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Research Article

Development of a Theoretical Delay Model for Heterogeneous and Less Lane-Disciplined Traffic Conditions

Abdhul Khadhir [1,2] **Ashish Bhaskar** [1,3] **Lelitha Vanajakshi** [1,2] **and Md. Mazharul Haque** [2]

[1] *Department of Civil Engineering, Indian Institute of Technology Madras, Chennai, Tamil Nadu 600036, India*
[2] *School of Civil and Environmental Engineering, Queensland University of Technology, Brisbane, QLD 4000, Australia*
[3] *Science and Engineering Faculty, School of Civil and Environmental Engineering, Queensland University of Technology, Brisbane, QLD 4000, Australia*

Correspondence should be addressed to Ashish Bhaskar; ashish.bhaskar@qut.edu.au

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In developing countries with limited or no availability of traffic sensors, theoretical delay models are the most commonly used tool to estimate control delay at intersections. The traffic conditions in such countries are characterised by a large mix of vehicle types and limited or no lane discipline (Heterogeneous, Less Lane-Disciplined (HLLD) traffic conditions), resulting in significantly different traffic dynamics. This research develops a queueing theory-based theoretical delay model that explicitly incorporates HLLD traffic conditions' characteristic features like lack of lane discipline, violation of the First-In-First-Out rule, and a large mix of vehicle types. A new saturation flow-based Passenger Car Equivalent (PCE) estimation methodology to address heterogeneity and a virtual lane estimation approach to address lack of lane-discipline are proposed. The developed model shows 64% lesser error in average control delay estimation compared to the in-practice delay estimation models under HLLD traffic conditions. The developed model is used for signal optimisation under HLLD traffic conditions and reductions of up to 24% in control delay in comparison to the in-practice signal timing approach are observed. The study also highlights the significance of knowing the variation of delay in addition to average delay and presents a simple approach to capture the variation in delay.

1. Introduction

Delay is the primary performance measure used to evaluate and quantify the performance of signalised intersections. The prevalence of delay as the Level of Service (LoS) measure for signalised intersection can be attributed to the fact that, unlike other performance measures like queue length and number of stops, delay is easily understandable by technical and nontechnical people alike. In addition, methods using aggregated network performance measures like Network Macroscopic Fundamental Diagram (NMFD) parameters are used by researchers for arriving at optimal signal timings [1, 2]. However, most of the conventional methods of signal design are based on minimising total intersection specific delay. A signal timing plan which minimises the overall intersection delay is generally chosen as the optimal signal timing plan. Hence, estimating delay is of high significance from both planning and operations perspectives.

Researchers have attempted different methods to estimate travel time and delay on urban arterials. An overview of the different methods available for delay estimation is presented in Section 3. Though diverse methods of delay estimation are reported in the literature, when the data availability is limited, delay estimation using theoretical models is widely used. Delay estimation is achieved using theoretical equations derived based on queuing theory, shockwave theory, or based on assumptions regarding the distribution of vehicle arrivals and departures. These theoretical delay models use traffic signal information and the prevailing traffic conditions as inputs and give a theoretical delay estimate.

Most of the conventional theoretical delay models reported in the literature are developed for homogeneous, lane disciplined traffic conditions. Researchers [3, 4] have shown that conventional delay models do not give a realistic estimate of delay under Heterogeneous, Less Lane-Disciplined (HLLD) traffic. This can be attributed to the characteristics of HLLD traffic conditions, such as a large mix of vehicle types in the traffic stream and poor or no lane discipline, leading to significantly different traffic dynamics. Due to the large mix of vehicle types with varying sizes, smaller vehicles seep through the gaps in between larger vehicles, thereby violating the first-in-first-out queue discipline. The lack of lane discipline further leads to multiple parallel movements within and across the lanes. Such behaviours are vastly different to those observed in homogeneous, lane-disciplined traffic conditions. Despite this, these conventional delay models are still used in countries with HLLD traffic conditions. The first part of this research evaluates the performance of existing delay models in HLLD traffic conditions and then develops a suitable delay estimation model specifically for such traffic conditions, overcoming the existing models' shortcomings. Unlike most other studies attempting to develop a delay model for HLLD traffic by performing mere calibration of conventional delay equations, this research aims to develop a theoretical delay model for such traffic considering the characteristic features of the HLLD traffic in the derivation. The developed delay model can be used to arrive at optimal signal control strategies for HLLD traffic conditions and be used as a tool to analyse the performance of signalised intersections. It is to be noted that the term HLLD used in this paper is synonymous with the terms heterogeneous or mixed and less lane-disciplined traffic conditions used in many other papers in the related literature.

In addition to the average delay experienced by vehicles, knowing the intervehicle variation of delay within a signal cycle is also crucial [5, 6]. The intervehicle variation in delay is more prominent under HLLD traffic conditions. This can be attributed to the fact that smaller vehicles percolate the gaps between larger vehicles due to the large mix of vehicle types, resulting in an inequitable distribution of delay among vehicle types. Hence, the study attempts to develop a model for the variability of delay in addition to the average delay.

The remainder of the paper is organised as follows. Section 2 presents the current state of the art in theoretical delay modelling. Section 3 provides the preliminary analysis adopted to evaluate the applicability of commonly used theoretical delay models reported in the literature, under HLLD traffic conditions. Section 4 elaborates the methodology adopted for developing a delay model for the HLLD traffic conditions. Section 5 implements and validates the developed delay model for HLLD traffic conditions. Section 6 presents an application of the developed theoretical delay model and Section 6 concludes the paper.

2. Literature Review

A common and simple method adopted in the literature for delay estimation is the use of arrival and departure counts of vehicles [7]. The area between the departure and the arrival cumulative count curves gives the total travel time of all the vehicles. When the arrival curve is shifted by the free-flow travel time, the area between the departure cumulative curve and the shifted arrival cumulative curve yields the total delay. However, the cumulative count-based method requires accurate arrival and departure information, as the errors in the cumulative count would result in significant shifting of the cumulative curves, which in turn results in erroneous delay estimates. Further, the delay estimates are sensitive to the information about the initial number of vehicles present within the section. To overcome these limitations, researchers have proposed different modifications, varying from manual adjustment [8] to probe data-based adjustment [9] of the cumulative count curves. Also, other analytical methods like Incremental Queue Accumulation (IQA) have been reported in the literature for delay estimation [10].

Researchers have also relied on dynamic approaches for delay estimation using state space modelling and filtering techniques [11–14]. Researchers have also used more data-intensive techniques like time series analysis, k-means clustering, and other artificial intelligence based solutions for delay estimation [15–20]. Researchers have also employed various metaheuristic approaches like genetic algorithm, differential evolution, harmony search, and artificial bee colony for estimating and optimising intersection delay [21–23]. The major limitation of all the methods listed above (cumulative count-based method, analytical methods, metaheuristics, and artificial intelligence-based data driven approaches) is that these methods require good estimates of the traffic demand data at reasonably high frequency to yield good delay estimates. This requires the use of location-based sensors, like loop detectors. However, such sensors are intrusive to the pavement and might render their installation and maintenance challenging in developing countries, where such sensors are not prevalent on existing roads. Also, the traffic conditions in developing countries, like India, are characterised by a large mix of vehicle types and lack of lane discipline, resulting in the occurrence of multiple parallel movements. Due to the heterogeneity and the lack of lane discipline, many commonly used location-based sensors like loop detectors, infrared, ultrasonic and microwave-based sensors are difficult to use in these developing countries [24].

In locations where location-based sensor data is not available, researchers have used probe vehicle data-based delay estimation techniques. Researchers have used GPS data [25, 26], cellular device data [27–29], Bluetooth data [30–32], and Wi-Fi sensors data [33–37] as probe data for estimating delay under conditions where location-based data is not available. Though the probe-based approaches yield good estimates of delay even on roads with no location-based sensors, they require additional sensors like GPS sensors, Bluetooth sensors, or Wi-Fi sensors. Such sensors are not readily available in most of the intersections in the developing countries. Further, even if the intersections are equipped with such sensors, the below par penetration rates of such sensors make delay estimates obtained unreliable.

Considering the above limitations, under conditions where data availability is limited, delay estimation using theoretical models is widely used. Delay estimation is achieved using theoretical equations derived based on queuing theory, shockwave theory, or based on assumptions regarding the distribution of vehicle arrivals and departures. These theoretical delay models use traffic signal information and the prevailing traffic conditions as inputs and give a theoretical delay estimate.

The pioneering work on theoretical delay estimation was proposed at the 7th annual meeting of the Highway Research Board in 1928, which had an assumption of deterministic arrivals [38]. Over the last century, delay estimation methods have evolved drastically to obtain more realistic delay estimates. The evolution can be grouped under three generations and is discussed below.

2.1. First-Generation Models. This family of delay models was based on queueing theory and assumed a particular distribution for the arrival rate. The arrival rate distributions assumed by many of the early models were the uniform distribution [38–42]. The uniform arrival assumption was found to be unrealistic as vehicles arrive randomly. To account for this randomness of arrivals, researchers used the binomial vehicle arrivals assumption [43–46]. Researchers also tried to model the vehicle arrivals to the intersection as a Poisson process. Kendall [47] was the first to propose a Poisson arrival-based average waiting time model for a random arrival, uniform service queueing system. A significant contribution in theoretical delay estimation models came from Webster [48]. The author used Kendall's model in the context of the signalised intersection queues by adding a random delay term to the uniform delay term to get the total delay per vehicle. This randomness was due to vehicle arrivals following a Poisson process. The author further added an empirical term obtained by simulating the traffic at a signalised intersection. The additional term usually ranged between 5% and 15% of the sum of the first two terms. Thus, if the sum of the two terms is denoted by "d," an approximation of $0.9d$ was shown to give the average delay [48]. Little [49], Darroch [50], and Robertson [51] also used the Poisson process assumption and further modified the Webster model. Researchers [41, 46, 52] also used a general arrival distribution to develop theoretical delay models. The term I, known as the variance-to-mean ratio, to take care of the stochasticity in the arrival process, was introduced. When I was close to one, Webster's model and the new model were shown to be in close agreement. But, as I became more than one, Webster's model was found to underestimate delay.

Out of the first-generation models, Webster's is the most commonly used delay estimation model. Despite other models like Newell's [41] being more accurate, Webster's model is widely used because of its algebraic simplicity without compromising accuracy. Overall, the first-generation models have the following shortcomings: (i) They overestimated delay at higher degrees of saturation; (ii) Resulted in unrealistic delay (infinite) when the degree of saturation reaches one; (iii) Failed to capture the transition in delay values between under and oversaturated conditions; and (iv) Assumed a steady-state traffic condition, which was unrealistic.

2.2. Second-Generation Models. Generation 2 models tried to overcome the limitations of Generation 1 models. These were predominantly time-dependent models. Time-dependency was introduced to overcome the inability of Generation 1 models to capture the transition between oversaturated and undersaturated conditions. The coordinate transformation technique was one of the techniques used to achieve this [53–56]. Researchers also attempted multistep analysis by splitting the time interval to be analysed into small stationary intervals and estimating the delay for each small time interval [57]. Researchers considered traffic intensity variation during peak hours by utilising a Peak Hour Factor (PHF). Many researchers attempted to improve the accuracy of the commonly used time-dependent Highway Capacity Manual (HCM) model of that time [55, 58]. Many of these suggestions were incorporated in the revised HCM. Use of Progression Factor (PF) to consider the effect of upstream signal on the arrival rate at a downstream intersection was also attempted in the second-generation delay estimation models [51, 59–63]. The findings from these studies shaped the use of PF in the different generations of HCM models.

The second-generation delay models were in general able to overcome most of the limitations of the first-generation models. However, these models failed in estimating delays for specific traffic conditions like queue spillback, turn lane clogging, etc. Though the introduction of PF tried to incorporate the reduction in delay due to signal coordination, it failed to give realistic results in conditions when initial queues were present.

2.3. Third-Generation Models. Newer models have come up since 2000 to overcome the shortcomings of Generation 2 models. The assumption of uniform arrival rates in the uniform delay term in HCM was inaccurate if intersections were present upstream, as different approaches have different vehicle arrival rates, resulting in a multifold arrival curve.

Benekohal and El-Zohairy [64] presented a new model accounting for various arrival types and eliminated the need for PF. Strong and Rouphail [65] proposed the Incremental Queue Accumulation (IQA) method, which used realistic arrival departure curves, thereby eliminating the need for PF. This method was incorporated in 2010 HCM as an alternative to the formulae-based approach. Ahmed et al. [66] and Xu et al. [67] studied delay estimation for the queue-spillback condition. Van Leeuwaarden [68] extended the idea of general arrival process by deriving probability generating functions for delay and queue length for different arrival processes. From the probability generating functions, the mean as well as variance of delay and queue length were estimated. Pacheco et al. [69] proposed formulae for computing the mean and variance of queue length and waiting time on a M/D/1 queuing system considering server vacations. The autors showed that considering the fixed time signal control problem as a server vacation problem increased the accuracy of the delay estimates compared to Webster's model and HCM delay model. The method showed higher superiority when the degree of saturation was less than 70%. Researchers also used Artificial Neural Networks (ANN), Fuzzy logic, and other Artificial Intelligence- (AI-) based techniques to estimate nonuniform delay for oversaturated conditions [16, 70–74]. These AI-based approaches performed better in comparison to the HCM 2000 model. The above discussed models were formulated for homogeneous and lane-following traffic conditions and hence have limited or no applicability under HLLD traffic conditions. Researchers have attempted to adapt the conventional theoretical delay models to better suit HLLD traffic conditions. Such studies are discussed next.

2.4. Delay Estimation Models for HLLD Traffic Conditions. All the models discussed above were derived assuming homogeneous traffic with lane discipline and may not be suitable for heterogeneous traffic conditions with less or no lane discipline. Arasan and Jagadeesh[75] developed a probabilistic method using the concepts of a first-order, second-moment method for estimation of saturation flow. To consider the characteristics of HLLD traffic, the intercorrelation between vehicle types was considered in the saturation flow rate estimation. Except for the saturation flow rate estimation, the study used Webster's delay estimation model as the basis for estimating delay under HLLD traffic conditions. Hoque and Imran[3] proposed modifications to Webster's model by adding a third term as Webster's adjustment derived using multilinear regression. Akgüngör [4] reported a study under heterogeneous (HLLD) traffic conditions and proposed a methodology to identify delay parameters to be included in the delay model. Minh et al. [76] estimated Passenger Car Unit (PCU) using multilinear regression, based on which average headways and saturation flow rate were calculated. The third term in Webster's delay model was modified using the saturation flow distribution, which was shown to be normally distributed. Preethi et al. [77] proposed a semiempirical adjustment term to Webster's model, based on field observations using Artificial Neural Network (ANN). GPS data was used for traffic stream parameters, and delay components were estimated with different field adjustment factors for different conditions. A major drawback of the all the above models for estimating delay under HLLD conditions was that these models were based on Webster's delay model, which was derived based on the assumption of homogeneous, lane-disciplined traffic conditions. Though the studies tried to adapt the Webster model to HLLD traffic conditions by introducing calibration constants and modification factors, the underlying limiting assumptions about the traffic conditions used in the derivation of the Webster model were not relaxed. Anusha et al. [13] proposed a hybrid data fusion-based approach for estimating urban arterial travel times using a hybrid method employing the HCM delay equation and Kalman filtering technique. Verma et al. [78] modified Webster's delay model by comparing it with delay values from the HCM field delay estimation method. The assumption of the M/D/1 queueing system in Webster's method was relaxed to an M/M/n queueing system to represent HLLD traffic better. Though the study tried to relax the assumption regarding the dissipation process, the authors failed to quantify the impact of the Probably First in Probably First Out (PFIPFO) approach that the study proposed as a replacement to the First-In-First-Out (FIFO) assumption. The developed model was validated using a field delay measurement technique proposed in HCM 2000. However, the applicability of the HCM delay measurement method under HLLD traffic conditions is limited.

From the above literature review, it is evident that most of the theoretical delay models reported were formulated for homogeneous and lane following traffic conditions, and their applicability under HLLD traffic conditions is limited as assumptions regarding the vehicle arrivals, the queueing discipline, and so on are violated. Thus, a straightforward adaptation of the conventional delay models to HLLD traffic would yield inaccurate and unrealistic delay estimates. Such incorrect delay estimates would result in suboptimal operation, management, and control of intersections. A few studies that attempted to estimate delay under HLLD traffic conditions by modifying the conventional delay models are highly site-specific. Other than mere calibration of the delay models developed for homogeneous conditions, the literature lacks a model incorporating the characteristic features of heterogeneous and less lane disciplined traffic.

Overcoming the above gaps in the literature, this research significantly advances the delay estimation for HLLD by explicitly incorporating the characteristic features of HLLD traffic conditions, like lack of lane discipline, violation of FIFO rule, and a large mix of vehicle types. To address the issue of heterogeneity of vehicle types, the study relies on a robust Passenger Car Equivalent (PCE) estimation approach based on the concept of equalising saturation flow across cycles. To tackle the effect of multiple parallel movements prevalent due to the lack of lane discipline, the study proposes the usage of virtual lane concept and multiserver queuing systems. The research also provides a model for estimating the variability of delay, in addition to the average delay model. Moreover, the proposed model requires minimal site-specific calibration, making it easy for field implementation.

3. Preliminary Analysis

Before developing a new model for HLLD traffic conditions, it is essential to evaluate the performance of some of the commonly used delay models under HLLD traffic conditions. Hence, this study starts with an evaluation of some of the conventional and in practice delay models under HLLD conditions. Based on the findings from this evaluation, a new

Table 1: List of notations.

Notation	Description
Signal control and traffic demand variables	
g	Effective green time for a phase (seconds)
R	Effective red time for a phase
C	Cycle length (seconds)
λ	Green time to cycle time ratio (g/C)
v	Traffic arrival rate (PCE per hour)
s	Saturation flow rate (vehicles per hour or PCE per hour)
c	Capacity (vehicles per hour or PCE per hour), $c = sg/C$
X	Volume-to-capacity ratio, $X = v/c$
Conventional theoretical delay model variables	
d	Total average control delay (seconds per vehicle or seconds per PCE)
d_1	Uniform delay (seconds per vehicle or seconds per PCE)
d_2	Random delay (seconds per vehicle or seconds per PCE)
PF	Progression factor
q	Traffic demand (vehicles/hour or PCE/hour)
k	Adjustment factor for type of controller
I	Upstream filtering/metering adjustment factor
T	Analysis period (hours)
$Q(0)$	Size of the initial queue at the start of the analysis period T (vehicles)
u	Delay parameter
X_0	Practical degree of saturation
Variables related to PCE estimation	
$count_c$	Observed number of cars in a cycle
$count_i$	Observed number of vehicles of class i
PCE_i	Passenger car equivalent corresponding to vehicle class i
Queuing theory-related variables	
ρ	Ratio of arrival rate to departure rate
n	Number of service channels/number of virtual lanes
CV_λ	Coefficient of variation of the arrival distribution
CV_μ	Coefficient of variation of the departure distribution
Additional variables	
f	Multiplicative adjustment factor for the theoretical delay equation
IFR	Intersection flow ratio
DSR	Demand split ratio
d_{StDev}	Standard deviation of average control delay (seconds per PCE)
MAE	Mean absolute error (seconds per PCE)
$MAPE$	Mean absolute percentage error (%)

model will be developed to cater to the characteristics of HLLD traffic conditions. For the ease of understanding, a list of all the notations used in this paper along with their descriptions is provided in Table 1.

3.1. Study Site and Data Collection. As getting accurate delay estimates from the field is cumbersome, a microsimulation-based delay estimation approach is adopted. Traffic demand data per signal cycle and the corresponding signal timings adopted in the field are extracted from video recordings of the southbound approach of a T-intersection located on the Rajiv Gandhi IT corridor, Chennai, India, as shown in Figure 1. The southbound approach has three lanes.

Traffic police manually control the signal timings in the field in accordance with their perception of traffic demand. Hence, the signal timings vary from cycle to cycle. The traffic demand and signal timing extracted from the video are presented in Table 2. The classified vehicle counts were also extracted from the video. The average vehicle composition was found to be 44.7% cars, 47.6% two-wheelers (motorbikes), 5.4% three-wheelers (auto-rickshaws), and 2.3% heavy vehicles (buses and trucks).

3.2. Evaluation of Conventional Delay Models under HLLD Traffic Conditions. Webster's delay model [48, 79], Akcelik's delay model [54], and the Indo-HCM delay model [80] are selected for the performance evaluation under HLLD conditions. Equations (1) to (4) show the respective delay equations of these models. The first three models are chosen as they are the most used delay models by practitioners and researchers worldwide, whereas the fourth was selected as it is suggested for Indian HLLD traffic conditions. It can be noticed that all these model equations have the same basic structure. The first term, known as the uniform delay term, is the same for all the models. The second term, known as the random delay term, is arrived at differently in each delay

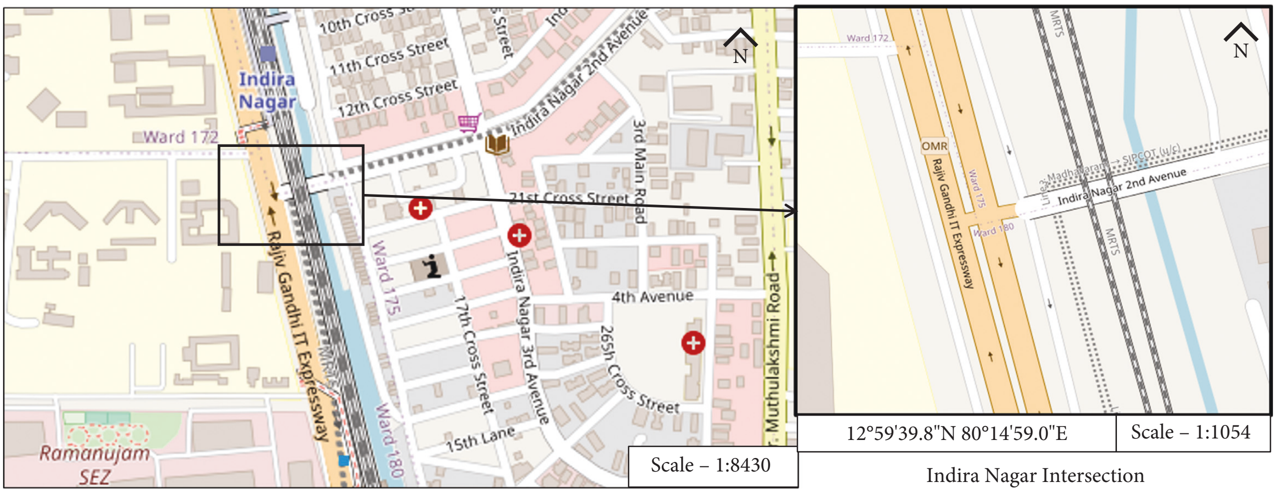

Figure 1: Location of the study intersection.

Table 2: Traffic demand and signal timings (from the field).

Cycle no.	Cycle time (s)	Green time for the southbound approach (s)	Approach demand for the southbound approach (veh/hr)
1	58	34	4097
2	117	47	3385
3	83	49	4381
4	153	75	3106
5	87	60	3310
6	143	53	3827
7	83	42	2949
8	105	28	3223
9	202	108	3778
10	170	87	2965
11	151	54	2551
12	147	64	2890
13	77	13	2899
14	125	63	2851
15	160	62	3803
16	103	46	3460
17	157	65	3875
18	139	63	3755
19	127	65	3798
20	114	60	3789
21	134	35	3573
22	134	57	3976
23	110	58	2978
24	138	36	3443
25	149	50	4277
26	132	58	4091
27	123	43	2985
28	119	48	3025
29	79	43	3919
30	146	43	3575
31	142	59	4183
32	144	57	3825
33	152	56	3813
34	138	56	3365
35	105	55	4594
36	145	51	3898
37	165	50	3033
38	115	57	3850
39	205	71	3600
40	151	53	2551

model. For the HCM delay model and Indo-HCM delay model, this term is very similar, except for the use of a constant PF value of 0.9 in the case of the Indo-HCM delay model.

$$d = \frac{C(1-\lambda)^2}{2(1-\lambda X)} + \frac{X^2}{2q(1-X)} - 0.65\left(\frac{C}{q^2}\right)^{1/3} x^{2+5\lambda}, \tag{1}$$

$$\begin{aligned} d = {} & \frac{C(1-\lambda)^2}{2(1-\min(1,X)\lambda}(PF) \\ & + 900T\left[(X-1) + \sqrt{(X-1)^2 + \frac{8kIX}{cT}}\right] \\ & + \frac{1800Q(0)(1+u)t}{cT}, \end{aligned} \tag{2}$$

$$\begin{aligned} d = {} & \frac{c(1-\lambda)^2}{2(1-\lambda x)} \\ & + 900T\left[(X-1) + \sqrt{(X-1)^2 + \frac{12(X-X_0)}{cT}}\right], \end{aligned} \tag{3}$$

$$\begin{aligned} d = {} & \frac{c(1-\lambda)^2}{2(1-\min(1,X)\lambda}(0.9) \\ & + 900T\left[(X-1) + \sqrt{(X-1)^2 + \frac{8kIX}{cT}}\right] \\ & + \frac{1800Q(0)(1+u)t}{cT}. \end{aligned} \tag{4}$$

To compare the performance of the chosen delay models, the delay estimates obtained from these theoretical delay models must be compared with the delay estimates from the field. As such data about intersection delay for each cycle is not easily measurable from field, the traffic conditions presented in Table 2 were simulated to obtain the equivalent ground truth delay values. As microscopic simulation helps to better reproduce the traffic dynamics [81] compared to macroscopic models, a microsimulation model was used to estimate ground truth delay values. The signal timings and traffic demands (Table 2) are given as inputs to a calibrated network in VISSIM version 11 [82]. As the driver behaviour under HLLD traffic conditions are drastically different from those under homogeneous and lane disciplined conditions, the microsimulation model must be calibrated. For instance, unlike in lane-based traffic, under less lane-disciplined traffic conditions drivers can choose among an infinite number of alternative travelling paths and speeds [83]. This makes the traffic dynamics under HLLD traffic conditions different from the traffic conditions with good lane discipline. For the current study, the calibration is performed based on the studies reported in the literature [84–86]. A summary of the calibrated parameters used is presented in Table 3.

To validate the calibrated VISSIM microsimulation model, field travel time values were obtained using a probe-based data source for the same data collection duration reported in Table 2. The average field travel time estimated for the study duration of 40 signal cycles was 103.45 seconds. The corresponding simulated travel time was obtained using the "Vehicle Travel Time Measurement" tool in VISSIM. The simulated average travel time was obtained as the average of five simulation runs, in which a mean absolute percentage error (MAPE) of 9.26% was shown. This indicates that the calibrated VISSIM microsimulation model can replicate the real-world HLLD traffic with reasonable accuracy.

Once the VISSIM model was calibrated and validated, five simulation runs were carried out for calculating the average control delay for each cycle. The average control delays obtained are compared with the average control delay values calculated using the delay models chosen for performance evaluation (Webster's, HCM, Akcelik's, and Indo-HCM delay models). The comparison of the cycle-by-cycle delay estimates of each of the delay models with the delay obtained from simulation is presented in Figure 2.

From Figure 2, it can be seen that none of the delay models can closely match the delay estimates from the simulation. Thus, these models are not adequate to estimate delay under HLLD conditions. The inadequacy of the conventional delay models to accurately estimate delay under HLLD traffic conditions can be attributed to the lack of consideration of the characteristic features of the HLLD traffic in the model development stage. Addressing these characteristic features of HLLD traffic in the present study is discussed next.

3.3. Challenges in HLLD Traffic. The major challenge in dealing with HLLD traffic is the need to incorporate the following characteristic features into the modelling framework.

(a) The vehicle stream consists of random arrival of vehicle types of varying size and dynamics.

(b) The vehicles do not follow lane discipline and thus do not obey the FIFO queue discipline. Smaller vehicles that arrived later can squeeze through the stopped vehicles and reach the front of the queue and can get serviced first [88, 89].

(c) Vehicles discharge in parallel without following lane discipline.

To cater to the heterogeneity of the vehicle stream, a new passenger car equivalent (PCE) calculation is proposed (in Section 4.1). The lack of lane discipline issue is addressed using the concept of multiple queue-serves in the same link using virtual lane concept and the Service in Random Order (SIRO) queue service discipline (details in Section 4.2). SIRO is chosen because, under HLLD conditions, vehicles overtake other vehicles in the same lane, and as a result, the FIFO queue discipline is violated. Also, control delay cannot be estimated for each lane separately as there are multiple parallel movements. The usage of multiserver queues and virtual lane concept is also discussed in Section 4.2.

Table 3: Summary of the calibration parameters used from literature [87].

S. no.	Parameter	Default value	Calibrated value
Driving behaviour parameters			
Look-ahead distance (m)			
1	Minimum	0	127.79
2	Maximum	250	500
3	Number of observed vehicles	4	7
Look-back distance (m)			
4	Minimum	0	50
5	Maximum	150	150
Wiedemann's 74-car following model			
6	Average standstill distance (m)	2	1.35
7	Additive part of safety distance	3	0.25
8	Multiplicative part of safety distance	3	0.35
Lane change			
9	Waiting time before diffusion (sec)	60	50
10	Minimum headway (min)	0.5	0.3
Lateral behaviour			
11	Desired position at free flow	Middle	Any
12	Diamond-shaped queuing	No	Yes
13	Consider next turning direction	No	Yes
14	Overtake on the same lane	No	On left and right
15	Minimum lateral distance at 0 and 50 kmph		
	Two-wheeler	1, 1	0.1, 0.3
	Three-wheeler	1, 1	0.3, 0.4
	Four-wheeler	1, 1	0.4, 0.6

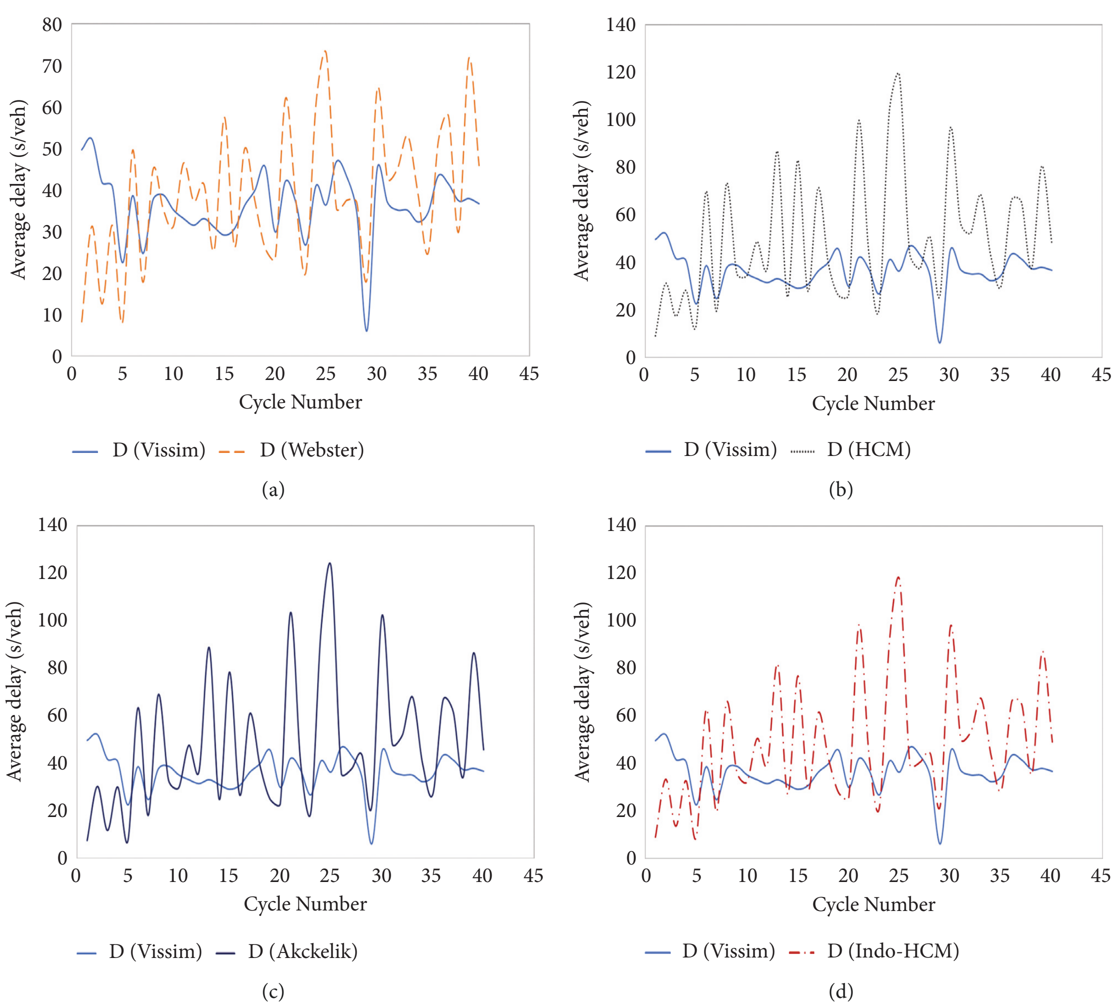

Figure 2: Comparison of delay estimates from delay models and simulation. (a) Webster's model (MAPE = 64.8%). (b) HCM model (MAPE = 79.9%). (c) Akcelik's model (MAPE = 80.2%). (d) Indo-HCM model (MAPE = 78.8%).

4. Methodology

The overall framework for developing a theoretical delay model for HLLD traffic conditions begins with a discussion on the methodology to tackle the two main characteristic features of HLLD traffic—heterogeneity and lack of lane discipline. These are discussed in detail in the following sections.

4.1. Addressing Heterogeneity. Though different approaches for PCE estimation based on vehicle size, vehicle dynamics and other performance measures (flow, travel time, etc.) are reported; they may not be the best for PCE estimation near intersections. Most of the PCE estimation methods for signalised intersections under HLLD traffic conditions reported in the literature were based on headway, delay, travel time, and queue discharge rate. These parameters are susceptible to queue formation and discharge patterns and vehicle acceleration and deceleration characteristics. Saturation flow-based PCE estimation is a viable alternative as saturation flow of an intersection is more of a characteristic of the intersection. Though researchers have attempted a variant of saturation flow-based PCE estimation, those methods relied on minimising the difference between the observed saturation flow and an ideal saturation flow. The current study proposes a pure saturation flow-based PCE estimation where the difference between the intercycle saturation flow values is minimised. The methodology adopted is shown in a flowchart form in Figure 3.

The methodology relies on the idea that irrespective of the vehicle composition, the saturation flow of an intersection should be constant. The saturation flow rate (expressed in PCE/hr) is a property of the intersection and should not be affected by vehicle composition. A set of PCE values that gives constant saturation flow values across different cycles is found by solving it as an optimisation problem. To ensure saturation flow throughout the green time, the demands presented in Table 1 were increased by 30%. The simulations were run for two different scenarios: (i) passenger car only and (ii) mixed traffic. For scenario (i), the demand is assumed to constitute only cars, whereas scenario (ii), the actual vehicle composition observed, is taken. The simulations are run for these two scenarios separately, and the discharge counts for each cycle were collected for both cases. As the intersection is saturated, the discharge count can be taken as the saturation flow. The counts from scenario (ii) are multiplied with the a priori estimate of the PCE values and compared with those from scenario (i). The sum of absolute differences of the two counts for all the cycles is taken as the objective function (see equation (5)).

$$z = \text{Minimize} \sum_{\text{allcycles}} \left\{ abs\left[\text{count}_c - \sum_i \text{count}_i * PCE_i \right] \right\}. \quad (5)$$

As PCE values cannot be negative, a nonnegativity constraint is applied in solving the minimisation problem (i.e., $PCE_i > 0$). Also, the PCE of a car is taken as one as it is the reference vehicle type.

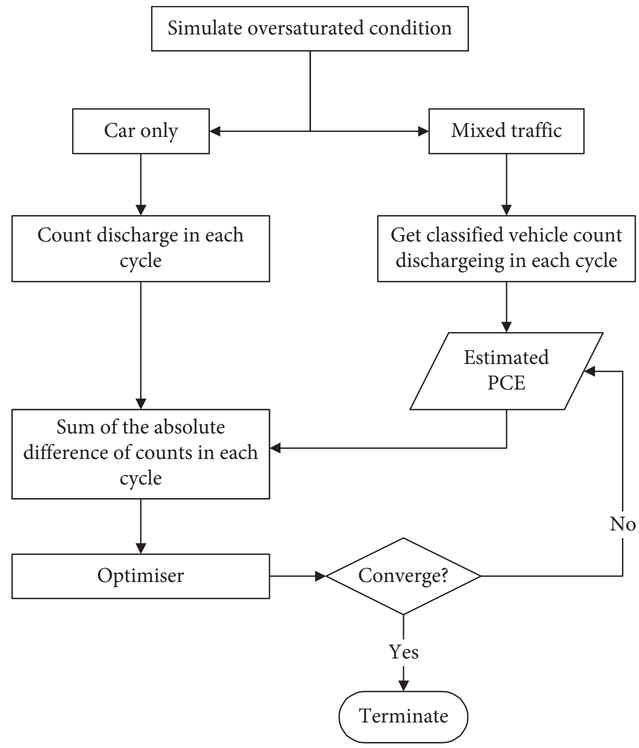

FIGURE 3: Methodology for saturation flow-based PCE estimation.

The optimisation problem was solved using a trust-region constrained algorithm [90] available in the SciPy Python library. The maximum number of iterations was fixed at 500, and the tolerance for the difference between the objective function values in successive iterations was taken as 0.001. The initial values for PCE were chosen as per the Indo-HCM [80] values, as shown in Table 4, column 2. The optimiser terminated after 78 iterations. The optimal PCE values for all vehicle types on termination are also presented in Table 4. It can be observed that, while the optimum PCEs are higher than the Indo-HCM Table 5 values for all categories, Three-Wheeler (3W)PCE valuewas found to bemore than that of cars. This may be due to the inferior vehicle acceleration-deceleration dynamics of most three-wheelers (auto-rickshaws) in comparison to a standard passenger car. These PCE values are used for further analysis.

4.1.1. Effect of Lane Configurations on PCE Values. Due to the combined effect of heterogeneity and lack of lane discipline, the PCE values may change with the increase or decrease in the number of lanes. Thus, to check the sensitivity of the estimated PCE values, the methodology was repeated for a four-lane and a two-lane scenario. The traffic demands were proportionally adjusted to ensure the same per lane demand.

Table 5 indicates the optimal PCE estimates do not drastically change with the increase or decrease in the number of lanes.

Table 4: Optimal PCE values.

Vehicle type	Initial PCE (Indo-HCM)	Optimal PCE
Car	1	1
2W	0.4	0.78
3W	0.5	1.92
HV	1.6	3.42

Table 5: Sensitivity of PCE estimates to the number of lanes.

Vehicle type	Optimal PCE (3 lanes)	Optimal PCE (4 lanes)	Optimal PCE (2 lanes)
Car	1	1	1
2W	0.78	0.75	0.72
3W	1.92	1.93	1.88
HV	3.42	3.44	3.46

Table 6: Variation of parallel movements during queue dissipation.

Cases	Parallel movements					Parallel servers
	Two wheelers	Three wheelers	Cars	Total vehicles	Total PCEs	
Beginning of queue	5	0	1	6	4.92	5
Middle of queue	2	1	1	4	4.49	5
End of queue	1	1	2	4	4.70	5

4.2. Addressing Lack of Lane Discipline. Due to the presence of parallel movement, unlike the lane disciplined traffic, the number of service channels would not be the same as the number of lanes. To find the number of service channels, a virtual lane concept is used. Based on this, the number of channels is taken as the number of vehicles passing the stop bar simultaneously at a given time. As the composition of vehicles passing the stop bar at every time instant is not the same, the number of vehicles passing the stop bar at different time instants during the queue dissipation process would also be different. To get an average value of number of service channels, the average number of parallel movements at three different time instances during the queue dissipation process is observed from the simulation. The three time instances considered are (i) beginning of queue dissipation (5th second from the beginning of green), (ii) middle of the queue (30% of the green time), and (iii) end of queue dissipation. The number of parallel PCE movements was measured for these three cases and are shown in Table 6. Based on these numbers obtained, the number of service channels is taken as 5.

The present study uses a queuing theory-based estimation of random delay. As the study assumes an undersaturated intersection, the overflow term is neglected. The SIRO queue discipline is considered as a better representation of the queuing process under HLLD conditions. However, when arrivals are memoryless (Poisson's arrivals) with independent and identically distributed (IID) service times, the mean waiting time is the same irrespective of the queue discipline [91]. Thus, the assumption of FIFO queue discipline is valid under HLLD traffic for average waiting time estimation. But the overall distribution of waiting time changes with the change in queue discipline. This implies that, though the mean waiting time (average delay) is the same, the delay variation would change with a change in queue discipline.

Using the above solutions to address heterogeneity and lack of lane discipline, formulation of the complete delay model for HLLD traffic considering its characteristic features explicitly in the model formulation stage was done as discussed next.

4.3. Model Formulation. The basic structure of any theoretical delay model has a uniform delay term, a random delay term, an overflow delay term, and other adjustment terms specific to the respective models [92]. The current formulation also follows the same structure. The first term corresponding to the uniform delay, which is the delay component based on the assumption of uniform vehicle arrivals with no individual cycle failures, is discussed first. This is followed by aqueuing theory-based estimation of random delay. As the study assumes an undersaturated intersection, the overflow term is neglected.

4.3.1. Uniform Delay. Compared to a delay model for homogeneous traffic conditions, the additional factor to be considered while using the first term (d_1 term or uniform delay term) for HLLD traffic condition is the representation of both arrival and departure flows in common units, taking into account the composition. For this, the method discussed in Section 4.1 to estimate PCE factors is used in the formulation.

Figure 4 shows cumulative arrival-departure curves for a saturated intersection. From Figure 4, total uniform delay can be obtained as the area of the triangle enclosed by the arrival and departure curves as given in the following equation:

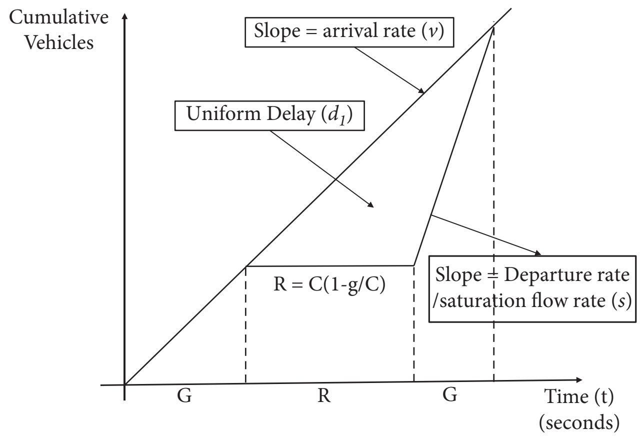

FIGURE 4: Components of delay-uniform delay.

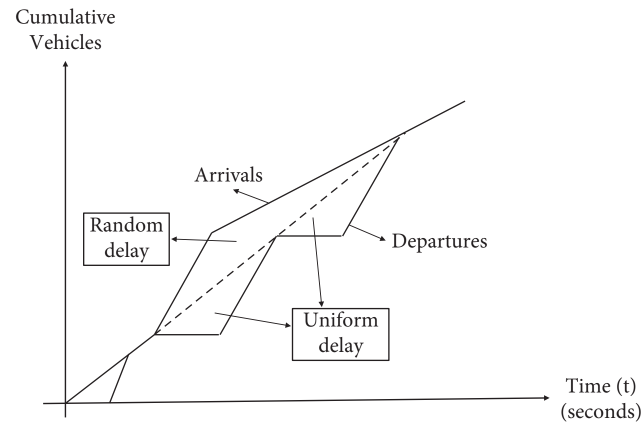

FIGURE 5: Components of delay-random delay.

$$\text{average uniform delay}, d_1 = \frac{C}{2}\frac{(1-(g/C))^2}{(1-(v/s))}. \tag{6}$$

Substituting $g/C = \lambda$ and $X = v/(sg/c)$ in equation (6), we get

$$d_1 = \frac{C(1-\lambda)^2}{2(1-\lambda X)}. \tag{7}$$

4.3.2. Random Delay. The second term corresponds to the random delay term to account for the additional delay, above and beyond uniform delay, due to random fluctuations in the arrivals. In Figure 5, the dotted line indicates the capacity flow rate. The area enclosed by the arrival curve and this dotted line is the random delay component.

Researchers have employed different techniques to estimate the random delay component, including queuing theory-based approaches, coordinate transformation techniques, and probabilistic approaches. The most commonly used approach to estimate the random delay component is the queuing theory-based approach. Based on this, the random delay term (d_2) is derived based on queuing theory concepts considering the system as an $M/D/n$ queuing system, where "n" is the number of service channels, M corresponds to random arrivals, and D corresponds to a deterministic departure. To derive the mean average delay, the $M/D/n$ queuing system properties are considered. For an $M/D/n$ queuing system, the cumulative distribution function (CDF) of waiting time is given as follows.

$$F(y) = \int_0^\infty (x+y-D)\frac{\rho^n x^{n-1}}{(n-1)!}e^{-\rho x}dx. \tag{8}$$

From equation (8), the expected value of waiting time (mean waiting time) can be ascertained as in the following equation.

$$\begin{aligned}\text{avg.waiting time} &= E(y)\\ &= \frac{1}{\mu}\sum_{j=1}^{\infty}\sum_{k=jn+1}^{\infty}\left[\frac{(jn\rho)^{k-1}}{(k-1)!} - \frac{(jn\rho)^k}{\rho k!}\right]e^{-jn\rho},\end{aligned} \tag{9}$$

where μ is the arrival rate and ρ is the ratio of arrival rate to departure rate.

As equation (9) is cumbersome to solve, further numerical approximations to obtain a closed-form solution for the average waiting time under $M/D/n$ queuing system were explored. In this regard, a widely used approximation for $G/G/n$ queuing system as presented in equation (10) [93] was selected.

$$W_q = \frac{X^{\sqrt{2(n+1)}}}{q(1-X)} \times \frac{CV_\lambda^2 + CV_\mu^2}{2}. \tag{10}$$

Equation (8) is the average waiting time for a $G/G/n$ queuing system (i.e., a general arrival and departure through n service channels). CV_λ and CV_μ represent the coefficient of variation for the arrival and departure distribution. For an $M/D/n$ queuing system, the arrivals follow a Poisson distribution with mean = standard deviation. Thus, $CV_\lambda = 1$. Similarly, the departures are deterministic, and hence the standard deviation is 0, which implies $CV_\mu = 0$. Thus, for $M/D/n$ queuing system, the average random delay can be simplified as

$$d_2 = \frac{X^{\sqrt{2(n+1)}}}{2q(1-X)}. \tag{11}$$

4.3.3. Total Average Control Delay. The overall average control delay under HLLD conditions can be obtained by adding the uniform and random delay terms as given in the following equation.

$$d = \frac{C(1-\lambda)^2}{2(1-\lambda X)} + \frac{X^{\sqrt{2(n+1)}}}{2q(1-X)}. \tag{12}$$

As discussed earlier, knowing the variability of delay, in addition to average delay, is important in understanding the performance of a signalised intersection under HLLD traffic conditions. This is because the smaller-sized vehicles seep through the gaps between the larger vehicles in the traffic stream under HLLD conditions, causing a disparity of delay between different vehicle types.

4.3.4. Control Delay Variability. Though it was found that a SIRO queue discipline would better represent the queue dissipation process under HLLD conditions, the average delays are independent of the queue discipline [91]. However, the actual distribution of delay would be different. That is, equation (8) would not hold under SIRO queue discipline as the underlying distribution of waiting time would be different. Thus, by knowing the distribution of waiting time for an *M/D/n* queuing system under SIRO queue discipline, one can ascertain the standard deviation of delay, which quantifies delay variability. However, there is no closed-form solution for the standard deviation of waiting time for *M/D/n*/SIRO queuing system to the best of the authors' knowledge. The current study develops an empirical model for the standard deviation of delay for the system under consideration.

To develop an empirical model for standard deviation (variability) of delay, multiple scenarios with different combinations of traffic demand and signal timing conditions are considered. A linear regression model is developed with standard deviation of delay as the depended variable and g/C and v/c as the independent variables. Section 5.2 details the empirical models derived and their performance.

The next section discusses the implementation of the methodology and results obtained under different traffic demand and signal timing scenarios.

5. Model Validation and Results

5.1. Average Delay. The developed model was implemented and tested under different traffic demands and signal settings. Overall, 36 combinations of λ (g/C) and x (v/c) were considered. The values of λ were varied from 0.2 to 0.7 in increments of 0.1, and the values of x were varied from 0.5 to 0.95. The combinations of λ and x were chosen in such a way as to replicate different traffic demand and signal timing scenarios that may be prevalent in an undersaturated intersection. These demand and signal timings were simulated on the same VISSIM model developed in Section 3.2. The cycle time was fixed as 120 seconds, and the saturation flow estimated from the simulation was 2900 PCE/hr/lane. The average delay for each λ, x combination was taken as the average of 5 simulation runs. The average control delays obtained from the simulation for 36 combinations are presented in Table 7.

For all the λ, x combinations, the corresponding delay values are calculated using the developed model and are compared with the values from Table 7. Results obtained are presented in Figures 6 and 7.

Table 7: Actual delay (simulated) for different λ, x combinations.

g/C	v/c					
	0.5	0.6	0.7	0.8	0.9	0.95
0.2	36.43	37.41	50.22	52.04	58.13	63.69
0.3	24.36	25.19	27.85	38.82	46.12	54.85
0.4	21.42	23.02	26.49	29.93	31.36	36.41
0.5	8.09	13.43	16.45	19.59	24.91	34.50
0.6	5.30	6.08	8.23	9.31	11.67	22.79
0.7	1.11	3.12	4.28	7.14	8.85	15.31

It can be observed that for some of the λ, x combinations, the average delay value is underestimated (top right of Figure 7), whereas for a large number of λ, x combinations, the average delay is overestimated. This is in line with Webster's finding that the first two terms of Webster's delay equation overestimated average delay. Webster had proposed a factor of 0.9 to be multiplied with the sum of the first two terms to take care of the overestimation. However, as can be observed from Figure 7, the model both underestimates and overestimates the average delay based on the λ, x combination. On closer observation, it can be seen that overestimation happens when x/λ is lesser than or equal to 3 and underestimation happens otherwise.

Thus, instead of a constant multiplication factor of 0.9, irrespective of the demand and signal timings, this study proposes a factor f that needs to be either multiplied or divided based on the x: λ ratio. Applying this, the final delay model for HLLD traffic condition can be expressed as follows:

$$d = \begin{cases} \left(\dfrac{C(1-\lambda)^2}{2(1-\lambda X)} + \dfrac{X^{\sqrt{2(n+1)}}}{2q(1-X)}\right) f, & \text{for } \dfrac{X}{\lambda} \leq 3, \\[2ex] \left(\dfrac{C(1-\lambda)^2}{2(1-\lambda X)} + \dfrac{X^{\sqrt{2(n+1)}}}{2q(1-X)}\right) \dfrac{1}{f}, & \text{for } \dfrac{X}{\lambda} > 3. \end{cases} \tag{13}$$

For the simulated data in the current study, the value of f that gave the least deviation among the estimated and actual delay values was 0.84. Using this, the delay values were recalculated, and the result obtained is shown in Figure 8.

It can be seen from Figure 8 that the estimated values are very close to the actual values with a mean absolute error (MAE) of 3.73 seconds/vehicle. Further, the R^2 value for the $x = y$ line has improved from 0.92 before the introduction of the multiplicative adjustment factor (Figure 7) to 0.95 after introducing a multiplicative adjustment factor (Figure 8). Despite the good accuracy obtained for delay estimation using equation (13), the model is not differentiable at $X/\lambda = 3$. The differentiability of delay models is vital as these models are widely used for arriving at optimal signal timings using optimisation methods. Considering this need for differentiability, an alternate approach for adjusting equation (12) is proposed next.

Figure 9 shows the variation of the delay estimation errors (obtained using equation (12)) with the X/λ ratio. It

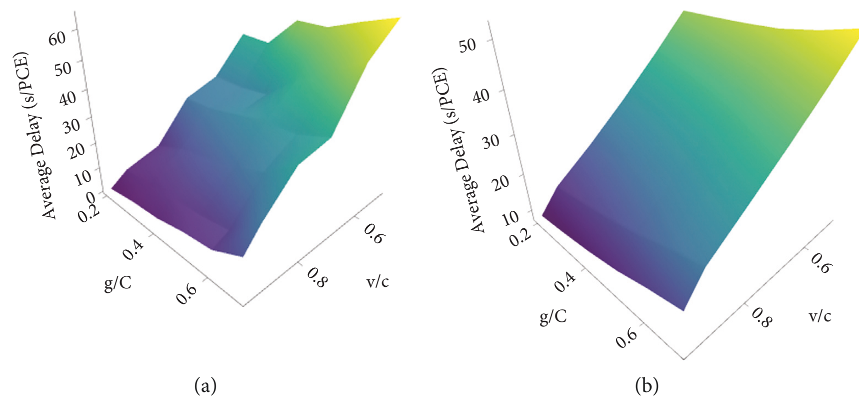

FIGURE 6: Variation of actual and estimated delay with v/c and g/C. (a) Actual delay. (b) Estimated delay.

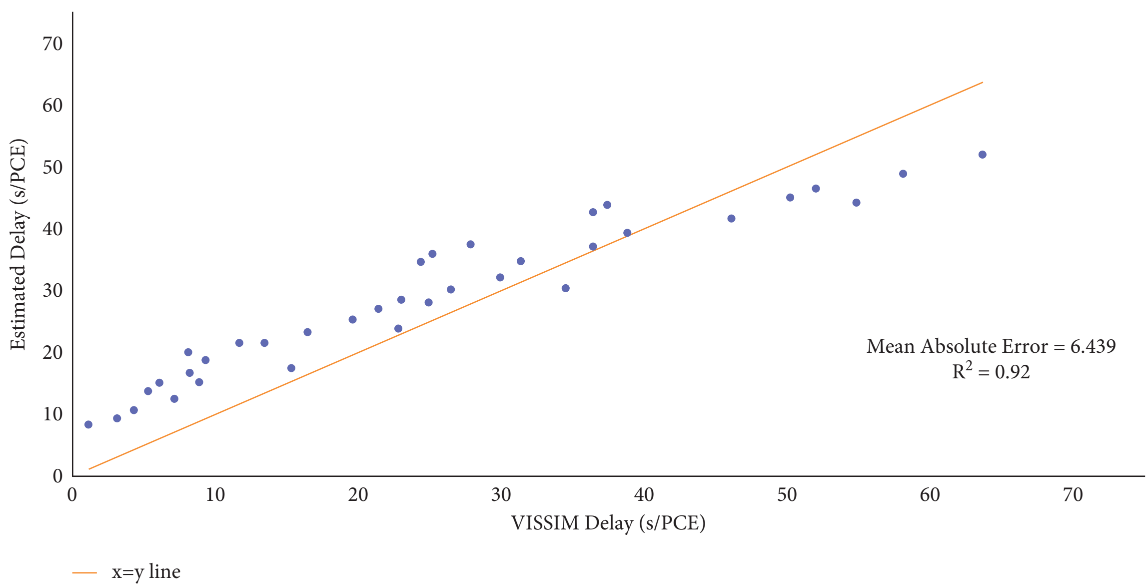

FIGURE 7: Comparison of actual and estimated delay.

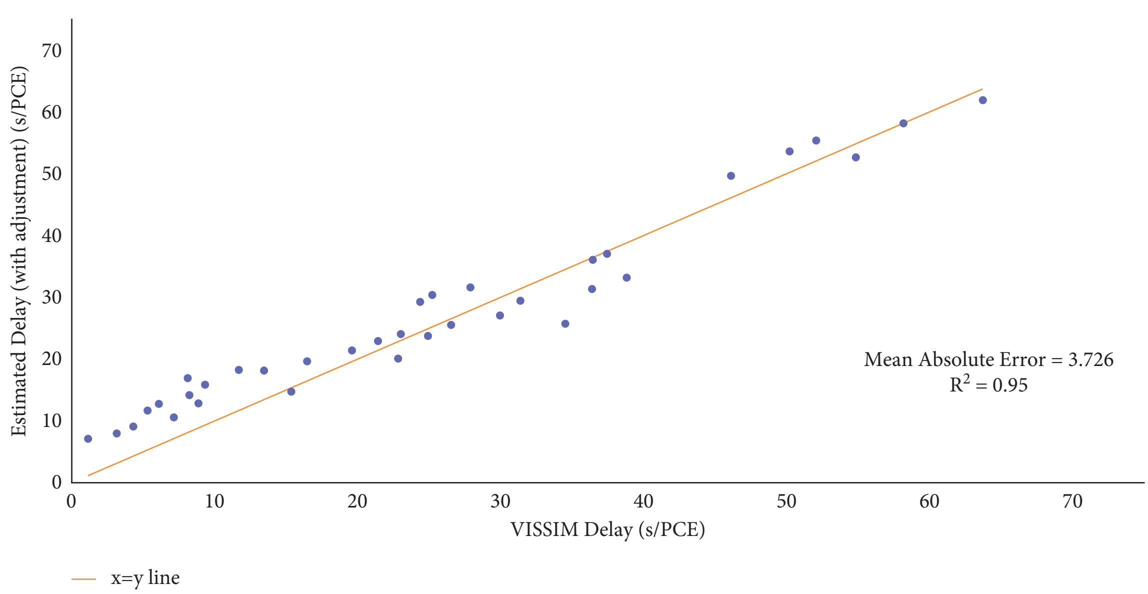

FIGURE 8: Comparison of actual and estimated delay with multiplicative adjustment factor.

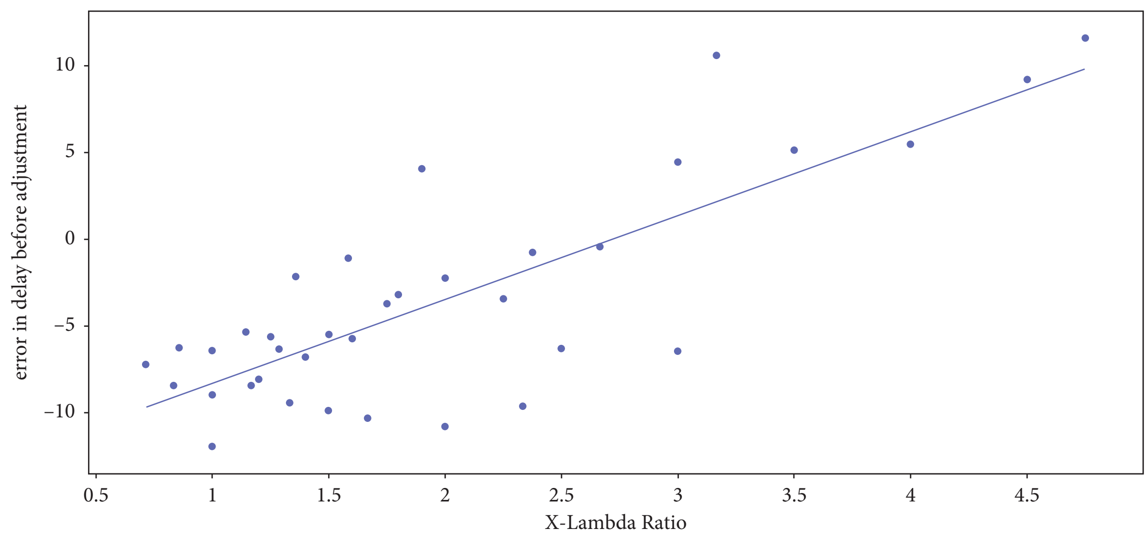

FIGURE 9: Variation of errors in delay estimates with X/λ ratio.

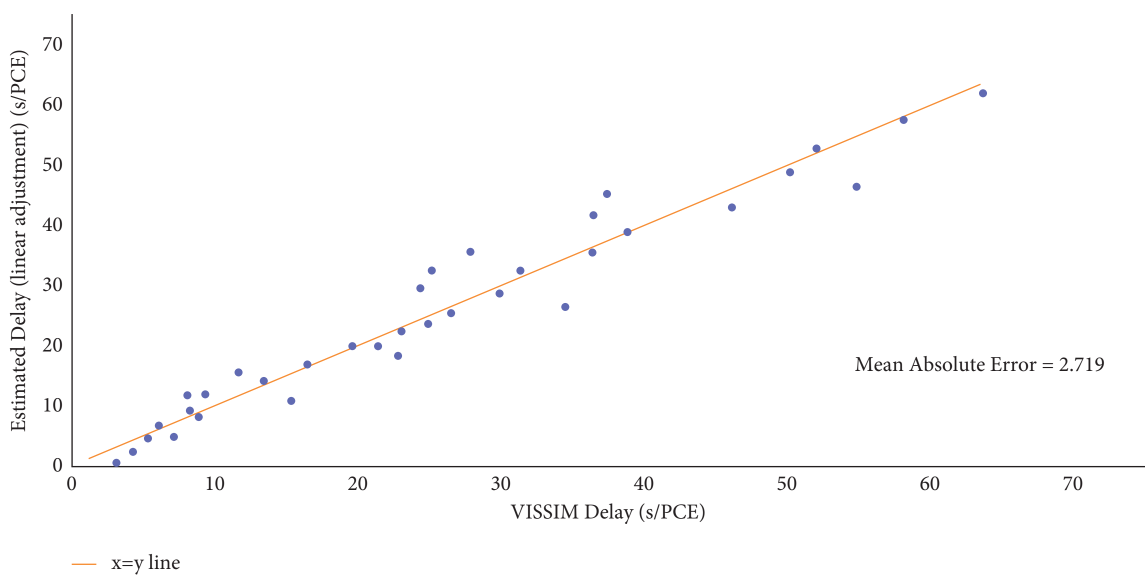

FIGURE 10: Comparison of actual and estimated delay with additive adjustment factor.

can be observed that the error in delay estimates obtained using equation (12) increases linearly as X/λ ratio increases. Thus, the error (e) can be written as

$$e = \text{delay}_{\text{actual}} - d = a\left(\frac{X}{\lambda}\right) + b. \tag{14}$$

On fitting a linear regression line using Ordinary Least Squares, a and b were found to be 4.84 and −13.15, with a resultant goodness of fit measure of $R^2 = 0.7$. Applying this correction to equation (12), the final theoretical delay model for HLLD traffic conditions can be rewritten as

$$d = \left(\frac{C(1-\lambda)^2}{2(1-\lambda X)} + \frac{X^{\sqrt{2(n+1)}}}{2q(1-X)}\right) + 4.84\left(\frac{X}{\lambda}\right) - 13.15. \tag{15}$$

Using equation (15), the delay values were recalculated, and the result obtained is shown in Figure 10.

It can be seen from Figure 10 that the estimated values are close to the actual values and the mean absolute error (MAE) (Table 8) was found to be 2.72 seconds/vehicles.

Table 8 summarises the accuracy measures for the delay estimates using the in-practice delay models and the developed delay equation for the HLLD condition with the above discussed multiplicative and additive adjustment factors.

5.2. Variation of Delay. The standard deviation of delay obtained from the 36 different combinations of traffic demand and signal timings is presented in Table 9.

Table 8: Summary of accuracy measures of delay models.

Model	MAPE (%)	MAE
Akcelik's delay model (with PCE)	77.2	18.4
HCM 2010 delay model (with PCE)	72.8	15.2
Indo-HCM delay equation (with Indo-HCM PCE values)	63.8	11.4
Webster's delay equation (using PCE) with 0.9 adjustment factor	59.4	10.4
Proposed delay equation for HLLD traffic—multiplicative adjustment factor	31.6	3.7
Proposed delay equation for HLLD traffic—additive adjustment factor	15.39	2.7

Table 9: Standard deviation of delay for different λ, x combinations.

g/C	v/c					
	0.5	0.6	0.7	0.8	0.9	0.95
0.2	9.05	9.31	10.19	10.11	10.75	10.44
0.3	10.10	10.27	10.48	11.55	11.46	12.28
0.4	10.37	10.74	11.46	11.81	12.80	13.00
0.5	11.87	12.15	12.39	12.74	14.01	13.75
0.6	12.89	12.88	13.75	14.37	14.02	14.72
0.7	13.20	14.28	14.35	14.17	15.60	15.68

Table 10: ANOVA results for linear regression.

Dependent variable	Delay st. dev	R-Squared	0.97			
Model	OLS	Adjusted R-squared	0.97			
Method	Least squares	F-statistic	557.6			
No. of observations	32	Prob (F-statistic)	0.00			
Degrees of freedom residuals	29	Log-likelihood	−7.63			
Degrees of freedom model	2					
	Coefficient	Std. error	t	$P>\|t\|$	[0.025	0.975]
Constant	4.70	0.28	16.59	0	4.13	5.28
X (v/c)	4.67	0.33	14.27	0	4.00	5.33
λ (g/C)	9.21	0.31	30.20	0	8.59	9.83

Of the above 36 observations, 32 data points were randomly chosen for training a linear regression model of the form

$$d_{StDev.} = a\lambda + bX + e. \tag{16}$$

Table 10 shows the ANOVA results of the linear fit for the training data, with an R^2 value of 0.97. It can be observed that both the dependent variables are statistically significant.

Thus, the resultant linear model for standard deviation of delay as a function of the g/C (λ) ratio and v/c ratio (X) can be written as

$$d_{StDev.} = 9.2\lambda + 4.7X + 4.7. \tag{17}$$

Equation (17) is further tested on the remaining four data points (testing dataset), and the MAPE and MAE were found to be 7.64% and 1.97, respectively. Equation (15) can, thus, be used to estimate the standard deviation of delay for a given pair of g/C and v/c values under a prevalent traffic condition.

6. Application of the Developed Theoretical Delay Model: Signal Timing Optimisation

In addition to being used as a means to ascertain the Level of Service (LoS) of a signalised intersection, most of the pioneering works on signal optimisation were based on the minimisation of analytical delay model. The conventional signal design procedures like Webster's method [48] and Akcelik's (ARRB 123) method [54] gave equations for optimal cycle time by minimising an analytical delay equation. On obtaining the optimal cycle time, the green splitting was carried out using the demand to saturation flow ratio method ("Equisat" method). Though the closed-form cycle length equations obtained from such methods are easy to use, they have used calibration constants and numerical approximations to derive them. The current study derives the optimal green timings directly from the derived delay equation using a simple trust region optimisation algorithm, as discussed next.

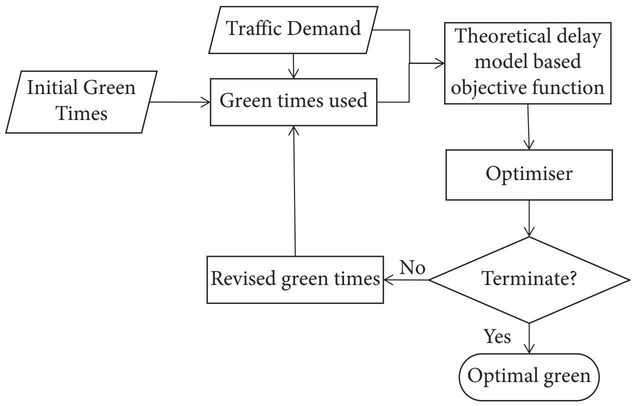

Figure 11: Methodology for obtaining optimal signal timings.

6.1. Optimal Green Time Calculation. To obtain the optimal green timings, which minimises the average intersection control delay, the methodology shown in Figure 11 is used. An initial green time estimate and the prevalent traffic demand are given as input to the theoretical delay model, derived in equation (15), to obtain the average approach wise control delays. Then, the approach-wise control delays along with the corresponding approach demands are used to calculate the value of the objective function (average intersection control delay) shown in the following equation.

$$\min d_{\text{avg}} = \frac{\sum_{i=1}^{m} v_i d_i}{\sum_{i=1}^{m} v_i}, \tag{18}$$

where v_i and d_i are the demand (PCE/hr) and the average control delay of the ith approach, respectively, and m is the number of approaches.

An optimisation problem with equation (15) as the objective function and the green times as the decision variable is set up. Minimum and maximum value constraints on the green values are applied and a simple trust region optimisation algorithm [94] is used to solve the optimisation problem.

6.2. Implementation and Results. For implementing the methodology, a four-legged intersection formed by the intersection of two three-lane-one-way roads is considered. This resulted in a two-phase signal with one phase each for N-S and E-W roads.

As the proposed signal optimisation approach aims at developing optimal signal timing plans for a given traffic scenario, the methodology was implemented for different traffic demand scenarios. To account for the different demand scenarios, two variables are used: (i) Intersection Flow Ratio (IFR) and (ii) Demand Split Ratio (DSR). IFR is a measure of the traffic demand level at the intersection. It is obtained as the summation of the volume to saturation flow ratios of all the approaches to the intersections (Equation (19)). DSR, on the other hand, is a measure of skewness in demand between opposing approaches. It is obtained as the ratio of the demand of one approach to the total demand (Equation (20)).

Table 11: Variation of percentage reduction in average intersection control delay.

		Demand split ratio (DSR)			
		0.5 (%)	0.6 (%)	0.7 (%)	0.8 (%)
Intersection flow ratio (IFR)	0.4	7	4	0	1
	0.5	15	9	0	1
	0.6	20	18	8	0
	0.7	9	14	15	7
	0.8	6	16	5	14
	0.9	24	21	21	18

$$\text{IFR} = \sum_{i=1}^{m} \frac{v_i}{s_i}, \tag{19}$$

$$\text{DSR} = \frac{v_i}{\sum_{i=1}^{m} v_i}. \tag{20}$$

Using the above two variables, multiple traffic demand scenarios are generated. By assuming a constant saturation flow value (s) of 2900 PCE/hour/lane (observed from simulation), the total demand ($\sum_{i=1}^{m} v_i$) of the intersection for a given IFR level can be found out as follows.

$$\sum_{i=1}^{m} v_i = \text{IFR} \times s. \tag{21}$$

On obtaining $\sum_{i=1}^{m} v_i$, the values forv_{E-W} (demand on East – West approach) and v_{N-S} (demand on North – South approach) can be found out for a given DSR value. For the current implementation, the IFR value is varied between 0.4 and 0.9 in increments of 0.1 and the SR value is varied between 0.5 and 0.8 in the increments of 0.1. Thus, 24 different demand scenarios are generated.

For each of the 24 demand scenarios, the methodology in Figure 11 is implemented with green timings obtained from IRC method [95] (in-practice signal design method under HLLD traffic conditions) as the initial solution. A minimum green constraint of 7 seconds was arrived at based on the minimum pedestrian requirements, and a maximum green constraint 113 seconds was arrived at by subtracting the minimum green for the opposing approach from the maximum allowable cycle time of 120 seconds [95]. The optimisation algorithm is run for all the demand scenarios and the optimal green timings, and the final objective function is noted. The percentage reduction in average intersection control delay for each of the 24 demand scenarios in comparison to the baseline control delay obtained using Webster's method is calculated and is shown in Table 11.

From Table 11, it can be observed that the reduction in control delay compared to Webster's method is maximum for high demand levels (high IFR) and low skewness in demand between the opposing approaches (low DSR) with a reduction in average intersection control delay of up to 24%. Thus, the proposed optimisation algorithm performs better than the baseline signal timing approach—Webster's method, which is most commonly used as the state of practice in most developing countries [95].

7. Conclusions

Theoretical delay models are the most widely used tools to estimate control delay at signalised intersections, especially when real-time traffic data collection sensors are not available. Literature is limited for theoretical delay estimation models underHLLD traffic conditions and the existing literature is empirical in nature, rendering its accuracy highly site specific. This research focused on developing a theoretical model for HLLD traffic. Towards this end, first, the performance of the conventional delay models under HLLD conditions is compared, which confirmed their limitations in capturing the characteristics of HLLD traffic. Thereafter, the principles of queueing theory are applied to develop a theoretical delay model by explicitly taking into account the characteristic features of HLLD traffic, namely, the heterogeneity and the lane indiscipline. The significant contributions of this study are as follows:

(a) A queueing theory-based theoretical control delay estimation model for HLLD traffic conditions by explicitly considering the characteristic features of HLLD traffic

(b) A comprehensive framework to develop an efficient delay model for HLLD traffic conditions with minimal calibration

(c) A saturation flow-based PCE estimation process

(d) A methodology to estimate the number of service channels using a combination of PCE and virtual lane concepts

(e) A signal optimisation approach using the proposed theoretical delay model for HLLD traffic conditions

This research also highlighted the significance of knowing the variation of delay in addition to average delay and presented a simple approach to capture the variation in delay.

The developed model was validated using simulated data for an isolated undersaturated intersection. The performance was compared with the in-practice methods for delay estimation in countries with HLLD conditions, such as India. The proposed model yielded 64% lesser mean absolute error for control delay estimates compared to the best-performing in-practice delay estimation model. An application of the developed theoretical delay model to optimise signal timings is also demonstrated under HLLD traffic conditions. A reduction in average intersection control delay of up to 24%, in comparison to in-practice signal design method under HLLD traffic conditions, was observed. Future research can focus on the extension of the study for oversaturated intersections and for the estimation of delay along an arterial corridor with multiple intersections.

Data Availability

The authors confirm that the data supporting the findings of this study, which are not available within this article, are available upon reasonable request from the corresponding author.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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