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Research Article

A Feedback Control Method with Connected Vehicles in a Lattice Hydrodynamic Model at Highway On-Ramps

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This paper presents a traffic flow control scheme of connected vehicles to stabilize a traffic system with on-ramps from a macromodel point of view. Firstly, this paper establishes a lattice hydrodynamic model with on-ramps, and an output feedback controller is designed based on the characteristics of connected vehicles. Then, by using the Lyapunov–Krasovskill functional analysis method, this paper studies the delay-dependent convergence conditions of the control method. String stability is defined as the nonamplification of the downstream disturbance of a traffic flow when it propagates upstream. The influence of the on-ramps is regarded as a stochastic disturbance in this paper. Through a theoretical analysis, the control conditions that can ensure the string stability of the traffic system under the influence of on-ramp are obtained. Finally, numerical experiments are carried out to compare the traffic states of the traffic system with and without control. The results show that the proposed control can effectively suppress the instability of the traffic system.

1. Introduction

With the rapid economic development and accelerating urbanization, the number of private cars is increasing. However, the speed of road development is far behind the increase of motor vehicles, resulting in frequent traffic problems. Therefore, these traffic issues have attracted many scholars' attention [1–3]. So far, scholars have proposed and studied traffic models from many aspects [4–51].

On the research of the traffic flow theory, scholars have used the macro hydrodynamic model to study and used the related partial differential equations to describe the relationship between traffic parameters [4–8]. In 1998, Nagatani [9] reasonably simplified the traditional hydrodynamic model and put forward the lattice hydrodynamic model for the first time. The lattice hydrodynamic model is composed of partial differential equations whose space is discrete and time is continuous, and its form is simple, so it is widely used to study traffic problems. Recently, the lattice hydrodynamic model has been used to study various traffic phenomena, and in this process, this model has also been developed [10–19]. On the problem of restraining traffic congestion, scholars have given the corresponding control scheme based on the lattice hydrodynamic model [10–14]. For example, Ge et al. [10] presented to use the feedback control of the flow difference between the front and back positions, and Redhu and Gupta [11] proposed to use the flow difference between the current state and the delayed state of the front position as the feedback control item and so on. These control schemes have a great control effect. Meanwhile, due to the complexity of the actual traffic, it is often affected by many factors, so researchers further develop the lattice hydrodynamic model [15–26]. In order to more accurately describe the traffic state, many practical factors, such as time-varying delay in sensing traffic flux [15], predictive effect [16], and memory effect [17], have been taken into account.

As we all know, due to the complexity of road conditions, the actual traffic environment is complex. In order to study the evolution of traffic state under on-ramp conditions, based on the lattice hydrodynamic model, Sun et al. [22] considered the influence of on-ramp and analyzed its stability. Different from Sun, in Tian's [23] model, the vehicle randomly enters the main road from the on-ramp, which is more in line with the actual traffic. Nagatani [24, 25] presented the two-dimensional traffic flow model in 1999. Based on the two-dimensional lattice model, Wang et al. [26] studied the impact of on-ramp on traffic and reproduced a variety of traffic congestion patterns. At the microlevel, there are many research studies on-ramp bottleneck. Based on the NaSch model, Diedrich et al. [28] studied the impact of on-ramp and off-ramp on the transportation system. Jiang et al. [29] investigated the influence of random slowing down, deterministic conditions, and other factors in the on-ramp system through the cellular automata model. After that, Jia et al. [30] studied the influence of the traffic rules of the ramp part on the on-ramp system. In addition, the impact of traffic bottlenecks caused by other traffic scenarios on the traffic system has also attracted scholars' attention [20, 33–38]. Zhou et al. [20, 34] proposed a macro model of the curve traffic system, and the factors that affect the maximal theoretical flux and velocity of traffic flow were studied. Based on the lattice hydrodynamic model, Zhang et al. [21] explored the evolution of traffic flow in the case of a partially reduced lane. These studies deepen our understanding of road bottlenecks.

Due to the progress of technology, vehicles can get useful information through communications, which makes many control schemes realized. Based on the microscopic model, many scholars have established the traffic model of connected vehicles and explored the corresponding control scheme to keep the fleet stable [39–42]. However, research studies on the control of traffic systems with on-ramp are rare, especially based on the macro hydrodynamic model. Meanwhile, vehicles entering the main road from the on-ramp will have randomness, which brings difficulties for the research. In this paper, we will establish a model for connected vehicles from a macro perspective and study the impact of on-ramp on the entire traffic flow. At the same time, we propose corresponding control strategies to keep the traffic system in a stable state and provide guidance for actual traffic.

Based on the lattice fluid dynamics model, this paper considers the characteristics of on-ramp and connected vehicles and proposes the corresponding macro model and control scheme. In the third part, the convergence analysis of the on-ramp traffic system is performed, and the control gain is determined. The fourth part of the paper illustrates the effectiveness of the control scheme through numerical experiments, and finally, we summarize the paper.

2. Modeling and Controller Design

In 1998, Nagatani [9] put forward the lattice hydrodynamic model for the first time, which is the simplified version of the macroscopic hydrodynamic model. As we all know, the lattice hydrodynamic model divides the road into many lattices, and the model is composed of a conservation equation and continuum equation. The two equations are expressed as follows:

$$\begin{aligned} \partial t\ \rho_j - \rho_0\left(q_{j-1} - q_j\right) &= 0, \\ \partial_t q_j &= a\rho_0 V\left(\rho_{j+1}\right) - a q_j. \end{aligned} \tag{1}$$

$V(\rho)$ is the optimal velocity function, and its form is as follows:

$$V(\rho) = \frac{V_{\max}}{2}\left[\tanh\left(\frac{1}{\rho} - \frac{1}{\rho_c}\right) + \tanh\left(\frac{1}{\rho_c}\right)\right], \tag{2}$$

where and ρ_j are q_j are the density and traffic flow of the lattice j, ρ_0 and ρ_c are the average density and temporary density, and a is the sensitivity coefficient.

Based on the lattice model, some researchers [23, 26] study the traffic characteristics of the on-ramp system and reproduce various traffic phenomena in the on-ramp system. Based on the macro model, Sun et al. [22] proposed corresponding control strategies for the traffic congestion caused by on-ramps but did not consider the influence of randomness of entering the trunk of the on-ramp for vehicles. Tian et al.'s [23] on-ramp model considers the randomness of on-ramp merging into the main road, but he does not propose corresponding control strategies to curb traffic congestion. Compared to the previous work, in this paper, we will consider the randomness of entering the main road to make the model more realistic, and according to the network environment, the corresponding control strategies are proposed to suppress the traffic congestion caused by the on-ramp. In the networked environment, drivers can obtain road information through road facilities and communication equipment and then achieve the purpose of regulating and controlling the traffic system. However, how to use the acquired information to control the traffic system is the key. In this regard, we use the flow difference between the front and rear positions and the difference between each position and the expected flow as the feedback control term. When the traffic system is stable, the flow will remain very constant. Thus, based on the characteristics of the on-ramp and connected vehicles, the lattice hydrodynamic model with on-ramp can be established, and the corresponding control scheme can be proposed:

$$\partial t\ \rho_j - \rho_0\left(q_{j-1} - q_j\right) = \rho_0 Q_j, \tag{3}$$

$$\partial_t q_j = a\rho_0 V\left(\rho_{j+1}\right) - a q_j + u_j, \tag{4}$$

$$\begin{aligned} u_j = {} & k_1\left(q_0 - q_j(t-\tau)\right) \\ & + k_2\left(q_{j+1}(t-\tau) - q_j(t-\tau)\right), \end{aligned} \tag{5}$$

$$Q_j = Q_{\max} \cdot f(t), \tag{6}$$

where $Q_{\max}$ is the maximum flow of the on-ramp into the main road, $f(t)$ represents the change of flow from the on-ramp to the main road with time, and in the sections without the on-ramp, $f(t)$ is 0. Here, k_1 and k_2 represent the feedback control gain. In this model, the delay time of communication is taken into account, and τ represents the control delay caused by communication delay.

This paper sets the expected density and flow as $\rho^* = \rho_0$ and $q^* = q_0 = \rho_0 \cdot V(\rho_0)$, respectively. The deviation between the actual density and flow rate and the expected density and flow rate is set as $\tilde{\rho}_j$ and $\tilde{q}_j$; then, $\tilde{\rho}_j$ and $\tilde{q}_j$ can be expressed as

$$\tilde{\rho}_j = \rho_j - \rho_0, \tag{7}$$

$$\tilde{q}_j = q_j - q_0. \tag{8}$$

Combining with (3)–(8) formulas, we can get the following nonlinear differential equations:

$$\partial t\, \tilde{\rho}_j = \rho_0\left(\tilde{q}_{j-1} - \tilde{q}_j\right) + \rho_0 Q_j, \tag{9}$$

$$\begin{aligned}\partial_t \tilde{q}_j = {} & a\rho_0 V'(\rho_0)\tilde{\rho}_{j+1} - a\tilde{q}_j + k_1\left(-\tilde{q}_j(t-\tau)\right) \\ & + k_2\left(\tilde{q}_{j+1}(t-\tau) - \tilde{q}_j(t-\tau)\right).\end{aligned} \tag{10}$$

Define $x(t) = \mathrm{col}[x_j]_{j=1}^n$, $u(t-\tau) = \mathrm{col}[u_j(t-\tau)]_{j=1}^n$, and $y(t) = \mathrm{col}[y_j]_{j=1}^n$ as state vector, control vector, and output vector, respectively. The j-th state variable is defined as $x_j = \mathrm{col}[\tilde{\rho}_j \tilde{q}_j]$ and the j-th output variable as $y_j = \mathrm{col}[-\tilde{q}_j \tilde{q}_{j+1} - \tilde{q}_j]$. On the basis of the equations.(8) and (9), The state-space equation of the whole traffic system can be expressed as

$$\dot{x} = Ax + Bu(t-\tau) + \rho_0 Q, \tag{11}$$

where

$$\begin{gathered}
B = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 1 \end{bmatrix}_{2n\times n}, \\
Q = \begin{bmatrix} Q_1 \\ 0 \\ Q_2 \\ 0 \\ \vdots \\ Q_n \\ 0 \end{bmatrix}_{2n\times 1}, \\
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & -a & a\rho_0 V' & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & \rho_0 & 0 & -\rho_0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -a & a\rho_0 V' & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \rho_0 & 0 & -\rho_0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -a & a\rho_0 V' & \cdots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \rho_0 & 0 & -\rho_0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -a & a\rho_0 V' & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \rho_0 & 0 & -\rho_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & a\rho_0 V' & -a
\end{bmatrix}_{2n\times 2n}.
\end{gathered} \tag{12}$$

According to the form vector and output vector, we can get the following formula:

$$y = Cx, \tag{13}$$

where

$$C = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \end{bmatrix}_{2n\times 2n}. \quad (14)$$

For the above traffic system, the feedback controller u_j is expressed as follows:

$$u_j(t-\tau) = K_j y_j(t-\tau), \quad (15)$$

where $K_i = [\,k_1 \;\; k_2\,]$.

Accordingly, the control vector can be written as

$$u(t-\tau) = BKCx(t-\tau), \quad (16)$$

where

$$K = \begin{bmatrix} K_1 & 0 & \cdots & 0 \\ 0 & K_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & K_n \end{bmatrix}. \quad (17)$$

Finally, we will get the model of the whole traffic system as follows:

$$\dot{x} = Ax + BKCx(t-\tau) + \rho_0 Q. \quad (18)$$

3. Convergence Analysis

If a traffic system is in a stable state, the density and flow of the traffic system will remain expected values. We regard the impact of the on-ramp in the transportation system as an external stochastic disturbance. In this section, we first study the conditions for ensuring the stability of the platoon without continuous disturbance, which is the condition to ensure that the error of traffic density and flow approaches zero. When the traffic system is disturbed, if the instantaneous error gradually expands upstream along the platoon, it will often lead to the collapse of the traffic system, so in order to prevent this situation, it is also necessary to analyze the string stability of the traffic system. In this section, in order to study the stability of the traffic system, the following lemmas will be very useful:

Lemma 1 (see [43–45], Newton–Leibniz formula). *The definite integral of a continuous function f(t) in the interval [t-τ, t] is equal to the increment of its original function F(t) in the interval [t-τ, t]:*

$$F(t-\tau) = F(t) - \int_{t-\tau}^{t} f(t)\mathrm{d}t. \quad (19)$$

Lemma 2 (see [46], fundamental inequality). $\forall a, b \in R^n, M = M^T > 0$, *the following inequality holds:*

$$-2a^T b \le a^T M a + b^T M b. \quad (20)$$

Theorem 1. *For the traffic system described by equation (15), if the influence of the on-ramp is not considered, the equation describing the traffic system is as follows:*

$$\dot{x} = Ax + BKCx(t-\tau). \quad (21)$$

For the abovementioned system, if there are positive definite matrices P, R_1, and R_2, the following inequality holds

$$\begin{aligned} &\tau\Big[R_1 + R_2 + \mathrm{PBKCAR}_1^{-1} A^T \big(\mathrm{BKC}^T\big) P \\ &\quad + \mathrm{PBKCBKCR}_2^{-1}\big(\mathrm{BKC}^T\big)^T\big(\mathrm{BKC}^T\big)^T P\Big] \\ &\quad + 2P(A + \mathrm{BKC}) < 0. \end{aligned} \quad (22)$$

Then, the traffic system will remain stable.

Proof. According to Lemma 1, we can change the equation (18) into the following form [43–45]:

$$\begin{aligned} \dot{x} = (A + BKC)x - BKC \int_{t-\tau}^{t} \\ (Ax(s) + BKCx(s-\tau))\mathrm{d}s. \end{aligned} \quad (23)$$

The selected Lyapunov–Krasovskill function [47] is as follows:

$$V = V_1 + V_2 + V_3, \quad (24)$$

where V_1, V_2, and V_3 are, respectively,

$$\begin{aligned} V_1 &= x^T P x, \\ V_2 &= \int_{-\tau}^{0}\int_{t+\theta}^{t} x^T(s) R_1 x(s)\mathrm{d}s\mathrm{d}\theta, \\ V_3 &= \int_{-2\tau}^{-\tau}\int_{t+\theta}^{t} x^T(s) R_2 x(s)\mathrm{d}s\mathrm{d}\theta. \end{aligned} \quad (25)$$

If we take the derivative of the function V, then, we need to take the derivative of the functions V_1, V_2, and V_3 first:

$$\dot{V}_1 = x^T\left[P(A+BKC)+(A+BKC)^T P\right]x - 2x^T PBKC\int_{t-\tau}^{t} Ax(s)\mathrm{d}s - 2x^T PBKC \int_{t-\tau}^{t} BKCx(s-\tau)\mathrm{d}s \dot{V}_2 = \tau x^T R_1 x - \int_{t-\tau}^{t} x^T(s)R_1 x(s)\mathrm{d}s \dot{V}_3 = \tau x^T R_2 x - \int_{t-2\tau}^{t-\tau} x^T(s)R_2 x(s)\mathrm{d}s. \tag{26}$$

Then, the following equation can be obtained:

$$\begin{aligned}\dot{V} = \dot{V}_1+\dot{V}_2+\dot{V}_3 = \phi+\eta_1+\eta_2 - \int_{t-\tau}^{t} x^T(s)R_1x(s)\mathrm{d}s\\ - \int_{t-2\tau}^{t-\tau} x^T(s)R_2x(s)\mathrm{d}s,\end{aligned} \tag{27}$$

where,

$$\begin{aligned}\phi &= x^T\left[P(A+BKC)+(A+BKC)^TP+\tau(R_1+R_2)\right]x,\\ \eta_1 &= -2\int_{t-\tau}^{t} x^T PBKCAx(s)\mathrm{d}s,\\ \eta_2 &= -2\int_{t-2\tau}^{t-\tau} x^T PBKCBKCx(s)\mathrm{d}s.\end{aligned} \tag{28}$$

Using Lemma 2, we can derive the following inequality:

$$\begin{aligned}\eta_1 &\le \tau x^T PBKCAR_1^{-1}A^T(BKC)^TPx + \int_{t-\tau}^{t} x^T(s)R_1x(s)\mathrm{d}s,\\ \eta_2 &\le \tau x^T PBKCBKCR_2^{-1}(BKC)^T(BKC)^TPx\\ &\quad + \int_{t-2\tau}^{t-\tau} x^T(s)R_2x(s)\mathrm{d}s.\end{aligned} \tag{29}$$

According to the Lyapunov stability theory, the following conditions must be satisfied for the system to converge:

$$\begin{aligned}\dot{V} \le x^T\big[&P(A+BKC)+(A+BKC)^TP+\tau(R_1+R_2)\\ &+\tau PBKCAR_1^{-1}A^T(BKC)^TP\\ &+\tau PBKCBKCR_2^{-1}(BKC)^T(BKC)^TP\big]x<0.\end{aligned} \tag{30}$$

According to the definition of the negative definite matrix, if equation (30) holds, then, the following must be satisfied:

$$\begin{aligned}&P(A+\mathrm{BKC})+(A+\mathrm{BKC})^TP+\tau(R_1+R_2)\\ &\quad+\tau PBKC\mathrm{A}R_1^{-1}A^T(BKC)^TP+\tau PBKCBKCR_2^{-1}\\ &\quad(BKC)^T(BKC)^TP<0.\end{aligned} \tag{31}$$

Consequently, we can derive that [41]

$$\tau < \frac{\left\|P(A+BKC)+\left((A+BKC)^TP\right\|\right.}{\left\|R_1+R_2+PBKCAR_1^{-1}A^T(BKC)^TP+PBKCBKCR_2^{-1}(BKC)^T(BKC)^TP\right\|}. \tag{32}$$

We regard the influence of the on-ramp as the continuous disturbance of the traffic system. The abovementioned theorem can ensure that the traffic system will eventually converge when disturbed, that is, in the traffic system, the error caused by the disturbance will tend to 0. However, whether the queue stability of the traffic system can be guaranteed, that is, whether the instantaneous error will gradually expand along the queue, is also a problem that must be considered in traffic control. The following theorem will discuss the chord stability of the traffic system. □

Theorem 2. *For the traffic system (15), due to the existence of the on-ramp, the traffic system will be affected by the on-ramp. If the system meets the following conditions, it can ensure that the disturbance caused by the on-ramp will not gradually expand to the upstream along the platoon:*

$$\sqrt{\frac{\left(-a\rho_0^2V'(\rho_0)+k_2\omega\cdot\sin(\omega\tau)\right)^2+\left(k_2\omega\cdot\cos(\omega\tau)\right)^2}{\left(-\omega^2-a\rho_0^2V'(\rho_0)+k_1\omega\cdot\sin(\omega\tau)+k_2\omega\cdot\sin(\omega\tau)\right)^2+\left(a\omega+k_1\omega\cdot\cos(\omega\tau)+k_2\omega\cdot\cos(\omega\tau)\right)^2}}\le 1. \tag{33}$$

Proof. A common method to study the string stability is to study the transfer function between the front and back errors of the traffic system. If the amplitude of the transfer function is less than 1 at all frequencies, the string stability can be guaranteed [48]. By Laplace transform of equation (8) and (9), the following equation can be obtained:

$$sP_j(s) = \rho_0\left(\vartheta_{j-1}(s) - \vartheta_j(s)\right) + \rho_0 Q_{\max} F(s), \tag{34}$$

$$\begin{aligned} s\vartheta_j(s) &= a\rho_0^2 V'(\rho_0) P_{j+1}(s) - a\vartheta_j(s) - k_1\vartheta_j(s)e^{-s\tau} \\ &\quad + k_2\vartheta_{j+1}(s)e^{-s\tau} - k_2\vartheta_j(s)e^{-s\tau}. \end{aligned} \tag{35}$$

Here, $P_{j+1}(s)$, $\vartheta_j(s)$, and $F(s)$ are Laplace transforms of $\widetilde{\rho}_j$, $\widetilde{q}_j$, and f. From the simultaneous equations (34) and (35), we can derive the following formula:

$$\begin{aligned} \vartheta_j(s) &= \frac{-a\rho_0^2 V'(\rho_0) + sk_2 e^{-s\tau}}{s^2 + as - a\rho_0^2 V'(\rho_0) + sk_1 e^{-s\tau} + sk_2 e^{-s\tau}} \vartheta_{j+1}(s) \\ &\quad + \frac{-a\rho_0^2 Q_{\max} F(s)}{s^2 + as - a\rho_0^2 V'(\rho_0) + sk_1 e^{-s\tau} + sk_2 e^{-s\tau}}, \end{aligned} \tag{36}$$

$$d(s) = s^2 + as - a\rho_0^2 V'(\rho_0) + sk_1 e^{-s\tau} + sk_2 e^{-s\tau}.$$

Here, $d(s)$ is called the characteristic polynomial. Through the Routh stability criterion, all coefficients of $d(s)$ are positive, so characteristic polynomial $d(s)$ is stable, which is in line with the theory of reference [42]. Correspondingly, we also obtain the transfer function $G(s)$:

$$G(s) = \frac{-a\rho_0^2 V'(\rho_0) + sk_2 e^{-s\tau}}{s^2 + as - a\rho_0^2 V'(\rho_0) + sk_1 e^{-s\tau} + sk_2 e^{-s\tau}}. \tag{37}$$

Because the vehicles on the on-ramp only enter the main road at the junction, the value of $F(s)$ in other areas is 0, which can be ignored. In this traffic system, if the disturbance is not amplified, the following conditions should be satisfied [42]:

$$\|G(s)\|_\infty = \sup|G(i\omega)| \le 1. \tag{38}$$

That is, the following formula holds for all frequencies ω:

$$\sqrt{\frac{\left(-a\rho_0^2 V'(\rho_0) + k_2\omega\cdot\sin(\omega\tau)\right)^2 + \left(k_2\omega\cdot\cos(\omega\tau)\right)^2}{\left(-\omega^2 - a\rho_0^2 V'(\rho_0) + k_1\omega\cdot\sin(\omega\tau) + k_2\omega\cdot\sin(\omega\tau)\right)^2 + \left(a\omega + k_1\omega\cdot\cos(\omega\tau) + k_2\omega\cdot\cos(\omega\tau)\right)^2}} \le 1. \tag{39}$$

The theorem is proved. □

4. Simulations

In this part, the effectiveness of the control method will be illustrated by numerical experiments. First, we perform numerical experiments on the model without control. In order to compare the simulation results, through the results of convergence analysis, we select the appropriate control parameters and time delay. It is assumed that the road is divided into 150 grids, and the initial density of the main road is set to 0.2. Finally, the parameters of the system are given as follows:

$$\begin{aligned} a &= 1, \\ \rho_0 &= 0.25, \\ \rho_c &= 0.25, \\ Q_{\max} &= 0.1, \\ k_1 &= 0.15, \\ k_2 &= 0.4, \\ \tau &= 0.3. \end{aligned} \tag{40}$$

Also, we randomly set the vehicle on the on-ramp entering into the main road; therefore, the function $f(t)$ is set as a random function.

In order to study the evolution of the traffic system without control, we set the control gain k_1 and k_2 to 0. In the numerical experiment, we set the on-ramp at the 120th lattice. Vehicles enter randomly from the on-ramp to the main road. Due to the impact of the on-ramp, the main road is constantly disturbed. Figure 1 shows a spatiotemporal pattern of changes in the traffic system density. From Figure 1, it can be seen that the disturbance continues to spread from the ramp to the upstream of the road. This causes a large oscillation in the upstream density, thereby forming traffic congestion. Figure 2 exhibits the short-term traffic density evolution of the 110th, 70th, 50th, and 10th lattices. It is found out that, as time goes on, the disturbance caused by the on-ramp continues to propagate upstream of the road. From Figure 3, it is observed that the final state of the traffic system is unstable, and the amplitude and frequency of the traffic density oscillation are very large. The result of Figure 4 clearly shows that the transient disturbance caused by the on-ramp gradually expands to the upstream along the platoon. The amplification of this disturbance will lead to instability in the intervehicle distance, which is easy to

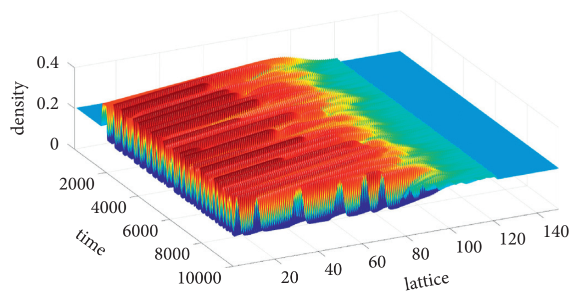

Figure 1: Spatiotemporal evolution pattern.

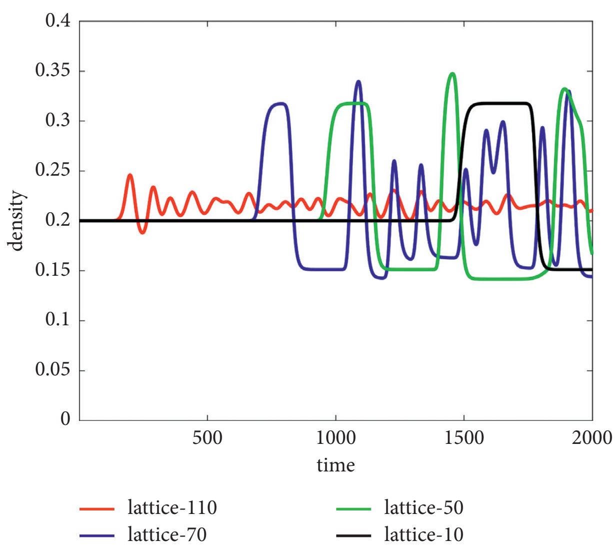

Figure 2: Density evolution diagram.

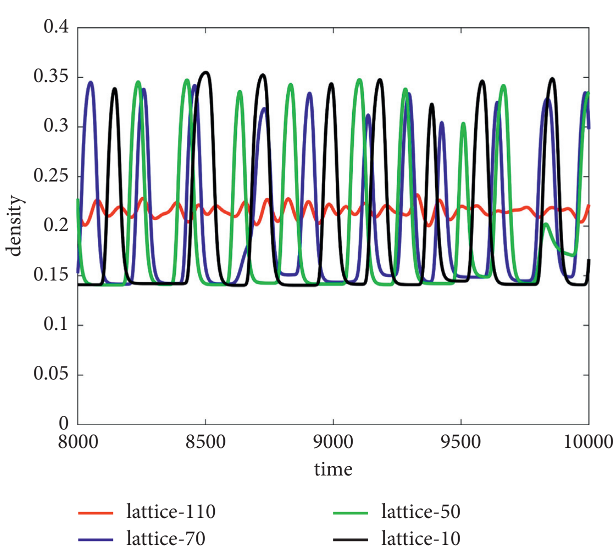

Figure 3: Density evolution diagram.

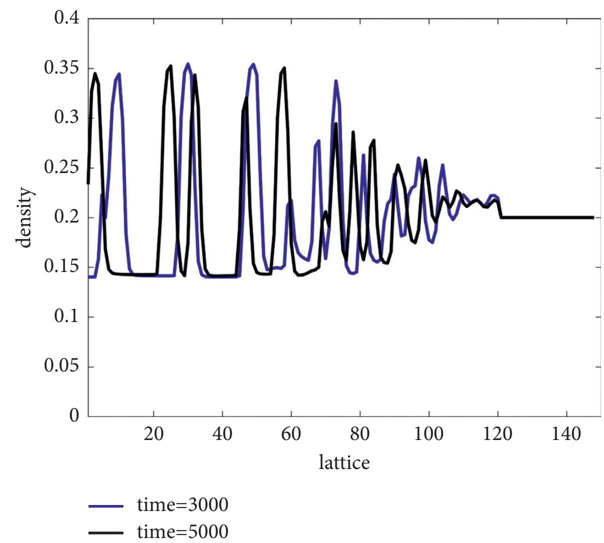

Figure 4: Density profile of all lattices.

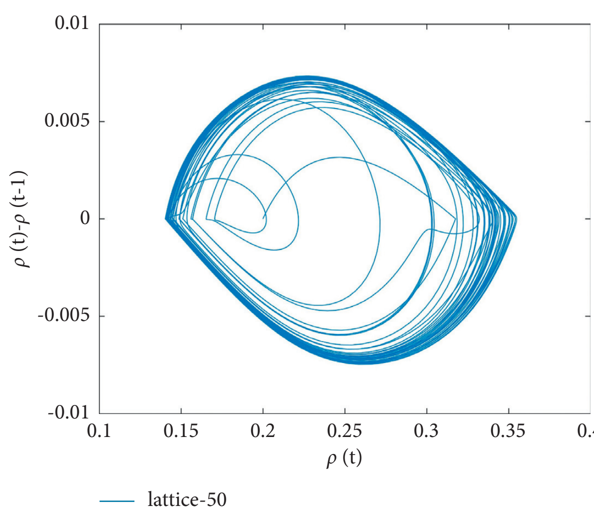

Figure 5: Phase trajectory.

induce accidents and lead to the disintegration of the traffic platoon. Correspondingly, chaotic behaviors also appear in the traffic system. In Figure 5, we select the 50th lattice upstream of the road and draw the phase trajectory of the traffic system at this location. It can be seen from Figure 5 that the topological structure of the phase trajectory is very complex, and it is limited in a limited area, which indicates that the state variables at this position show a chaotic behavior. Not only that, this phenomenon also appears in other upstream places of the road.

When we add control to the transportation system, the evolution of the transportation system will be very different. From Figure 6, it can be seen that although vehicles continue to enter the main road from the on-ramp, there is no violent oscillation in the upstream area of the main road. In order to compare the results of numerical experiments, we also selected the 110th, 70th, 50th, and 10th lattice as the research objects. Based on Figure 7 and Figure 8, it can be found that, as on-ramp vehicles converge into the main road, the traffic density in the upstream will increase, but the traffic density will remain stable. When the evolution time reaches 8000, the traffic density will remain stable. This shows that the state

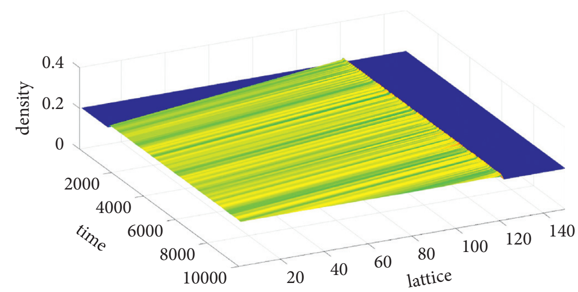

FIGURE 6: Spatiotemporal evolution pattern.

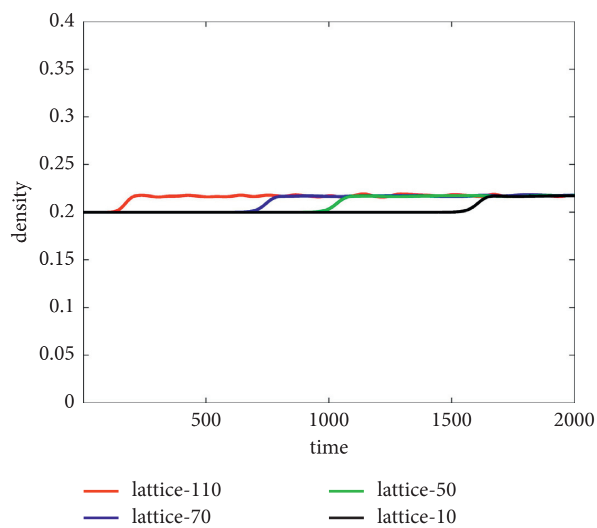

FIGURE 7: Density evolution diagram.

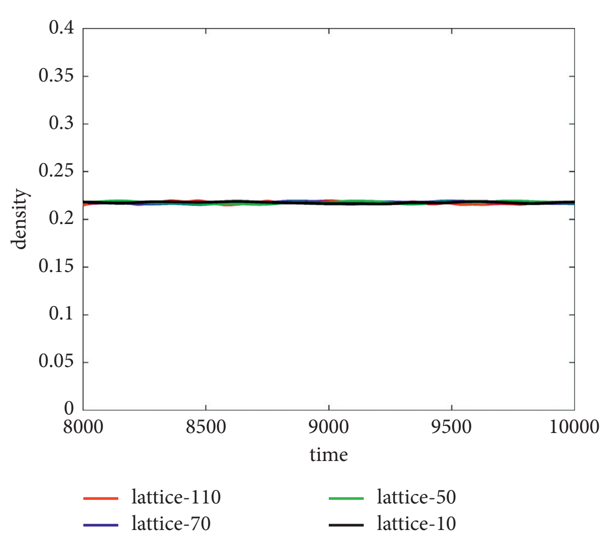

FIGURE 8: Density evolution diagram.

of the traffic will eventually converge to a stable state after the traffic system joined the control and will remain unchanged. At the same time, it can be seen from Figure 9 that the instantaneous disturbance will not be amplified. The results show that the traffic system with control is less prone

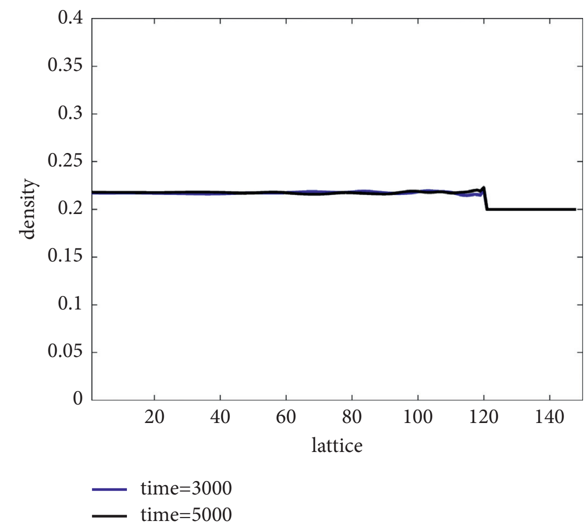

FIGURE 9: Density profile of all lattices.

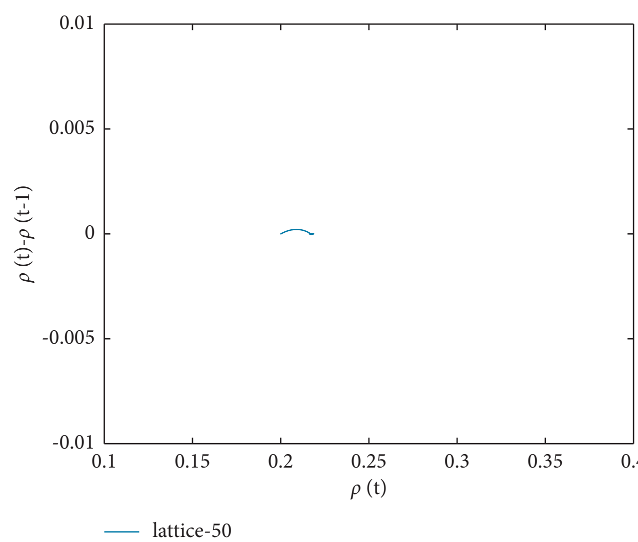

FIGURE 10: Phase trajectory.

to accidents and safer. Correspondingly, by observing the phase trajectories of the 50th lattice in Figure 10, the phase trajectory can almost be regarded as a point, which corresponds to the previous simulation, indicating that in the stable state, the traffic system has no chaotic characteristic.

Similarly, when there are two on-ramps in the traffic system, similar results will appear. If there is an on-ramp at the 120th and 60th lattices of the main road, the main road will be disturbed at two locations. Obviously, the traffic system is unstable in the upstream area of the road without control through Figure 11. However, when the same control is added, the traffic system will be stable. Compared with the traffic system with only one on-ramp, the evolution of the traffic is different. It can be seen from Figure 12 that there are two stable areas in the upstream section of the main road. At the same time, we can also find that the traffic system eventually evolves into two parallel and stable traffic density evolution curves by observing Figure 13. This is because of

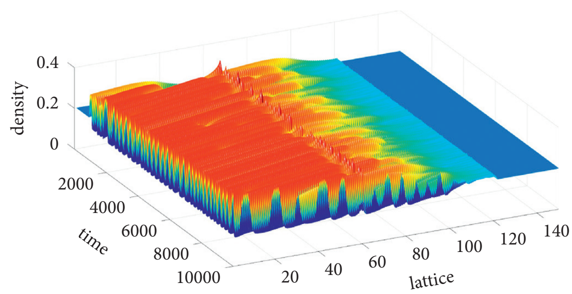

Figure 11: Spatiotemporal evolution pattern.

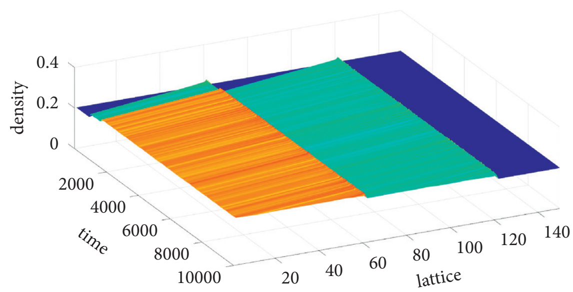

Figure 12: Spatiotemporal evolution pattern.

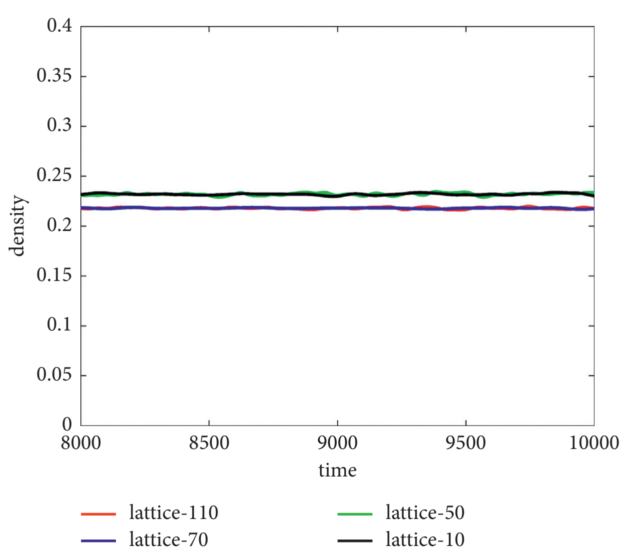

Figure 13: Density evolution diagram.

the existence of the second on-ramp. Although the disturbance caused by the second on-ramp does not make the traffic system unstable, it increases the traffic density in the upstream area of the main road of the second on-ramp.

5. Conclusion

In this study, under the condition of network connection, we proposed a control strategy for a traffic system with the on-ramp. The convergence of the controller is analyzed by the Lyapunov stability analysis method. In addition, in order to ensure that the transient disturbance generated by the on-ramp does not amplify in the process of transmission, the stability of the traffic system is analyzed by using the transfer function method. Through the convergence analysis, we can obtain the time delay and control gain which meet the stability condition. In the numerical experiment, we can see that if there is no control of the traffic system with the on-ramp, the disturbance will be amplified, and the traffic state in the upstream area of the on-ramp entrance will be unstable, resulting in congestion. However, for the traffic system with control, the disturbance will not be amplified and will be quickly suppressed, and the whole traffic system is in a stable state. The results show that, for the on-ramp traffic, the control strategy proposed in this paper can effectively suppress the disturbance and improve the traffic efficiency of the traffic system. However, the actual traffic needs to consider the problem of multilane lane changing, but the main research object in this paper is a single lane. Besides, the time-varying communication delay in the actual setting should be more practical. These problems indicate that the paper needs further improvement in theory, which is also our further research work in the future.

Data Availability

The data are available by contacting the corresponding author.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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