Research Article

Optimum Intervention in Transportation Networks Using Multimodal System under Fuzzy Stochastic Environment

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Received 8 December 2021; Revised 11 April 2022; Accepted 11 May 2022; Published 27 June 2022

Academic Editor: Alain Lambert

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Multimodal transport refers to the transportation of goods under a single contract but performed with at least two different modes of transport. This research designs a new method for solving the Transportation Problem (TP) by introducing multimodal transport systems under fuzzy-stochastic environment, which we refer to as Fuzzy-Stochastic Multimodal Transportation Problem (FSMMTP). An algorithm is developed to reduce FSMMTP to a deterministic TP, which is mainly based on α-cut of fuzzy numbers, and uses a signed distance function based on the mean expectation of the fuzzy-stochastic cost parameters. We derive the optimal solution as well as optimal selection of mode for transporting the goods in our proposed model. A numerical example justifies the effectiveness of our proposed study. The paper ends with a conclusion and an outlook to future studies.

1. Introduction

In the earlier days in operations research, the Transportation Problem (TP) was mainly used to minimize the total transportation cost for distributing items from sources to numerous destinations. In most daily-life problems, there are several decisions to be made such as fixing the cost of goods, profit for sellers, and multiple objective functions, which can be generated by TP. Nowadays, in competitive market scenarios, minimizing the transportation cost in a business economy and in government policies is one of the most important matters.

Multimodal transport, which is also known as combined transport, allows to transport goods under a single contract, but it is performed with at least two modes of transport. The carrier is liable (in the usual sense) for the whole conveyance even though several/different modes of transport are considered like road, sea, and train. The carrier does not have to possess all the means of transport, and in practice usually this is not valid. The carrier often contracts subcarriers, which are known as actual carriers in common language. Under the situation in which a single type of commodity is required to be transported to the destinations by more than a single type of carrier in a transportation situation, the different types of carriers are termed as Multimodal Transport Operators. In a transportation network, if there exists at least one supplementary origin, then it is described as MMTTP. To accommodate the real-life transportation problem, it is not always possible to fulfill the demand of the customers at the destination points using a single mode of transportation. There are sometimes restrictions for transporting the goods, and so it is required to consider multimodes of transportation from different nodes. Thus, the transportation is not a simple TP but becomes MMTTP.

In real-world situations, sometimes data are not precisely available. This imprecision may occur in a stochastic
or nonstochastic (i.e., fuzzy) sense or for both fuzzy and stochastic environments. Here, a fuzzy-stochastic environment on a cost parameter is considered for solving the Multimodal Transportation Problem (MMTP) and selecting the mode of transportation corresponding to the optimal solution under the prescribed uncertain situations. Probability and mathematical expectations have often been used during the formulation of stochastic models to deal quantitatively with random data. We incorporate a simple and effective way for defuzzification and derandomization of the fuzzy-stochastic cost parameter of our proposed method. We use the concept of \( \alpha \)-cut of the fuzzy numbers, take the expectation separately on both the lower and upper \( \alpha \)-cuts, and then calculate the mean expectation with the help of signed distance. Thereafter, we solve the reduced Linear Programming Problem (LPP) using the simplex method to derive the mode of transportation as well as optimal solution.

Now, we present the main contributions of our study as follows:

- Design FSMMTP, a new class of TP under the consideration of multimodal transport systems
- Incorporate uncertainty through fuzzy-stochastic cost parameters
- Introduce a new algorithm to solve FSMMTP
- On solving FSMMTP, we establish the superiority of our approach compared with classical TP

The rest of the paper is set out as follows. In the next section (Section 2), the review of related research is presented. Section 3 considers the preliminary concepts related to fuzzy and random numbers. The detailed problem background is presented in Section 4. Section 5 designs a new mathematical model for the MMTP under fuzzy-stochastic environment. Section 6 discusses the solution procedure. To show the application of proposed mathematical model of MMTP, Section 7 considers a numerical example including a subsection (c.f., Section 7.1) which depicts the result and discussion regarding the proposed study. Finally, Section 8 describes the conclusions with avenues for future study.

2. Review of Related Research

The traditional transportation model was first initiated by Kantorovich [1], who prescribed an incomplete algorithm for calculating the solution of the transportation problem. Hitchcock [2] studied the problem of minimizing the cost of product distribution from several warehouses to a number of purchasers. Various studies on transportation safety planning were developed by Ergun et al. [3], Luathepet al. [4], Zhi-Chun et al. [5], and Norouzi et al. [6]. Recently, the multiobjective transportation problem has been discussed in different directions by several researchers such as Ehrgott et al. [7], Gupta et al. [8], Maity et al. [9], Castellanos and Frueutz [10], Gupta et al. [11], and Roy et al. [12].

There are several research studies available in the literature to accommodate the transportation problem through the unimodal system, such as Nanry and Wesley [13] who presented a detailed study for solving the unimodal TP. Catalani [14] considered a statistical study to improve intermodal freight transport in Italy, by using road-ship and road-train transports. Qu and Chen [15] posed MMTP as a Multicriteria Decision-Making (MCDM) problem. Recently, Maity et al. [16] studied on MMTP and solved an optimization problem involving neural network.

Several research works (cf. [17, 18]) are available in the literature that analyze the transportation problem, but to the best of our knowledge, until now no one has used multimodal transport systems in uncertain environments for solving TP. Li and Lai [19] presented a fuzzy compromise programming approach to MOTPs. In order to measure a fuzzy event, Zadeh [20] proposed the concept of a possibility measure. Since then, the possibility theory has been studied by many researchers such as Zimmermann [21], Dubois and Prade [22], and Liu [23]. Although the possibility measure has been widely used, it has no self-duality property. However, a self-dual measure is absolutely needed in both theory and practice. A study on bi-level MOTP under fuzziness has been incorporated by Mardanya et al. [24], while Ebrahimnejad et al. [25] examined the shortest path routing problem under a fuzzy environment. The multiobjective multi-item transportation problem has been analyzed through just-in-time by Mardanya et al. [26]. Several researchers analyzed TP in stochastic (cf. [17, 27]) and fuzzy (cf. [21, 28, 29]) environments. Xu and Huang [30] pointed out that the time windows in a vehicle routing problem with soft time windows contain both fuzzy and stochastic information simultaneously. The concept of fuzzy-random variable was introduced by Kwakernaak [31].

The contributions of various researchers related to this field are shown in Table 1. To the best of our knowledge, no attempt has been made so far to solve MMTP under fuzzy-stochastic environment. Therefore, in this study, we oppose FSMMTP and justify its utility considering a real-life problem.

3. Preliminary Concepts

Consider a triangular fuzzy number \( \tilde{A} = (a_1, a_2, a_3) \). An approximation on Triangular Fuzzy Number (TFN) \( \tilde{A} \) is presented in many different ways. According to the proposition of Kaufmann and Gupta [37], the approximated value of \( \tilde{A} \) is denoted by \( \tilde{ap}(\tilde{A}) \) and is defined as

\[
\tilde{ap}(\tilde{A}) = a_1 + 2a_2 + a_3/4.
\]

The algebraic operations on TFNs are as follows:

(i) Addition. The addition of two TFNs \( \tilde{A} = (a_1, a_2, a_3) \) and \( \tilde{B} = (b_1, b_2, b_3) \) using max-min convolution on fuzzy sets is defined as follows:

\[
\mu_{\tilde{A} \oplus \tilde{B}}(z) = \max \{ \mu_{\tilde{A}}(x) \land \mu_{\tilde{B}}(y) : z = x + y \}.
\]

Therefore, \( \tilde{A} \oplus \tilde{B} = (a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \) and \( \tilde{A} \oplus \tilde{B} = (a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1) \).
Table 1: Contributions of different authors related to TP under uncertainty.

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Crisp cost</th>
<th>Fuzzy cost</th>
<th>Stochastic cost</th>
<th>Fuzzy-stochastic cost</th>
<th>Single mode of transport</th>
<th>Multiple mode of transport</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kantorovich [1]</td>
<td></td>
<td></td>
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<tr>
<td>Puri and Ralescu [32]</td>
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<tr>
<td>Mahapatra et al. [17]</td>
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<tr>
<td>Mathur et al. [29]</td>
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</tr>
<tr>
<td>Maity et al. [33]</td>
<td>√</td>
<td>√</td>
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</tr>
<tr>
<td>Maity et al. [16]</td>
<td>√</td>
<td>√</td>
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<tr>
<td>Khalifa et al. [34]</td>
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<tr>
<td>Biswas et al. [35]</td>
<td>√</td>
<td>√</td>
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<tr>
<td>Mardanya et al. [24]</td>
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<tr>
<td>Chhibber et al. [36]</td>
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<tr>
<td>Our paper</td>
<td>√</td>
<td>√</td>
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</tr>
</tbody>
</table>

Here, the symbols $\oplus$ and $\ominus$ are addition and subtraction of fuzzy numbers, respectively.

(ii) Scalar Multiplication. Multiply by a scalar $k$ with TFN, $A$ whose membership function is $\mu_A(x)$ which gives a TFN that has the following membership function: $\mu_{kA}(z_1) = \mu_A(kz_1)$, Therefore,

$$kA = \begin{cases} (ka_1, ka_2, ka_3), & \text{if } k \geq 0, \\ (ka_3, ka_2, ka_1), & \text{if } k < 0. \end{cases}$$

(iii) Signed Distance between Two Fuzzy Numbers. According to the concept of Yao and Wu [28], if two fuzzy numbers $A$ and $B$ are represented by $\cup_{a \in (0,1)} [a_L^\alpha, a_U^\alpha]$ and $\cup_{b \in (0,1)} [b_L^\beta, b_U^\beta]$, then the signed distance between $A$ and $B$ is the distance between the midpoints $M(a(a)) = 1/2(a_L^\alpha, a_U^\beta)$ of $[a_L^\alpha, a_U^\beta]$ and $M(b(\beta)) = 1/2(b_L^\beta, b_U^\beta)$ of $[b_L^\beta, b_U^\beta]$ over the set of $\alpha$ values in $(0,1)$, which is defined as follows:

$$d(\tilde{A}, \tilde{B}) = 1/1 - 0 \int_0^1 [M(a(a)) - M(b(\beta))] \, d\alpha = 1/2 \int_0^1 [a_L^\alpha + a_U^\beta - b_L^\beta - b_U^\beta] \, d\alpha.$$

(vi) Expectation of Fuzzy Random Variable (Xu and Huang [30]). The expectation of fuzzy random variable $X$ is a unique fuzzy number $E(X)$, which is

$$E(X) = \mu_{\mu_X}^{-1}(\omega) = \sup_{w \in \mathbb{R}} \left\{ a : \mu_X(w) \right\},$$

where $\mu_X(w)$ is the membership function of fuzzy random variable $X$. The Fuzzy Random Variable forms a Borel measurable function $\tilde{X}$: $(\mathbb{S}, \Omega, \mathbb{P}) \rightarrow (\mathbb{F}, \rho)$, where $\mathbb{S}$ is considered as the sample space, $\Omega$ is a Borel measurable subsets of $\mathbb{S}$, and $\mathbb{P}$ refers to a probability measure.
4. Problem Background

The term *multimodal* is defined for several modes of transport in a TP, and as a whole, it is referred to as the multimodal transportation problem. Goods can be transported via several modes, and people also use different modes of transportation for their journey. Therefore, multiple modes of transportation are considered as follows:

(i) Roadway/highway automobiles (including taxi), truck, motorcycle, and so on
(ii) Passenger rail and traditional freight train, bus, car, and so on
(iii) Air-passenger service and freight transportation
(iv) Water-ferry, barge, transatlantic vessel, cruise ship, and so on
(v) Bicycling, nonmotorized-walking, and so on

Under the consideration that in a transportation system, there exist multiple types of transportation vehicles and goods are transported from origins to destinations by use of multiple steps using different types of vehicles, then MMTP is used to formulate and solve the mathematical problem. There are important perspectives in MMTP such as follows:

(i) To increase economic productivity and effectiveness, thereby enhancing the nation’s competitiveness globally
(ii) To decrease total transportation costs by allowing each mode to be used for the portion of the trip in which it is the best suited
(iii) To generate higher returns from public and private investments
(iv) To reduce overcrowding and burden of a transportation management
(v) To improve mobility for the elderly, isolated, disabled, and economically disadvantaged
(vi) To reduce energy consumption and apply in sustainable development (cf. Das et al. [38])

To represent the MMTP, we introduce the following useful definitions.

**Definition 1 (Starting Origin (STO)).** In a transportation problem, the sources that have the capacity to supply the goods only but there are no capacity to gather the goods treated as Starting Origins.

**Definition 2 (Final Destination (FD)).** In a transportation problem, the destinations that have the capacity of gathering the goods only but no such capacity to supply the goods are considered as Final Destinations.

It is not possible to supply the goods according to the requirement of the final destinations from the starting origins because of the vehicle capacity/multiple routes of transport. In that case, some destination nodes are required that have the capacity of supplying and receiving the goods simultaneously. These nodes are known as supplementary origins.

**Definition 3 (Supplementary Origin (SO)).** In a transportation problem, the destinations that have the capacity of collecting the goods as well as the capacity of delivering the goods are denoted as Supplementary Origins.

In Figure 1, $O_1, O_2, \ldots, O_m$ are the ground origins; $SO_{11}, SO_{12}, \ldots, SO_{1m}$ are the supplementary origins of label 1, and $D_1, D_2, \ldots, D_n$ are referred as the final destinations.

In a transportation network, if there exists at least one supplementary origin, then it is described as MMTP. To accommodate the real-life transportation problem, it is not always possible to fulfill the demand of the customers at the destination points using a single mode of transportation. There are sometimes restrictions for transporting the goods, and so it is required to consider multimodes of transportation from different nodes. Thus, the transportation is not a simple TP but becomes MMTP.

This study also considers real-life practical uncertain phenomena and incorporates a fuzzy-stochastic cost parameter in MMTP, which is termed as the Fuzzy-Stochastic Multimodal Transportation Problem (FSMMTP). When there are several agents of transportation corresponding to a particular mode of transportation with different probabilities of selection by a transportation agent, then corresponding to each mode of transportation, there is some ambiguous information on the transportation cost. Hence, the selection of an agent for transportation in MMTP is considered to be stochastic in nature, and again each agent provides a fuzzy cost to complete the transportation due to the seasonal effect. As a whole, the cost parameter in MMTP is formulated in fuzzy-stochastic environment. The mathematical model of the proposed FSMMTP is described in detail in the next section.

5. Mathematical Model

A fuzzy-stochastic transportation problem is a typical one in which the main objective is to minimize the total transportation cost and is defined as follows.

**Model 1**

\[
\begin{align*}
\text{minimize } & \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}, \\
\text{subject to } & \quad \sum_{j=1}^{n} x_{ij} \leq a_i \ (i = 1, 2, \ldots, m), \\
& \quad \sum_{i=1}^{m} x_{ij} \geq b_j \ (j = 1, 2, \ldots, n), \\
& \quad x_{ij} \geq 0 \forall i \text{ and } j,
\end{align*}
\]
where $\tilde{C}_{ij}$ ($i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$) is the fuzzy-stochastic transportation cost per unit commodity from the $i^{th}$ origin to the $j^{th}$ destination. Here, $a_i$ ($i = 1, 2, \ldots, m$) and $b_j$ ($j = 1, 2, \ldots, n$) are availability and demand at the $i^{th}$ origin and at the $j^{th}$ destination, respectively, and $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j$ is the feasibility condition.

The presence of SO in a fuzzy-stochastic TP reduces Model 1 to FSMMTP. To formulate the mathematical model of FSMMTP, we use the following notations (Table 2).

The complete FSMMTP model (c.f., Maity et al. [16]) is described as follows.

\textbf{Model 2}

\begin{equation}
\begin{align*}
\text{minimize } & z = z^1 + z^2 + \cdots + z^r, \\
z^1 & = \sum_{i=1}^{m_1} \sum_{j=1}^{n_1} a^1_i \tilde{C}_{ij} x^1_{ij} + \sum_{i=1}^{m_2} \sum_{j=1}^{n_2} a^2_i \tilde{C}_{ij} x^2_{ij} + \cdots + \sum_{i=1}^{m_r} \sum_{j=1}^{n_r} a^r_i \tilde{C}_{ij} x^r_{ij}, \\
z^2 & = \sum_{i=1}^{m_1} \sum_{j=1}^{n_1} a^1_i \tilde{C}_{ij} x^1_{ij} + \sum_{i=1}^{m_2} \sum_{j=1}^{n_2} a^2_i \tilde{C}_{ij} x^2_{ij} + \cdots + \sum_{i=1}^{m_r} \sum_{j=1}^{n_r} a^r_i \tilde{C}_{ij} x^r_{ij} - 1, \\
& \vdots \\
z^r & = \sum_{i=1}^{m_1} \sum_{j=1}^{n_1} a^1_i \tilde{C}_{ij} x^1_{ij},
\end{align*}
\end{equation}

subject to the constraints representing storing capacity at STO and SO of all labels:
Table 2: Different notations for designed mathematical model.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_i ):</td>
<td>Number of Starting Origins (STOs)</td>
</tr>
<tr>
<td>( n_f ):</td>
<td>Number of Final Destinations (FDs)</td>
</tr>
<tr>
<td>( m_i ):</td>
<td>Number of Supplementary Origins (SOs) at ((t - 1))th level, (t = 2, 3, \ldots, r)</td>
</tr>
<tr>
<td>( r ):</td>
<td>Number of labels for origins</td>
</tr>
<tr>
<td>( a_i^{1,t} ):</td>
<td>Availability of goods at (t)th origin of STO</td>
</tr>
<tr>
<td>( a_i^{1,t} ):</td>
<td>Availability of goods at (t)th origin of ((t - 1))th level SO, (t = 2, 3, \ldots, r)</td>
</tr>
<tr>
<td>( b_j ):</td>
<td>Demand at (j)th node of FD</td>
</tr>
<tr>
<td>( g_{ij}^{t} ):</td>
<td>Carrying capacity of a vehicle from STO to FD</td>
</tr>
<tr>
<td>( c_{ij}^{t} ):</td>
<td>Transportation cost per unit commodity from (t)th origin to (j)th destination from STO to FD where (t = 2, 3, \ldots, r)</td>
</tr>
<tr>
<td>( z^t ):</td>
<td>Transportation cost per unit commodity for transportation from (t)th origin to (j)th destination from SO of ((t - 1))th level to FD</td>
</tr>
<tr>
<td>( x_{ij}^{1,t} ):</td>
<td>Decision variable of transportation from (t)th origin of STO to (j)th destination of FD</td>
</tr>
<tr>
<td>( x_{ij}^{2,t} ):</td>
<td>Decision variable of transportation from (t)th origin to (j)th destination from SO of ((t - 1))th level to FD</td>
</tr>
<tr>
<td>( x_{ij}^{3,t} ):</td>
<td>Decision variable of transportation from (j)th node of TO of ((t - 1))th level to (j)th node of TO of ((s - 1))th level, (t = 2, 3, \ldots, r - 1; s = 2, 3, \ldots, r) with (t &lt; s)</td>
</tr>
<tr>
<td>( a_i^{2,t} ):</td>
<td>Carrying capacity of vehicle from SO of ((t - 1))th level to FD</td>
</tr>
<tr>
<td>( a_i^{3,t} ):</td>
<td>Carrying capacity of vehicle from SO of ((t - 1))th level, (t = 2, 3, \ldots, r), to SO of ((s - 1))th level label, (s = r, r - 1, \ldots, 2)</td>
</tr>
<tr>
<td>( z^t ):</td>
<td>Objective function for minimizing the transportation cost to the final destination from STO and all SO</td>
</tr>
<tr>
<td>( x_{ij}^{1,t} ):</td>
<td>Decision variable of transportation from (t)th origin of STO to (j)th destination of FD</td>
</tr>
<tr>
<td>( x_{ij}^{2,t} ):</td>
<td>Decision variable of transportation from (t)th origin to (j)th destination from SO of ((t - 1))th level, (t = 2, 3, \ldots, r) to FD</td>
</tr>
<tr>
<td>( x_{ij}^{3,t} ):</td>
<td>Decision variable of transportation from (j)th node of TO of ((t - 1))th level to (j)th node of TO of ((s - 1))th level, (t = 2, 3, \ldots, r - 1; s = 2, 3, \ldots, r) with (t &lt; s)</td>
</tr>
</tbody>
</table>

\[
\sum_{j=1}^{n_f} a_i^{1,t} x_{ij}^{1,t} + \sum_{j=1}^{n_f} a_i^{2,t} x_{ij}^{2,t} + \cdots + \sum_{j=1}^{n_f} a_i^{r-1,t} x_{ij}^{r-1,t} \leq a_i^{1,t} (i = 1, 2, \ldots, m_i), \tag{6}
\]

the constraints regarding least demands at FD:

\[
\sum_{i=1}^{m_i} a_i^{1,t} x_{ij}^{1,t} + \sum_{i=1}^{m_i} a_i^{2,t} x_{ij}^{2,t} + \cdots + \sum_{i=1}^{m_i} a_i^{r-1,t} x_{ij}^{r-1,t} \geq b_j \quad (j = 1, 2, \ldots, n_f), \tag{7}
\]

and the constraints regarding storing and distributing of goods at the nodes of, SO of all labels:

\[
\sum_{j=1}^{n_f} a_i^{1,t} x_{ij}^{1,t} + \sum_{j=1}^{n_f} a_i^{2,t} x_{ij}^{2,t} + \cdots + \sum_{j=1}^{n_f} a_i^{r-1,t} x_{ij}^{r-1,t} \leq a_i^{1,t} (i = 1, 2, \ldots, m_i), \tag{8}
\]

\[
\sum_{j=1}^{n_f} a_i^{1,t} x_{ij}^{1,t} + \sum_{j=1}^{n_f} a_i^{2,t} x_{ij}^{2,t} + \cdots + \sum_{j=1}^{n_f} a_i^{r-1,t} x_{ij}^{r-1,t} \leq a_i^{1,t} (t = 1, 2, \ldots, m_2), \tag{8}
\]

\[
\sum_{j=1}^{n_f} a_i^{1,t} x_{ij}^{1,t} + \sum_{j=1}^{n_f} a_i^{2,t} x_{ij}^{2,t} + \cdots + \sum_{j=1}^{n_f} a_i^{r-1,t} x_{ij}^{r-1,t} \leq a_i^{1,t} (t = 1, 2, \ldots, m_3), \tag{8}
\]

\[
\vdots
\]

\[
\sum_{j=1}^{n_f} a_i^{1,t} x_{ij}^{1,t} + \sum_{j=1}^{n_f} a_i^{2,t} x_{ij}^{2,t} + \cdots + \sum_{j=1}^{n_f} a_i^{r-1,t} x_{ij}^{r-1,t} \leq a_i^{1,t} (t = 1, 2, \ldots, m_r), \tag{8}
\]

\[
x_{ij}^{(s)} \geq 0 \forall i, j, s and p.
\]
To obtain the feasible solution of Model 2, it is again essential to satisfy that the amount of goods required at the nodes of FD be less or equal to the sum of their availability at the nodes of STO. Therefore, the feasibility condition of Model 2 is considered as ∑m l=1 a l ≥ ∑n j=1 b j.

In Model 2, the number of decision variables is (m1 × m2 × · · · × m r × n1).

The feasible region of Model 2 is constructed by considering the following assumptions:

(i) There are m1 availability constraints (6) for the ground origins.

(ii) There are n1 number of demand constraints (6) for the final destinations.

(iii) There are restrictions of storing items in the supplementary origins, and so we introduce (m2 + m3 + · · · + m r ) number of inequalities from (6) to (7).

(iv) Again, the delivered amount of goods from the supplementary origins does not exceed the supplied amount of goods to the respective supplementary origins. To do this, we introduce (m2 + m3 + · · · + m r ) number of inequalities from (8).

Model 2 thus consists of (m1 × m2 × · · · × m r × n1) variables and [2(m2 + m3 + · · · + m r ) + m1 + n1] constraints along with the nonnegativity conditions. It is noted that, if there does not exist any path between two nodes in the proposed FSMMTP, then we consider the value of decision variable to be 0 and remove the variable form the proposed model, which reduces the number of variables in the mathematical model. In the next section, we present the solution procedure for solving FSMMTP.

6. Solution Procedure

The formulated FSMMTP consists of fuzzy-stochastic cost parameters, and so the objective function is now in a nondeterministic form. First, we discuss the reduction of fuzzy-stochastic cost parameters to crisp parameters, and thereafter we present an algorithm to solve FSMMTP.

Reduction of fuzzy-stochastic parameter into crisp form is as follows.

Proposition 2. Assume an exponential function f(x) = Ae−|x−μ|, x ≥ 0, where A is a constant; then, f is a probability density function if A = 1/(2 − e−μ); here, μ > 0 is a real constant.

Proof. For continuous case, the probability density function f satisfies ∫−∞ +∞ f (x)dx = 1.

Here, f (x) = Ae−|x−μ|, x ≥ 0.

Therefore,

\[ A \int_{0}^{\infty} e^{-|x-\mu|} dx = 1, \]

or \[ A \left( \int_{0}^{\mu} e^{-x-\mu} dx + \int_{\mu}^{\infty} e^{-x+\mu} dx \right) = 1, \]

or \[ A \left( e^{-\mu} \int_{0}^{\mu} 1 dx + e^{\mu} \int_{\mu}^{\infty} -1 dx \right) = 1, \]

or \[ A \left( 1 - e^{-\mu} + 1 = \frac{1}{2 - e^{-\mu}} \right) \]

This completes the proof of the proposition.

Let C ij denote the fuzzy-stochastic unit transportation cost from i-th source to j-th destination in t-th label of FSMMTP. These costs can be represented in the following way.

Let C ij follow a fuzzy-stochastic density function f (C ij) with \( \frac{x}{\bar{C} ij} \geq 0 \), which has the membership function \( \mu_{\bar{C} ij} \) : \( R \rightarrow (0, 1) \), and the membership function \( \mu_{\bar{x}ij} \) is defined as follows:

\[ \mu_{\bar{x}ij} (x) = \begin{cases} \frac{x - \bar{C} ij + d_1}{d_1}, & \text{if } \bar{C} ij - d_1 \leq x \leq \bar{C} ij \\ \frac{\bar{C} ij + d_2 - x}{d_2}, & \text{if } \bar{C} ij \leq x \leq \bar{C} ij + d_2 \\ 0, & \text{if elsewhere} \end{cases} \] (10)

Here, \( d_1 \) and \( d_2 \) are left and right deviations with respect to the mean of the fuzzy-stochastic cost parameter. The probability density function can be taken according to the decision maker’s choice, which may be Normal, Gamma, Weibull, and so on. For many real-life situations in TP, the DM may consider the previous record of transportation cost, and from them the DM can easily calculate the mean cost. It is also easy to formulate the corresponding exponential function. In this paper, we consider the exponential distribution as follows:

\[ g(C ij) = \frac{1}{(2 - e^{-\mu})} e^{-|C ij - \mu|}. \] (11)

The mean corresponding to the pdf can then be calculated as follows:

\[ \frac{1}{2 - e^{-\mu}} (2\mu + e^{-\mu}). \] (12)

Again, the variance of the pdf is defined as

\[ 1/(2 - e^{-\mu})^2 (e^{-2\mu} - (2\mu + 4\mu + 8)e^{-\mu} + 8). \]

The objective function of Model 2 is not in deterministic form as the fuzzy-stochastic cost components are present in it. Hence, we need to convert the objective function into deterministic form by reducing the fuzzy-stochastic cost parameter into a crisp parameter in the following way.

First, we treat the fuzzy nature of the cost parameter and defuzzify it by considering the α-cut of the cost parameter with the corresponding probability distribution. Second, we
take an expectation to reduce it to the corresponding crisp parameter. Following the concept of Yao et al. [28], the membership function \( \mu_{\tilde{C}_{ijr}}(x) \) of the fuzzy probability density function \( f(\tilde{C}_{ijr}) \) corresponding to \( \tilde{C}_{ijr} \) is obtained by the extension principle. The \( \alpha \)-cut of \( \tilde{C}_{ijr} \) and \( f(\tilde{C}_{ijr}) \) is, respectively,

\[
(\tilde{C}_{ijr})_\alpha = [(C_{ijr})^\alpha, (C_{ijr})^{\alpha \bot}],
\]

where \( (C_{ijr})^\alpha = C_{ijr} - (1 - \alpha)d_1 \), \( (C_{ijr})^{\alpha \bot} = C_{ijr} + (1 - \alpha)d_2 \), and \( f(\tilde{C}_{ijr})_\alpha = [f(C_{ijr}^\alpha), f(C_{ijr}^{\alpha \bot})] \), where \( f(C_{ijr}^\alpha) \) and \( f(C_{ijr}^{\alpha \bot}) \) can be obtained by using equation (11).

Using the signed distance function, we reduce the objective function with the fuzzy-stochastic cost to a crisp objective function described as follows:

\[
E(z^*') = E(z^*') + E(z^*') \cdots + E(z^*')
\]

where \( E(z^*') = \sum_{i=1}^{m} \sum_{j=1}^{n} \alpha_i^1 x_{ij}^1 \frac{1}{2} \int_0^\infty [E\left(\left(C_{ijr}^{\alpha_1}\right)_a\right) + E\left(\left(C_{ijr}^{\alpha_2}\right)_a\right) ] d\alpha
\]

\[
+ \sum_{i=1}^{m} \sum_{j=1}^{n} \alpha_i^2 x_{ij}^2 \frac{1}{2} \int_0^\infty [E\left(\left(C_{ijr}^{\alpha_3}\right)_a\right) + E\left(\left(C_{ijr}^{\alpha_4}\right)_a\right) ] d\alpha
\]

\[
+ \cdots + \sum_{i=1}^{m} \sum_{j=1}^{n} \alpha_i^r x_{ij}^r \frac{1}{2} \int_0^\infty [E\left(\left(C_{ijr}^{\alpha_r}\right)_a\right) + E\left(\left(C_{ijr}^{\alpha_{r+1}}\right)_a\right) ] d\alpha
\]

\[
E(z^*') = \sum_{i=1}^{m} \sum_{j=1}^{n} \alpha_i^1 x_{ij}^1 \frac{1}{2} \int_0^\infty [E\left(\left(C_{ijr}^{\alpha_1}\right)_a\right) + E\left(\left(C_{ijr}^{\alpha_2}\right)_a\right) ] d\alpha
\]

In particular, let us consider the fuzzy probability density function:

\[
f(C_{ijr}) = 1/2 - \mu_{ijr} \left| \tilde{C}_{ijr} \right|, \forall \tilde{C}_{ijr} \geq 0.
\]

We then have
\[ f(C_{ij}^d) = \frac{1}{2 - \mu_{ij}} e^{-[(c_{ij})^d]], C_{ij} \geq (1 - \alpha)d_1,} \]

\[ f(C_{ij}^c) = \frac{1}{2 - \mu_{ij}} e^{-[(c_{ij})^c]], C_{ij} \geq -(1 - \alpha)d_2,} \]

\[ E\left(\left(C_{ij}^d\right) \alpha\right) = \frac{1}{2 - \mu_{ij}} \lim_{B \to \infty} \int_{(1-a)d_1}^B \left[ C_{ij}^d - (1 - \alpha)d_1 \right] e^{-\left(C_{ij}^d - \mu_{ij}\right)} dC_{ij} \]

\[ + \frac{1}{2 - \mu_{ij}} \lim_{B \to \infty} \int_{(1-a)d_2}^B \left[ C_{ij}^c + (1 - \alpha)d_2 \right] e^{-\left(C_{ij}^c - \mu_{ij}\right)} dC_{ij} \]

\[ = 2\mu_{ij}^d - 2(1 - \alpha)d_1 + e^{-(\mu_{ij}(1-a)d_1)} \int E\left(\left(C_{ij}^d\right) \alpha\right) \]

\[ = 2\mu_{ij}^c + 2(1 - \alpha)d_2 + e^{-(\mu_{ij}(1-a)d_2)} \int E\left(\left(C_{ij}^c\right) \alpha\right) \]

The integral in the expression of \( z^t, z^{t'}, \ldots, z^{t'} \) now reduces to the following form:

\[ \int_0^1 \left[ E\left(\left(C_{ij}^d\right) \alpha\right) + E\left(\left(C_{ij}^c\right) \alpha\right) \right] d\alpha \]

\[ = \frac{1}{2 - \mu_{ij}} \int_0^1 \left[ 2\mu_{ij}^d - 2(1 - \alpha)d_1 + e^{-(\mu_{ij}(1-a)d_1)} \right] \]

\[ + \left[ 2\mu_{ij}^c + 2(1 - \alpha)d_2 + e^{-(\mu_{ij}(1-a)d_2)} \right] d\alpha \]

\[ = \frac{1}{2 - \mu_{ij}} \left[ 4\mu_{ij}^d - (d_1 - d_2) + e^{-\mu_{ij}} \left( \frac{e^{d_1} - 1}{d_1} + \frac{e^{d_2} - 1}{d_2} \right) \right]. \]

We thus present an algorithm to solve our proposed FSMMTP problem (Algorithm 1).

7. Numerical Example

A numerical example is presented here to justify the utility of FSMMTP. A reputed Mineral Water Production Company (MWPC) has three factories producing mineral water at three locations S1, S2, and S3 (here treated as the ground origins). There are two cities, D1 and D2 (here taken as final destinations), with serious drinking water problems. MWPC supplies drinking water to these two cities. MWPC has three storage sites, namely, B1, B2 (considered as the first-level supplementary origins), and C1 (considered as the second-level supplementary origins). There is only one way to supply water from B1 or B2 to C1. There are different modes of transportation from origins to destinations. Company wants to determine the transportation plan with the minimum transportation cost. The graphical representation of the numerical example is shown in Figure 2.

The traditional approach of TP cannot provide any such mathematical model to solve the proposed problem. To solve the problem, we here formulate the mathematical model known as FSMMTP.

The following notations and assumptions are considered to formulate the mathematical model of FSMMTP:

(i) The decision variables for transporting the goods are as follows:

When S1, S2, and S3 to D1 and D2 are considered as \( x_{ij}^{t'}, \) use ship-way with vehicle capacity \( a_t = 500 \) unit
When $B_1$ and $B_2$ to $D_1$ and $D_2$ are taken as $x_{ij}^1$, use rail-way with vehicle capacity $\alpha_1 = 100$ unit.
When $C_1$ to $D_1$ and $D_2$ are considered as $x_{ij}^3$, use road-way without any vehicle capacity restriction, i.e., $\alpha_3 = 1$ unit.
When $S_1$, $S_2$, and $S_3$ to $C_1$ are considered as $x_{ij}^2$, vehicle restriction $\alpha_2 = 250$ unit.
When $B_1$ and $B_2$ to $C_1$ are considered as $x_{ij}^2$, there is no vehicle restriction.
When $S_1$, $S_2$, and $S_3$ to $B_1$ and $B_2$ are considered as $x_{ij}^3$, there is not any vehicle restriction.

(ii) The optimal feasible region of FSMMTP corresponding the numerical example consists by twelve constraints.

The supply capacity at the ground origins $S_1$, $S_2$, and $S_3$ is introduced by three constraints. The demand at the final destinations $D_1$ and $D_2$ is considered by three constraints. Storing capacity at the supplementary origins $B_1$, $B_2$, and $C_1$ provides three constraints. Amount of goods distributed from the supplementary origins $B_1$, $B_2$, and $C_1$ does not exceed the amount of storing items and produces three constraints. Hence, the number of constraints in MMTP of the numerical example is 12.

Since the seasonal effects of the cost parameters (e.g., production costs and carrying costs) are uncertain, MWPC considered fuzzy-stochastic costs, which follow a triangular fuzzy exponential distribution. Therefore, MWPC sets the expected cost as the exponential mean (i.e., $\mu_{ij}$) of fuzzy-stochastic cost parameters for different modes of transportation, which as presented in Tables 3 to 8. MWPC also assumes that $d_1 = 3$ and $d_2 = 4$.

The availability of goods at each Ground Origin $S_1$, $S_2$, and $S_3$ is 1250, 1250, and 1250 units, respectively. The maximum capacity of storing at the Supplementary Origins

### Algorithm 1: Steps for solving FSMMTP.

1. **Step 1:** according to the given problem, first set up the level of the supplementary and other nodes of the decision making problem.
2. **Step 2:** consider the fuzzy-stochastic cost with respective membership functions and distribution functions for each transportation route by the decision maker’s choice.
3. **Step 3:** construct the objective function corresponding to each level of transportation along with the main objective function and formulate the restrictions as per choice of the DM, described in the constraints (6)–(8) of Model 2.
4. **Step 4:** reduce FSMMTP to crisp TP using Section 5.
5. **Step 5:** solve the deterministic model obtained in Step 4 by using a simplex algorithm, and in case of a number of variables, one can use MATLAB or LINGO software to solve the problem.
6. **Step 6:** from the obtained solution, we get the optimal value of the objective function as well as the optimal selection transportation mode. Thus, DM can get information about the amount of goods run over the supplementary origins throughout the procedure.
7. **Step 7:** stop.

![Figure 2: Graphical representation of the numerical example.](image)
Table 3: Exponential mean cost from S1, S2, and S3 to D1 and D2 (in $).

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>14.12</td>
<td>12.25</td>
</tr>
<tr>
<td>S2</td>
<td>14.25</td>
<td>17.20</td>
</tr>
<tr>
<td>S3</td>
<td>13.10</td>
<td>12.20</td>
</tr>
</tbody>
</table>

Table 4: Exponential mean cost from B1 and B2 to D1 and D2 (in $).

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>7.25</td>
<td>9.85</td>
</tr>
<tr>
<td>B2</td>
<td>8.20</td>
<td>6.12</td>
</tr>
</tbody>
</table>

Table 5: Exponential mean cost from C1 to D1 and D2 (in $).

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>5.40</td>
<td>4.25</td>
</tr>
</tbody>
</table>

B1, B2, and C1 is 1100 units, 1200 units, and 1000 units, respectively. The demands at the destinations D1 and D2 are 1755 and 1975, respectively. Using the procedure described in Section 5, we obtain the mathematical model as follows.

\[
\begin{align*}
\text{minimize } & z_1 + z_2 + z_3, \\
& z_1 = 500 \left( 14.37x_{111}^1 + 12.50x_{121}^1 + 14.50x_{211}^1 + 17.45x_{221}^1 + 13.35x_{311}^1 + 12.45x_{321}^1 \right) \\
& + 100 \left( 7.50x_{111}^2 + 10.10x_{121}^2 + 8.45x_{211}^2 + 6.37x_{221}^2 \right) + 5.65x_{111}^3 + 4.51x_{121}^3, \\
& z_2 = 250 \left( 10.4x_{112}^1 + 10.50x_{122}^1 + 7.45x_{212}^1 \right) + 8.40x_{112}^2 + 6.50x_{212}^2, \\
& z_3 = 4.40x_{113}^1 + 5.51x_{123}^1 + 6.45x_{213}^1 + 5.11x_{223}^1 + 7.50x_{313}^1 + 6.40x_{323}^1, \\
& 500(x_{111}^1 + x_{121}^1) + 100x_{112}^1 + 250(x_{113}^1 + x_{123}^1) \leq 1250, \\
& 500(x_{111}^2 + x_{121}^2) + 100x_{112}^2 + 250(x_{113}^2 + x_{123}^2) \leq 1250, \\
& 500(x_{111}^3 + x_{121}^3) + 100x_{112}^3 + 250(x_{113}^3 + x_{123}^3) \leq 1250, \\
& 500(x_{111}^1 + x_{211}^1 + x_{311}^1) + 100(x_{112}^1 + x_{212}^1) + x_{111}^3 \geq 1755, \\
& 500(x_{111}^1 + x_{211}^1 + x_{311}^1) + 100(x_{122}^1 + x_{222}^1) + x_{121}^3 \geq 1975, \\
& x_{113}^1 + x_{123}^1 + x_{131}^1 \leq 1100, \\
& x_{113}^2 + x_{123}^2 \leq 1200, \\
& 250(x_{112}^1 + x_{212}^1 + x_{312}^1) + x_{112}^1 + x_{212}^1 \leq 1000, \\
& x_{113}^1 + x_{213}^1 + x_{313}^1 \leq 100(x_{111}^1 + x_{211}^1) + x_{112}^2 + x_{212}^2, \\
& x_{123}^1 + x_{223}^1 + x_{323}^1 \leq 100(x_{121}^1 + x_{221}^1) + x_{122}^2 + x_{222}^2, \\
& 250(x_{112}^1 + x_{212}^1 + x_{312}^1) + x_{112}^2 + x_{212}^2 \leq x_{111}^3 + x_{121}^3, \\
& x_{ijp}^p \geq 0 \text{ (all are taken integers); } \forall i, j, s, p.
\end{align*}
\]

Model 3 is simply a LPP that can be solved through any simplex algorithm. As Model 3 contains a large number of variables, we use LINGO software to obtain the solution of Model 3.
7.1. Result and Discussion. The minimum value of the objective function is $46,082$. The optimal solution of Model 3 is shown in Tables 9-12.

Solution of the numerical example in absence of supplementary origins is as follows.

In the absence of supplementary origins (cf. B1, B2, and C1), the problems become a classical TP. Then, the mathematical mode is as follows:

Model 4

\[
\begin{align*}
\text{minimize } z_1, \\
& z_1 = 500(14.37x_{111}^1 + 12.50x_{121}^1 + 14.50x_{211}^1 + 17.45x_{221}^1 + 13.35x_{311}^1 + 12.45x_{321}^1), \\
& 500(x_{111}^1 + x_{121}^1) \leq 1250, \\
& 500(x_{211}^1 + x_{221}^1) \leq 1250, \\
& 500(x_{311}^1 + x_{321}^1) \leq 1250, \\
& 500(x_{111}^1 + x_{211}^1 + x_{311}^1) \geq 1755, \\
& 500(x_{121}^1 + x_{221}^1 + x_{321}^1) \geq 1975, \\
& x_{ij}^p \geq 0.
\end{align*}
\]

The minimum value of the objective function is $49,495$. The optimal solution of Model 4 is shown in Table 13.

However, the company would have to pay the transportation cost for the following amount of goods (see Table 14) because the carrying cost of vehicle is not reduced by the empty space in the vehicle.

As a result, the total transportation cost significantly exceeds the computed cost $49,495. Furthermore, sometimes the transportation of goods between the nodes is limited due to vehicle capacity. To minimize the transportation costs, different appropriate types of vehicles need to be used at different nodes.

To validate the proposed mathematical model of MMTP, we describe various possibilities for the numerical example as follows:

(i) Consider that the routes among the supply points S1, S2, and S3 to destination points D1 and D2 are sea-way. Delivering the goods is thus made via ship. Obviously, a large amount of goods can be delivered by ship, and the amount is 500 units. In that situation, if there are no other nodes available like B1, B2, and C1, then the formulated TP is a classical TP. In this case, we see that there exists a feasible solution of the proposed problem, but the transportation cost is not minimized. In each destination the minimum requirements are 1755 units and 1975 units of goods, which indicate at least two ships are required for delivering the goods in each node D1 and D2. Thus, traditional TP is not enough to give a definite conclusion without considering the supplementary origins as we considered in our proposed study.

(ii) We again assume that there is a connection through rail-way between B1 and B2 to D1 and D2. The capacity of transports in each time by rail-way is large, and so we consider that a single transport it needs 100 units of goods. In that situation, we solve the problem without considering the supplementary origin C1 (i.e., using the value for the variables as “0” for those that are taken for C1) and add the total transportation cost of Dollar 47, 604; clearly, this is a larger cost than the original solution presented by our method. The amount of transported goods to nodes D1 and D2, respectively, is 2000 units and 1800 units. The amount of goods supplied at the supplementary origins B1 and B2 is 1100 units and 1200 units, respectively.

(iii) In a similar way, if we formulate the mathematical model without considering the supplementary origins B1 and C1 or B2 and C1, then the transportation cost will increase.

(iv) Another important factor in the study is that it considers transportation parameters as fuzzy-stochastic types which accommodate the real situations of MMTP where data are not given in a crisp way. The fuzzy-stochastic parameters are handled in our study and which will be helpful in real-life uncertain multimodal transportation systems.

According to our discussion, we have analyzed that the introduction of a multimodal system in TP is very much.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$x_{111}^1$</th>
<th>$x_{121}^1$</th>
<th>$x_{211}^1$</th>
<th>$x_{221}^1$</th>
<th>$x_{311}^1$</th>
<th>$x_{321}^1$</th>
<th>$x_{212}^1$</th>
<th>$x_{222}^1$</th>
<th>$x_{312}^1$</th>
<th>$x_{322}^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0</td>
<td>1 $\times$ 500</td>
<td>0</td>
<td>0</td>
<td>1 $\times$ 500</td>
<td>0</td>
<td>8 $\times$ 100</td>
<td>0</td>
<td>0</td>
<td>12 $\times$ 100</td>
</tr>
</tbody>
</table>

Table 9: The amounts of transported goods to final destinations D1 and D2.
Transported to final destinations \( D_1 \) and \( D_2 \).

Supplementary origins \( B_1 \) and \( B_2 \).

Origin \( C_1 \).

Mode of transportation.

Procedure to calculate the optimal value as well as optimal parameter in MMTP and we have discussed a solution in certain and uncertain environments. Pëhe proposed model of FSMMTP may be applied to formulate the mathematical model under multiple-mode transportation, and its solution finds the lowest cost transportation route. Pëhe's study has introduced the multimodal system into TP planning should be integrated such as networks, stations, user information, and fare payments systems under different uncertain environments. The proposed model of FSMMTP can thus be used for selecting the modes in a variety of transportation improvement policies such as mobility management strategies, pricing reforms, and smart growth land use policies.

### Data Availability

All data used to support the findings of this study are included within the article.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

### Acknowledgments

The work of the second author was partially supported by the Ministry of Science and Technology of the Republic of China (Taiwan) under Grant MOST 108-2221-E-011-051-MY3 and the Center for Cyber-Physical System Innovation from the Featured Areas Research Center Program within the Framework of the Higher Education Sprout Project by the Ministry of Education (MOE) in Taiwan.

### References


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### Table 10: The amounts of transported goods to supplementary origin \( C_1 \).

<table>
<thead>
<tr>
<th>Variable ( x_{112} )</th>
<th>( x_{122} )</th>
<th>( x_{132} )</th>
<th>( x_{212} )</th>
<th>( x_{222} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0</td>
<td>0</td>
<td>3 \times 250</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 11: The amounts of transported goods to supplementary origins \( B_1 \) and \( B_2 \).

<table>
<thead>
<tr>
<th>Variable ( x_{111} )</th>
<th>( x_{121} )</th>
<th>( x_{131} )</th>
<th>( x_{211} )</th>
<th>( x_{221} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>750</td>
<td>0</td>
<td>50</td>
<td>1200</td>
</tr>
</tbody>
</table>

### Table 12: The amounts of transported goods stored at all supplementary origins.

<table>
<thead>
<tr>
<th>Node</th>
<th>B1</th>
<th>B2</th>
<th>C1</th>
<th>D1</th>
<th>D2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>800</td>
<td>1200</td>
<td>730</td>
<td>1755</td>
<td>1975</td>
</tr>
</tbody>
</table>

### Table 13: The quantity of goods transported to final destinations \( D_1 \) and \( D_2 \).

<table>
<thead>
<tr>
<th>Variable ( x_{111} )</th>
<th>( x_{121} )</th>
<th>( x_{131} )</th>
<th>( x_{211} )</th>
<th>( x_{221} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0</td>
<td>2.5 \times 500</td>
<td>2.46 \times 500</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 14: Transportation costs need to be paid for the goods transported to final destinations \( D_1 \) and \( D_2 \).

<table>
<thead>
<tr>
<th>Variable ( x_{111} )</th>
<th>( x_{121} )</th>
<th>( x_{131} )</th>
<th>( x_{211} )</th>
<th>( x_{221} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0</td>
<td>1500</td>
<td>1500</td>
<td>0</td>
</tr>
</tbody>
</table>


