Cooperative Tracking Control of the Multiple-High-Speed Trains System Using a Tunable Artificial Potential Function

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It is a challenge to maintain a safe and efficient tracking for multiple high-speed trains under the moving block operational mode. In this paper, a novel cooperative tracking control based on a consensus algorithm and artificial potential field theory is proposed to realize the train tracking within a distance range. A tunable artificial potential function is first designed to dynamically adjust the distance between adjacent high-speed trains with real-time train states. By regulating the parameters of the artificial potential function, the safety distance can be adjusted according to the required tolerance deviation of the actual distance. Under the proposed strategy, each high-speed train operates with the desired speed and tracks the preceding one with an adjustable distance range. Numerical train operational cases are investigated to illustrate the effectiveness of the proposed methods.

1. Introduction

Based on an advanced wireless communication technology and positioning technology, a wireless moving block system is currently being developed and proved to be an effective mode to improve the capacity of railway lines [1]. In the moving block system, a virtual block is formed between adjacent trains with a safe tracking interval [2]. The interval of the virtual block is computed based on the real-time speed and position of the preceding train, which enables a smaller tracking distance among trains and more efficient utilization of railway lines [3–5]. Therefore, the moving block system is a promising mode to promote the high-speed train performance, while it brings some new difficulties and challenges in achieving safe and efficient cooperative tracking of the multiple high-speed trains.

It is vital to control multiple high-speed trains to maintain a safe and efficient tracking of the moving block system in future European Train Control System (ETCS-3) and Chinese Train Control System (CTCS-4). Recently, there are numerous studies conducted to realize safe and efficient tracking. Based on nonsingular terminal sliding mode technology and radial basis function neural network, two chatter-free control strategies are proposed to achieve stable tracking control [6]. An event-triggered model predictive control algorithm is designed to solve the tracking control problem with random switching topologies [7]. Energy consumption is also taken into consideration to achieve energy-efficient tracking operation [8–10]. The above-mentioned studies are mostly based on centralized control of the railway control center, which may reduce the robustness and reliability of the system.

With the development of communication technology, train-to-train communication appears to allow direct information exchange between trains, making virtual coupling and autonomous driving possible [11, 12]. Based on real-time train-to-train communication, the cooperative control theory can further shorten the tracking interval between trains [13]. In the cooperative control system, the moving decision of high-speed trains no longer depends on the control center but on its own and neighboring states. It can react more quickly to state variation of adjacent trains, ensuring the safety of trains running at the small interval.
On the train cooperative control based on train-to-train communication, many important contributions have been proposed [14–18]. Using local neighboring information among trains, Gao et al. propose a decentralized adaptive cooperative control method, under which the rear train tracks the preceding one with a minimum separation distance [14]. In the study of [15], a distributed control law is designed by virtue of the cluster consensus theory such that each train tracks the target speed and a linear disturbance observer-based compensator is proposed to enhance the disturbance rejection ability. In the study of [16], a prescribed performance control strategy is presented to achieve the cooperative tracking operation and the speed deviation and position deviation are bounded in the dynamic adjustment process. A distributed cooperative control strategy is designed to achieve consensus of cars at the desired distance while keeping the distance between trains to avoid collisions in the study of [17]. These studies control the distance between adjacent trains to converge to a desired constant value.

Nowadays, some scholars tried to utilize cooperative control to constrain the distance within a safe range rather than a constant value. A multigroup parallel differential evolution algorithm is developed, in which a resilience set is introduced to guarantee that the distance is within the safe range [19]. Based on the potential function and LaSalle’s invariance principle, a cooperative control strategy is designed to ensure a safe tracking operation [20]. An adaptive cooperative control strategy with input saturation and uncertain parameters is designed, in which a potential function is introduced to ensure the safety distance between adjacent trains [21]. However, in the literature mentioned above, the boundary of the safe range is fixed and the initial interval distance is required to be within the range. In fact, the safe range should be related to the operation state of high-speed trains, which is more efficient with speed change.

This paper addresses the cooperative tracking control equipped with the train-to-train communication system. Our previous work was reported in the study of [22], which restrains the distances within a fixed interval. In this paper, a new cooperative control strategy is proposed to adjust the distance dynamically based on the real-time train states. The artificial potential function is introduced to control the high-speed train to track the preceding one with a safety distance, which can be dynamically adjusted by regulating the parameters of the artificial potential function. Under the proposed strategy, a safe and efficient tracking operation is achieved. The simulation results illustrate the effectiveness of the proposed control strategy. Compared with the existing literature, which is listed in Table 1, the main contributions of this paper are listed as follows:

1. A novel cooperative tracking control strategy is proposed, under which each train operates with the desired speed and tracks the preceding train with a safety distance, improving the railway capacity while ensuring safe operation.

2. The stable safety distances between adjacent high-speed trains are distributed within a controllable range, which can be adjusted by regulating the parameters of artificial potential function according to the required tolerance deviation of actual distance.

The remainder of this paper is organized as follows: in Section 2, the dynamic model and communication topology for multiple high-speed trains are constructed. In Section 3, a cooperative tracking control strategy based on the consensus algorithm and the artificial potential field theory is designed. In Section 4, numerical cases are conducted to illustrate the effectiveness of the proposed methods. Finally, the conclusions are presented in Section 5.

2. System Model

In this section, the system model of multiple high-speed trains is built. Firstly, the longitudinal dynamic characteristics of the high-speed train are established. Next, the communication topology is modelled based on the graph theory. And then, the cooperative tracking control problem is formulated.

2.1. Dynamic Characteristics of the High-Speed Train.

Similar to reference [7, 20, 21], because the safety distance is much longer than the length of the train, it is reasonable to ignore the train length. Regardless of the train length, all the forces suffered by the train during operation are focused on a rigid particle. The longitudinal forces of the high-speed train mainly include three kinds of forces, namely, the traction force $F_{tr}$, the braking force $B$, and the resistance $R$. The direction of $F$ is the same as the running direction of the high-speed train. The other two forces $B$ and $R$ are acting in the opposite direction of the train operation. The running resistance $R$ is composed of basic resistance and additional resistance. The basic resistance is affected by many factors, so it is difficult to establish an accurate mathematical model. The Davis equation is used to describe the basic resistance, which is composed of mechanical resistance and air resistance [23]. The additional gravitational resistance and curvature resistance are considered. The total resistance is expressed as follows:

$$R(t) = m(c_0 + c_1 v(t) + c_2 v^2(t)) + mgsin\theta + 0.004m D,$$  \hspace{1cm} (1)

where $m$ is the mass of the high-speed train, $v(t)$ is the speed at time $t$, $c_0$, $c_1$, and $c_2$ are positive coefficients related to the type of high-speed train and the number of cars, $\theta$ is the slope angle of the train, and $D$ is the degree of curvature of the train track segment. In the previous equation, $c_0 + c_1 v(t)$ denotes the mechanical resistance, proportional to speed $v(t)$, which is the main component of the basic resistance at low speed. And, $c_2 v^2(t)$ is the air resistance, proportional to $v^2(t)$, which gradually becomes the main resistance with the increase of speed. The gravitational and curvature resistance forces experienced by the train are $mgsin\theta$ and $0.004m D$, respectively.

According to Newton’s second law, the longitudinal dynamic characteristics of the train are formulated as follows:
trains. Note that the communication relationship between high-speed trains is abstracted into a graph [26]. Based on algebraic graph theory, the weighted adjacency matrix $A$ is given as follows:

$$
A = \begin{bmatrix}
0 & 1 & 0 & \ldots & 0 \\
1 & 0 & 1 & \ldots & 0 \\
0 & \ldots & 1 & 0 & 1 \\
0 & \ldots & 0 & 1 & 0
\end{bmatrix}
$$

Assume that no trains leave and no new trains join during operation. The communication relationship is fixed, so the weighted adjacency matrix $A$ remains unchanged.

2.3. Problem Formulation. The high-speed train operation process typically involves four working conditions: traction, cruising, coasting, and braking. The cruising phase occupies most of the time during the entire operation process. This paper focuses on the cruise operation of the high-speed train. The cruise control mechanism is aimed to control the current speed of the high-speed train to reach the desired speed $v_r$. According to the deviation between the desired speed and the current speed, an effective control algorithm is applied to generate the corresponding control variable. And, the power device of the high-speed train is controlled to generate traction or braking force. The power device generates traction to accelerate the high-speed train when the actual speed is lower than the desired speed. Otherwise, braking force is generated to slow the train. The train continuously adjusts the speed until it runs stably at the desired speed, i.e., $v = v_r$, so as to realize the cruising operation of the train.

The cooperative tracking control of multiple high-speed trains is a macro-operation based on single-train cruise control. The high-speed train is given a certain degree of autonomy in operation control. The high-speed train can also set its own operation strategy according to the real-time information of adjacent trains, so as to improve the overall operational efficiency of the railway. Each high-speed train in the cooperative tracking control system still needs to track the desired speed. In the steady-state, $v_1 = v_2 = \cdots = v_n = v_r$. In addition, keeping a distance $d_r$ between adjacent high-speed trains is required to ensure the operation safety. The actual distance between trains is calculated as follows:

$$
d_{ij}(t) = |x_i(t) - x_j(t)| \quad (5)
$$

where $i, j \in \{1, 2, \ldots, n\}$. We define $d_e = v_e(t) \cdot h^* + d_0$, in which $h^*$ is the desired time headway between adjacent

### Table 1: The comparisons between related works and our work.

<table>
<thead>
<tr>
<th>Research work</th>
<th>Method</th>
<th>Safety distance</th>
<th>Can one change safety distance with velocity?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Literature [14–18]</td>
<td>Consensus based strategy</td>
<td>Constant value</td>
<td>No</td>
</tr>
<tr>
<td>Literature [19]</td>
<td>Resilience set</td>
<td>Fixed range</td>
<td>No</td>
</tr>
<tr>
<td>Literature [20, 21]</td>
<td>Potential function</td>
<td>Fixed range</td>
<td>No</td>
</tr>
<tr>
<td>Our work</td>
<td>Tunable potential function</td>
<td>Controllable range</td>
<td>Yes</td>
</tr>
</tbody>
</table>

where $x(t)$ is the position of the high-speed train at time $t$. $u(t)$ is the traction force $F$ or the braking force $B$, which is the control variable we are going to design later.

For riding comfort, the control variable of the high-speed train is a macro-operation based on single-train cruise control. Based on algebraic graph theory, the communication topology is abstracted into a graph [26]. Ignoring the physical meaning, each high-speed train is considered a node. The set of all nodes in the graph is denoted as $N = \{1, 2, \ldots, n\}$. An edge $e_{ij}$ exists if the node $i$ receives information from the node $j$. For node $i$, the node $j$ is its neighbor. All edges in the graph are denoted as $e = \{e_{ij}, (i, j) \in V \times V\}$. Generally, a coefficient $a_{ij} > 0$ is set on the edge to indicate the weight of the information. A weighted adjacency matrix $A = [a_{ij}]_{nn}$ is adopted to represent the communication relationship between high-speed trains. Note that $a_{ii} = 0$ if there is no communication between nodes $i$ and $j$. Suppose that each high-speed train gets information from the neighboring trains. Each high-speed train knows its own information and does not need to communicate with itself, i.e., $a_{ii} = 0$. The weighted adjacency matrix $A$ is given as follows:

$$
A = \begin{bmatrix}
0 & 1 & 0 & \ldots & 0 \\
1 & 0 & 1 & \ldots & 0 \\
0 & \ldots & 1 & 0 & 1 \\
0 & \ldots & 0 & 1 & 0
\end{bmatrix}
$$

\[ \begin{align*}
\dot{x}(t) &= v(t), \\
\dot{m}v(t) &= u(t) - m(c_0 + c_1v(t) + c_2v^2(t)) - mg\sin\theta - 0.004mD.
\end{align*} \]
trains and $d_0$ is the minimum safety distance [27]. The desired distance will be properly adjusted according to the speed. When the speed increases, the desired distance also increases, thus ensuring the operation safety. When the desired speed decreases, the distance decreases to improve the capacity of the railway.

To sum up, the goal of this paper is to design a cooperative tracking control strategy to ensure that all trains operate with the desired speed $v_r$ and the trains $i \in 2, \ldots, n$ track the preceding one at a safety distance.

3. The Cooperative Tracking Control Strategy Design

In this section, a cooperative tracking control strategy is designed to achieve the above-mentioned control objectives. The artificial potential field theory is introduced to control the distance between adjacent high-speed trains. The speed of high-speed trains is aligned based on the consensus algorithm. By assigning a weight coefficient to each control component, it is integrated into the final cooperative control strategy.

3.1. The Artificial Potential Function. The artificial potential field theory is introduced to control the distance between high-speed trains. The artificial potential field theory proposed by Khatib is a real-time programming method, which is widely used in the path planning and obstacle avoidance of agents [28, 29]. The agent is regarded as a particle moving in a two-dimensional space. The goal of the agent is to move to a specified point and avoid obstacles during the movement. An artificial potential field is established to represent the environment where the agent is located. The target point area has low potential energy, and the obstacle area has high potential energy. Agents tend to move in the direction of low potential energy, so that the target point is attractive to the agent and obstacles are repulsive to the agent [30]. The agent will reach the point eventually with the smallest potential energy, i.e., the target point.

In this paper, the goal is to control the distance between high-speed trains to approach the desired distance. When the actual distance is large, the potential field generates a force that accelerates the train behind to reduce distance. When the actual distance is small, a repulsive force in the opposite direction will be generated so that the high-speed train behind will decelerate. The force $F_p$ generated by the artificial potential field is defined as follows:

$$F_p(d_{ij}) = \text{sgn}(d_{ij} - d_r) \frac{10^a (d_{ij} - d_r)}{10^a - 1} \cdot \Pi,$$

where $a$ is a positive parameter and $\Pi$ is the maximum traction or braking force that the power device can output. We temporarily do not consider the actual value of $\Pi$, i.e., $\Pi = 1$, and just analyse the nature of $F_p(d_{ij})$. If $d_r = 10$, the graph of $F_p(d_{ij})$ with different $a$ values is shown in Figure 1. It can be seen that when the actual distance is greater than the desired distance, $F_p(d_{ij}) > 0$, which means that the power device generates traction force to accelerate the train. Otherwise, the train slows down. If $d_{ij} = d_r$, $F_p(d_{ij})$ is equal to 0. And, the larger the parameter $a$, the wider the range of $F_p(d_{ij})$ equal to 0.

The corresponding potential function can be obtained by integrating $F_p(d_{ij})$.

$$P(d_{ij}) = \int_0^{d_{ij}} F_p(d) \, d(d_{ij}).$$

The graph of $P(d_{ij})$ is plotted in Figure 2. The potential energy is minimum when the actual distance is equal to the desired distance. When the actual distance is greater than or less than the desired distance, the potential function has greater potential energy. The larger the parameter $a$, the wider the range of the low potential energy area.

Remark 1. The shape of the artificial potential function is related to the parameter $a$. If the parameter $a$ is relatively small, the potential function only has low potential energy at the desired distance. Otherwise, the potential energy is low in a range near the desired distance. And, the larger the parameter $a$, the wider the range. Within this range, the corresponding control component of the potential function is almost zero, that is, the steady-state distance is allowed to be distributed within this range. A suitable value can be set for the parameter $a$ according to the tolerance of the actual distance deviating from the desired distance.

High-speed trains can obtain information about the front and rear trains, which can be fully utilized to design control strategies. If the distance between the train and the preceding train is large, the potential field produces a positive attraction to accelerate the train. Otherwise, it generates a force in the opposite direction. When the distance between the train and the rear train is large, the train should slow down, and vice versa. Considering the influence
of front and rear trains simultaneously, the corresponding control strategy is designed as follows:

\[ u_i(t) = -\sum_{j=1}^{n} a_{ij} \text{sgn}(i-j) \cdot \Delta d_{ij} \cdot \frac{10^{a \Delta d_{ij}} - 1}{10^a - 1}, \]

where \( \Delta d_{ij} = d_i - |x_i - x_j| \). If the actual distance deviates from \( d_i \), it returns to the desired distance owing to \( u_i(t) \). \( a_{ij} \) denotes the communication relationship between train \( i \) and train \( j \). If \( a_{ij} = 1 \), which means train \( i \) can get the speed and position information of train \( j \), then there will be a force between train \( i \) and train \( j \) to drive them close to the safety distance. \( \sum_{j=1}^{n} a_{ij} \) means for any train \( i \), all the other trains are considered if they can transmit the information to train \( i \). The direction of force \( u_i(t) \) is determined by \( \text{sgn}(i-j) \cdot \Delta d_{ij} \). There are four cases: when \( i < j \) and \( \Delta d_{ij} < 0 \), train \( i \) is in front of train \( j \) and their distance is greater than the reference and then the force \( u_i(t) \) should be negative to reduce their distance; when \( i < j \) and \( \Delta d_{ij} > 0 \), train \( i \) is at the back of train \( j \) and their distance is less than the reference and then the force \( u_i(t) \) should be positive to increase their distance; when \( i > j \) and \( \Delta d_{ij} > 0 \), train \( i \) is at the back of train \( j \) and their distance is greater than the reference and then the force \( u_i(t) \) should be positive to decrease their distance. In summary, the term \( i-j \) presents the order of train \( i \) and train \( j \), and \( \Delta d_{ij} \) presents whether the current distance is greater than the reference distance. Thus, operation safety is ensured by introducing the artificial potential function.

3.2. The Consensus Algorithm. As mentioned above, the multiple high-speed trains system is regarded as a multi-agent system with the ability of information interaction. The consensus problem, to achieve a consistent state among agents, is a basic problem in the multiagent system, which is usually implemented by a consensus algorithm [31]. The consensus algorithm is a local interaction rule, which describes the process of information exchange between adjacent agents. The goal of multiagent system consensus control is described as follows:

\[ \lim_{t \to \infty} x_i(t) = 0, \forall i \in V, \]

where \( x_i \) is the state of the agent \( i \).

The first-order consensus algorithm of multiagent systems has three forms. The first algorithm controls the state \( z_i \) to track the desired state \( z_{ir} \).

\[ \dot{z}_i = \ddot{z}_{ir} - a_i(z_i - z_{ir}), i = 1, 2, \ldots, n. \]

Here, \( a_i > 0 \).

The second algorithm ensures the state of the agents \( i \) and \( j \) is consistent.

\[ \dot{z}_i = -\sum_{j=1}^{n} a_{ij}(z_i - z_j), i = 1, 2, \ldots, n, \]

where \( a_{ij} \) is the communication relationship between agents \( i \) and \( j \).

The third algorithm controls all agents to approach the desired state \( z_{ir} \).

\[ \dot{z}_i = \ddot{z}_{ir} - a_i(z_i - z_{ir}) - \sum_{j=1}^{n} a_{ij}(z_i - z_j), i = 1, 2, \ldots, n. \]

This paper aims to control the speed of all high-speed trains to track the desired speed \( v_r \). Thus, according to (12), the third consensus algorithm form is adopted as follows:

\[ u_{12}(t) = m_i \left( \dot{v}_{ir} - a(v_i(t) - v_r) - \sum_{j=1}^{n} a_{ij}(v_i(t) - v_j(t)) \right), \]

where the term \( \dot{v}_r - a(v_i(t) - v_r) \) makes the speed of the train \( i \) to track the desired speed \( v_r \) and the term \( \sum_{j=1}^{n} a_{ij}(v_i(t) - v_j(t)) \) makes the speed of all trains approach consensus.

At the end of the design process, in order to overcome the effects of basic resistance and gravitational and curvature resistance force, we can get the following equation:

\[ u_{13}(t) = m_i \left( c_0 + c_1 v_i^2(t) + c_2 v_r^2(t) \right) + m_i g \sin \theta + 0.004m_i D_i. \]

According to the above-mentioned discussion, to render all trains to track the desired speed and maintain a controllable distance range between neighboring trains, the final cooperative control strategy \( u_i(t) \) for the train \( i \) is designed as an integration of Equations (8), (13), and (14).

\[ u_i(t) = k_1 u_{11}(t) + k_2 u_{12}(t) + u_{13}(t), \]

where \( k_1 \) and \( k_2 \) are positive parameters.

Remark 2. In the proposed cooperative tracking control strategy, Equation (14), the first term \( u_{11}(t) \) controls the
distance between neighboring trains using a tunable artificial potential function. It is worth noting that the desired safety distance is adjusted according to the real-time speed and the distance range is controllable by designing the parameter \( a \) of the potential function. The second term \( u_2(t) \) ensures the speed consensus by utilizing the consensus algorithm. The third term \( u_3(t) \) overcomes the basic resistance, gravitational, and curvature resistance force of each train. Under the proposed cooperative tracking control strategy, each train operates with the desired speed \( v_r \) and tracks the preceding train at an adjustable distance.

4. Simulation Results

In this section, the simulation experiments are carried out to evaluate the effectiveness of the proposed control strategy. Firstly, the experimental setup is introduced. Secondly, the potential function parameter \( a \) is set as different values to study the variation of the steady-state distance range. Finally, the simulation is carried out to study how the distance between trains changes with the real-time speed and simulation results of, respectively, applying the control strategies of literature [17] and literature [20] are presented for comparison.

4.1. The Experimental Setup. Consider 5 high-speed trains running on the railway. Assume that all trains have the same system parameters. The mass of the high-speed train is set as 400 t. The desired speed is \( v_r = 70 \text{ m/s} \). By referring to the studies of [27, 32], to achieve the balance of efficiency and safety, the desired headway is \( h^* = 60 \text{ s} \) and the minimum safety distance \( d_0 = 3.2 \text{ km} \). Then, the desired distance \( d_i = v_r h^* + d_0 = 7.4 \text{ km} \). The maximum acceleration is \( 1 \text{ m/s}^2 \), and the maximum deceleration is \(-1 \text{ m/s}^2 \). Assumed that the train runs in a straight line, the positive resistance coefficients \( c_0, c_1, \) and \( c_2 \) obtained by the wind tunnel test are given in Table 2 [33]. The initial states of the multiple high-speed trains are presented in Table 3.

4.2. Experimental Validation for the Controllability of Distance Range. In this section, we carry out simulations to study the controllability of distance range when the desired speed is a fixed constant. Let \( a = 1 \). The simulation time horizon is \([0, 500] \text{ s}\). By applying the cooperative tracking control strategy to multiple high-speed trains, the running status is shown in Figure 3. Figure 3(a) plots the speed curves of the high-speed trains. It can be seen that the high-speed trains accelerate to the desired speed with different accelerations at first, which is owing to the different initial states. At about 300 s, all trains reach desired speed of \( 70 \text{ m/s} \) simultaneously and operate at \( 70 \text{ m/s} \) within \([300, 500] \text{ s}\). Figure 3(b) shows the distance curves of multiple high-speed trains, where \( d_i \) represents the distance between the \( i \)-th train and \((i + 1)\)-th train. The distances between adjacent trains are adjusted within the range \([0, 300] \text{ s}\) due to the action of the potential function \( u_{i1}(t) \). And, the distances are kept near 7.4 km in \([300, 500] \text{ s}\). The simulation results illustrate that the high-speed train can track the desired speed and keep a safety distance from the preceding train under the proposed tracking control strategy.

Table 2: Resistance coefficients of the high-speed train.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_0 )</td>
<td>0.01176</td>
<td>( N/kg )</td>
</tr>
<tr>
<td>( c_1 )</td>
<td>0.00077616</td>
<td>( N \cdot \text{s/(m \cdot kg)} )</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>0.00016</td>
<td>( N \cdot \text{s}^2/(\text{m}^2 \cdot \text{kg}) )</td>
</tr>
</tbody>
</table>

Table 3: The initial states of the high-speed trains.

<table>
<thead>
<tr>
<th>( i )</th>
<th>( v_{i0}(\text{m/s}) )</th>
<th>( x_{i0}(\text{km}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68</td>
<td>57.3</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>49.8</td>
</tr>
<tr>
<td>3</td>
<td>45</td>
<td>42.7</td>
</tr>
<tr>
<td>4</td>
<td>49</td>
<td>35.3</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>28.5</td>
</tr>
</tbody>
</table>

4.3. Experimental Validation for the Distance Adjustability with Speed. In this section, simulation is carried out to study the adjustability of distance when the desired speed changes with time. The simulation time is set as \([0, 6000] \text{ s}\). The desired speed curve is presented in Figure 5, which is divided into three intervals. The desired speeds within \([0, 2000]\text{ s}\), \([2000, 4000]\text{ s}\), and \([4000, 6000]\text{ s}\) are, respectively, \(70 \text{ m/s}\), \(65 \text{ m/s}\), and \(70 \text{ m/s}\).

With the same initial states and system parameters, the cooperative control strategy based on the consensus algorithm [17] is applied to multiple high-speed trains. The desired distance is set as 7.4 km. The running states of all trains during the simulation are shown in Figure 6. Figures 6(a) and 6(b), respectively, represent the speed curve and distance curve. At the early stage of simulation, the running deviation of each train is large and different, so the speed, acceleration, and distance between trains are greatly adjusted. At about 430 s, each train accurately tracks the desired speed and the distance between trains stabilizes at 7.4 km. When the expected speed decreases from 70 m/s to 65 m/s, each train adjusts the control variables according to its own state and the state of adjacent trains, until the train runs stably at 65 m/s. When the expected speed changes back to 70 m/s, the adjustment process of the multiple high-speed trains is contrary to that of the previous stage, finally returning to the stable state of the first stage. In the whole simulation process, the running state of multiple trains only has obvious inconsistency in the initial stage. When the state reaches consistency, the running state of all trains remains consistent even if the desired speed changes.
Li et al. designed a cooperative control strategy to control the motion of multiple high-speed trains, under which the distances between adjacent trains are kept at a safe range [20]. Using the same initial states and parameters in the previous experiment, the simulation results are given in Figure 7. The safety distance is [6.5, 8] km. Figure 7(a) plots the speed curves under the control strategy. The high-speed trains achieve consistent speed at about 80 s before they reach the desired speed. During this period, the trains accelerate with different accelerations, so the distances will change correspondingly. After that, all high-speed trains accelerate to the desired speed with the same acceleration, during which the distances do not change. Within [100, 2000] s, the high-speed trains run at the desired speed of 70 m/s. At 2000 s, the desired speed changes to 65 m/s. All high-speed trains decelerate to 65 m/s with a consistent state.

Figure 3: The simulation results of applying the proposed cooperative tracking control strategy, in which the potential function parameter $a = 1$. (a) The speed curves. (b) The distance between adjacent trains, in which $d_i$ represents the distance between the $i$th train and the $(i + 1)$th train, respectively.

Figure 4: The simulation results of applying the proposed cooperative tracking control strategy, in which the potential function parameter $a = 5$. (a) The speed curves. (b) The distance between adjacent trains.

Figure 5: The desired speed curve, which is divided into three intervals. The desired speeds within [0, 2000] s, [2000, 4000] s, and [4000, 6000] s are, respectively, 70 m/s, 65 m/s, and 70 m/s.
The distances do not need to be adjusted because the boundaries of the safety distance are the same. The situation is the same when the desired speed becomes 70 m/s at 4000 s. This cooperative control strategy is more tolerant to the distance because the distance is allowed to be within a safe range. Once all the high-speed trains reach the consensus state, the distance between adjacent trains will not change no matter how the desired speed changes.

In this paper, a tunable artificial potential function is designed to dynamically adjust the distance with the change of speed. Using the same initial states, parameters, and reference speed, the proposed cooperative tracking control strategy is applied to the multiple high-speed trains system. The potential function parameter \( a = 1 \). The speed and distance curves are shown in Figure 8. Due to the deviation between the initial state and the desired state, the acceleration or braking rate of the high-speed train will be adjusted in the initial stage. At about 300 s, all high-speed trains reach the desired speed and the distances stabilize at about 7.4 km. The high-speed trains run at the desired speed of 70 m/s within [300, 2000] s. The convergence time is longer because the distance needs to be adjusted. At 2000 s, the desired speed becomes 65 m/s. The desired distance changes accordingly, so the trains are not accelerated with a consistent state. After about 1000 s, the speeds reach the desired speed of 65 m/s and the distances stabilize at about 7.1 km. When the desired speed changes back to 70 m/s at 4000 s, the distance between trains finally stabilizes at about 7.4 km. The simulation results prove that the distance between high-speed trains can be adjusted dynamically.

To compare the three control strategies, four performance indices are selected, namely, convergence time, maximum speed overshoot, maximum distance overshoot,
and maximum acceleration. The comparison results are given in Table 4. It is shown that the cooperative control strategy in the study of [17] has the maximum acceleration among the three strategies, which leads to a fast rise at the beginning in Figure 6, but a larger overshoot and thus a long convergence time. The performance indices of the cooperative control strategy in [20] are significantly reduced and also the most optimal among the three strategies. In our proposed control strategy, the convergence time, maximum speed overshoot, and maximum distance overshoot are a little higher than the strategy in the study of [20], which is an acceptable cost to get the dynamical adjustment of distance. Moreover, from Figure 8, it can be found that the convergence curve is smoother with smaller fluctuation. In summary, our proposed cooperative control strategy renders each train running with the desired speed and tracking of the preceding train with a controllable safety distance, while the performances are a little reduced but acceptable.

### 5. Conclusions

In this paper, the cooperative tracking control issue of multiple high-speed trains is investigated. A dynamics equation is established to describe the longitudinal motion of the high-speed train. Based on the consensus algorithm and the artificial potential field theory, a cooperative tracking control strategy is designed to ensure safe and efficient operation. By tuning the parameters of the artificial potential function, the distance between trains can be adjusted dynamically and the distribution range of safety distance is controllable. The simulation results are presented to verify the effectiveness of the proposed cooperative tracking control strategy. The future work will extend the proposed control strategy which can adapt to the situation of dynamically changing communication delays with asymmetrical potential functions.

### Data Availability

The data used to support the findings of this study are included within the article.

### Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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