

## Research Article

# Scheduling Synchronization for Overlapping Segments in Bus Lines: Speed Control and Green Extension Strategies

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Overlapping bus lines are ubiquitous in bus networks, particularly in metropolitan areas. The overlapping of bus lines can provide convenience for passengers who wish to transfer. However, it also tends to cause bus bunching at overlapping segment stops. Moreover, overlapping of bus lines introduces additional complexity to the operation of bus systems. This study aimed to dispatch bus vehicles entering overlapping segments dynamically by adopting speed control and green light extension strategies. This ensures that transferring passengers experience less transfer waiting time and reduced bus bunching at overlapping segments. The proposed model considers environmental constraints on vehicle speed and the stochastic factors of passenger arrivals at a stop. Synchronization is maximized by controlling the speed of vehicles along a roadway and determining whether a green light extension strategy is enabled. The effectiveness of the proposed model was verified by applying it to a real overlapping segment in Harbin, China. The results demonstrate that the proposed model can more than double the opportunity for synchronization in overlapping segments while reducing bus bunching at the stops in overlapping segments.

## 1. Introduction

Public transportation plays a major role in daily travel for numerous people. An increasing number of bus lines are being planned in cities to meet the growing travel demands of urban residents. Despite the overlapping of bus routes in most cities, conventional bus systems still cannot provide door-to-door transportation. Some bus riders must transfer between buses to complete their trips [1]. Transferring results in additional waiting time and inconvenience to passengers, making the transit system less attractive [2]. As a result, many studies have begun to investigate seamless transfers between buses through scheduling and control strategies [3–6].

The bus scheduling problem for minimizing passenger waiting times at transfer stops has been extensively analyzed in the literature in the form of the synchronization bus timetabling problem [3, 6, 7]. A number of optimization models have been developed to balance the benefits of bus agencies with those of transfer passengers. Silva-Soto and Ibarra-Rojas developed a biobjective mixed-integer linear

optimization model with the optimization objectives of minimizing transfer waiting times for transfer passengers and minimizing bus line operating costs. An improved genetic algorithm was proposed to solve this model [8]. Ibarra-Rojas et al. developed a nonlinear mixed-integer model to describe a timetable scheduling problem that minimizes the costs for passengers and operators considering passenger transfers under the conditions of constant passenger travel demand. In this model, the optimization objective was implemented through bus route departure frequency determination and passenger route assignment. The authors also designed an iterative heuristic algorithm to solve the proposed model. The validity of the model and algorithm was verified on the Santiago Chile transportation network [9]. Liu and Ceder developed a biobjective integer optimization model that maximizes the benefits of both bus passengers and bus operators to achieve schedule synchronization. A deficit-function-based search method was designed to obtain the Pareto optimal solution of the model [10]. To obtain a profitable trade-off between passenger service and public operating costs, Peterson et al. developed

an integer programming model for synergistic bus vehicle scheduling and solved the model using a large neighborhood search approach. Tests on a bus network in the Greater Copenhagen area demonstrated that their approach can lead to a 20% reduction in passenger transfer waiting time while ensuring essentially constant vehicle dispatch costs [11]. Ceder et al. proposed a mixed-integer planning model to develop a timetable that maximizes the number of vehicles arriving at a transfer stop simultaneously [4].

Based on the complexity of the synchronization transfer schedule optimization model, there have been many approaches to solve the scheduling model using a simpler method. Xiao et al. improved bus transfer efficiency by optimizing the slack times and bus vehicle arrival times in timetables [12]. Liu et al. developed a new deficit function method to solve the schedule synchronization and bus vehicle scheduling problems. An optimal solution is generated using a graphical optimization technique, and this method can be applied to generate efficient solutions in large-scale bus networks [13]. Wu et al. developed a timetable with the objective of minimizing the total transfer waiting times for transferring passengers. Additionally, an efficient recursive quasi-linear-time algorithm was proposed to obtain a synchronization timetable [3]. Lai et al. proposed a method for flexible schedule development for overlapping buses. The effects of passenger boarding and alighting delays and bus priority strategies on the synchronization transfer between vehicles were analyzed. A case study on bus networks in the Shanghai bus corridor found that the proposed bus schedule measure could improve the adaptability of bus systems [14]. Fonseca et al. proposed a biobjective mixed-integer programming model [15]. This model combines the timetable formulation and vehicle scheduling problems with the optimization objective of minimizing transfer passenger costs and bus operating costs. The model was applied to the Greater Copenhagen area, and the results demonstrated that it is able to reduce transportation costs. Konstantinos weighted the conflict between reducing passenger transfer times and the regularity of bus operations by introducing a weighting factor. A case study in Stockholm found that this approach can reduce passenger transfer times by 13% at the expense of 2.8% of bus operation regularity [16].

Several researchers have also considered other aspects of the impact of synchronization on public transportation systems. The impact of overlapping route layouts and traffic signal priority strategies on transit system operation has been investigated. The synchronization between overlapping bus lines and the frequency of vehicles on each route has been identified with the goal of maximizing benefits for passengers and operators [17]. To address the problem of long passenger transfer times caused by low-frequency bus services in rural areas of Japan, Takamatsu and Taguchi proposed a mathematical model to revise current bus schedules without increasing the number of bus services, resulting in a reduction in passenger transfer waiting times. The application of this method in the Tohoku region of Japan demonstrated that it has significant practicality [18]. Wu et al. focused on demand allocation and passenger rerouting after passenger transfer failures and designed a bus schedule

based on interval times and slack times. They also developed a transfer passenger path selection model [19]. Ibarra-Rojas et al. proposed a measure of bus synchronization using the time deviations of vehicles arriving at stops in overlapping segments as a penalty function. A speed control strategy was used to optimize the scheduling of buses operating in overlapping segments. The proposed model was validated on a bus network in Santiago, Chile, and the results demonstrated that it is beneficial for reducing passenger transfers between overlapping bus lines [20]. Gao et al. proposed a bus scheduling model that can limit the spread of infectious diseases during COVID-19 [21]. Chu et al. proposed a mixed-integer linear programming model that can optimize both bus schedules and passenger travel path selection. Additionally, a heuristic algorithm was proposed for solving the model. The proposed schedule maximizes simultaneous passenger transfers [22]. It is also possible to maximize passenger transfer rates and minimize bus congestion in bus networks by planning different timetables for each bus line during different planning periods in the day [5].

Typically, based on the technical limitations of buses and the road environment, the bus synchronization transfer problem is formulated as a static schedule optimization problem. However, the optimization models that are typically developed are difficult to solve [23]. Although a large number of solution methods have been studied, they are still less than ideal. Overlapping operations between transfer bus lines further increase the complexity of synchronization schedule development. This is because vehicles in overlapping segments may experience bunching at common stops [24]. The synchronization of bus schedules is performed not only to facilitate passenger transfers between different bus lines but also to avoid the bunching of bus vehicles belonging to different bus lines at common stops. The development of novel technologies such as intelligent communication technology and vehicle positioning technology has led to the maturation of vehicle dynamic control strategies, which facilitate the control of vehicle bunching and cooperative interchange.

This study introduces the speed control and green light extension strategies to provide transfer passengers with more transfer opportunities in an overlapping segment. These control strategies have been extensively analyzed in the last decade to improve the bus system's regularity [25, 26]. However, this study is focused on the effects of these strategies on the transfer options among overlapping routes. An overlapping segment is formed when there are more than two consecutive common stops on several routes. Transferring passengers can complete their transfers at any stop in an overlapping segment. We developed a synchronization scheduling optimization model for overlapping bus routes. By using speed control and green light extension strategies, vehicles entering overlap intervals are dynamically dispatched in an overlapping segment. This provides ideal interchange opportunities for passengers while mitigating bus bunching as much as possible.

The remainder of this paper is structured as follows. Section 2 introduces the concepts of speed control and the green light extension strategy in overlapping segments.

Section 3 describes the development of the proposed optimization model. A case study is presented in Section 4. Finally, conclusions and future research outlooks are discussed in Section 5.

## 2. Control Strategies in Overlapping Segments

Passenger transfers at the stop in an overlapping segment are illustrated in Figure 1. When the origin and destination of a passenger's trip are not on one bus line, a transfer is required to complete the trip. The overlapping of bus lines in a city facilitates passenger transfers. As shown in Figure 1, the three bus lines Line<sub>1</sub>, Line<sub>2</sub>, and Line<sub>3</sub> share common stops from cs<sub>1</sub> to cs<sub>4</sub> and form an overlapping segment. If passengers wish to depart from the stop s<sub>1,1</sub> on Line<sub>1</sub> to reach stop s<sub>3,2</sub> in Line<sub>3</sub>, they can ride the bus running on Line<sub>1</sub> first and transfer to the bus running on Line<sub>3</sub> at the stop in the overlapping segment to complete the trip.

Note: this is an example of three overlapping bus lines sharing four common stops in the overlapping segment. The general formulation can be easily obtained if Line<sub>3</sub> is replaced by Line<sub>N</sub>, where N is the dimension of the subset of bus lines; stop cs<sub>4</sub> is replaced by stop cs<sub>M</sub>, where M is the total number of common stops in the overlapping segment.

For overlapping bus lines, to prevent bus bunching between vehicles belonging to different bus lines at overlapping segment stops, vehicles are scattered into overlapping segments as much as possible. However, having vehicles enter an overlapping segment in a scattered manner is not conducive to the transfer of passengers between different lines. To facilitate transfers for passengers, the proposed method dynamically adjusts the vehicle operation status after a vehicle enters an overlapping segment to facilitate transfers between vehicles.

The time when the  $m$ -th vehicle online  $p$  arrives at the first stop in an overlapping segment is  $t_0(p, m)$ , and the serial number for the vehicle in the overlapping bus line  $i$  is  $\alpha(p, m)$ . If the  $m$ -th vehicle on route  $p$  is the  $k$ -th vehicle entering the overlapping segment during the optimization time period, then

$$\alpha(p, m) = k. \quad (1)$$

For two vehicles  $v_{\alpha(p,m)}$  and  $v_{\alpha(q,n)}$  that enter an overlapping segment consecutively, as shown in Figure 2, the following relationship is obtained:

$$\alpha(p, m) + 1 = \alpha(q, n). \quad (2)$$

Vehicle  $m$  online  $p$  enters the overlapping segment before vehicle  $n$  online  $q$ . Figure 3 presents the trajectories of the vehicles before and after the implementation of the proposed transit control strategy, where trajectory<sub>1</sub> represents the trajectory of the vehicle  $v_{\alpha(p,m)}$  before the implementation of the transit control strategy, trajectory<sub>2</sub> represents the trajectory of the vehicle  $v_{\alpha(q,n)}$  before the implementation of the transit control strategy, and trajectory<sub>3</sub> represents the trajectory of the vehicle  $v_{\alpha(o,l)}$  before the implementation of the transit control strategy. For the purpose of synchronization transfer, the green extension

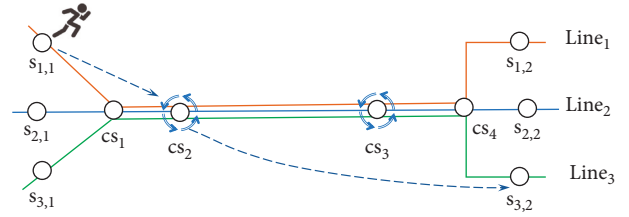


FIGURE 1: Passenger transfers at stops in an overlapping segment.

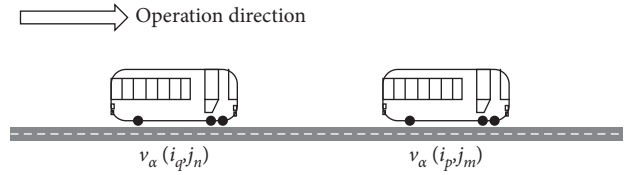


FIGURE 2: Diagram of vehicle serial numbers.

strategy can be implemented for vehicle  $v_{\alpha(q,n)}$  at intersection<sub>1</sub> such that the vehicle  $v_{\alpha(q,n)}$  arrives at stop<sub>2</sub> earlier than planned in the timetable and completes a synchronization transfer with the vehicle  $v_{\alpha(p,m)}$ . Additionally, to reduce the waiting time for transferring passengers between  $v_{\alpha(q,n)}$  and  $v_{\alpha(o,l)}$ , the speed control strategy can be implemented for vehicle  $v_{\alpha(q,n)}$  after it leaves the stop<sub>2</sub> such that the vehicle  $v_{\alpha(q,n)}$  arrives at stop<sub>3</sub> later than planned in the timetable. Thus, to provide more transfer opportunities for passengers riding on the vehicle  $v_{\alpha(q,n)}$ , it speeded up before arriving at stop 2 to collaborative interchange with the vehicle  $v_{\alpha(p,m)}$  at stop 2. However, to collaborative interchange with the vehicle  $v_{\alpha(o,l)}$  at stop 3, the speed of the vehicle  $v_{\alpha(q,n)}$  should be reduced after leaving stop 2. The green extension strategy can be implemented for vehicle  $v_{\alpha(o,l)}$  at intersection<sub>1</sub> and intersection<sub>2</sub> such that vehicle  $v_{\alpha(o,l)}$  arrives at stop<sub>3</sub> earlier than planned in the timetable. As shown in Figure 3, trajectory<sub>2</sub>' represents the trajectory of the vehicle  $v_{\alpha(q,n)}$  after the implementation of the transit control strategy, and trajectory<sub>3</sub>' represents the trajectory of the vehicle  $v_{\alpha(o,l)}$  after the implementation of the transit control strategy.

## 3. Synchronization for Overlapping Segments

**3.1. Assumptions.** To make this study more focused on the role of synchronization transfers and bus vehicle dynamic control strategies, the following assumptions were adopted:

- (1) Vehicles are dispersed as much as possible when entering an overlapping segment, and the green light extension strategy is implemented at intersections in the overlapping segment.
- (2) The green light extension strategy can be adapted according to the location of a bus and can be controlled to extend the duration of the street lights in the direction of the road on which the bus is traveling.
- (3) Without considering speed control conditions, the vehicle runs at the maximum speed allowed by the current traffic environment. The speed control strategy can only enable the vehicle speed to be less

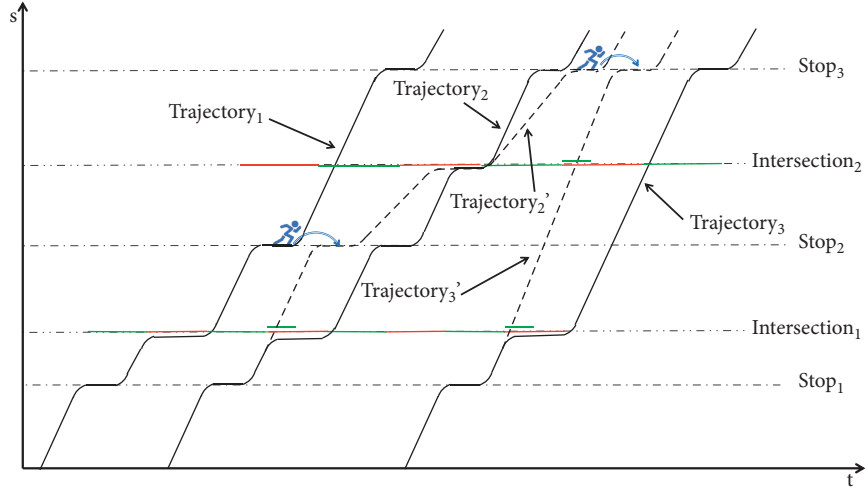


FIGURE 3: Vehicle trajectories under different control strategies.

than the normal speed, while speeds faster than the normal speed are not allowed.

- (4) Passengers cannot tolerate longer waiting times but can accept more in-vehicle time. The reasonableness of this assumption can be illustrated by the passenger's perceived time in literature [27]. In terms of perceptions of public transportation service quality, transit riders place more importance on less wait time than on less journey time.
- (5) Overtaking is allowed between vehicles belonging to different bus lines. And multiple loading/unloading platforms are available at a stop along the overlapping segment to serve buses arriving at the same bus stop simultaneously.

#### 4. Model Formulation

Vehicle  $v_{\alpha(p,m)}$  arrives at the bus stop  $s_a$  at time  $AS(a, p, m)$  and departs at time  $DS(a, p, m)$ . Let the term  $dw\ ell(a, p, m)$  denote the dwell time of the vehicle  $v_{\alpha(p,m)}$  at bus stop  $s_a$ . Then, the following equation is obtained:

$$DS(a, p, m) = AS(a, p, m) + dw\ ell(a, p, m). \quad (3)$$

Vehicle  $v_{\alpha(p,m)}$  arrives at the intersection  $\phi$  at time  $AI(\phi, p, m)$  and departs at time  $DI(\phi, p, m)$ . When vehicle  $v_{\alpha(p,m)}$  is about to reach the intersection  $\phi$ , the traffic light at intersection  $\phi$  must be checked to determine whether it is in the green or red phase. If the traffic light at intersection  $\phi$  is in the green phase at the arrival time of the vehicle  $v_{\alpha(p,m)}$ , then we have

$$DI(\phi, p, m) = AI(\phi, p, m). \quad (4)$$

Otherwise, if the traffic light at the intersection  $\phi$  is in the red phase at the time  $AI(\phi, p, m)$ , then the vehicle must wait at the intersection for a period of time  $e(\phi, p, m)$  until the traffic light turns to the green phase. In this case, the departure time  $DI(\phi, p, m)$  of the vehicle  $v_{\alpha(p,m)}$  from intersection  $\phi$  is formulated as follows:

$$DI(\phi, p, m) = AI(\phi, p, m) + e(\phi, p, m). \quad (5)$$

Let  $C_\phi$  be the total traffic light cycle at the intersection  $\phi$ , and let  $\epsilon_\phi$  denote the maximum percentage of the total traffic light for green extension allowed at intersection  $\phi$ . The maximum extension time of the green phase allowed at intersection  $\phi$  is denoted as  $G_\phi$  and is expressed as

$$G_\phi = C_\phi \times \epsilon_\phi. \quad (6)$$

There are  $L_a$  intersections between two consecutive bus stops  $s_a$  and  $s_{a+1}$ , and

$$AI(\phi_{a,1}, p, m) = DS(a, p, m) + \frac{\text{dis}(a, \phi_{a,1})}{x(p, m, a, \phi_{a,1})}, \quad (7)$$

$$AI(\phi_{a,l}, p, m) = DI(\phi_{a,l-1}, p, m) + \frac{di\ s(\phi_{a,l-1}, \phi_{a,l})}{x(p, m, \phi_{a,l-1}, \phi_{a,l})}, \quad (8)$$

$$AS(a+1, p, m) = DI(\phi_{a,L_a}, p, m) + \frac{di\ s(\phi_{a,L_a}, a+1)}{x(p, m, \phi_{a,L_a}, a+1)}, \quad (9)$$

where  $\phi_{a,1}$  is the first intersection between the two consecutive bus stops  $s_a$  and  $s_{a+1}$ ,  $di\ s(a, \phi_{a,1})$  is the distance between the bus stop  $s_a$  and the first intersection  $\phi_{a,1}$ , and  $x(p, m, a, \phi_{a,1})$  is the cruising speed for the vehicle  $v_{\alpha(p,m)}$  between bus stop  $s_a$  and the first intersection  $\phi_{a,1}$ . The term  $\phi_{a,l}$  represents the  $l$ -th intersection between the two consecutive bus stops  $s_a$  and  $s_{a+1}$ , and  $\text{dis}(\phi_{a,l-1}, \phi_{a,l})$  is the distance between intersections  $\phi_{a,l-1}$  and  $\phi_{a,l}$ .

The cruising speed for a vehicle between stop and intersection and between two continuous intersections is a control variable that controls the arriving time of the vehicle at each intersection. Appropriate vehicle arrival times at intersections can promote the use of green light signal extension strategies. This further increases the opportunity for

coordinated interchanges. The speed control strategy is adopted for the cruising speed for vehicles between the stop and intersection or between two adjacent intersections. Recall that we assume that speed control can only reduce the speed. Therefore, the following constraints can be obtained:

$$0 < x(p, m, a, \phi_{a,1}) \leq X(a, \phi_{a,1}), \quad (10)$$

$$0 < x(p, m, \phi_{a,l-1}, \phi_{a,l}) \leq X(\phi_{a,l-1}, \phi_{a,l}), \quad (11)$$

$$0 < x(p, m, \phi_{a,L_a}, a + 1) \leq X(\phi_{a,L_a}, a + 1), \quad (12)$$

where  $X(a, \phi_{a,1})$  and  $X(\phi_{a,L_a}, a + 1)$  are the maximum cruising speeds between a stop and the adjacent intersection and  $X(\phi_{a,l-1}, \phi_{a,l})$  is the maximum cruising speed between two adjacent intersections.

When vehicle  $v_{\alpha(p,m)}$  arrives at the bus stop  $s_a$ , the number of passengers waiting at the stop  $s_a$  to alight at stop  $s_b$  is denoted as  $\theta_{a,b}^{\alpha(p,m)}$  and formulated as

$$\theta_{a,b}^{\alpha(p,m)} = \int_{DS(a,p,m-1)}^{AS(a,p,m)} \lambda_{a,b}^p(t) dt, \quad (13)$$

where  $\lambda_{a,b}^p(t)$  is the arrival rate function of passengers traveling from stop  $s_a$  to stop  $s_b$  by taking the bus belonging to the overlapping bus line  $p$ .

Let  $\tau_a$  represent the average alighting time per passenger and  $\tau_b$  represent the average boarding time per passenger. The dwelling time of the vehicle  $v_{\alpha(p,m)}$  at bus stop  $s_a$  depends on the number of passengers boarding and alighting. The dwelling time  $dwell(a, p, m)$  for vehicle  $v_{\alpha(p,m)}$  at stop  $s_a$  can be expressed as

$$dwell(a, p, m) = \max \left( \sum_{s_c \in S(p-, a)} \theta_{c,a}^{\alpha(p,m)} \tau_a, \sum_{s_b \in S(p+, a)} \theta_{a,b}^{\alpha(p,m)} \tau_b \right), \quad (14)$$

where the set  $S(p+, a)$  denotes the set of all stops upstream (without  $s_a$ ) of station  $s_a$  on bus line  $p$  and the set  $S(p-, a)$  denotes the set of all stops downstream of the stop  $s_a$  (without  $s_a$ ) on bus line  $p$ .

For any intersection  $\phi_{a,l}$  in an overlapping bus line segment, when vehicle  $v_{\alpha(p,m)}$  arrives at intersection  $\phi_{a,l}$ , the phase state of the traffic signal is expressed as a percentage of the total traffic light cycle and denoted as

$$\mu(\phi_{a,l}, p, m) = \frac{AI(\phi_{a,l}, p, m) + TS_0(\phi_{a,l})}{C_\phi} - \left[ \frac{AI(\phi_{a,l}, p, m) + TS_0(\phi_{a,l})}{C_\phi} \right], \quad (15)$$

where  $TS_0(\phi_{a,l})$  is the time that had elapsed in the traffic signal cycle at the intersection  $\phi_{a,l}$  when the optimization started and  $[AI(\phi_{a,l}, p, m) + TS_0(\phi_{a,l})/C_\phi]$  is the integer part of  $AI(\phi_{a,l}, p, m) + TS_0(\phi_{a,l})/C_\phi$ .

We defined that a traffic light cycle starts with the red phase and ends with the green phase for overlapping bus line segments for all intersections. The sum of red light and

amber light as a proportion of the entire traffic light cycle at the intersection  $\phi_{a,l}$  is  $\omega_{a,l}$ .

If  $\mu(\phi_{a,l}, p, m)$  is no larger than the maximum percentage of the total traffic light cycle for green extension allowed at intersection  $\phi_{a,l}$  (i.e.,  $\mu(\phi_{a,l}, p, m) \leq \epsilon_{a,l}$ ), then we can adopt the green extension strategy when vehicle  $v_{\alpha(p,m)}$  approaches intersection  $\phi_{a,l}$  to increase the average speed of vehicle  $v_{\alpha(p,m)}$  and enable synchronization interchanges with the vehicles ahead.

If  $0 \leq \mu(\phi_{a,l}, p, m) < \omega_{a,l}$  and there is no green extension strategy, when vehicle  $v_{\alpha(p,m)}$  approaches intersection  $\phi_{a,l}$ , the traffic signal at the intersection  $\phi_{a,l}$  is in the red phase, and the vehicle must stop before the intersection and wait for a time  $e(\phi_{a,l}, p, m)$  defined as

$$e(\phi_{a,l}, p, m) = C_\phi \times (\omega_{a,l} - \mu(\phi_{a,l}, p, m)). \quad (16)$$

If  $0 \leq \mu(\phi_{a,l}, p, m) < \omega_{a,l}$  and the green extension strategy is adopted, when vehicle  $v_{\alpha(p,m)}$  approaches intersection  $\phi_{a,l}$ , the traffic signal at intersection  $\phi_{a,l}$  is extended in the green phase, and the vehicle does not need to stop and can continue straight through the intersection.

$$e(\phi_{a,l}, p, m) = 0. \quad (17)$$

If  $\omega_{a,l} \leq \mu(\phi_{a,l}, p, m) < 1$ , when vehicle  $v_{\alpha(p,m)}$  approaches intersection  $\phi_{a,l}$ , the traffic signal at intersection  $\phi_{a,l}$  is in the green phase, and the vehicle does not need to stop and can continue straight through the intersection. In this case, (17) is activated.

The green extension parameter is defined to determine whether the green extension strategy must be adopted when vehicle  $v_{\alpha(p,m)}$  approaches intersection  $\phi_{a,l}$  and is denoted as  $y(\phi_{a,l}, p, m)$ . This parameter is a binary variable. If the green extension strategy is employed, then  $y(\phi_{a,l}, p, m) = 1$ . Otherwise,  $y(\phi_{a,l}, p, m) = 0$ .

$$e(\phi_{a,l}, p, m) = (1 - \hat{\mu}(\phi_{a,l}, p, m)) \times (1 - y(\phi_{a,l}, p, m)) \times C_{\phi_{a,l}} \times (\omega_{a,l} - \mu(\phi_{a,l}, p, m)). \quad (18)$$

Here,  $\hat{\mu}(\phi_{a,l}, p, m)$  is a term defined to transform the fractional  $\mu(\phi_{a,l}, p, m)$  into a binary (0, 1) variable, which is expressed as follows:

$$\hat{\mu}(\phi_{a,l}, p, m) = \begin{cases} 1, & \text{if } \omega_{a,l} \leq \mu(\phi_{a,l}, p, m) < 1, \\ 0, & \text{if } 0 \leq \mu(\phi_{a,l}, p, m) < \omega_{a,l}. \end{cases} \quad (19)$$

The passengers riding in vehicle  $v_{\alpha(p,m)}$  transferring to bus line  $q$  should transfer at a stop in an overlapping segment to minimize the arrival time difference between vehicle  $v_{\alpha(p,m)}$  and the vehicle belonging to the bus line  $q$ . We denote a stop as  $s_{a^*}$ , and the following equation is used to identify  $s_{a^*}$ :

$$(a^*, n^*) = \operatorname{argmin} (AS(a, p, m) - AS(a, q, n)), \quad (20)$$

$$AS(a, p, m) \geq AS(a, q, n). \quad (21)$$

In (20), we use the function  $\operatorname{argmin}$  to obtain the argument of the minimum function. The value of  $n^*$  obtained

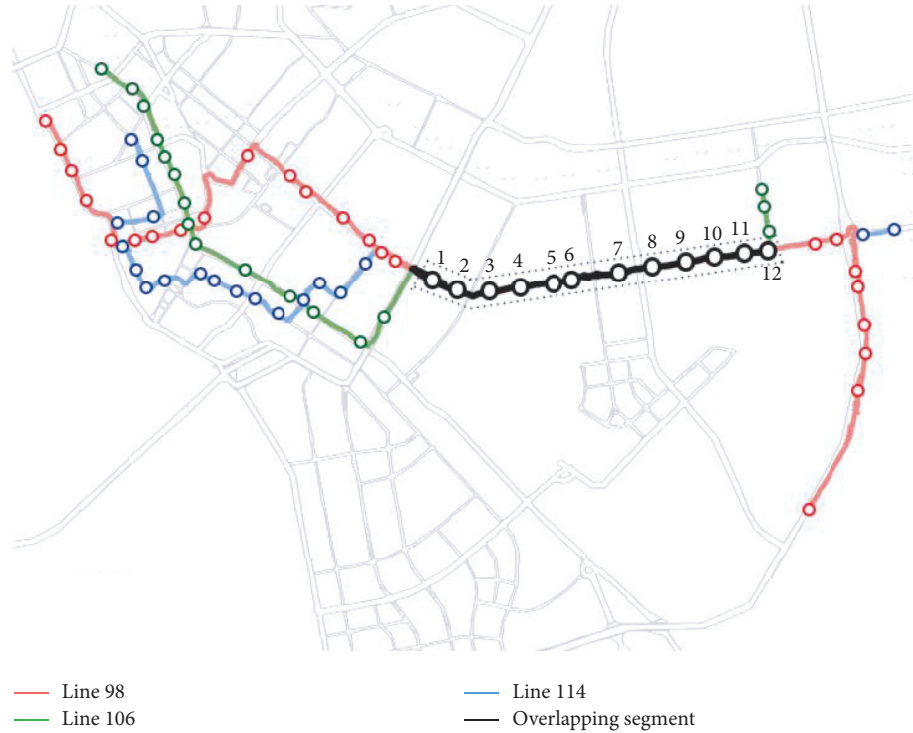


FIGURE 4: Overlapping segments in a bus line network in Harbin, China.

in (20) must obey the time order of vehicle arrival, which is expressed in (21)

Let  $TF(p, m, q, n)$  denote the set of effective transfer opportunities for vehicles  $v_{\alpha(p,m)}$  and  $v_{\alpha(q,n)}$ . If the waiting time for passengers riding in vehicle  $v_{\alpha(p,m)}$  and transferring to vehicle  $v_{\alpha(q,n)}$  is less than  $W_e$ , then  $TF(p, m, q, n)$  is equal to one. Otherwise,  $TF(p, m, q, n)$  is equal to zero.  $W_e$  is the acceptable transfer waiting time threshold.

$$TF(p, m, q, n) = \begin{cases} 1 & \text{if } |AS(a^*, p, m) - AS(a^*, q, n)| \leq W_e, \\ 0 & \text{otherwise.} \end{cases} \quad (22)$$

The target optimization problem aims to maximize the number of effective transfer operations.

$$Z = \max \sum_{\substack{p, q \in I \\ m \in J_p, n \in J_q}} TF(p, m, q, n). \quad (23)$$

Here,  $I$  is the set of overlapping bus lines,  $J_p$  is the set of vehicles belonging to line  $p$ , and  $J_q$  is the set of vehicles belonging to line  $q$ .

## 5. Solution Method

The synchronization model proposed in this study is a mixed 0-1 integer programming model constraint to equations (3)–(23). The objective function is defined in (23), which is neither convex nor concave. The decision variables are the speed control variable (i.e.,  $x(p, m, a, \phi_{a,1})$ ;  $x(p, m, \phi_{a,l-1}, \phi_{a,l})$ ;  $x(p, m, \phi_{a,L_a}, a + 1)$ ), defined in equations (10)–(12),

and green extension variable  $\gamma(\phi_{a,l}, p, m)$ , defined as binary variables. The addressed problem is clearly an NP-hard problem, which has an exponential computational cost, and it cannot be solved to global optimality in near real time for bus lines with regular sizes. To rectify this, a genetic algorithm (GA) was adopted to solve the optimization model proposed in this article. GA is a genetics-based approach to solving optimization problems and has been applied to a diverse range of problems. Its basic unit of operation is the chromosome. Optimization is achieved by selection, crossover, and mutation to produce offspring to accomplish genetic operations. The details of these processes in GA can refer to particular articles provided in these references [28, 29]. In this study, equation (23) is used as a fitness function in GA. The variables corresponding to control strategies form a chromosome. A chromosome is a set of feasible control strategies. Different values of these variables produce different chromosomes, which then constitute an initial group, that is, a set of model solutions. The chromosomes of a group go through crossover, mutation, and selection, eventually producing a chromosome with the highest fitness. The set of control strategies associated with the highest-fitness chromosome is the optimal model solution.

## 6. Case Study

In this section, three overlapping lines in Harbin, China, were selected to illustrate the validity of the proposed traffic control strategies. Equations (1) to (23) were implemented in the Python programming language on a laptop computer with a 2.60 GHz CPU and 16 GB of RAM. The overlapping lines are presented in Figure 4. There are three overlapping



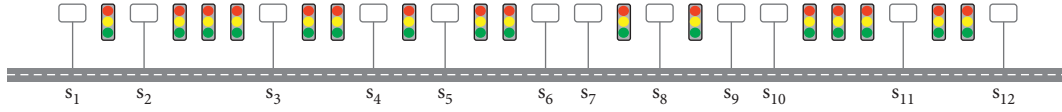


FIGURE 5: Layout of stops and intersections in the target overlapping segment.

TABLE 1: Timetable for vehicles entering the overlapping segment between 8:00 and 9:00 AM.

Arrive time for $i_1$ ( $H_1 = 3$ min)	Arrive time for $i_2$ ( $H_2 = 4$ min)	Arrive time for $i_3$ ( $H_3 = 5$ min)
08:00:00	08:01:17	08:02:34
08:03:51	08:05:08	08:07:42
08:06:25	08:10:16	08:12:50
08:08:59	08:14:07	08:17:58
08:11:33	08:16:41	08:23:06
08:15:24	08:20:32	08:26:57
08:19:15	08:25:40	08:32:05
08:21:49	08:29:31	08:38:30
08:24:23	08:33:22	08:43:38
08:28:14	08:37:13	08:47:29
08:30:48	08:41:04	08:53:54
08:34:39	08:44:55	08:59:02
08:35:56	08:50:03	
08:39:47	08:52:37	
08:42:21	08:56:28	
08:46:12		
08:48:46		
08:51:20		
08:55:11		
08:57:45		

bus lines (lines 98, 106, and 114) with 12 common stops. For simplicity, line 98 is labeled as line  $i_1$  and is operated with a headway  $H_1 = 3$  min, line 106 is labeled as  $i_2$  and is operated with a headway  $H_2 = 3$  min, and line 114 is labeled as  $i_3$  and is operated with a headway  $H_3 = 5$  min. The stops on the overlapping segment are numbered from  $s_1$  to  $s_{12}$  from left to right in Figure 4.

The maximum cruising speed between two consecutive intersections and between intersections and stops is 40 km/h. The average alighting time per passenger is  $\tau_a = 2$  s, and the average boarding time per passenger is  $\tau_b = 2$  s. The distributions of intersections and bus stops on the overlapping segment are presented in Figure 5. There is one traffic signal intersection between stops  $s_1$  and  $s_2$ , and three traffic signal intersections between stops  $s_2$  and  $s_3$ , as shown in Figure 5. We assume that all of the intersections with a traffic signal have  $C_\phi = 120$  s, and the ratio of the red phases plus the amber phases that accounts for the entire traffic light cycle at all of the intersections is 0.5. The maximum portion of the total traffic light cycle for green extension allowed at the intersections is 0.15. The acceptable transfer waiting time threshold is equal to one-third of the interval of the vehicles arriving at the first stop in the overlapping segment; that is,  $W_e = 77/3 \approx 25.7$ .

Our model was implemented in the Python programming language. The max generation in GA was set to 200. The crossover probability was set to 0.9, and the mutation probability was set to 0.05. To explore how the transfer passenger waiting time at the transfer stop on the

overlapping segment was influenced by the synchronized traffic light green extension and speed control strategies, 47 vehicles entering the overlapping segment between 8:00 and 9:00 AM were optimized using these control strategies. The timetable for all 47 vehicles arriving at the overlapping segment is presented in Table 1.

In the timetable presented in Table 1, according to the headways for all three overlapping bus lines, there are  $60\text{min}/H_1 = 20$  vehicles belonging to bus line  $i_1$  entering the overlapping segment in one hour,  $60\text{min}/H_2 = 15$  vehicles belonging to bus line  $i_2$  entering the overlapping segment in one hour, and  $60\text{min}/H_3 = 12$  vehicles belonging to bus line  $i_3$  entering the overlapping segment in one hour. In this timetable, there should be 47 vehicles entering the overlapping segment within one hour. So, the ideal headway for these vehicles should be  $H = 3600\text{s}/47 \approx 77\text{s}$ , if they arrive at the first stop in the overlapping segment with the same time interval. However, in this situation, the headway among vehicles operating the same line will be extremely irregular. In order to balance the arrival time interval for vehicles belonging to different bus lines at the first stop in the overlapping segment and the headway of the same line, the arrival time at the first stop in the overlap area was optimized by using the optimization method. The details of the optimization method are described in literature [30].

Figure 6 presents the traveling trajectories of 47 vehicles entering the overlapping segment from 8:00 to 9:00 AM before and after synchronization optimization. As shown in Figure 6(a), before synchronization optimization, two or

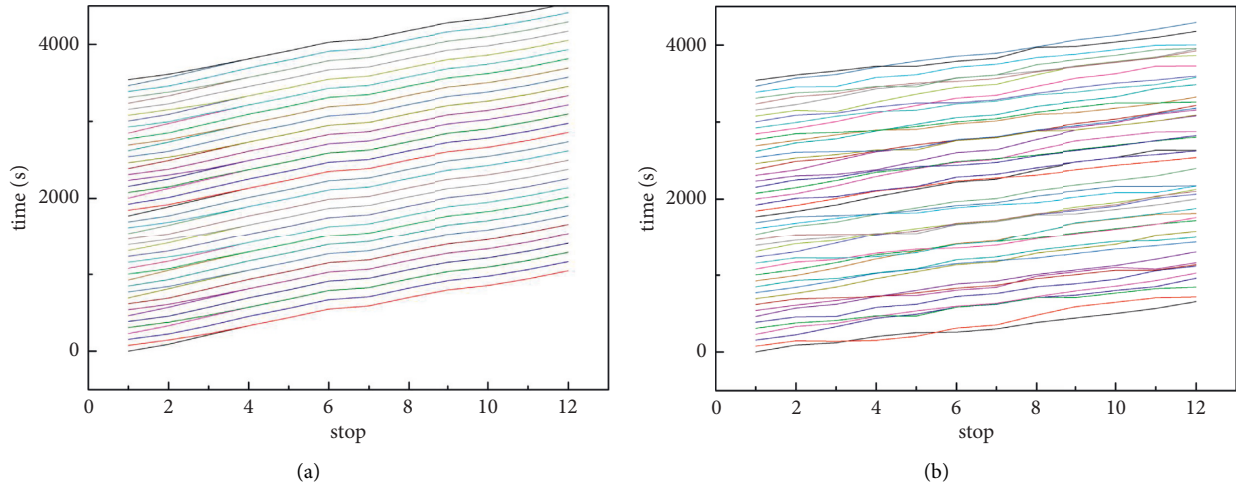


FIGURE 6: Vehicle trajectories in the overlapping segment (before and after optimization). (a) Vehicle trajectories before optimization. (b) Vehicle trajectories after optimization.

three vehicles entering the overlapping segment sequentially operate in pairs with each other, and their traveling trajectories overlap after several stops until leaving the overlapping segment. This is caused by two or three vehicles encountering the red phase of a traffic light when they arrive at an intersection and waiting for some time. These vehicles leave the intersection simultaneously when the traffic light shifts to the green phase. This scenario is very beneficial for passengers who transfer between the vehicles in small groups. However, the waiting times for passengers who wish to transfer between vehicles in different groups are significant. Figure 6(b) presents the traveling trajectories of 47 vehicles after synchronization optimization. After adopting the speed control and green extension strategies, numerous trajectories of the vehicles operating in the overlapping segment cross or almost overlap at one of the stops in the overlapping segment, providing significant convenience for transfer passengers moving between different vehicles.

Figure 7 presents the effective transfer opportunities between vehicles operating in the overlapping segment. The transfer between two vehicles in this study was considered to be directional. The red circles and blue multiplication symbols correspond to the occurrences of effective transfers. There are 45 effective transfer opportunities among the 47 vehicles entering the overlapping segment within one hour.

The horizontal coordinates correspond to the stop numbers at which effective transfers occur. The vertical coordinates represent the vehicle number in which effective transfers occur. For an effective transfer opportunity at stop  $s$  in the horizontal coordinates and vehicle number  $k$  in the vertical coordinates in Figure 7, a red circle indicates that passengers in vehicle  $k$  can transfer to vehicle  $k + 1$  within the desired time, while the blue multiplication symbols indicate that passengers in vehicle  $k + 1$  can transfer to vehicle  $k$  within the desired time. Of these, the majority of effective transfer opportunities occurred at stop 4, with 20 times. Consistent with the vehicle trajectories after synchronization optimization in Figure 6(b), a large number of trajectories begin to cross with each other and operate in

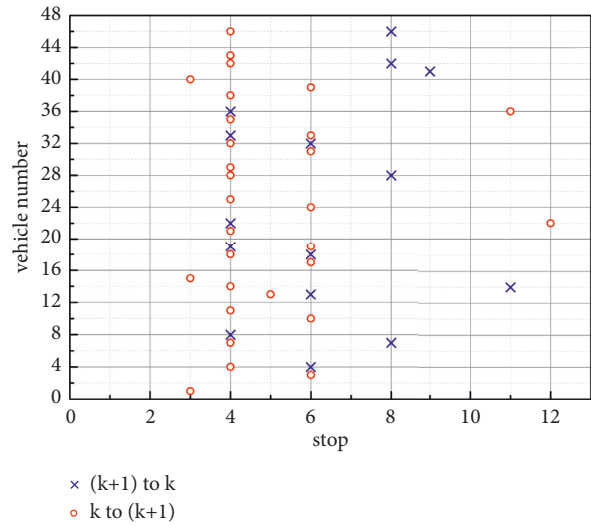


FIGURE 7: Synchronization distribution map.

pairs around stop 4. These observations revealed that the cumulative effect of the traffic light becomes effective at stop 4 (after six intersections) in this case. In practice, the city traffic designer should consider expanding the topology of stop 4 to allow more vehicles to interchange at this stop.

### 7. Conclusions and Future Directions

In a bus overlapping segment, the synchronization scheduling of vehicles is required to balance bus bunching and passenger transfer between different bus lines. This study investigated the dynamic synchronized scheduling of vehicles running on overlapping segments using a speed control strategy and green light extension strategy. This enabled us to mitigate bus bunching as much as possible while increasing the effective transfer opportunities for transfer passengers.



For overlapping bus lines, if the lines depart too frequently (the headway is too small), it is easy to form pairs of adjacent vehicles to run with each other. This is caused by the effects of intersection traffic lights. In the case where an intersection light is in the red phase, the vehicles arriving in succession have to wait together at the intersection and leave together only after the light turns green.

This makes it easy for adjacent vehicles to form pairs to run with each other, arrive at stops and intersections together, and leave together. This is detrimental to the operation of bus systems. A segment of overlapping bus lines in Harbin, China, confirmed this. This is also consistent with the phenomenon of several buses waiting in line at intersections that we often see in our daily lives.

In this study, we developed a synchronization scheduling model. Considering the convenience of passenger transfer between different lines in an overlapping segment, a speed control strategy and green light extension strategy were used to achieve synchronization. The effectiveness of the proposed model was verified by modeling three overlapping bus lines in Harbin, China, as an example. The results demonstrated that, by implementing the speed control strategy and green light extension strategy, the number of effective synchronization transfers between 12 stops in the overlapping segment was increased significantly (more than double).

The control strategies proposed in this paper aim to improve the convenience of passenger transfers between overlapping segments while avoiding bus bunching on different bus lines. These control strategies can increase passenger transfer opportunities and provide more convenient door-to-door bus service, but this comes at the cost of extending the travel time for some passengers. Thus, this control strategy is not recommended for situations where interchange demand is low. The benefits of this control strategy can only be maximized if the majority of passengers' trip origins and destinations are not distributed in overlapping intervals. Thus, this in turn leads to a future research program on these studies, which is to quantitatively study the effect of passenger interchange demand on interchange strategies. In addition, with the development of intelligent technologies in transit systems, future research can be conducted to enhance the effectiveness of transit operations through the simultaneous implementation of multiple transit control strategies.

### Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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