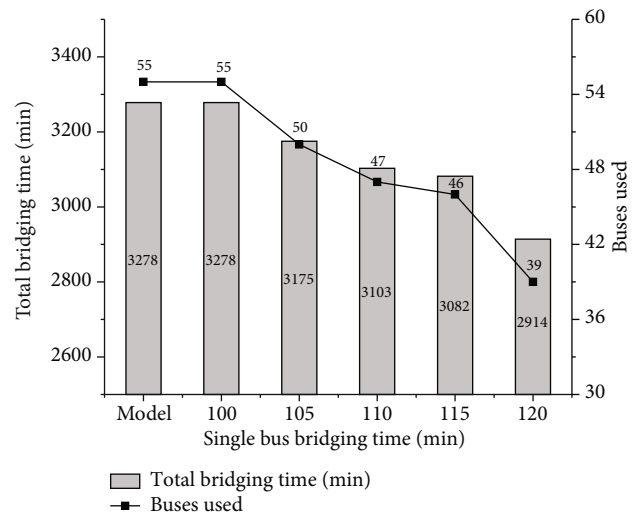


FIGURE 7: Changes in total bridging time and buses used with different bus evacuation time.

other routes are DBR with average bridging time of 59.6 minutes.

4.4. Sensitivity Analysis

4.4.1. Bridging Time of a Single Bus. The experiment focuses on the changes in total bridging time and the number of buses used as bridging time of a single bus increases. The result is presented in Figure 7.

Figure 7 indicates that total bridging time and the number of buses utilized are reduced significantly when the bridging time of a single bus increases. When bridging time of a bus increases from 90 to 120 minutes, the total bridging time correspondingly decreases by 364 minutes, while the number of buses required reduces by 16. Overall, the model proposed would balance the single bus bridging time and available bus resources.

When designing the bus bridging service, passengers' time requirements should be given a high priority if there are sufficient buses. On the condition that there is a limited number of buses, some passengers have to wait for long time to obtain a feasible solution of a bus bridging plan.

4.4.2. Passengers' Demand. This paper analyzes changes of total bridging time and the number of buses used in different demand levels.

As shown in Figure 8, it is obvious that total bridging time and the number of buses used increase in correspondence with demand. When demand is reduced by half, the total bridging time reduced by 1517 minutes; meanwhile, the number of buses in use reduced by 24. The current bridging bus model with parameters set could satisfy 1.4 times of passengers' demands at most. When the number of passengers exceeds the maximum load capacity of the bus bridging service system, the extra passengers have to get to destination by other transport modes such as taxi and online ride-hailing.

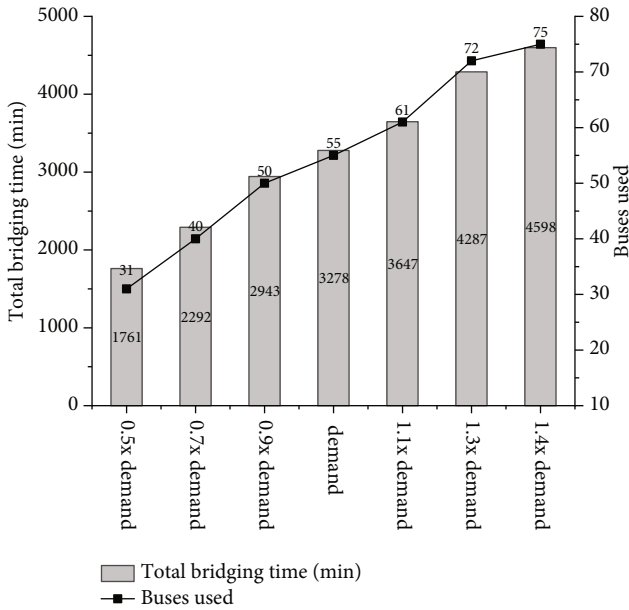


FIGURE 8: Changes in total bridging time and buses used under different passenger demands.

4.4.3. *Bus Capacity.* The capacity of a single bus is increased in order to observe any changes in the total bridging time and the number of buses used. The result is shown in Figure 9.

It could be seen from Figure 9 that as the capacity of a single bus grows, total bridging time and the number of buses used decrease sharply. However, total bridging time drops slower with the increasing of the bus capacity. For instance, total bridging time decreases by 335 minutes accordingly when the bus capacity increases from 90 to 100. As total bridging time reduced by 138 minutes in response to bus capacity increase from 140 to 150, it can be stated that a bus with a large capacity over 130 could not effectively reduce bridging time. In regard to the number of buses used, increasing the bus capacity is effective. In a word, the more passengers a single bus could transport, the fewer buses will be used. When the bus capacity is below 90, the model in this numerical example becomes ineffectual. From the above analysis, we can conclude that it is critical to select a suitable bus type in a disruptive event. Therefore, it is better for URT authorities to choose an appropriate bus type according to passengers' space-time requirements.

4.5. *Comparative Experiments.* The computational results of releasing show the necessity to simultaneously minimize the total bridging time and passenger waiting time. On the one hand, Equations (15) and (16) indicate that there is no limit on the maximum bridging time of a single bus. On the other hand, Equation (19) indicates that there is no restriction on the maximum waiting time of passengers.

The comparative result reveals that the number of buses used and bridging time are reduced significantly regardless of passengers' time requirements. However, the maximum single bus bridging time is 174 minutes with consideration of passengers' time requirements, which exceeds the dura-

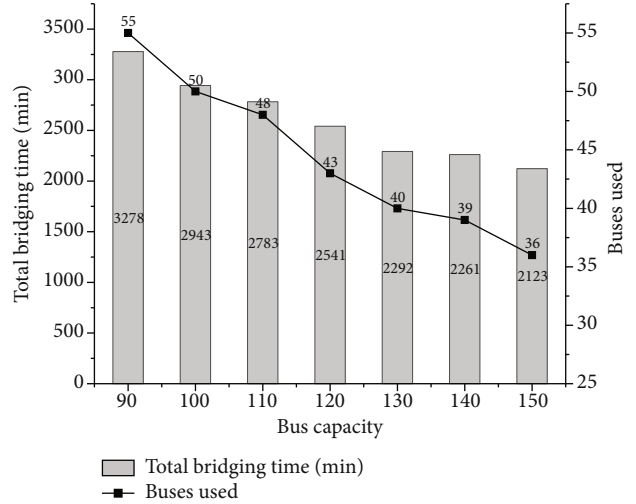


FIGURE 9: Changes in total bridging time and buses used under different bus capacity.

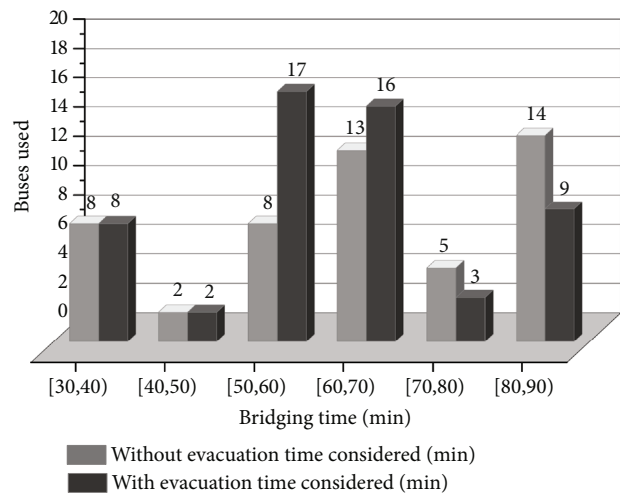


FIGURE 10: Bus bridging time and buses used with and without evacuation time considered.

tion of the disruption. Consequently, it is essential to consider passengers' time requirements in the bus bridging service.

In addition, this paper analyzes the effect of bus evacuation time on the total passenger bridging time and the number of buses used based on bus dispatch time. When considering evacuation time, a small increase in the number of bridging buses reduces the operation time of most bridging buses.

According to Figure 10, considering both of the dispatch time and the evacuation time, the number of buses with a total bridging time exceeding 70 minutes is reduced, the bridging time of the majority of the number of buses is 50-70 minutes, and the number of bridging buses used increases by 10%, which infers that considering both of dispatch time and evacuation time could reduce the bridging time of most buses while increasing the number of buses used. Therefore,

it is necessary to trade off the number of buses used and bridging time of buses in practical work.

5. Conclusions

Prompt and safe passenger evacuation is a huge challenge for URT in a disruptive event. This paper investigates the bus bridging service under a URT disruptive event and proposes a bus bridging service model which takes the space-time requirements of passengers into consideration. Two kinds of bridging routes are generated for affected passengers, including LBR and DBR. CPLEX Component Libraries are invoked by Python in order to solve the problem. The model is validated by a disruptive event in Shanghai URT network. The main findings of the study are summarized as follows:

- (a) The model proposed in this research is able to meet not only the requirements of passenger between the disruption stations and the turnover stations but also the demand of passengers whose origins and destination happen to be the disrupted URT stations
- (b) The constraints of travel time requirements for passengers have great significance in relation to improve the service level, although it would increase the number of bridging buses. It is necessary to trade off passenger travel time requirements and the costs related to increasing the number of bridging buses for URT operators
- (c) There is a threshold for bus capacity to reduce the bridging time. Our results reveal that a big capacity does not necessarily equate to a better service. A large-capacity bus more than 130 is unable to effectively reduce the system bridging time. Thus, it is important to select suitable bus capacity
- (d) URT operators must have well-prepared plans to deal with potential disruptive events. According to the distribution of URT passenger flow, the operator can estimate the required number of bridging buses, dispatch, and evacuation route scheme based on the model of this paper

There are still many aspects of this model that need to be improved. This paper only considers a disruption that occurs in a URT single route. In the future, the model will provide a solution for network disruption in URT. A further issue of interest is the way to determine the location of bus depots. Dynamic passenger flow during URT disruptive events should also be considered.

Data Availability

No real data was used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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