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Research Article

A Queuing Model for Mixed Traffic Flows on Highways considering Fluctuations in Traffic Demand

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Modelling the mixed traffic flows of autonomous vehicles (AVs) and human-driven vehicles (HVs) on highways is challenging. Randomness, fluctuations, and congestion exist in the mixed traffic flows. This paper extends the current literature by proposing an M/G(n)/c/c state-dependent queuing model operating in a random environment. The fluctuating traffic demand is addressed by arrival rates modulated by the random environment. Meanwhile, a Markovian arrival process (MAP) is incorporated to describe the platoons. We investigate the performance of the mixed traffic flow under the I policy (AVs and HVs travel together in all lanes) and the D policy (one lane is designated to AVs). Numerical experiments reveal the following interesting findings: (1) the fluctuation degree of traffic demand, the traffic intensity, and the penetration rate of AVs play essential roles in determining the performance of mixed traffic flows. (2) The I policy should always be adopted if the travel time is more valuable. In terms of output rate, the choice between the I and the D policies depends on the traffic intensity, SCV of arrival rates and penetration rate. (3) A larger penetration rate is required to completely eliminate congestion on a longer highway segment.

1. Introduction

With the development of sensing and communication techniques, rapid progress has been achieved in the field of autonomous vehicles (AVs) in recent years. Compared with regular human-driven vehicles (HVs), AVs are expected to be capable of increasing road capacity, relieving congestion, and improving the stability of traffic flows [1–6]. However, it is generally recognized that there is a long way to go before full automation is reached. Therefore, the mixed traffic flow of AVs and HVs will become mainstream in the following twenty to thirty years [7].

Intensive attention has been paid to the mixed traffic flow of AVs and HVs. Research in this field mainly focuses on two aspects: mixed traffic flows modelling [7, 8] and optimal traffic management [9, 10]. Modelling the mixed traffic flows of AVs and HVs, which aims to capture the interaction between different vehicles and the interaction between vehicles and road, is the basis for the analysis and control of mixed traffic flows. It belongs to the scope of collaborative optimization and modeling of multi-transport modes and is one of the hot research topics in the field of traffic and transportation.

Generally, two basic categories of traffic flow are analyzed, interrupted traffic flow and uninterrupted traffic flow. The former focuses on traffic flows affected by external elements (such as traffic signals), whereas the latter analyzes vehicles that only interact with other vehicles and roads. The scope of this paper is limited to the uninterrupted traffic flows. Specifically, we analyze the mixed traffic flows on highways.

Modelling the mixed traffic flows on highways is full of challenges due to the randomness and fluctuations in traffic flows. Randomness, or stochastic, in traffic flows mainly results from the uncertain behaviors of drives. Fluctuations of traffic parameters, such as flow rate and velocity, is the result of factors such as tidal phenomenon, lane closure, traffic accidents and weather conditions. Randomness and fluctuations in traffic flows have been well documented in the literature [7, 11]. And various approaches and models have been proposed, such as queuing techniques and the cellular automaton (CA) based models. Congestion and platoons also exist in highway traffic flows. When the number of vehicles on a highway segment exceeds a particular value, congestion occurs due to limited land space. The velocity of vehicles declines as the number of vehicles increases. In addition, vehicles on highways usually travel in platoons, which allows a smaller headway.

To accurately model and assess the mixed traffic flows on highways, one should consider all the characteristics in reality. Mirzaeian et al. [12] applied an $M/G(n)/c/c$ queuing model incorporated with a Markovian arrival process (MAP) for the mixed traffic flows on highways. In the model, the random arrival followed a Poisson process. The effect of congestion was addressed by a state-dependent service rate (service rate depends on the number of vehicles n). The MAP was applied to describe the platoons. However, fluctuations in traffic demand were ignored.

This paper aims to provide a more realistic model for the mixed traffic flow on highways. The main contribution of this paper is that we present an $M/G(n)/c/c$ queuing model operating in a random environment. In this model, the fluctuations in traffic demands are treated as arrival rates modulated by a continuous time Markov chain. And the fluctuation degree of arrival is quantized by the squared coefficient of variation (SCV) for arrival rates. Although we only consider varying traffic demand, the proposed model can deal with fluctuations in velocity or capacity, which may be caused by adverse weather conditions, lane closure, or accidents.

Following Mirzaeian et al. [12], we analyze two policies: the I policy (AVs and HVs travel together in all lanes) and the D policy (one lane is designated to AVs). The second contribution of the study is that the D policy applied in this paper is more rational. The D policy in this paper works as follows (Figure 1): a newly arrived AV enters the designated lane if the designated lane is not fully occupied, or it enters the rest lanes if the designated lane is fully occupied while the rest lanes are not.

The main difference between this paper and Mirzaeian et al. [12] is as follows. First, we address the fluctuations in traffic demands by arrival rates modulated by a random environment. In contrast, the fluctuations in traffic demands were ignored in Mirzaeian et al. [12] and the arrival rate was a fixed value. Second, the D policy in this paper is more rational compared with that in Mirzaeian et al. [12]. In Mirzaeian et al. [12], the AVs are only allowed to use the designated lane in the D policy. If the designated lane is fully occupied, a newly arrived AV cannot enter the rest lanes even though they are unsaturated.

Compared with the general homogeneous traffic flow model (such as the traffic wave model), the $M/G(n)/c/c$ state-dependent queuing model in a random environment proposed in this paper is superior in the following aspects. First, the randomness in traffic demand and vehicle velocity is addressed by the queuing model. Second, the random environment of traffic flows is represented by a continuous time Markov chain. Third, the effect of the platoons on velocity is also considered by incorporating a MAP and applying state-dependent velocity. Fourth, we can obtain the second, third and higher moment for the performance measures of traffic flows using the proposed queuing model.

The proposed model in this paper, which is novel in literature, helps us gain deeper insights into the influence of AVs on the mixed traffic flows on highways. Numerical experiments reveal that the fluctuation degree of arrival (SCV for arrival rates) plays an essential role in determining the performance. The choice between the I policy and the D policy depends on the traffic intensity and the SCV of arrival rates. As the influence of the SCV of arrival rates is taken into account, and the AVs rejected by the designated lane are allowed to use lanes for HVs, the policy recommendations in this paper are different from that in Mirzaeian et al. [12]. In addition, it is found that a larger penetration rate is required to completely eliminate congestion on a longer segment. These findings are of great value when making a decision related to AVs management and control, such as time-sharing priority or time-sharing pricing for AVs.

The rest of the paper is organized as follows. In Section 2, we review the literatures in the related fields. The queuing model for the mixed traffic flows is presented in Section 3. Section 4 displays the numerical experiments and Section 5 concludes the paper.

2. Literature Review

The related literatures in traffic flow modelling, mixed traffic flows and AVs are reviewed in this section.

Various approaches have been proposed to model traffic flows. According to the level of detail, there are microscopic, mesoscopic and macroscopic models. As a mesoscopic method, queuing techniques have been used to model traffic flows in different scenarios, including uninterrupted flows on highways [13, 14]. Queuing models fall into two groups: stationary model [15–17] and transient model [18–20]. The former focuses on capturing the steady state performance and is applicable in facility design and policy decision. The latter, on the contrary, calculates the dynamic performance measures and is often used for management and control. This paper belongs to the former category. Heidemann [21] applied the basic M/M/1 and M/G/1 queuing models to capture the speed-flow-density relationships of traffic flows. To address the effect of congestion, Jain and Smith [15] proposed an $M/G(n)/c/c$ state-dependent queuing model, in which the service rate depends on the number of vehicles n (n is also defined as the state of the queuing system). Smith and Cruz [16] also applied the $M/G(n)/c/c$ state-dependent queue to model traffic flows.

The arrival and service rates are fixed in the above queuing models. However, traffic flows are affected by many other factors that may cause fluctuations in traffic demand, vehicle velocity or road capacity. Agarwal et al. [22] analyzed the impact of various adverse weather on the capacity and speed of highways. Baykal-Gürsoy et al. [23] presented an M/M/C queue in a random environment for traffic flows interrupted by incidents. Yang et al. [24] proposed a passenger–taxi matching queueing model that considered the fluctuating arrival of passengers and taxis. Gerum and Baykal-

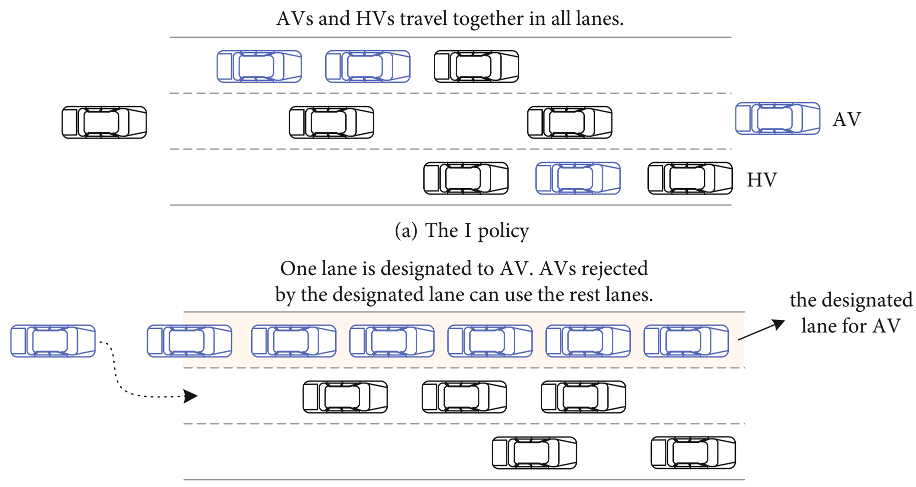


Figure 1: The I policy and the D policy.

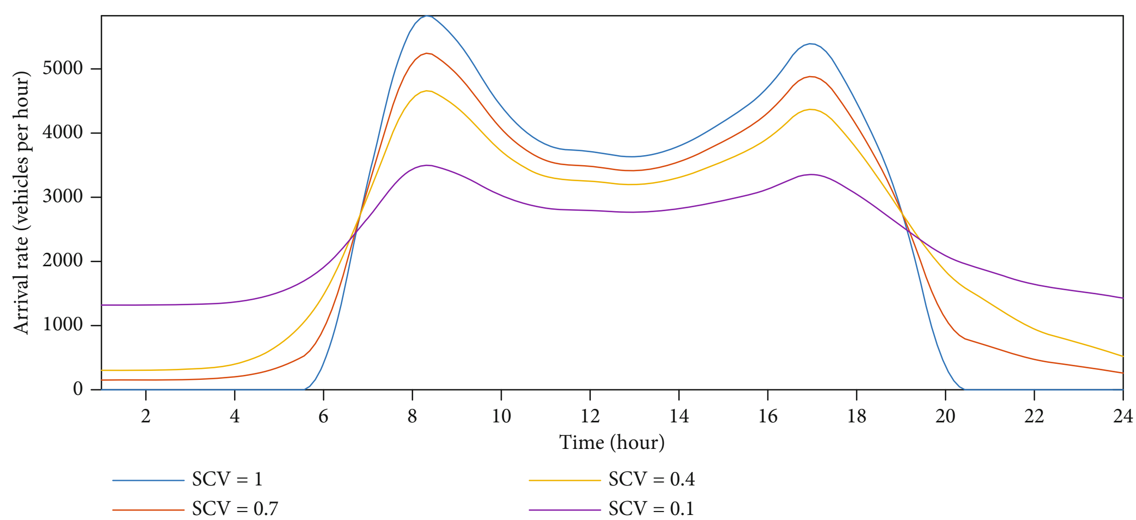


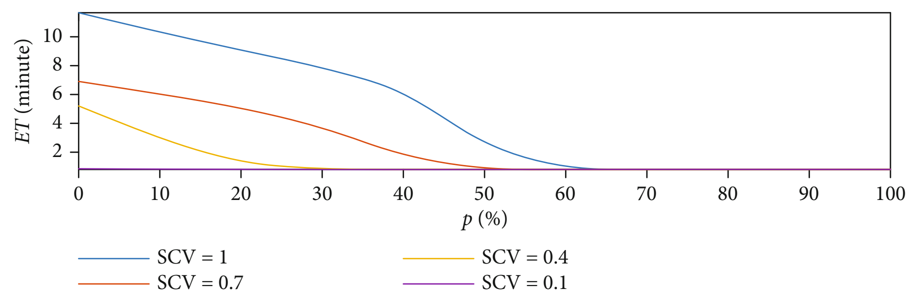
Figure 2: Varying arrival rates for the low traffic intensity.

Gürsoy [25] used tandem queues to analyze the traffic density in roadways where the service is affected by random incidents.

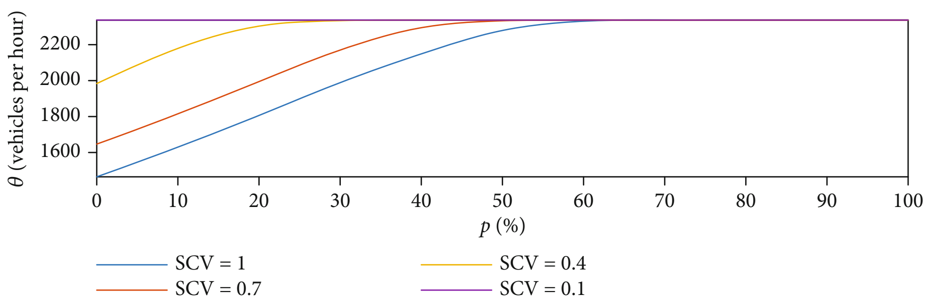
Platooning is another typical feature of the traffic flows on highways. Alfa and Neuts [26] presented a discrete time MAP to model the platooned arrivals in traffic flows. Readers can find more details on MAP in Neuts [27]. Breuer and Alfa [28] further presented an EM-based procedure to estimate the parameters for the platoon arrival process. Based on these researches, Mirzaeian et al. [12] also applied the MAP to model the platoons on highways. Other research on platoons includes Jin et al. [29] who introduced a fluid model to investigate the interaction between the AVs platoons and the non-AVs and proposed platoon coordination strategies. From a macroscopic level, Sala and Soriguera [4] provided a generalized model to estimate the platoon length in mixed traffic flows.

Research on the mixed traffic flows of AVs and HVs also has rich achievements. Mahmassani [30] established a microsimulation framework to examine the stability and throughput of mixed traffic flows under varying market penetration rates of autonomous connected vehicles. To analyze the impact of AVs on the mixed traffic flows of HVs and AVs, Zheng et al. [7] developed a stochastic Lagrangian model which considered human drivers' heterogeneous behaviors.

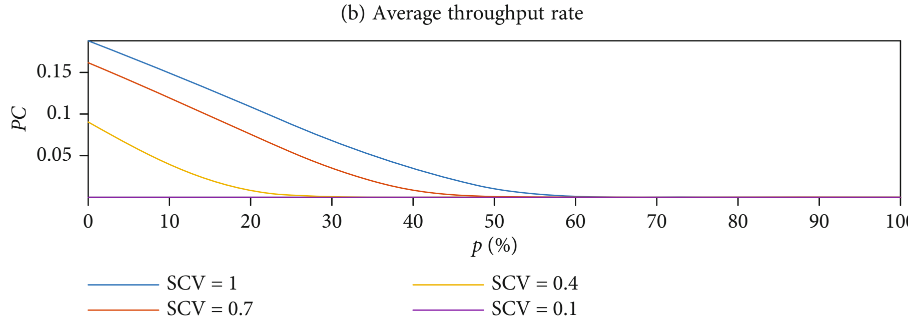
The paper by Mirzaeian et al. [12] is the most related research to this study. Compared to Mirzaeian et al. [12], our model is more realistic as it addresses the fluctuations in traffic flows by applying a queuing model operating in a



(a) Average duration time



(b) Average throughput rate



(c) Average blocking probability

Figure 3: Average performance measures under a low traffic intensity.

random environment. Meanwhile, the D policy in this paper is more rational as it allows the AVs rejected by the designated lane to enter the rest lanes if they are not fully occupied.

3. Modelling Mixed Traffic Flows

In this section, we present the queuing model for the mixed traffic flows on highways. First, we introduce the $M/G(n)/c/c$ state-dependent queuing model operating in a random environment. Second, the MAP used to describe the platoons is presented. Then, we give the steady state performance measures of the queuing model.

3.1. The M/G(N)/c/c Queuing Model Operating in a Random Environment. A highway segment can be viewed as a queuing system, where the vehicles are customers, and the road segment is servers that provide passage service. For a highway segment of length l (in miles) with w lanes, its queuing capacity c is equal to the number of vehicles the road can accommodate, which is also the number of servers of the queuing system.

$$c = Klw \tag{1}$$

where K is defined as the jam density, which means the maximum number of vehicles a lane of one mile can hold.

Vehicles arrive according to a Poisson process with an arrival rate λ. When a newly arrived vehicle finds the road fully occupied (the number of vehicles on the road reaches the capacity c), it turns away. In reality, a driver may take an alternative route when he finds a severe traffic jam ahead. The process that a vehicle passes through the highway is viewed as the service process. Therefore, the service time is determined by the velocity. Due to the effect of congestion, the velocity varies with the number of vehicles on the road. The more vehicles travel on the road simultaneously, the slower the velocity tends to be. The number of vehicles n is defined as the state of the queuing system. When $n = 1$, there is only one vehicle on the highway, travelling with a free velocity v_1. When there are c vehicles on the road segment, the velocity decreases to 0. That means the velocity v_n, for $n = 1, 2, \cdots, c$, is state-dependent. Therefore, the service rate (of a single server in the queuing system) $\mu_n = v_n/l$ is also state-dependent.

The traffic flows on highways are affected by many factors, such as the tide phenomenon, traffic accidents and weather conditions, which may cause changes in traffic demand and velocity. We can introduce a random environment represented by a finite state Markov process to model the influence of these factors. Consider an irreducible continuous time Markov chain i_t, $t \geq 0$, in the state space {1, 2, …, N}, $N \geq 2$. The infinitesimal generator $\mathbf{Q}$ for i_t is as follows:

$$\mathbf{Q} = \begin{bmatrix} Q_{11} & \cdots & Q_{1N} \\ \vdots & \ddots & \vdots \\ Q_{N1} & \cdots & Q_{NN} \end{bmatrix}, \quad (2)$$

where $Q_{ii} = -1/t_i \ (1 \leq i \leq N)$ and t_i is the mean duration time at state i for Markov chain i_t; $Q_{ij} \ (1 \leq i, j \leq N, i \neq j)$ represents the transition intensity of the Markov chain i_t from state i to state j. The invariant probability vector of $\mathbf{Q}$ is $\boldsymbol{x}$ and it satisfies $\boldsymbol{x}\mathbf{Q} = 0$, $\mathbf{xe} = 1$. The duration time of each environment state is a random number which may follow an exponential, Erlang, or pH distribution. For the sake of simplicity, exponential duration time is considered in this paper. When a queuing model operates under a random environment, its parameters, such as the arrival rate, the service rate or the capacity may change with the random environment.

In this study, the M/G(n)/c/c queuing model is modulated by the random environment as follows. When the Markov chain i_t is at state i, the arrival rate of the queuing system is λ_i. When the state of the Markov chain changes, the arrival rate of the queuing system changes synchronously. Although we only consider arrival rate changing with the random environment in this paper, the model can also deal with varying service rate and varying capacity.

The M/G(n)/c/c queuing model operating in a random environment can be described by a continuous time level-dependent QBD (quasi-birth-and-death) process $\xi_t = \{n_t, i_t\}$, $t \geq 0$, where $n_t = 0, 1, \cdots, c$. The infinitesimal generator of $\xi_t, t \geq 0$ is given by $\mathbf{Q}^*$

$$\mathbf{Q}* = \begin{bmatrix} \mathbf{D}_0 & \mathbf{F}_0 & & & \\ \mathbf{E}_1 & \mathbf{D}_1 & \mathbf{F}_0 & & \\ & \cdots & \cdots & \cdots & \\ & & \mathbf{E}_{c-1} & \mathbf{D}_{c-1} & \mathbf{F}_0 \\ & & & \mathbf{E}_c & \mathbf{D}_c \end{bmatrix}, \quad (3)$$

where
$\mathbf{F}_0 = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$,
$\mathbf{E}_n = \text{diag}(n\mu_n, n\mu_n, \ldots, n\mu_n)$, $n = 1, \ldots, c$,
$\mathbf{D}_n = \mathbf{Q} - \text{diag}(\lambda_1 + n\mu_n, \lambda_2 + n\mu_n, \ldots, \lambda_N + n\mu_n)$, $n = 1, \ldots, c$-1,
$\mathbf{D}_0 = \mathbf{Q} - \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$,
$\mathbf{D}_c = \mathbf{Q} - \text{diag}(c\mu_c, c\mu_c, \ldots, c\mu_c)$.

The stationary probability vector for $\mathbf{Q}^*$ is $\boldsymbol{\pi}$, and it satisfies the following global balance equation:

$$\boldsymbol{\pi}\mathbf{Q}* = 0, \boldsymbol{\pi}\boldsymbol{e} = 1, \quad (4)$$

where $\boldsymbol{\pi} = (\boldsymbol{\pi}_0, \boldsymbol{\pi}_1, \cdots, \boldsymbol{\pi}_n, \cdots, \boldsymbol{\pi}_c)$ and $\boldsymbol{\pi}_n = (\pi_{n1}, \pi_{n2}, \cdots, \pi_{nN})$. π_{ni} is the steady-state probability for the system state is n and the environment state is i. Meanwhile, the stationary probability satisfies $\boldsymbol{\pi}_{n+1} = \boldsymbol{\pi}_n\mathbf{R}_n \ (0 \leq n \leq c-1)$, where $\mathbf{R}_n$ is a level dependent rate matrix and $\boldsymbol{\pi}_0(\mathbf{D}_0 + \mathbf{R}_0\mathbf{E}_1) = \mathbf{0}$, $\mathbf{R}_n = -\mathbf{F}_0/(\mathbf{D}_{n+1} + \mathbf{R}_{n+1}\mathbf{E}_{n+2})$, $\mathbf{R}_c = \mathbf{0}$.

3.2. MAP for the Platoons of Traffic Flows. Vehicles on the highway always form platoons. The mean headway when there are n vehicles on the road segment is denoted as h_n. Traffic density k on the highway segment is expressed as $k = n/(lw)$. According to the relationship between the traffic flow, density and speed, the flow equals to $nv_n/(lw)$, which is also equal to the inverse of the mean headway h_n. Hence, we have

$$v_n = \frac{lw}{nh_n}, \quad (5)$$

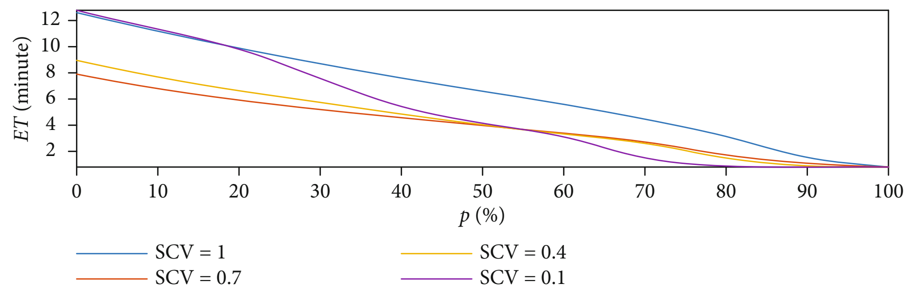
We follow Mirzaeian et al. [12] to apply a MAP to describe the platoons in highway traffic flows. A MAP is defined by two $m * m$ matrices, $\mathbf{C}_n^0$ and $\mathbf{C}_n^1$ when the number of vehicles is n. The irreducible generator matrix of the MAP is $\mathbf{C}_n = \mathbf{C}_n^0 + \mathbf{C}_n^1$, with the corresponding stationary probability vector $\mathbf{m}_n$. The mean headway equals the mean of the MAP

$$h_n = \frac{1}{\boldsymbol{m}_n\mathbf{C}_n^1\mathbf{e}}, \quad (6)$$

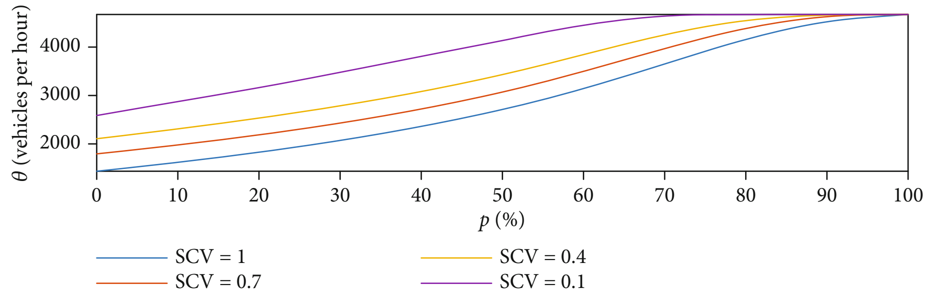
where $\mathbf{e}$ is a column vector of all ones. Readers can refer to Mirzaeian et al. [12] for more details on the MAP. The distributions of three elements, platoon size (with mean $1/\delta$), intraplatoon headway (with mean $1/\xi$), and interplatoon headway (with mean $1/\varphi$), are necessary to calibrate $\mathbf{C}_n^0$ and $\mathbf{C}_n^1$.

Following Mirzaeian et al. [12], two policies can be adopted for the AVs. One is the designated-lane policy (referred to as the D policy) and the other is the integrated policy (referred to as the I policy). The penetration rate of AVs is denoted as p. Under the I policy, the AVs and the HVs arrive at all lanes with an arrival rate λ, and the capacity of the queuing system is Klw. Under the D policy, the AVs arrive at the designated lane with an arrival rate $p\lambda$, forming a queue with a capacity Kl. And the HVs arrive at the rest lanes with arrival rate $(1-p)\lambda$, forming another queue with a capacity $Kl(w-1)$. The AVs cannot enter the designated lane if the number of AVs already in the lane reaches its capacity. The AVs rejected by the designated lane are allowed to join the rest lanes if they are not fully occupied. In this case, the rest lanes essentially work as the I policy, where the arrival rate is $(1-p)\lambda + p\lambda p_c^{\mathrm{D}}$, and the penetration rate is $p\lambda p_c^{\mathrm{D}}/((1-p)\lambda + p\lambda p_c^{\mathrm{D}})$, where p_c^{D} is the blocking probability of the designated lane.

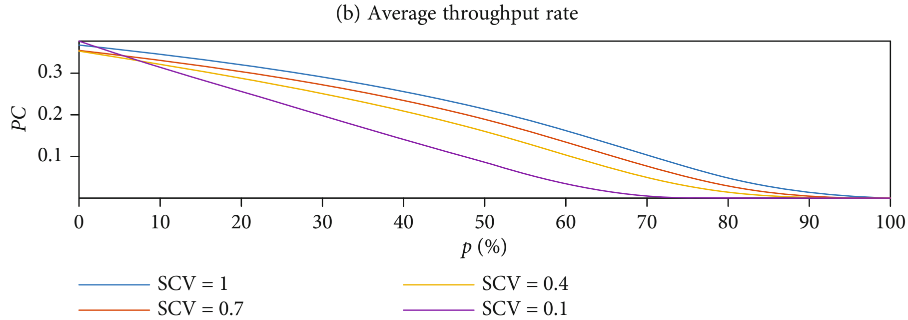
The distribution and the formulation for $1/\delta$, $1/\xi$, and $1/\varphi$ for the D policy and the I policy are presented in detail in Mirzaeian et al. [12]. Based on them, we can



(a) Average duration time



(b) Average throughput rate



(c) Average blocking probability

Figure 4: Average performance measures under a high traffic intensity.

obtain the state-dependent velocity v_n according to Equations (5) and (6).

3.3. Performance Measures. Given the queuing system in Section 3.1, we can calculate the stationary performance measures of interest based on the steady state distribution $\boldsymbol{\pi}$. For queuing systems with fixed parameters, we focus on the average performance measures, such as the average duration time ET, the average blocking probability PC, and the average output rate θ.

$$ET = \sum_{n=1}^{c} \frac{\pi_n \mu_n^{-1}}{1 - \boldsymbol{\pi}_0 \mathbf{e}},$$
$$PC = \boldsymbol{\pi}_c \mathbf{e}, \tag{7}$$
$$\theta = \sum_{n=1}^{c} n\mu_n \pi_n \mathbf{e}.$$

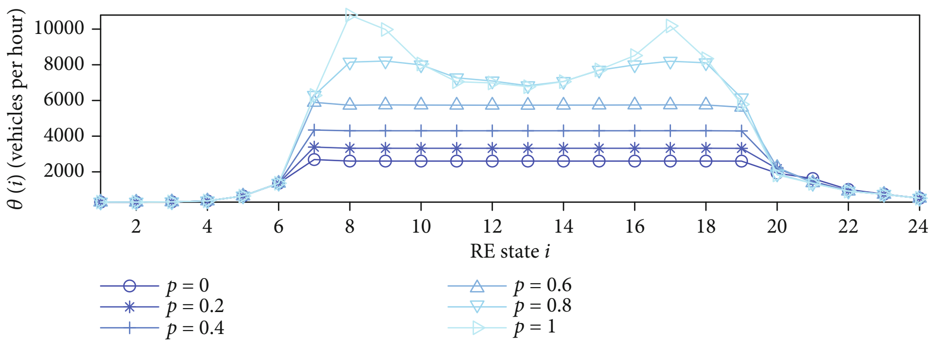
However, the performance measures change with time for queuing systems with varying parameters. For these queues, the conditional performance measures provide more valuable information. To calculate the conditional performance measures, we introduce a conditional probability $p_n(i)$, which represents the conditional probability that the system state is n on the condition that the environment state is i
,

$$p_n(i) = \frac{\pi_{ni}}{x_i}. \tag{8}$$

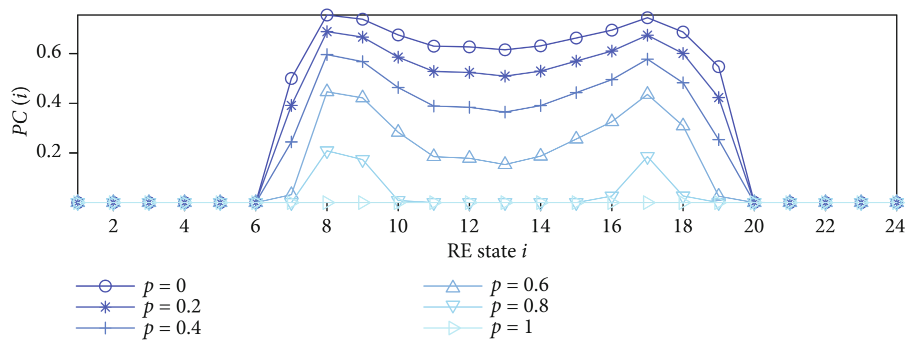
Then the conditional performance measures, such as the conditional duration time $ET(i)$, the conditional blocking probability $PC(i)$, and the conditional output rate $\theta(i)$ at environment state i can be expressed as

$$ET(i) = \sum_{n=1}^{c} \frac{p_n(i)}{[\mu_n(1 - p_0(i))]},$$
$$PC(i) = p_c(i), \tag{9}$$
$$\theta(i) = \sum_{n=1}^{c} np_n(i)\mu_n.$$

Due to the introduction of the random environment and the state-dependent service rate, it is very difficult to obtain the explicit expression for the steady-state distribution $\boldsymbol{\pi}$. Following Zhu et al. [17], we apply the Matrix Analytic

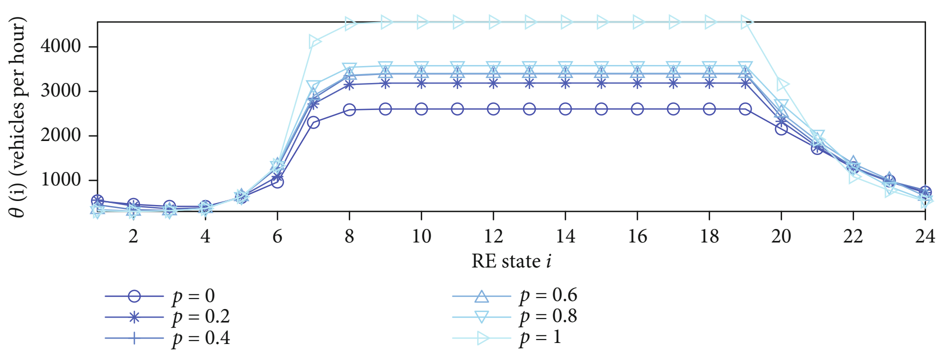


(a) Conditional output rate

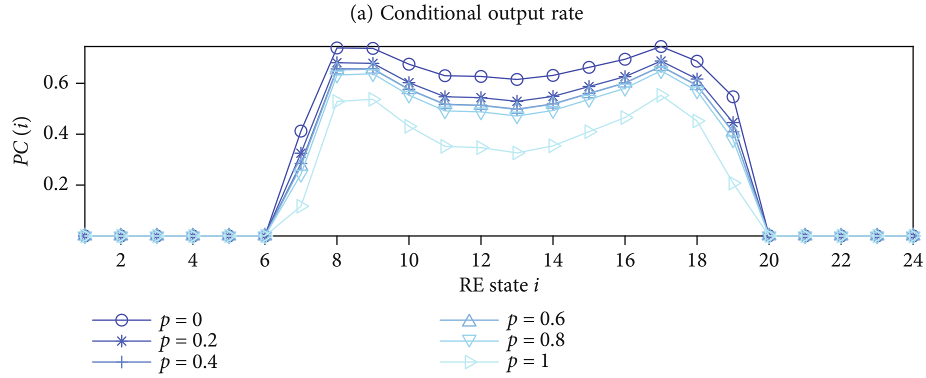


(b) Conditional blocking probability

Figure 5: Varying performance measures when $l = 1$ mile.

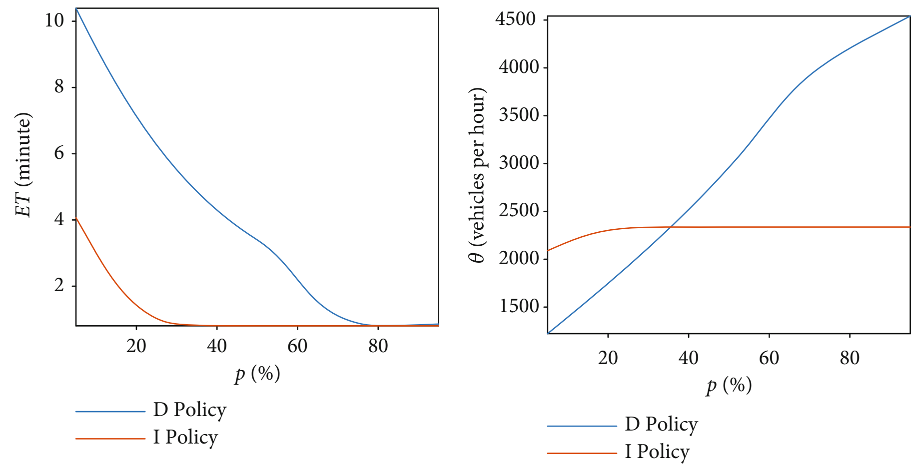


(a) Conditional output rate



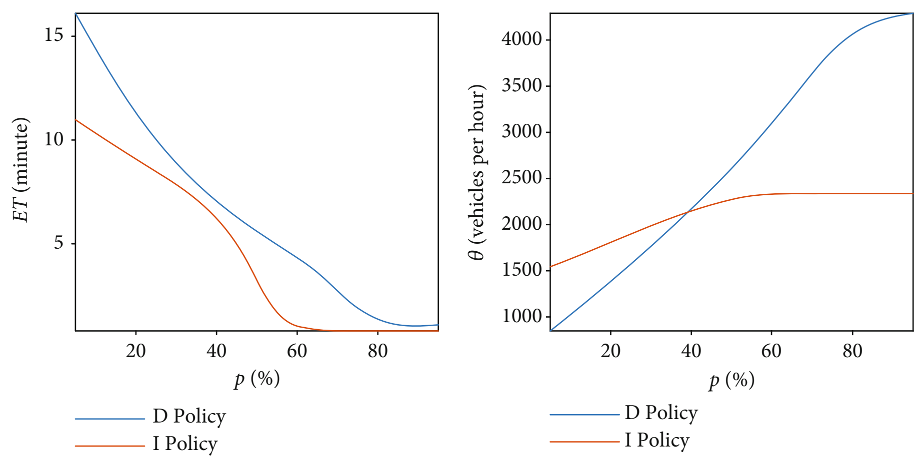
(b) Conditional blocking probability

Figure 6: Varying performance measures when $l = 4$ miles.



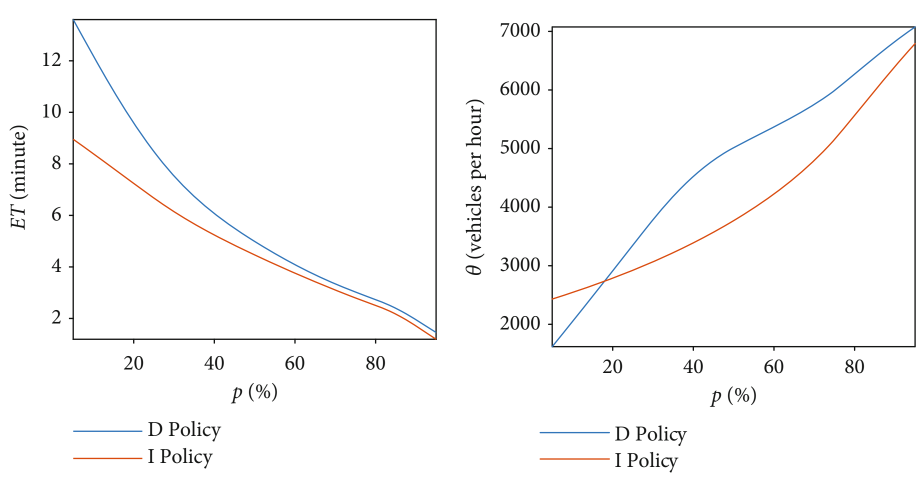
(a) Average duration time (b) Average output rate

Figure 7: Comparison between the D policy and the I policy when SCV = 0.4 under a low traffic intensity.



(a) Average duration time (b) Average output rate

Figure 8: Comparison between the D policy and the I policy when SCV = 1 under a low traffic intensity.



(a) Average duration time (b) Average output rate

Figure 9: Comparison between the D policy and the I policy when SCV = 0.4 under a high traffic intensity.

Scheme (MAS) algorithm in Baumann and Sandmann [31] to solve the proposed model. Using the MAS algorithm, the explicit calculation of the stationary distribution $\boldsymbol{\pi}$ is unnecessary. Hence, the space complexity of the algorithm is significantly reduced. In this paper, the time complexity of the MAS algorithm is a linear function of cN, and the space complexity of the algorithm is a linear function of N.

4. Numerical Experiments

In this section, we present numerical experiments on mixed traffic flows. First, the calibration of the model is presented. Then, we analyze the average and varying performance measures affected by the fluctuation degree of arrival rate and the penetration rate under the I policy. Finally, we compare the D policy with the I policy.

4.1. Model Calibration. To describe the highway traffic flow as an M/G(n)/c/c queuing model operating in a random environment, we need the following parameters: the infinitesimal generator $\mathbf{Q}$, the invariant probability vector $\boldsymbol{x}$, the arrival rate λ_i for each Markov state i, and the state-dependent velocity v_n for each system state n. It is worth noting that Markov state (environment state) i represents one of the N states for the Markov chain i_t, while system state n refers to the number of vehicles on the highway.

The arrival rate of vehicles changes with time due to the tidal phenomenon. 24 hours of a day is treated as a random environment. The Markov chain i_t, $t \geq 0$ has a state space $\{1, 2, \ldots, N\}$ and $N = 24$. For the infinitesimal generator $\mathbf{Q}$, we have $Q_{ii} = -1\ (1 \leq i \leq N)$, $Q_{i,i+1} = 1\ (1 \leq i \leq N-1)$, and $Q_{N,1} = 1$, while the rest of the entries in matrix $\mathbf{Q}$ is 0. And $\boldsymbol{x} = \mathbf{e}_{1\times N}/N$.

We use the open data from the Department for Transport to calibrate the varying arrival rate. The total number of vehicles that arrived in 24 hours for a highway segment with three lanes is 56080. Four curves for varying arrival with the same average arrival rate but different SCVs of arrival rates are presented in Figure 2. When SCV = 0.1, the fluctuation degree is relatively small; when SCV = 1, the fluctuation degree is large. In this figure, the line for SCV = 0.4 reflects the actual data on-site. Note that when SCV = 0, there is no fluctuations in arrival (it shows a horizontal line), which corresponds to the arrival rate in Mirzaeian et al. [12]. We also analyze a high traffic intensity by increasing the arrival rate by one time or two times. In the following content, we use low and high traffic intensity to refer to the two traffic load scenarios.

Let the length l of the segment be one mile and the number of lanes be 3. The value of K typically ranges from 185 to 265 veh/mile-lane according to Jain and Smith [15]. In this paper, it is set to be 185. For the I policy, AVs and HVs form a mixed traffic flow and travel together in three lanes, so the capacity of the queue is 555 ($w = 3$). In the D policy, one specific lane is assigned to the AVs, therefore the capacity for the AVs queue is 185 ($w = 1$) and that for the HVs queue is 370 ($w = 2$).

According to Mirzaeian et al. [12], the state-dependent velocity v_n^{DH} for the HV queue and v_n^{DA} for the AV queue under the D policy are as follows,

$$v_n^{\mathrm{DH}} = 66e^{-\left(n^{3.4}/5215902\right)} + 2, n = 1, 2, \cdots, 370.$$
$$v_n^{\mathrm{DA}} = \min\{74.7, (3600 + 2.16n)/0.855n\}, n = 1, 2, \cdots, 185, \tag{10}$$

where 74.7 miles/hour is the free speed.

For the I policy, the state-dependent velocity $v_n^{\mathrm{I}}(p)$ is

$$v_n^{\mathrm{I}}(p) = \min\{74.7, 10800w/[n((2-1.7p)A+(1+1.7p)B)]\},$$
$$n = 1, 2, \cdots, 555, p\epsilon[0, 1], \tag{11}$$

where

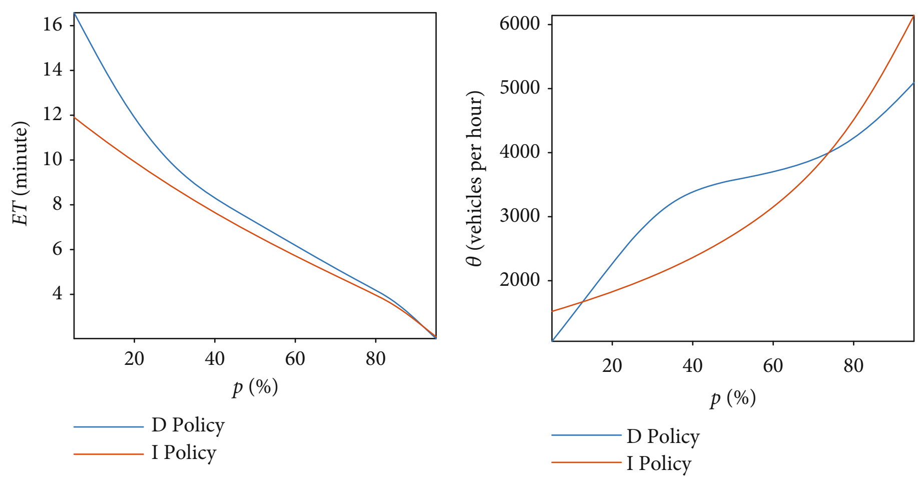
$$A = \frac{\left[(10800 - 7.56n)p + 4.59np^2\right]}{3000 + 0.6n} + \frac{10800(1-p)}{nE} - 0.55(1-p),$$
$$B = 0.55p^2 + 1.4p(1-p) + 1.1(1-p), \tag{12}$$
$$E = 46.67e^{-\left(n^2/21049\right)} + 3.13.$$

Readers can refer to Mirzaeian et al. [12] for the details of the calibration work.

4.2. Average Performance Measures under the I Policy. In this experiment, we analyze the influence of the AVs under different SCVs of arrival rates. Four values for the SCV of arrival rates, 1, 0.7, 0.4, and 0.1, and penetration rates from 0 to 100% are analyzed. Both the low traffic intensity and the high traffic intensity (two times of the low traffic intensity) are tested. The average duration time ET, the average blocking probability PC, and the average output rate are displayed in Figures 3 and 4.

The two figures show that the increase in penetration rate does not influence the average performance measures under certain circumstances, specifically, when the average blocking probability is 0 (which means there is no congestion). When the average blocking probability is larger than 0, the increase in penetration rate remarkably improves the average performance measures. Note, the improvement of performance measures refers to the decrease in duration time, the reduction in blocking probability or the increase in output rate, and vice versa. In Figure 3, the maximum increase in the average output rate is 60%, and the maximum decrease in the average duration time is 93%, while that in Figure 4 is 225% and 94%, respectively. This result makes sense as the AVs help to alleviate congestion.

Meanwhile, Figures 3 and 4 illustrate that the SCV of arrival rates remarkably affect the performance of the mixed traffic flow. When the SCV of arrival rates increases, a larger proportion of AVs are required to achieve stable average performance, that is, to completely eliminate congestion in traffic flows. Under a low traffic intensity, the average performance measures become stable at a penetration rate of 30% when the SCV of arrival rate is 0.4. In contrast, a penetration rate of 60% is required when the SCV of arrival rate is 1. Under a high traffic intensity, the average blocking



(a) Average duration time

(b) Average output rate

Figure 10: Comparison between the D policy and the I policy when SCV = 1 under a high traffic intensity.

probability is larger than 0 until the penetration rate reaches 70% for SCV = 0.1 and 100% for SCV = 1.

From the figures, we can see that if we apply SCV = 0 (as in Mirzaeian et al. [12]), the average performance measures will be improved compared to SCV = 0.1. It implies that the performance measures of the mixed traffic flows are overestimated if we ignore the fluctuations in traffic demand. The findings of this experiment are novel to the AV literature as the SCV of arrival rates was ignored in research to date.

Notably, the average blocking probability for SCV = 0.1 is slightly larger than that for SCV = 1 in Figure 4. There are two opposite effects when the SCV of arrival rates grows. As Figure 4 analyzes a high traffic intensity, congestion already exists in traffic flows for SCV = 0.1. Increasing the SCV of arrival rates results in more vehicle loss during the peak periods (increase the average blocking probability) and lower arrival rate during the off-peak periods (decrease the average blocking probability). The effective arrival rate gets smaller. Therefore, the average output rate gets smaller from SCV = 0.1 to SCV = 1.

4.3. Varying Performance Measures under the I Policy. In this section, we focus on the varying performance measures under the I policy. The high traffic intensity with an SCV = 0.7 is applied. We tested two length values for the segments 1 mile and 4 miles. The conditional output rate and the conditional blocking probability are presented in Figures 5 and 6.

Figures 5 and 6 show that the conditional performance measures improve distinctly with the penetration rate increase at the environment state with a conditional blocking probability larger than 0. However, Figures 5 and 6 also show different phenomena. In Figure 5, when the length of the segment is 1 mile, a penetration rate of 1 eliminates the congestion completely. The conditional blocking probability for all environment states drops to 0 for $p = 1$ in Figure 5. It means the congestion due to the peak hours is completely dissipated. By contrast, the conditional blocking probability is about 53% during the morning and evening peak hours for $p = 1$ in Figure 6. It is because the number of vehicles stuck in congestion are larger in a longer segment. In this case, the congestion due to the morning peak hours is not effectively dissipated and lasts to the end of the evening peak hours. It is not difficult to understand as the congestion of several vehicles dissipates much easier than severe traffic jams that last for miles. In Figure 6, it takes more time for vehicles to leave the highway. Therefore, it is observed that the conditional throughput rate at 20:00 for $p = 1$ is 1828 in Figure 5, whereas it is 3167 in Figure 6.

In addition, from the two figures, one can easily conclude that improving the penetration rate during peak hours tremendously improves the performance of highways. This finding provides evidence for highway management, such as dynamic toll pricing.

4.4. Comparison of the D Policy and the I Policy. In this section, we present the comparison between the D policy and the I policy. We conducted experiments on the low and high (three times of the low traffic intensity) traffic intensities with different SCVs of arrival rates (SCV = 0.4 and SCV = 1). In this experiment, the high traffic intensity is set to be three times of the low traffic intensity. The result is displayed in Figure 7–10.

Figure 7–10 indicate that the arrival rates and the SCV of arrival rates play an important role in determining which policy should be adopted. Regarding travel time, the D policy always outperforms the I policy. As for the output rate, the situation is much more different. In Figure 7–9, the I policy has a larger output rate when p is smaller than certain values. Specifically, under a low traffic intensity, the I policy is superior when p is smaller than 36% when SCV = 0.4 and when p is smaller than 39% when SCV = 1. Under a high traffic intensity, the I policy has a larger output rate when

p is smaller than 17% when SCV = 0.4. The explanation is the arrival rate to the designated lane in the D policy is too small under these situations, and the capacity of the designated lane is not fully utilized. When p exceeds certain values, the D policy yields a larger output rate. This is because the capacity of the designated lane is fully utilized and it yields a high output rate.

In Figure 10, when the SCV = 1 under a high traffic intensity, the output rate of the I policy is larger when $p < 12\%$ and $p > 73\%$. When $12\% < p < 73\%$, the D policy has a larger output rate. In this case, the arrival rate of HVs to the designated lane is moderate and the designated lane has a high throughput rate. The explanation is as follows: when $p > 73\%$, the arrival rate to the designated lane in the D policy is too large. As serious congestion occurs, the designated lane's output rate decreases dramatically due to the state-dependent service rate. Therefore, the overall output rate of the D policy is surpassed by that of the I policy as p grows.

Note that the output rate of the D policy grows (Figures 7, 9 and 10) or tends to be stable (Figure 8) when the penetration rate is close to 1. This is different from that in Mirzaeian et al. [12], where there is a drop in the output rate for the D policy. This is because the AVs rejected by the designated lane enter the rest lanes for HVs as long as they are not fully occupied in this paper.

As the influence of the SCV of arrival rates is taken into account, and the rejected AVs are allowed to use lanes for HVs, the policy recommendations in this paper are different from that in Mirzaeian et al. [12]. Generally, if the travel time is more valuable, the I policy should be adopted. However, if the output rate is more important, the decision should be made based on the traffic intensity and the SCV of arrival rates.

5. Conclusion

This paper explores the influence of AVs on highway performance in a more realistic situation. The mixed traffic flow of AVs and HVs is modelled as an $M/G(n)/c/c$ state-dependent queue operating in a random environment. The randomness and fluctuations in traffic demand is addressed by a Possion process with the arrival rate modulated by a continuous time Markov chain. The state-dependent velocity describes the effect of congestion and the MAP is applied to describe the platoons. The proposed model is more coincident with the reality compared with existing queuing models for mixed traffic flows on highways.

We investigate the conditional and average stationary performance measures under the I policy and the D policy. For the D policy, we allow the AVs rejected by the designated lane to join the rest lanes for HVs as long as they are not fully occupied.

The numerical experiments yield conclusions that are consistent with existing research. More importantly, this paper reveals the following interesting findings:

(1) The fluctuation degree of arrival (SCV of arrival rates), as well as the traffic intensity (average arrival rate) and the penetration rate play important roles in determining the performance of the mixed traffic flows on highways. This finding is novel to the AV literature as the SCV of arrival rates was ignored in related research to date

(2) Increasing the penetration rate can remarkably improve the performance only when there are congestions (the blocking probability is larger than 0) in traffic flows. This result provides evidence for result [22] as congestion in traffic flows is determined not only by the average arrival rate but also by the SCV of arrival rates

(3) A larger penetration rate is required to completely eliminate congestion on a longer segment

(4) The I policy should always be adopted if the travel time is more valuable. If the output rate is more important, the choice between the I policy and the D policy depends on both the traffic intensity and the SCV of arrival rates

(5) Improving the penetration rate during peak hours can tremendously improve the performance of highways

These findings provide evidence for highway management and control, such as time-sharing priority or time-sharing pricing. Only AVs and HVs are considered in this paper. In the future, there will be many types of mixed traffic flow, such as the mixed traffic flow of connected automated vehicles (CAVs), connected vehicles (CVs), AVs and HVs. The method proposed in this paper can be applied to model mixed traffic flows of three, four, or even more different vehicle types. For the integrated traffic flows in the I policy, it can be modelled as a queue. If a lane is designated to a special type of vehicle when applying the D policy, the traffic flow on the designated lane should be modelled as a separate queue. Calibrate work should be conducted for each type of traffic flow.

In this paper, we only analyze arrival rate that changes with the random environment. In reality, vehicles' velocity and the segment's capacity may also vary due to adverse weather conditions or lane closure. In addition, the proposed queuing model is loss-based as the newly arrived vehicles immediately leave the queue (take another route) if the segment's capacity is reached. However, in most actual situations, these vehicles have to wait and try to enter the queue repeatedly as there is no other option. This will considerably influence the performance of the traffic flows. We leave these topics to our future research.

Data Availability

The data that support the findings of this study are available from the corresponding author.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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