Research Article

Optimisation of Airline Dynamic Multileg Capacity Control Problem considering Competition from High-Speed Rail

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This paper develops an optimisation model to address the dynamic multileg capacity control problem for the airline, taking into account the competition from the high-speed rail (HSR). Conventional capacity control models assume that the passenger demand is independent and that airlines control capacity independently. The proposed model considers the semi-independent demand, i.e., a customer of certain fare class arrives and makes choices from available alternatives provided by both airline and HSR. If the customer’s first option is rejected, their booking request will be diverted both horizontally to the other operator and vertically to parallel flights within the same operator. Making use of this assumption, the customer’s second and third choices are captured. With this model, airlines can control the sales process to determine whether or not to accept an initial booking request or diverted booking request to maximise their revenue. The deterministic linear programming (DLP) method is proposed as a model solution, enabling the computation of partitioned allocation and the bid price of each product. The optimal capacity control strategy is then obtained based on bid price control. To validate the model, numerical experiments are conducted to simulate customers’ actual arrival and to show that the model is capable of producing better revenue, especially in the event that demand exceeds supply. Moreover, the demand diversion effect brings more revenue for both airline and HSR, and this potential revenue source has been ignored by previous research studies.

1. Introduction

Nowadays, rapid developments of high-speed rail (HSR) offer customers more choices between air and HSR transportation. Statistics from the International Union of Railways shows that as of June 2021, the total mileage of high-speed lines in operation worldwide was 56,129 km, covering 44 countries [1]. High-speed railway traffic increased from 245.1 billion passenger-km in 2010 to 1,029.4 billion passenger-km in 2019, to which China contributed an increase from 18.9% to 75.3% [2]. Table 1 compares the operational characteristics of air transportation and HSR transportation. The advantages of air transportation are that it is faster, safer, and can reach greater areas. Nonetheless, taking into account the vulnerability to the adverse weather conditions, restrictions on cabin space, strict security check processes, and often distant location of airports, air transportation is less favourable than HSR transportation in terms of punctuality, comfortability, and accessibility. HSR transportation provides easier accessibility and larger capacity to carry passengers with higher punctuality rate; however, the construction cost is huge and cabin services are limited. Although scholars have different opinions about the range of distance in which HSR can compete with airlines, total travel time is the most significant factor in determining passengers’ choice. Research shows that the watershed total travel time is about 3-4 hours [3]. Chen [3] demonstrated that for journeys with a 3 to 4-hour duration, HSR competes fiercely with airlines. Assume that travelling to/from the airport plus the security check in the airport accounts for one hour, and the journey distance is equivalent to 500–750 km if the speed is 250 km/h or 700–1,050 km if the speed is 350 km/h. In other words, HSR has the clear advantage for short-distance journeys (less than 500 km) and offers a competitive service for middle-distance journey (500–1,050 km). For long-distance journey (above 1,050 km), air transportation is more
favourable for passengers. In fact, the prosperity of HSR network places great pressure on air transportation. The operation of Eurostar from London to Paris caused airlines to lose 56% of passengers in 1994 [4]. In China, the operation of Wuhan-Guangzhou HSR reduced the daily frequency of airline flights from 32 to 17 in 2010 [5].

Faced with the competition from HSR, airlines have adopted various management techniques in order to improve revenue, such as marketing, fleet management, flight crew management, cost control management, and revenue management. These techniques have helped airlines to increase their income and reduce the expenditure from some operational aspects. The revenue management (RM) technique is applied based on customer behaviour predictions at micro-market level and is aimed at selling the right product to the right customer at the right time for the right price. Unlike other techniques, it optimises revenue based on current operation conditions. Therefore, the RM technique has been widely applied to air transportation since the deregulation of the airline industry in 1978 in the U.S. [6], prior to which, airline prices were strictly regulated by the U.S. Civil Aviation Board (CAB) based on standardised pricing and profitability targets. Following the application of RM, the revenue of the whole industry is estimated to increase by 4-5% [7]. There are increasing concerns about airlines’ RM strategy considering the pressure from HSR transportation. Traditionally, airlines set different fare classes for identical physical seats and by controlling how these fare classes are offered, they obtain higher revenue. Optimising the allocation of limited capacity of resources to different classes of demand is known as capacity control—the quantity-based RM. However, the conventional capacity control model assumes that the demand for different fare classes is independent, i.e., that a customer of a certain fare class arrives and buys that product, and then the operator decides whether to accept or reject this booking request. Given the situation of intense competition with HSR, this assumption is no longer accurate because the reality is that a customer with given fare class (full or discounted) arrives and makes a booking request based upon their first option (i.e., direct or connecting flight or HSR train for any given OD pair) based on the preference probability quantified by the multinomial logit (MNL) model. If the customer’s booking request has been rejected by operator’s RM system, the selection can then be diverted to the customer’s second option, as long as the RM system accepts this diversion request, until it reaches the last option.

Classical quantity-based RM theory has developed from a static single-resource model to a dynamic network model. In recent research, customer choice behaviour has drawn more attention and become one of the hot issues [8]. However, critically, the element of competition, especially competition from HSR, has seldom been included in airline RM. Based on this context, we have developed a dynamic multileg, multiclass capacity control model for both airline and HSR. This is different from previous research wherein operators independently controlled the capacity, and this model considers the demand diversion, including the horizontal diversion between airline and HSR, and vertical diversion among operator’s other available flights. The customer’s demand diversion captures their second and even third choice, which contributes to the improvement of revenue and corresponds more appropriately to the reality.

In this research, we introduce HSR into the air travel market as a competitor and focus on the optimisation of airlines’ revenue. The major factors are as follows. (1) The demand is semi-independent. A customer from the independent submarket arrives and makes choices from alternatives offered by airlines and HSR. Traditionally, the RM model assumes that the demand is independent of the availability of other products, and that customers only consider one specific product. (2) Rejected booking requests can be both horizontally shifted to another operator and vertically transferred within one operator’s different flights. Based on the assumption of semi-independent demand, customers can choose between alternatives provided by both airline and HSR. (3) The assumption that a maximum of one booking request for a product can arrive has been moderated. This is in accordance with reality because the possibility of a customer buying two tickets simultaneously might happen but cannot be considered as group arrival.

The rest of the paper is organised as follows. Section 2 reviews the related literature from the perspectives of the airline, HSR, and customers’ choice behaviour. Section 3 introduces the research problem and outlines the process from the customer arrival to capacity control and demand diversion, as well as the solution method. Section 4 applies the model to numerical experiments with different scenarios in order to test its performance and analyse the output.

<table>
<thead>
<tr>
<th>Table 1: Comparation of air transportation and HSR.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air transportation</td>
</tr>
<tr>
<td>Network coverage</td>
</tr>
<tr>
<td>Speed</td>
</tr>
<tr>
<td>Capacity</td>
</tr>
<tr>
<td>Safety level</td>
</tr>
<tr>
<td>Construction cost</td>
</tr>
<tr>
<td>Comfortability</td>
</tr>
<tr>
<td>Security check</td>
</tr>
<tr>
<td>Cabin service</td>
</tr>
<tr>
<td>Punctuality rate</td>
</tr>
<tr>
<td>Location of airport or railway station</td>
</tr>
</tbody>
</table>
results. Section 5 summarises this research and draws conclusions for the operators.

2. Literature Review

RM involves a wide range of techniques, decisions, methods, and processes. It predicts customer behaviour at the micro-market levels and optimises product availability and leverages price elasticity in order to maximise revenue, as a result of the deregulation of the airline industry in 1978 in the U.S. [6]. This definition suggests that one of the main optimisation methods is capacity control. Littlewood [9] proposed the first single-resource capacity control model that involved two fare classes and devised the optimal protection level for the high-fare class; this was known as *Littlewood’s rule*. Later, Belobaba [10–12] extended the single-resource capacity control problem to multiple fare classes and proposed two heuristic methods, EMSR-a and EMSR-b.

All the above research focuses on static capacity control models. They are all founded on the important assumption that requires low-class demand to arrive before high-class demand. The dynamic models, first proposed by Lee and Hersh [13], relaxed this assumption. They developed a dynamic programming (DP) model to decide whether to accept or reject the booking request, in which they assumed that the demand would follow the Poisson distribution. Barz and Gartner [14] applied the DP model to air cargo network revenue management. Utilizing techniques like linear programming, approximate dynamic programming, and decomposition, they identified the upper bounds and the relationships between these bounds.

With the expansion of airline’s hub-and-spoke network, the initial optimisation of the single-resource problem cannot satisfy the need of the network in this situation. Therefore, the network RM emerges. Due to the increasing complexity and volume of data, approximation methods are adopted for solving network capacity control problems, which can be further categorised into two basic strategies. The first is to address the problem from the perspective of mathematical programming. Glover et al. [15]; Dror et al. [16]; and Wong et al. [17] applied the deterministic linear programming (DLP) model to simplify the network problem. Demand was treated as deterministic with its mean. By solving the DLP model, partitioned allocation for each product was obtained. Wollmer [18] proposed the probabilistic nonlinear programming (PNLP) model, which treated the demand as the expected sales of the product under the partitioned allocation. Though the demand in PNLP is considered more as randomness, the assumption of partitioned allocation of the product can lead to worse performance than DLP [19]. The second strategy is to deconstruct the network problem into the single-resource problems. Williamson [20] investigated the prorated EMSR method and allocated the revenue of each product into its used resources according to the prorating scheme. However, this method is highly dependent on the prorating scheme and there are no general prorating schemes for all cases. Another popular decomposition method is displacement-adjusted virtual nesting (DAVN), proposed by Smith and Penn [21]. DAVN uses displacement-adjusted revenue to approximate the net benefit of accepting a product and then solves single-resource problems on each resource. By comparing two kinds of approximation strategies, it is clear that the approximation based on mathematical programming has the disadvantage that it generates partitioned allocation, which is inferior to nested allocation generated by approximation based on decomposition. However, mathematical programming methods provide the displacement cost, which is required in decomposition methods, and as a result, the two methods are usually applied in combination.

The quantity-based RM has expanded from a single resource into a network and has transformed from a static model into a dynamic one. New trends begin to focus on customer choice behaviour, and the modelling of customer choice can be categorised into parametric and nonparametric models. Parametric models are based on random utility theory. Abdallah and Vulcano [22] introduced the minorisation-maximisation (MM) algorithm to MNL parameter estimation to accelerate the convergence of the EM approach. In addition to the MNL model, the nested logit model and the mixed MNL model are also popular parametric models used to overcome the disadvantages of MNL, including unobservable membership of segment and IIA assumption. Though the parametric models have been successfully used for many years, they have several disadvantages. For instance, the accuracy decreases with censored demand data and choice probability does not change with time and variation in offered products. Therefore, these disadvantages stimulated the development of the nonparametric models. Sharif Azadeh [23] proposed a mixed integer nonlinear program to estimate the utilities as well as capture the seasonal effect of demand. Jörg and Cleophas [24] employed the finite mixture model to describe the demand for different segments. This approach was concerned about the specification of the demand structure and specific distribution functions.

Apart from the choice model itself, choice-based RM relaxes the assumption that the demand for different products is independent. Customers can make choices from available alternatives. Therefore, the customer’s demand is dependent. If customer’s preferred product is unavailable, this rejected or unmet demand can “spill” or be “recaptured” [25]. Spill means this demand is redirected to a competitor or no-purchase alternative; recapture means this demand is redirected to another available product. In the context of dependent demand, the core problem becomes dynamic availability control, which refers to the decision about the optimal offer set of available products for customers at each time period. If we address this problem within one time period, the dynamic availability control problem degenerates into a static assortment optimisation problem. Normally, the assortment optimisation is solved by heuristic methods. The first is the revenue-ordered approach, in which all sets are nested by displacement-adjusted revenues and then searched to find the maximum revenue set. Berbeglia and Joret [26] showed that revenue-ordered assortment provides provable guarantees for a general regular discrete choice model.
Another heuristic method is the greedy approach, which adds revenue-maximising products to an initially empty set. Hekimoğlu et al. [27] proposed a greedy heuristic approach to solve assortment optimisation with log-linear demand and generated better results than mixed integer nonlinear programming. For the dynamic availability control problem, Feldman and Topaloglu [28] considered three classes of problems, static assortment optimisation, single-resource availability control, and network availability control. They solved these problems by using linear programming approximation and showed that the linear program can be reduced to an equivalent with a smaller size.

With the increased choices available to customers, the service suppliers are not confined to one airline. Researchers have begun to focus on parallel flights and even multiple airlines. Le and Lu [29] developed a modified sales-based linear program (M-SBLP) to address the network choice-based revenue management problem with overlapping segments. They applied the model to both a network with parallel flights and a hub-and-spoke network and found that M-SBLP performs quickly and produces similar results to those generated by other optimisation models. Zhu and Hu [30] studied the seat allocation and price competition game between duopoly airlines under the mandate of government and established the existence of pure-strategy Nash equilibriums. They showed that airlines should reserve more high-fare tickets and increase the price of high-fare products in order to cope with the impact of the government mandate.

Table 2 summarises the variation in the existing literature. The majority of works mainly focus on the airline itself without considering competition as a factor, especially the competition from other modes of transportation, such as HSR. In fact, HSR severely threatens the revenue of airlines, especially in short and middle-distance transportation. Besides, the research on customer choice behaviour mainly focuses on the modelling of choice behaviour itself and on the availability control problem with total dependent demand. The capacity control problem under restricted customer choice has never been investigated. This research aims to fill the gap by assuming that the demand is semi-independent and capturing the customer’s second and third choices when the first choice is rejected. Finally, the optimal control strategy is developed for airline to maximise its revenue in context of the competition from HSR.

3. Methodology

3.1. Model Description. Figure 1 presents a hypothetical directed transportation network with 3 nodes and 3 links. A, B, and C represent 3 cities. The airline operates two parallel flights, i.e., direct flight A2 from A to C and connecting flight A1 from A to C, via B. HSR company provides a stop-by-stop train service H from A to C, via B. Both flights and train journeys provide two fare classes in each origin and destination (OD), full fare and discount fare, each offering different rights of refund and exchange. Therefore, a 3-node network and 2 fare classes form 6 submarkets and 14 origin-destination itinerary fares (ODIFs), which are presented in Table 3.

The selling horizon is denoted by \([0, T]\), in which each time period is indexed by \(t\). The flight or train will depart immediately after time period \(T\). In this instance, we do not consider cancellation and group arrival and assume that both operators are risk neutral. This whole process is described in Figure 2. In each time period, customers from different submarkets arrive and make choices from available alternatives, i.e., A1, A2, and H, according to a discrete choice model.
Based on the known pattern and parameters of the choice behaviour, both operators can forecast the demand and generate predictive data on the demand. The predicted demand data and remaining capacity data will be inputted into the capacity control model. By solving the capacity control model, the displacement cost is generated and used to generate a control strategy. Meanwhile, if the booking request is rejected, it will be diverted to the second and third choices. If the control strategy accepts this diverted booking request, the remaining capacity will be updated; otherwise, this diverted booking request is finally rejected. Specifically, the research problem is how the airline would control the sales process to determine whether or not to accept booking requests from a specific ODIF in order to obtain maximum revenue.

### Table 3: Customer arrival of different submarkets and ODIFs.

<table>
<thead>
<tr>
<th>ODIF K</th>
<th>Submarket j</th>
<th>OD</th>
<th>Alternatives i</th>
<th>Fare class</th>
<th>Proportion $\tau$</th>
<th>Demand $\mu$</th>
<th>$\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>AB</td>
<td>A1</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>BC</td>
<td>A1</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>ABC (AC)</td>
<td>A1</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>AB</td>
<td>A1</td>
<td>D</td>
<td>$\tau_j$</td>
<td>$\mu_j$</td>
<td>$\sigma_j^2$</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>BC</td>
<td>A1</td>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>AB</td>
<td>H</td>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>BC</td>
<td>H</td>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>ABC (AC)</td>
<td>H</td>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>A2</td>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>A2</td>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.2. Customer Arrival and Choice Behaviour. The customer arrives in each submarket, following independent normal distribution, the parameters of which are listed in Table 3. Notations used are listed in Table 4.

In each time period $t$, customers from submarket $j$ arrive and make choices from available alternatives according to the MNL model. The utility function $U(i, j)$ is composed of the observable parts $V(i, j)$ and random parts $\epsilon(i, j)$.

$$V(i, j) = \alpha - \beta_j \frac{p_{ij}}{\max p_j} - \gamma_j \frac{h_{ij}}{\max h_j}$$

$$\beta_j = \begin{cases} \beta_S & j = 1, 2, 4, 5 \\ \beta_L & j = 3, 6 \end{cases}, \gamma_j = \begin{cases} \gamma_S & j = 1, 2, 4, 5 \\ \gamma_L & j = 3, 6 \end{cases}$$ (1)

The observable parts $V(i, j)$ of utility function consider two factors: ticket fare $p_{ij}$ and total travelling time $h_{ij}$, for which the coefficients are $\beta_j$ and $\gamma_j$, respectively. These two coefficients are further extended by the travel distance and distinguished by the subscripts $S$ and $L$, which are shown in (1). Generally, the customer would obtain lower utility with the increase of both fare and total travelling time. In addition, there is a constant $\alpha$, which is the attribute of alternative $i$. The constant is used to describe the customer’s overall evaluation of an alternative in addition to fare and travelling time. The random parts $\epsilon(i, j)$ describe the unobservable factors that may influence customers’ choice to the researcher. Each $\epsilon(i, j)$ is assumed to follow independent Gumbel distribution. As such, the results of customer choice are revealed in the following equation:

$$q_i(p_j) = \frac{e^{V(i, j)}}{\sum_{i=1}^{3} e^{V(i, j)}} \quad i = 1, 2, 3$$ (2)

where $q_i(p_j)$ is the probability that a customer from submarket $j$ chooses alternative $i$.

3.3. Demand Forecast. Across the whole selling horizon, customers arrive and make choices from available alternatives, and the airline and HSR have to forecast the demand in different submarkets according to the known arrival parameters and the demand split into each ODIF according to the choice model. The predicted aggregate demand of different ODIF in each time period will be important as input data for the capacity control optimisation model.

3.3.1. Predicted Demand. Airline and HSR predict customer demand in different submarkets based on the parameters of demand distribution stated in Table 3 and the choice probability given by (2). Both demand curves and arrival curves are a series of normal distribution curves with parameters $\mu_j$ and $\sigma_j^2$. The demand rate, $\lambda_j'$, is equal to the probability density function of normal distribution. Arrival rate, $\lambda_j'^{'}$, is defined by (3). The difference is that the arrival rate considers both the relative amount calculated by normalising the demand rate and the absolute amount quantified by the total demand of submarket $j$.

$$\lambda_j' = \frac{D \tau_j \epsilon_j}{\sum_i \epsilon_j}$$ (3)

By introducing the choice probabilities $q_i(p_j)$ and aggregating the arrival rate, $\lambda_j'$, the expected arrival of different submarkets can be further expanded to the expected arrival of customer’s first option. (4) demonstrates the predicted aggregate demand of the customer’s first option in ODIF $k$. 
3.3.2. Actual Arrival. Actual arrival sequences are the results of simulated customer arrival together with choice behaviour, which records the amount of arrival of ODIF $k$ in time period $t$. The sequences are generated using the following procedures.

$$
\mathcal{A}_k(t) = \begin{cases}
q_1(\mathbf{p}_k) \sum_t \lambda^t_k & k = 1, \ldots, 6 \\
q_2(\mathbf{p}_{k-4}) \sum_t \lambda^t_{k-4} & k = 7 \\
q_2(\mathbf{p}_{k-2}) \sum_t \lambda^t_{k-2} & k = 8 \\
q_3(\mathbf{p}_{k-8}) \sum_t \lambda^t_{k-8} & k = 9, \ldots, 14.
\end{cases} \quad (4)
$$

**Step 1.** Generate actual arrival time series of each submarket.

**Step 1.1.** Calculate $\omega_j$ with the Box–Muller transformation shown in equation (5), and $U_1 \sim U(0, 1) \text{ and } U_2 \sim U(0, 1)$ follow independent standard uniform distribution, $\omega_j \in \Omega_j$.

$$
\omega_j = \sigma_j \sqrt{-2 \ln U_1 \cos(2\pi U_2)} + \mu_j. \quad (5)
$$

**Step 1.2.** Validate the set of arrival time and get $\Omega_j = \{\omega_j| 0 < \omega_j < T\}$.

**Step 1.3.** Get the set of arrival occurring in time period $t$, $\Omega_j(t) = \{\omega_j| t - 1 < \omega_j < t\}$.

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**Figure 2:** Flowchart of the proposed model.
Table 4: Notations used in this model.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Alternative set, ( i \in I, i = 1 ) represents ( A_1 ), ( i = 2 ) represents ( A_2 ), ( i = 3 ) represents ( H )</td>
<td>Set, parameter</td>
</tr>
<tr>
<td>J</td>
<td>Submarket set, ( j \in J )</td>
<td>Set, parameter</td>
</tr>
<tr>
<td>K</td>
<td>ODIF set, ( k \in K, k = 1, 2, \ldots, 6 ) belongs to ( A_1 ), ( k = 7, 8 ) belongs to ( A_2 ), ( k = 9, 10, \ldots, 14 ) belongs to ( H )</td>
<td>Set, parameter</td>
</tr>
<tr>
<td>T</td>
<td>Selling horizon</td>
<td>Scalar, parameter</td>
</tr>
<tr>
<td>( p_{ij} )</td>
<td>Fare of alternative ( i ) in submarket ( j )</td>
<td>Scalar, parameter</td>
</tr>
<tr>
<td>( r_{ij} )</td>
<td>Total travelling time of alternative ( i ) in submarket ( j )</td>
<td>Scalar, parameter</td>
</tr>
<tr>
<td>( \beta_j )</td>
<td>Elasticities of fare in submarket ( j )</td>
<td>Scalar, parameter</td>
</tr>
<tr>
<td>( \gamma_j )</td>
<td>Elasticities of time in submarket ( j )</td>
<td>Scalar, parameter</td>
</tr>
<tr>
<td>( \alpha_i )</td>
<td>Overall evaluation or satisfaction to alternative ( i )</td>
<td>Scalar, parameter</td>
</tr>
<tr>
<td>( D )</td>
<td>Total demand of the transportation network</td>
<td>Scalar, parameter</td>
</tr>
<tr>
<td>( \tau_j )</td>
<td>Share of submarket ( j )</td>
<td>Scalar, variable</td>
</tr>
<tr>
<td>( \epsilon_j^t )</td>
<td>Probability that one customer arrives in submarket ( j ) in time period ( t )</td>
<td>Scalar, variable</td>
</tr>
<tr>
<td>( \lambda_j^t )</td>
<td>Total expectation of customer arrives in submarket ( j ) in time period ( t )</td>
<td>Scalar, variable</td>
</tr>
<tr>
<td>( d_k(t) )</td>
<td>Predicted aggregate demand of ODIF ( k ) from time period ( t ) to ( T )</td>
<td>Scalar, variable</td>
</tr>
<tr>
<td>( \omega_j(t) )</td>
<td>Customer’s actual arrival time in submarket ( j )</td>
<td>Scalar, variable</td>
</tr>
<tr>
<td>( \Omega_j^t )</td>
<td>Actual arrival time set</td>
<td>Set, variable</td>
</tr>
<tr>
<td>( \Omega_j(t) )</td>
<td>Valid actual arrival time set</td>
<td>Set, variable</td>
</tr>
<tr>
<td>( d_k(t) )</td>
<td>Actual arrival of ODIF ( k ) in time period ( t )</td>
<td>Set, variable</td>
</tr>
<tr>
<td>( arr_j(t) )</td>
<td>Amount of actual arrival in submarket ( j )</td>
<td>Scalar, variable</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Minimum allocation percentage of HSR</td>
<td>Scalar, constant</td>
</tr>
<tr>
<td>( A )</td>
<td>Resource occupation matrix</td>
<td>Matrix, constant</td>
</tr>
<tr>
<td>( x(t) )</td>
<td>Resource capacity vector in time period ( t )</td>
<td>Vector, variable</td>
</tr>
<tr>
<td>( V_t(x(t)) )</td>
<td>Value function when remaining capacity is ( x(t) ) in time period ( t )</td>
<td>Scalar, objective function</td>
</tr>
<tr>
<td>( y(t) )</td>
<td>Partitioned allocation of each product in time period ( t )</td>
<td>Vector, decision variable</td>
</tr>
<tr>
<td>( \pi(t, x(t)) )</td>
<td>Displacement cost when remaining capacity is ( x(t) ) in time period ( t )</td>
<td>Vector, variable</td>
</tr>
<tr>
<td>( c_q(t) )</td>
<td>Bid price of product ( k ) in time period ( t )</td>
<td>Scalar, variable</td>
</tr>
<tr>
<td>( u_{kn}(t) )</td>
<td>Indicator of whether the ( n )th booking request for product ( k ) is accepted or refused in time period ( t )</td>
<td>0–1 variable</td>
</tr>
<tr>
<td>( s_k(t) )</td>
<td>Total number of accepted booking requests for product ( k ) in time period ( t )</td>
<td>Scalar, variable</td>
</tr>
</tbody>
</table>

Step 1.4. Recognise the amount of arrival of submarket \( j \) in time period \( t \), as shown in equation (6), and the notation \( \text{card} \) represents the cardinality of a set.

\[
arr_j(t) = \begin{cases} 
\text{card}[\Omega_j^t] \cdot \Omega_j^t(t) \neq \emptyset \\
0 \\
\Omega_j^t(t) = \emptyset.
\end{cases} \tag{6}
\]

Step 2. Customer of submarket \( j \) chooses their preferred option within available alternatives.

Step 2.1. Generate uniformly distributed random number in the interval (0, 1).

Step 2.2. Compare the random number with known choice probability of available alternatives, as shown in Figure 3.

Step 2.3. Return the results of customer preferred option.

Step 2.4. Recognise the amount of arrival of ODIF \( k \) in time period \( t \).

Then, the actual arrival sequence of \( A_1 \) in ODIF \( k \) in time period \( t \) is demonstrated in (7) and (8).

\[
\hat{c}_n = \begin{cases} 
1 & \text{rnd} (t) < q_1(p_j) \\
0 & \text{rnd} (t) > q_1(p_j) \text{ or } arr_j(t) = 0
\end{cases}, \quad 0 \leq q_1(p_j) \leq 1 \tag{7}
\]

\[
d_j(t) = \sum_{n=1}^{\text{card}[\Omega_j^t]} \hat{c}_{n,j} \tag{8}
\]

The actual arrival sequence of \( A_2 \) in ODIF \( k \) in time period \( t \) is demonstrated in the following equation:

\[
\hat{c}_{n,j} = \begin{cases} 
1 & q_1(p_j) \cdot \text{rnd} (t) < q_2(p_j) \\
0 & \text{rnd} (t) > q_2(p_j) \text{ or } \text{rnd} (t) < q_1(p_j), \text{ or } arr_j(t) = 0
\end{cases}, \quad 0 \leq q_1(p_j) \leq 1 \tag{9}
\]

\[
d_{j+1}(t) = \sum_{n=1}^{\text{card}[\Omega_j^t]} \hat{c}_{n,j}, \quad j = 3, \tag{9}
\]

\[
d_{j+2}(t) = \sum_{n=1}^{\text{card}[\Omega_j^t]} \hat{c}_{n,j}, \quad j = 6. \tag{9}
\]
3.4. Capacity Control Optimisation. In this transportation network, there are \( r \) resources and \( k \) products. Resources refer to the seats occupied in each leg of flight or train journey. For example, resource 1 refers to the seats of AB, resource 2 refers to the seats of BC, and resource 3 refers to the seats of AC. Resource 1 and resource 2 are used by flight A1 and HSR, and resource 3 is used only by flight A2. Products refer to the ticket of an ODIF, which is a combination of a bundle of resources and fares with certain refund rights and purchasing restrictions. The resource occupation matrix \( \mathbf{A} = (a_{rk}) \), \( a_{rk} = 1 \) in the event that ODIF \( k \) uses resource \( r \), and uses \( a_{rk} = 0 \) otherwise. Resource capacities are indicated by vector \( \mathbf{x}(t) = [x_1(t), x_2(t), x_3(t)] \), \( r = 1, 2, 3 \). The first element represents resource AB, the second element represents resource BC, and the third element represents resource AC. By inputting the current time period \( t \), remaining capacity \( \mathbf{x}(t) \), and predicted aggregate demand \( \tilde{d}_k(t) \), the capacity control model makes decisions with regard to whether or not to accept the current booking request from ODIF \( k \).

For the network capacity control problem, it is very difficult to get the exact solution of its value function \( V_t(x) \), especially for the large-scale network. Instead, the approximation method is more suitable. The DLP method is applied to solve the value function in the proposed capacity control problem. For each ODIF \( k \), the demand is treated as deterministic and equal to the predicted aggregate demand \( \tilde{d}_k(t) \), which is why it is named deterministic linear programming. By solving this approximation with the DLP method, it is possible to obtain the optimal partitioned allocation strategy and displacement costs of each resource in time period \( t \). The airline and HSR optimise their revenues based on the predicted arrival of ODIF \( k \) in time period \( t \) and remaining capacity \( \mathbf{x}(t) \) in time period \( t \). The fares of all products are known and indicated by vector \( \mathbf{p} = (p_k), = (p_{1j}, p_{23}, p_{26}, p_{3j}): j = 1, 2, \ldots, 6 \).

3.4.1. Airline Capacity Control Optimisation (DCCM). Flight A1 and A2 are operated by airline as parallel flights, so the airline controls these two flights’ capacity simultaneously and generates revenue from both A1 and A2. The resource capacities of the airline are denoted by \( \mathbf{x}^A(t) = [x_1^A(t), x_2^A(t), \ldots, x_r^A(t)] \), \( r = 1, 2, 3 \), and the fare vector is \( \mathbf{p} = (p_k), k = 1, 2, \ldots, 8 \). The decision variables are denoted by vector \( \mathbf{y}^A(t) = [y_1^A(t), \ldots, y_k^A(t)] \), \( k = 1, 2, \ldots, 8 \), which results in the optimal partitioned allocation of capacity, or booking limit, for each of the airline’s products. Therefore, the dynamic capacity control model (DCCM) of the airline with the DLP method is laid out by the following equation:

\[
V_{A_t}^{\text{DLP}}[x_1^A(t), x_2^A(t), x_3^A(t)] = \max \sum_{k=1}^{8} p_k y_k^A(t),
\]

\[
s.t. \sum_{k=1}^{8} d_k y_k^A(t) \leq x_1^A(t), r = 1, 2, 3;
\]

\[
0 \leq y_k^A(t) \leq \tilde{d}_k(t), k = 1, 2, \ldots, 8;
\]

\[
y_k^A(t) \in \mathbb{Z}.
\]

The objective function \( V_{A_t}^{\text{DLP}}[x^A(t)] \) denotes the airline’s maximum expected revenue of the remaining selling horizon and provides an upper bound of the real revenue [31]. The second line of equation (12) describes that the resources required by allocation plan for all products of airline should not exceed the remaining resource capacities. The third line of equation (12) requires that the allocation plan is a nonnegative integer and should not exceed the predicted mean aggregate demand.

3.4.2. HSR Capacity Control Optimisation. HSR operates one train, and thus it controls the train’s capacity and generates revenue. The resource capacities of HSR are denoted by \( \mathbf{x}^H(t) = [x_1^H(t), \ldots, x_r^H(t)] \), \( r = 1, 2, 3 \), and the fare vector is \( \mathbf{p} = (p_k), k = 9, 10, \ldots, 14 \). The decision variables are denoted by vector \( \mathbf{y}^H(t) = [y_k^H(t)] \), \( k = 9, 10, \ldots, 14 \) to present the optimal partitioned allocation of capacity for individual products of HSR listed in Table 3. Therefore, the dynamic capacity control model (DCCM) of HSR using the DLP method is demonstrated by the following equation:

\[
V_{H_t}^{\text{DLP}}[x_1^H(t), x_2^H(t), x_3^H(t)] = \max \sum_{k=9}^{14} p_k y_k^H(t),
\]

\[
s.t. \sum_{k=9}^{14} d_k y_k^H(t) \leq x_1^H(t), r = 1, 2, 3;
\]

\[
0 \leq y_k^H(t) \leq \tilde{d}_k(t), k = 9, 10, \ldots, 14;
\]

\[
y_k^H(t) \in \mathbb{Z}.
\]
3.4.3. Solution Method. The capacity control model with DLP approximation is solved by MATLAB program with LINDO solver. By solving the DLP model, the optimal partitioned allocation of capacity for each product of airline and HSR is obtained, expressed here by (14) and (15).

\[ y^A(t) = [y^A_k(t)], \quad k = 1, 2, \ldots, 8, \] (14)

\[ y^H(t) = [y^H_k(t)], \quad k = 9, 10, \ldots, 14. \] (15)

Although the partitioned allocation can be used directly to control the capacity, this type of control is inflexible. Because it sets strict booking limit for each product, one product can only be transferred to the other products. (The notation \( Z_{he} \) capacity control model with partitioned allocation of capacity for each product of airline and HSR is obtained, expressed here by (14) and (15).

\[ \pi^H(t, x^H(t)) = \begin{pmatrix} \pi^H_1(t) \\ \pi^H_2(t) \\ \pi^H_3(t) \end{pmatrix}, \] (16)

\[ \pi^H(t, x^H(t)) = \begin{pmatrix} \pi^H_1(t) \\ \pi^H_2(t) \\ \pi^H_3(t) \end{pmatrix}. \] (17)

The bid price of product \( k \) is calculated by summing the displacement costs of required resources of product \( k \), as shown in (18). The notation \( \text{supp}(f) \) represents the support of \( f \), the set of points in \( X \) where \( f \) is nonzero: \( \text{supp}(f) = \{ x \in X : f(x) \neq 0 \} \). This capacity will only be sold if the fare of product \( k \) is higher than the sum of displacement cost of required resources and the capacity is available. Therefore, a simpler capacity control strategy for the airline and HSR is generated based on the bid price, which only stores a single price instead of a series of capacity limits for each class.

\[ c_k(t) = \sum_{i \in \text{supp}(A_k)} \pi^H_i(t). \] (18)

The number of customers who requested product \( k \) in time period \( t \) is denoted as \( d_k(t) \), for each customer; \( u_{k,n}(t) \) records the result of whether a booking request is accepted or refused according to the remaining capacity and bid price. \( s_k(t) \) records the total number of accepted booking requests. \( f_k(t) \) records the refused booking requests which will be transferred to the other products.

\[ u_{k,n}(t) = \begin{cases} 1 & \text{if } p_k \geq c_k(t), \quad A_k \leq x^A(t), \\ 0 & \text{otherwise} \end{cases} \]

\[ s_k(t) = \sum_{n=1}^{d_k(t)} u_{k,n}(t). \] (19)

\[ f_k(t) = d_k(t) - s_k(t). \]

After making decisions for the arrived customers in the first round, the remaining capacity and bid price may change, so the updated capacity is denoted in (20). The bid price will be updated to \( c_k'(t) \) by resolving the capacity control model of equations (12) and (13) with updated capacity.

\[ x'(t) = x(t) - A_k \cdot s_k(t). \] (20)

3.5. Demand Diversion. Once the first option of a customer’s booking request has been refused, the booking request will be diverted to the second option. The operator should make decisions to accept or reject the diverted booking request according to the remaining capacity and the bid price of resources it requires.

The transfer-out matrix \( M = [m_{k,i}] \) describes all the available alternatives a booking request can be transferred to for ODIF \( k \). This matrix is obtained by checking the service availability of other alternatives in each submarket. The element \( m_{k,i} = 1 \) if the service of alternative \( i \) is available to accept the transfer of booking request from other alternatives in ODIF \( k \), and \( m_{k,i} = 0 \) otherwise.

\[ M = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}. \] (21)

The transfer-in matrix \( Z(t) = [z_{k,k'}(t)] \) describes whether a booking request for ODIF \( k \) can successfully be transferred to ODIF \( k' \). The value of \( z_{k,k'}(t) \) is calculated by (22), which depends on the remaining capacity \( x(t) \) and bid price \( c_k'(t) \) of ODIF \( k' \). \( k \) denotes the origin of a refused booking request that needed to be transferred out, and \( k' \) denotes the destination the booking request would have been transferred to. A booking request can only be transferred to the corresponding destinations. The mapping of \( k \) and \( k' \) is depicted in Figure 4.
which alternative the booking request has been transferred to. (24) and (25) demonstrate $\bar{R}(t)$, $\tilde{r}_k(t) = 0$ if the booking request for ODIF $k$ failed to transfer, $\tilde{r}_k(t) = i'$ if the booking request for ODIF $k$ is successfully transferred to alternative $i'$ in ODIF $k'$. The nonzero value of $\tilde{r}_k(t)$ expresses to which alternative the booking request for ODIF $k$ will be transferred.
Therefore, the whole process of demand diversion starts from a rejected booking request for alternative \( i \) in ODIF \( k \) for the first time and finishes by being transferred to alternative \( i' \) in ODIF \( k' \). (26) and (27) show the second-round decision on transferred demand, for each transferred-out booking request in the first round. \( \hat{u}_{k,n}(t) \) records the result of whether the transferred booking request is accepted or refused. \( s'(t) \) updates the total number of accepted booking requests, including initial booking requests and transferred booking requests to ODIF \( k' \). \( s(t) \) is the vector of products sold in time period \( t \). The remaining capacity is updated to \( x(t + 1) \) as denoted by (28).

\[
s'_k(t) = s_k(t) + \sum_{n=1}^{j_k(t)} \hat{u}_{k,n}(t),
\]

\[
s(t) = [s_k(t)],
\]

\[
x(t + 1) = x'(t) - A \cdot s(t).
\]

4. Numerical Experiments and Analysis

In this section, the capacity control model with demand diversion is applied to a numerical experiment. Customers of different submarkets arrive according to the parameters listed in Table 5. The parameters used in DCCM are listed in Table 6.

Figure 5 demonstrates the demand and arrival rates of customers from different submarkets. Figure 5(a) presents the demand of customers in different submarkets. The customers of discount-fare class arrive throughout the whole selling horizon, and the customers of full-fare class arrive just before the end of the selling horizon. From the perspective of demand intensity, the customer’s demand for discount-fare class is moderate and evenly distributed throughout the whole selling horizon. The demand for full-fare class on the tail end of the selling horizon has stronger intensity. This is because business customers, who are usually more willing to buy a full-fare ticket, are not as sensitive to price and not inclined to advance purchase. By contrast, leisure customers are more sensitive to price and willing to purchase in advance to benefit from the discount. Figure 5(b) presents the distribution of customer arrival. Considering the difference of absolute quantity between the two kinds of customers, the peak arrival rate of discount-fare classes is equivalent to that of full-fare class.

The results of numerical experiments are analysed according to the framework in Figure 6. First of all, when airline and HSR both adopt DCCM strategy, the basic control strategy as well as the whole sales process and optimal revenue is generated. Secondly, the performance of DCCM is compared in different scenarios. Thirdly, the demand diversion effect is analysed when the airline adopts DCCM strategy and HSR adopts DCCM or TA strategy. These two parts provide the most significant results to show the model’s performance and reveal the demand diversion effect. On top of this, the computational efficiency is analysed as a subsidiary to improve the model’s performance.

### Table 5: Customer arrival parameters.

<table>
<thead>
<tr>
<th>Submarket ( j )</th>
<th>OD</th>
<th>Ticket class</th>
<th>Proportion ( \tau_j )</th>
<th>Demand ( \mu_j ), ( \sigma^2_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AB</td>
<td>F</td>
<td>0.084</td>
<td>0.9<em>T 0.1</em>T</td>
</tr>
<tr>
<td>2</td>
<td>BC</td>
<td>F</td>
<td>0.09</td>
<td>0.91<em>T 0.15</em>T</td>
</tr>
<tr>
<td>3</td>
<td>ABC (AC)</td>
<td>F</td>
<td>0.056</td>
<td>0.8<em>T 0.21</em>T</td>
</tr>
<tr>
<td>4</td>
<td>AB</td>
<td>D</td>
<td>0.304</td>
<td>0.55<em>T 0.45</em>T</td>
</tr>
<tr>
<td>5</td>
<td>BC</td>
<td>D</td>
<td>0.298</td>
<td>0.45<em>T 0.5</em>T</td>
</tr>
<tr>
<td>6</td>
<td>ABC (AC)</td>
<td>D</td>
<td>0.168</td>
<td>0.35<em>T 0.4</em>T</td>
</tr>
</tbody>
</table>

### Table 6: Parameters used in DCCM.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = [\alpha_1, \alpha_2, \alpha_3] )</td>
<td>[1 1 0.95]</td>
</tr>
<tr>
<td>( \beta = [\beta_1, \beta_2] )</td>
<td>[0.3 0.2]</td>
</tr>
<tr>
<td>( \gamma = [\gamma_1, \gamma_2] )</td>
<td>[0.1 0.2]</td>
</tr>
<tr>
<td>( h_{ij} = -3.5 -3.5 )</td>
<td>[2.3 1.75 4.5 2.3 1.75 4.5]</td>
</tr>
<tr>
<td>( p_{ij} = -2.3 -3.5 )</td>
<td>[5.5 4 9.75 5.5 4 9.75]</td>
</tr>
<tr>
<td>( \eta = 0.03 )</td>
<td>[1,350 1,050 2,150 830 650 1,310]</td>
</tr>
<tr>
<td>( x^1(0) = 250 )</td>
<td>[520 460 860 470 410 690]</td>
</tr>
<tr>
<td>( x^H(0) = 120 )</td>
<td>[400 400 0]</td>
</tr>
<tr>
<td>( T = 1,000 )</td>
<td></td>
</tr>
<tr>
<td>( D = 1,500 )</td>
<td></td>
</tr>
</tbody>
</table>

4.1. Control Strategy and Displacement Cost. Figure 7 demonstrates the partitioned allocation of flight A1 with all its products and displacement costs of resources 1 and 2. The displacement costs of resources 1 and 2 are fixed at ¥830 and ¥650 until the discount-fare classes of AB and BC are closed from time period 718 and 750, respectively. Because the discount fare of AC is ¥1,310, less than the sum of the minimum displacement costs of resources 1 and 2, that is ¥1,480, the discount-fare class of AC is closed within the whole selling horizon. Then, displacement cost of resource 1 climbs to ¥1,350, while displacement cost of resource 2 follows and climbs to ¥800. This threshold price rejects booking request from any discount-fare class. Finally, displacement cost of resource 2 increases to ¥1,050 since time period 901. Under this price, the full-fare class of AC is closed. Notably, the displacement cost of resource 2 switches between ¥800 and ¥650 in certain time periods between 751 and 900, during which the discount-fare class of BC would be reopened if resource 2’s displacement cost drops to ¥650. Similarly, the displacement cost of resource 1 switches between ¥1,350 and ¥1,100 in certain time periods after 891, during which the full-fare class of AC would be reopened if the displacement cost of resource 1 drops to ¥1,100. The reopen action indicates the worry that the demand with higher revenue may be inadequate in the remaining selling horizon. For example, the reopening of the full-fare class of AC comes from the worry of inadequate demand of full-fare class of AB and BC. Obviously, if we have just one seat...
inventory, selling to the customers of full-fare class of AB and BC makes more revenue than selling to the customer of full-fare class of AC. The full-fare classes of AB and BC are always opened until their capacities are used up. The dynamic changes of displacement costs in the later selling horizon make it possible for airline to control their capacities according to customers’ arrival and remaining capacities in real time.

Figure 8 demonstrates the partitioned allocation of flight A2 for the full-fare class and discount-fare class of AC as well as the displacement cost of resource 3. Since time period 523, the discount-fare class of AC is closed but not permanently, and it would be reopened irregularly until time period 769. The full-fare class of AC is always opened until its capacity is used up. The variation of displacement cost confirms the control strategy, i.e., the displacement cost rises from ¥1,600 to ¥2,300 after the discount-fare class of AC is closed and drops back to ¥1,600 when it is necessary to reopen this class according to customers’ arrival and remaining capacity.

Figure 9 demonstrates the partitioned allocation of HSR for all its products and displacement costs of resources 1 and 2. Both full and discount-fare classes of long-distance products are closed earlier than those of short-distance products. The discount-fare class of AC is closed in time period 678, and full-fare class of AC is closed in time period 786. Then, the discount-fare classes of AB and BC are closed from time period 861 to 882 but reopen irregularly until the end of the selling horizon, to supplement the absence of demand of full-fare classes. The full-fare classes of AB and BC are always opened until the remaining capacities are used up. Notably, the displacement costs of resources 1 and 2 ($\pi_1^H$, $\pi_2^H$) switch from array (¥470, ¥220) to array (¥410, ¥280) before the time period when the discount-fare class of AC closes. However, the sum of the displacement cost of two resources is ¥670 in both combinations during those time periods, which means that the booking requests of all products are accepted. In fact, the control strategy does not change during those time periods. The frequent switch can
Figure 7: Partitioned allocation and displacement cost of flight A1.

Figure 8: Partitioned allocation and displacement cost of flight A2.
be attributed to the increasing difference in remaining capacity. For example, the consumption of resource 2 is faster than that of resource 1, and the changes of displacement cost reflect the increasing value of resource 2 and decreasing value of resource 1. By contrast, when the difference narrows, the displacement cost returns to array (¥470, ¥220).

Figures 7–9 demonstrate the optimised capacity control strategies of the airline and HSR. Based on these strategies, Figure 10 presents the whole sales process, in which a large marker denotes that more than 2 tickets of this product are sold in a time period. According to the control strategy, though customers of discount-fare classes arrive throughout the selling horizon, the airline rejects all booking requests of discount-fare class of AC and accepts booking requests of discount-fare class of AB and BC in the early and middle stage of the selling horizon. In the late stage of selling horizon, the airline mainly accepts the full-fare class booking requests. For HSR, the discount-fare classes are closed later than airline, especially for the AB-discount product. In the late stage of the selling horizon, HSR still accepts booking requests of discount-fare classes, which is different from the airline’s control strategy.

4.2. Model Performance in Different Scenarios. To further investigate the performance of the proposed dynamic capacity control model in different scenarios—a strategy combination of the airline and HSR—a comparison analysis is conducted, and the results are presented in Table 7. The strategies are described as follows.

S1: theoretically static optimal allocation based on complete predicted demand sequence without considering the demand diversion.

S2: theoretically static optimal allocation based on complete actual arrival sequence without considering the demand diversion.

FCFS: first-come-first-serve strategy. Operator accepts any booking request as long as the remaining capacities are available.

TA: current capacity control mechanism of HSR in China, in which tickets are allocated for each product without updating during the selling horizon.

DCCM: proposed model in this research, which considers the demand diversion.

The numerical experiment shows that this capacity control model with demand diversion achieves better performance on revenue maximisation. The airline achieves the highest revenue of ¥652,210 in the scenario that it applies DCCM* strategy and HSR applies TA strategy. Meanwhile, HSR achieves the revenue of ¥344,060. In this scenario, HSR only optimises the ticket allocation plan once, without any updates during the selling horizon, so the performance of HSR is weakened. HSR achieves the highest revenue of ¥344,610 among the three realistic strategies (i.e., FCFS,
DCCM, and TA) in the scenario when both airline and HSR apply the DCCM strategy. With the application of DCCM strategy, HSR improves its revenue compared to when it uses the TA strategy. Meanwhile, the airline achieves a revenue of ¥651,210—slightly lower than the revenue achieved in the scenario when HSR applies TA strategy. This result shows that the DCCM strategy achieves better performance for both the airline and HSR. However, highest revenue HSR achieved (with DCCM strategy) is lower than the theoretical revenue achieved in the scenario when both the airline and HSR apply S1 or S2 strategy. This phenomenon can be explained by the difference of resource supply and demand.

**Table 7: Revenue comparison.**

<table>
<thead>
<tr>
<th>Operator</th>
<th>ODIF</th>
<th>Strategies</th>
<th>S1</th>
<th>S2</th>
<th>DCCM</th>
<th>FCFS</th>
<th>DCCM+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airline</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>59</td>
<td>64</td>
<td>60</td>
<td>6</td>
<td>64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>64</td>
<td>67</td>
<td>61</td>
<td>13</td>
<td>65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>28</td>
<td>24</td>
<td>23</td>
<td>8</td>
<td>19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>158</td>
<td>162</td>
<td>167</td>
<td>171</td>
<td>167</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>28</td>
<td>37</td>
<td>16</td>
<td>33</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>84</td>
<td>79</td>
<td>94</td>
<td>79</td>
<td>94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total (¥)</td>
<td>643,840</td>
<td>627,760</td>
<td>651,210</td>
<td>554,230</td>
<td>652,210</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 8: Resource supply and demand.**

<table>
<thead>
<tr>
<th>Resource</th>
<th>Predicted demand</th>
<th>Supply</th>
<th>Actual sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airline</td>
<td>HSR Airline HSR Airline HSR</td>
<td>1 394 413 250 400</td>
<td>250 400</td>
</tr>
<tr>
<td>2</td>
<td>396 411 250 400</td>
<td>250 400</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>112 0 120 0 120 0</td>
<td>120 0</td>
<td></td>
</tr>
</tbody>
</table>

DCCM+ is used to differentiate airline’s performance with HSR’s different strategy.
According to Table 8, the airline’s predicted demand requires 394 units of resource 1, 396 units of resource 2, and 112 units of resource 3—the demand is about 1.5 times the supply, except for the case of resource 3. HSR’s predicted demand for resources 1 and 2 is 413 and 411, respectively—slightly higher than its supply. The demand intensity of HSR is inferior to that of the airline, though the absolute total demands are equivalent. The proposed DCCM achieves good performance for both the airline and HSR operators, especially in high demand scenarios. The DCCM improves revenue by 17.5% for airline compared with using the FCFS strategy. If HSR adopts the TA strategy, the DCCM improves the revenue by 17.7% for the airline compared with using the FCFS strategy.

4.3. Demand Diversion Effect. Compared with individual capacity optimisation, the demand diversion increases both the chance of acquiring more customers from competitors and the uncertainty of capacity control. Because the optimisation is based on predicted demand, demand diversion can be regarded as a disturbance to the planned control strategy. Table 9 compares the performance of DCCM with and without demand diversion. Column 3 and column 4 are the final sales of different products and revenues in the scenario that both the airline and HSR apply DCCM strategy with and without considering the diversion. When not considering demand diversion, the final revenue of the airline is ¥626,610—lower than the original model with demand diversion. HSR achieves a similar result insofar as the revenue for the no diversion scenario is lower than that of the scenario with diversion. Column 5 and column 6 compare the results in the scenario that the airline applies DCCM strategy and HSR applies TA strategy with and without diversion. With diversion, the airline achieves revenue of ¥652,210—higher than that without diversion. HSR achieves higher revenue ¥341,450 in the scenario with diversion than that without diversion. Therefore, demand diversion accelerates the flow of demand between operators and creates more revenue for both operators. The consideration of demand diversion captures the process that customers turn to their second or third option when their prior option is rejected. The numerical experiments show that this demand diversion effect provides more revenue for both the airline and HSR operators. In essence, this part of increased revenue is ignored if we do not consider the demand diversion effect.

4.4. Computational Efficiency. As a dynamic capacity control model, the control strategy is updated frequently within the remaining time periods. Though the duration of each individual optimisation is less than 1 s, the total time consumption increases significantly with multiple updates of bid prices. The computational efficiency is optimised as shown in Table 10. The computing time of the original model with update interval 1 is 1,327 s, which is regarded as the benchmark for other efficiency optimisations. Then, with the update interval set as 2, the computation time decreases to 624 s with identical revenues for both the airline and HSR, improving the computation time by 53%. When the update interval is set as 10, the computation time further decreases to 130 s; though this saves 90% of the computation time, the accuracy decreases. The revenue of the airline declines to
5. Conclusions

In this paper, we have discussed the development of a dynamic capacity control model (DCCM) for airline and HSR travel, while considering the demand diversion both horizontally and vertically. The DCCM uses the MNL model to describe customer arrival and choice behaviour, constructs multiple diversion matrices to describe the process of demand diversion behaviour, and solves the value function and optimal control strategy by making use of the DLP method. With frequent updates for bid prices, the dynamic capacity control process is generated. To test the performance of this model, a numerical experiment is conducted, in a simulation with random-generated customer arrival sequences from different submarkets; the model gives the optimal control strategy of each time period. By comparing the performance in different scenarios with different strategies, we are able to draw the following conclusions:

(1) The primary capacity control strategies vary significantly over the selling horizon. In the early stage, the majority of booking requests are accepted, although a certain fare class is always closed. In the middle stage, the key capacity control strategy centres around deciding when to close certain discount-fare classes to reserve space for full-fare class customers. Before the first discount-fare class is closed, it is convenient for the operators to update the capacity control strategies, replacing them with a longer interval without affecting the revenue. After that, operators should update their capacity control strategies more frequently because the customers from full-fare class arrive intensively in the middle and late stage of the selling horizon. In the later stage, the key capacity control strategy is to strike a balance between the full-fare class’s demands for short distance and long distance. Reopening some discount-fare class in a timely manner will compensate for the possible revenue loss brought about by potentially over-estimated demand of the full-fare class.

(2) Displacement cost is the foundation for generating the control strategy because it reflects the scarcity of resources. When one resource is consumed faster than another, its displacement cost will increase. Besides, the displacement cost increases with the reduction of the remaining time and capacity, which explains why the capacity control strategy usually closes the discount-fare classes in the middle stage of the selling horizon and keeps the full-fare classes open until the later stage. For operators, implementing capacity control with the bid price instead of partitioned allocation—even nested allocation—has the benefit of flexibility in adjusting the control strategy and it is easier to store with just price.

(3) The DCCM’s performance relies on the relationship of supply and demand. In peak season or hot market, demand exceeds supply, and thus the operator has more opportunity to accept the booking requests of full-fare classes and reject the requests of discount-fare classes. However, in slack season or unsaturated market, supply equals or exceeds demand; ergo, the operator is obliged to accept as many booking requests as possible, including the rejected requests from rivals. Therefore, the application of DCCM will facilitate more effective performance in a market with higher demand. In lower demand markets, though the DCCM still secures a more positive performance than FCFS or TA strategies (for HSR), the impact will decrease.

(4) The introduction of demand diversion brings more revenue for both airline and HSR operators. Conventional capacity control models only consider the optimisation within one operator and assumes that customers arrive and buy tickets. With demand diversion, the customers’ second choice, and even third choice, behaviour is captured and applied when their first choice is rejected by operators. In terms of the operator, demand diversion provides more opportunities to acquire customers, especially for operators facing lower demand. The comparison between scenarios with and without demand diversion shows that demand diversion brings more revenue for both airline and HSR operators; it is...
significant to employ demand diversion within the capacity control model to capture this ignored part of revenue.

The limitations of this research lie in two aspects. For one thing, the research scale is limited to the simplest network with one HSR and two flights. In reality, airlines operate a more complicated hub-and-spoke network across a region, while HSR operates multiple trains across lines. Designing a complex regional transportation network that is compatible with both air and HSR transportation and evaluating the proposed model’s performance in this complex network will be the new direction for this research. For the other, the parameters used in the numerical experiments are specifically based on the case of Beijing-Shanghai corridor—a 1,200 km long busy line with abundant demand. The model’s performance in other cases, such as insufficient demand and longer or shorter distance, merits similar analysis [31].

Data Availability
The data used to support the findings of this study are included within the article.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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