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Research Article

Hybrid Scheduling Model Based on Fare Incentives for Peak Time Interval of the Metro: The Harbin Metro System

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Considering the operation efficiency of the metro system, the boarding queuing problem is a complex and intractable dilemma caused by the security and ticket checking process in urban metro stations, especially for the mass-transit metro system in China. In this study, a novel scheme of fare differentials based on demand control and congestion management for reducing congestion in the metro boarding process was proposed. In addition, we extended Vickrey's point-queue model into a boarding congestion model incorporating the bidesired departure time considering the security and ticket checking process in stations as a continuum bottleneck. The train-run and departure time choosing behaviors of passengers under different schedule gaps of the adjacent train runs are explored when the boarding system achieves user equilibrium in the proposed model. Then, we examined the demand regulatory mechanisms of fare incentives (fare differentials) in reducing the queuing boarding time in metro stations when the optimization of the schedule gaps is ineffective in the pattern of mass-scale travel demand. The analytical solutions of these two optimal methods (schedule gaps and fare differentials) for boarding congestion management are presented. After comparing the two congestion-reduced methods, the fare incentive rule has a better regulation effect on the smoothing of travel demand. The results of the sensitivity analysis using numerical simulations reveal the regulatory mechanism of fare differentials in reducing the queuing time and increasing the incremental revenue. (1) The demand threshold is only related to the boarding capacity and schedule gaps (i.e., the greater the boarding capacity and schedule gaps, the greater the passenger capacity of the metro station). (2) The effect of fare incentives in reducing the boarding congestion is better if the lower fare is implemented in the later train runs. (3) A lower fare differential between two adjacent metro runs can be used to regulate the proportion of staggered passengers in the queuing line to reduce the crowd gathering in the metro station hall when travel demand is high, meanwhile, a higher fare differential between two adjacent metro shuttles can increase incremental revenue effectively. (4) The measure of fare differentials causes worse results in both the reduction of the queuing time and the increase of the incremental revenue when the metro travel demand is lower than the demand threshold. This conclusion is consistent with the pattern of reality and experience. Therefore, the definition and judgment conditions of demand thresholds, introduced in this study, can provide theoretical guidance when implementing fare incentive policies in aviation and metro networks.

1. Introduction

With the rapid increase in travel demands in urban areas, high passenger flow becomes a common phenomenon in the metro systems of some large cities, especially during morning rush hours. Scholars from various countries generally believe that adopting a dynamic and diversified ticket strategy can effectively change the times at which passengers travel, increase travel during nonpeak hours, and alleviate peak passenger flow congestion. At the same time, the analysis of passenger flow data shows that the passenger flow of metro transit not only has peaks and troughs throughout the day but also has obvious peaks and troughs during the peak period itself. To ensure safety and improve the operational efficiency of the metro system, it is necessary to explore the boarding queuing problem in metro systems.

The boarding queuing problem is a complex and intractable issue in the transportation system, owing to its significant influence on the travel choice behavior of passengers at the macro level and the operation efficiency of the transport network at the microlevel. The boarding process of passengers is composed of various related links, such as ticket purchase, security checking, lining for boarding, headway of metro, and luggage handling. In the past few years, the different focuses of the studies on the boarding queuing problem can be broadly classified into three types: (1) modeling, optimization, and simulation of the boarding process in different transportation scenes (e.g., [1–6]); (2) analyzing the effects of boarding congestion on travel behavior and departure time choice (e.g., [5, 7, 8] and (3) improving the operation efficiency of boarding congestion management (e.g., [6, 9, 10].

1.1. Different Modeling Perspectives of the Boarding Process. In the modeling and simulation of boarding flows, the boarding characters of passengers can be analyzed at the (1) macroscale (no-individual differences) [2, 11] (2) microscale (individual differences) [7, 10, 12, 13], or (3) middle-scale (group behaviors of similar passengers) [8, 14–16], with respect to the evaluation scale of factors affecting the boarding behavior, including passenger's motion and previous experience. Boarding models considering passengers with no-individual differences are usually developed to provide an effective method for the optimization of the planning timetable by calculating the bus/metro dwell time [7, 17, 18]. It can be used to evaluate the efficiency of operation measures in public transit or other transportation systems such as metro [19] and aviation [9, 20] systems. Boarding models considering the individual preference of passengers can better describe the effect of congestion-reduced strategies on individual boarding behavior. However, boarding bottleneck models are usually presented using numerical simulations because the analytical solutions cannot be obtained easily [13]. As a result, boarding models considering group behaviors of similar passengers formed in the boarding process have gained attention. For example, some measures of passenger classification are developed based on the similar characteristics of seat number allocation and the size of hand-luggage to speed up the metro boarding process [21, 22]. In addition, the study of [8] established a group-based boarding scheme using a similarity analysis of the individual properties of four influencing factors. They found that the positive impacts of group-based boarding behavior on the operation efficiency can be more significant with an increasing number of groups. Moreover [15], developed a bigroup (leader-follower) boarding model to simulate the boarding process in a metro station. Numerical examples show that the group-based principle is suitable for the realistic modeling and simulation of boarding flows.

1.2. Impact of Boarding Congestion on Passenger Distribution. Many studies have found that the service time of passengers caused by peak-hour boarding congestion can affect the travel behavior of passengers and their departure time choice [6, 19], which occupies a significant position in the dwell schedule of public transport [23, 24], and affects the capacity design of service facilities such as security and ticket checking equipment in metro stations [25]. [26, 27] investigated the equilibrium behavior of the departure time choice of passengers when the security checking process in rail stations and airports is regarded as boarding congestion. [25] modeled the boarding congestion from a mixed nonlinear type (simulation one) to explore the effect of boarding congestion on the arrival time distribution of passengers and achieved an optimal result of both operation efficiency improvement and passenger boarding time savings.

Recently, a date-oriented analytical method has been developed to obtain arrival time distribution equilibrium of passengers considering the boarding process in metro stations [23, 28]. [28] imported the date of the arrival time of passengers collected in three metro stations into eight existing model of distributions and found that the Hyper-Erlang distribution is most suited to the behavior of departure time choice of passengers under boarding congestion. The study of [23] established a model of the estimated boarding time to explore the behavior of passengers arriving on schedule and collected large-scale travel information from the Greater Copenhagen Area covering the regional metro network. The simulation results show that the actual timetable and high service frequencies help reduce 43% of the boarding congestion.

1.3. Operation Efficiency of the Boarding Demand Management. The boarding demand management aims to redistribute the concentrated travel demand in the waiting station from the temporal and spatial perspectives and improve the operation performance in different transport systems. Two alternative methods of boarding congestion management have been widely studied and can be roughly classified into two categories: optimization-based boarding process management [29–33] and incentive-based boarding process management [2, 5, 34–36].

Passenger classification, which is a commonly used measure of optimization-based boarding process management, is developed for the process of security and boarding check to reduce the total boarding time. Most studies on passenger classification based on different individual characteristics concentrate on boarding issues, especially in the airport boarding process [33, 37, 38]. The family-based boarding in cliques [33], the setting of fast checking queues [14], and the hand-luggage based boarding in cliques [8] have been proven to have a positive impact on efficiency in the air boarding process.

The policy of boarding demand management in airports can also be useful in subway passenger management. Moreover, the timetabling optimization [23, 32, 39, 40] and the design of the dwell time [17, 24] can direct passengers and obtain a better transport service by avoiding boarding congestion. Results show that the loading of peak-hour demand can be effectively transferred to downstream until the demand of the system is solved. Therefore, optimization- and incentive-based boarding process management are used collaboratively in metro systems to smoothen the centralized travel demand of passengers. However, the optimization-based methods for metro boarding management involve high planning and management costs because they require real-time monitoring and optimization for passenger flow. In addition, these methods are strongly dependent on the capacity of the boarding and checking facility, such as the size of the boarding and waiting platform of the metro station, the number of ticket or security checking facilities, the headway of the metro, and the capacity of the designated entrance and exit. Therefore, they are considered as less-practical approaches due to the limited funds and space in the boarding process.

The incentive-based demand control strategies have more flexible extensive application scenarios and low operating costs compared to optimization-based methods. In addition, incentive-based demand control strategies can utilize trade-offs that exist between passenger preferences for departure and arrival times, comfort and crowding, and travel cost [2, 35]. Three alternative incentive-based strategies, which are all widely studied in the literature, include the staggered shifts or early bird [41], fare differentials [5, 36], and real-time information based methods [42]. Various studies regarding these three strategies usually exploit the theory of user equilibrium (UE) and system optimal (SO) under certain conditions to balance the relationship between the disutility of the schedule delay and the willingness to pay quantitatively [43]. Many studies [44] have indicated that the measure of stagger shift such as flexible working mechanism can bring potential direct benefits to the efficient operation and sustainable development of urban transit systems. The measure of staggered shifts implemented in Singapore [45], Melbourne [46], Beijing [47], and Tehran [48] (usually in the form of free "early bird tickets") have been proved to be an effective policy for congestion mitigation when the transit system reaches its capacity [5, 49]. The study of [41] introduced an interesting lottery-based staggered incentive scheme to smoothen the demand during commuting peak hours. The results of an experimental economic analysis indicate that this lottery-based scheme not only helps reduce the boarding congestion in peak hours but also boosts the usage rate of public transit during off-peak periods. Considering that the use of off-peak hours to travel normally has a high schedule delay cost for passengers without strong flexible working hours, [35] developed a biobjective design scheme of fare differentials at two neighboring train stations when the total travel demand is uncertain and obtained two analytical results for reducing boarding time: the SO solution and equity-based UE solution. The study of [50] explored how the policy of fare incentives affects the peak-hour travel demand and choice behavior of passengers given Australia's public transit smart card data of fare incentives imposed in the southeast area of Queensland. A statistical analysis shows that the PT travel demand increases as fare prices decrease, which results in overall revenue gain. Meanwhile, the study of [42] established a simulation-based model with three aspects of guidance information (in this study, all passengers are assumed to obtain all details of boarding congestion and know all information of timetable and the crowding level on each run.) to explore the impact of real-time information on passenger travel choice behavior and obtain an ideal simulation result.

1.4. Approaches to Modeling Boarding Congestion in Metro System. The economic bottleneck model [51], which is elaborated by [52, 53], is extensively used as a common approach in economic analysis to model the boarding congestion (crowding) and fare incentive schemes [5, 6, 34, 54, 55]. There are five main types of fare incentive methods that reduce the boarding or crowding congestion in metro systems, including the flat fare scheme [56, 57], the step fare scheme [58], the time-varying fare [2, 59], the trial-and-error fare scheme (e.g., [34, 35], the fare reward scheme [5], and the hybrid fare scheme [55, 60]. Over the past decades, the initial measures of the flat fare scheme developed by [61, 62] is prevalent in the early development of metro demand management due to its low-cost characteristic. However, this method is gradually replaced by other differentiated fare methods due to the disadvantage of inequitable social welfare (especially for passengers with short-distance and off-peak demand) [36, 63]. In a previous study by [2, 59], they derived a biobjective fare scheme for a metro system to explore the equilibrium patterns of a three fare scheme (no-fare, flat fare, and time-varying fare), and the optimal system capacity and improved social welfare were analyzed. In addition, they discovered that the time-varying fare scheme can smoothen the overconcentrated travel demand and generate more revenue than other metro fare schemes. The study of [58] proposed an analysis model of the differentiated fare including single-step and multi-step pricing strategies that aim to reduce the total cost of a metro transit system. Here, they discussed the equilibrium arrival time choice of passengers and obtained the optimal schedule gap of metro runs under a certain demand pattern. The numerical simulation demonstrates that the more fare steps, the more distribution of arrival times of passengers and the lower the system cost. [34, 35] applied the traffic toll theory of Vickrey's bottleneck model to the studies of boarding/alighting congestion and developed a novel fare measure including a trial-and-error scheme to smoothen the peak-hour commuting metro demand. The study of [5] described the complimentary tickets scheme to encourage commuters to travel during off-peak hours, which they called the "fare-reward scheme." The optimal fare differentials and the optimal reward ratio were determined by an analytic solution under demands. Considering the heterogeneity of the preferred schedules of passengers, the study of [60] proposed a hybrid metro fare scheme by mixing the fare-reward and uniform fare scheme. The analytical results (including UE and SO solutions) and numerical simulation show that this novel fare scheme can significantly reduce travel cost.

1.5. Objectives and Contributions of This Study. In this study, we adapted a novel fare incentive scheme to model the boarding process, regarding the security and ticket checking process as a limited bottleneck in the metro station. In addition, we extended the Vickrey's classical model into a boarding congestion model with a bidesired departure time (hereafter called a biboarding bottleneck model). The train-runs selecting and departure time choosing behaviors of passengers under different schedule gaps of adjacent train runs are explored when the boarding system achieves UE in the proposed boarding congestion model. Then, we examine the demand regulatory mechanisms of fare incentives for reducing the queuing boarding time in metro stations when the optimization of schedule gaps is ineffective in the pattern of mass-scale travel demand. Meanwhile, we introduce a concept of demand threshold (N^*) in the extended boarding congestion model to determine whether the current travel demand has already exceeded the capacity of the boarding service. The contribution of this study can be summarized as follows:

(1) Considering the boarding congestion in the security checking and boarding process, a method is presented to determine the threshold of the travel demand in metro stations. This method can be used to confirm the condition when introducing the measure of fare incentives to balance the boarding congestion.

(2) Sensitivity analyses are conducted to investigate the effect of fare incentives on boarding efficiency and incremental revenue in analytical solutions and numerical simulations. The results answer the question, "how should we use the measure of fare incentives to balance the boarding congestion?"

(3) The result of the fare differentials shows that the flat fare scheme is not beneficial for some passenger travel patterns.

This study aims to find an optimal way in reducing the boarding congestion. This study also answers the following questions:

(i) Can we design appropriate measures to help reduce the boarding congestion without infrastructure expansion

(ii) When can we introduce the measure of fare incentives balancing the boarding congestion

(iii) How do we design a fare incentive scheme to smoothen the peak-hour demand

(iv) Can these policies achieve the expected results

The rest of the paper is organized as follows: in Section 2, we present the review of the literature. In Section 3, we discuss the descriptions of the boarding bottleneck model and cost formulations for staggered commuters. In Section 4, the sensitivity analysis of the staggered shifts is conducted. Here, the evolution of the boarding process in a metro station is also shown in the analysis. In Section 5, we examine the effectiveness of fare incentives in the boarding model when the queuing system achieves UE, and the numerical illustrations are shown. Finally, the conclusions are presented in Section 6.

2. Model Framework

Considering the security and ticket checking process as the main boarding bottleneck, we focus on introducing a novel biarrival time-based boarding congestion model to analyze and evaluate the operational effectiveness of fare incentives for the boarding congestion problem in metro stations. The notations used in this study are listed as follows.

2.1. Notations. Model parameters (all positive scalars)

i: Set of metro service runs $i/i \in \{1, 2\}$

$i = 1$: Metro train cluster during rush hours

$i = 2$: Metro train cluster during flat hours

α: Unit cost of walking time from entrance of metro station to platform

β: Unit cost of early arrival penalty

τ_i: The fare of the i-th metro service run

t_i^*: Departure time of the i-th metro service

Δt: Schedule gap between metro train clusters during rush hours and flat hours

$\Delta \tau$: The fare differentials of metro train clusters during rush hours and flat hours

s: Boarding (checking) capacity of metro station (person/gate/min)

ω: Number of metro trains during rush hours and flat hours

N: Total number of passengers

N_i: Total number of passengers of different metro service runs

Time-varying variables

$q_i(t)$: Queuing length for passengers on i-th metro service run with arrival time t

$T_i(t)$: Total travel time for passengers on i-th metro service run with arrival time t

$T_i^w(t)$: Total queuing time for passengers on i-th metro service run with arrival time t

$T_i^f(t)$: Total free-flow walking time for passengers on i-th metro service run with arrival time t

$T_i^b(t)$: Boarding time of passengers on i-th metro service run with arrival time t

$E_i(t)$: Early boarding delay for passengers on i-th metro service run with arrival time t

$r_i(t)$: Arrival rate of passengers on i-th metro service run with arrival time t

$c_i(t)$: Boarding cost of passengers on i-th metro service run with arrival time t

t_i^a: Earliest arrival time for commuters who take i-th metro service run

t_i^b: Latest arrival time for commuters who take i-th metro service run

2.2. Model Description and Main Assumption. Considering the metro transport line that connects a residential city and a destination, a continuum of N homogeneous passengers go to the metro station to take their desired train runs during rush hour, forming a boarding

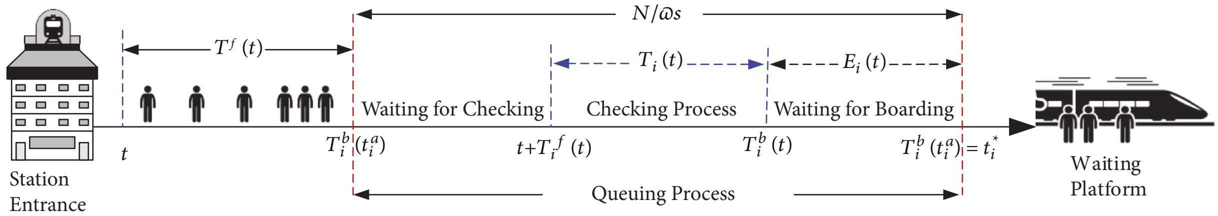

Figure 1: The temporal and spatial boarding process in a metro station.

queuing bottleneck with capacity s in consideration of the requirements in security, identity, and ticket checking; the details can be seen in Figure 1. Supposing that there are two different metro service runs in the same line during rush hour, denoting their departure time as t_1^*, t_2^*, respectively, and there are also two groups of corresponding passengers, N_1, N_2, regarding the schedules above as their desired boarding time. Let $T_i^f(t)$ be the free-flow walking time of different groups of passengers from the entrance gate to the checking infrastructures who arrive at the entrance of the station at time t. Here, we assume that passengers differ only by the departure time t_i^* and fare differentials $\Delta\tau$ of their desired metro service runs. Therefore, without the loss of generality, we have $T_1^f(t) = T_2^f(t) = 0$. In addition, the following assumptions are made in this study:

(i) In general, after the security check, passengers will walk to the waiting hall to conduct the ticket checking process. Then, they will board on the desired metro service runs. We assume that the waiting time between security checking and ticket checking is zero due to a fact that passengers must take the desired runs if they were already in the waiting hall, but they must miss their desired train if they fail to pass the security process punctually. Therefore, the waiting time before ticket checking can be ignored in this study.

(ii) The walking time from the checking infrastructures to the waiting platform is ignored in this study since there is no queuing congestion during this process.

(iii) The desired departure time t_1^*, t_2^* satisfies $t_1^* - t_2^* > 0$. Therefore, the timetable interval of these two neighboring metro runs is $\Delta t = t_2^* - t_1^*$.

2.3. Estimating the Queuing Congestion: Vickrey's Classic Bottleneck Model. The pedestrian flow forms a queue before entering the checking bottleneck when their arrival rate $r_i(t)$ exceeds the service capacity based on the Vickrey's classic bottleneck model (without considering the pattern caused by the spillover). The boarding congestion in the metro station occurs during the interval $t \in [t_1^a, t_2^b]$ by combining assumptions (i), (ii), and (iii). In addition, t_1^a and t_2^b can be also regarded as the passengers' earliest and the latest time to encounter queuing, respectively.

Here, $A_i(t)$ and $D_i(t)$ are the cumulative arriving and boarding curves of different groups of passengers who arrive to the metro station at time t, respectively. The passengers' queuing length for the i-th metro service run can be modeled using the following:

$$q_i(t) = D_i(t) - A_i(t) = \int_t^{t+T_i(t)} s\mathrm{d}x, \tag{1}$$

$$q_i(t) = \begin{cases} \int_{t_i^a}^{t} (r_i(x) - s)\mathrm{d}x, & \text{if} \quad q_i(x) > 0, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Here, $T_i(t)$ is the passengers' total commuting time for the i-th metro service run. In this model, we regarded the waiting time $T_i^w(t)$ as the walking free-flow time $(T_1^f(t) = T_2^f(t) = 0)$. The change rate of the queuing length can be simplified as an ordinary differential equation.

$$\frac{dq_i(t)}{dt} = r(t - t_i^a) - \begin{cases} s, & if\ q_i(t) > 0, \\ \min\{r(t - t_i^a), s\}, & \text{otherwise.} \end{cases} \tag{3}$$

Based on (1), (2), and (3), the total travel time for different groups of passengers who arrive to the station at time t and take the i-th metro service run is

$$T_i(t) = T_i^w(t) = \frac{q_i(t)}{s}. \tag{4}$$

2.4. Estimating Queuing Cost: Vickrey's Time Schedule Penalty Scheme. Based on the classical ADL model of the morning commute problem [52, 64], the generalized passengers' queuing cost $c_i(t)$ is formulated in this subsection when the staggered passengers' desired departure time is t_i^* and their arrival time is t.

For passengers choosing the i-th metro service, their travel cost includes the boarding time cost, schedule early penalty, and fare cost of the metro service. Let $c_1(t)$,, $c_2(t)$ be the total boarding cost for passengers with the desired departure time t_1^*, t_2^* with arrival time t, respectively.

$$c_1(t) = \alpha T_1(t) + \beta(t_1^* - t - T_1(t)) + \tau_1, \tag{5}$$

$$c_2(t) = \alpha T_1(t) + \beta(t_2^* - t - T_1(t)) + \tau_2, \tag{6}$$

where α and β are the unit cost of queuing time and schedule early penalty, respectively. In (5) and (6), τ_1 and τ_2 are the fares for passengers with desired departure times t_1^* and t_2^*, respectively (We assume $\tau > \alpha > \beta$ in this study adopted from the parameter estimation in Vickrey's bottleneck model by [65] and the research of optimal pricing in rail transport considering bottleneck congestion [66]). The fare is constant between the same OD pair in the operational metro service line. Moreover, we introduce a factor of fare differentials

($\Delta\tau = \tau_2 - \tau_1$) in different metro service runs to get a better effect of the boarding congestion control. According to assumptions (i), (ii), and (iii), the desired boarding time for different groups of passengers can be regarded as the departure time of the corresponding metro service runs.

In our boarding congestion model, passengers prefer to select the optimal departure times to make trade-offs among the queuing time, schedule delay time, and ticket price. Finally, the system will result in a dynamic user equilibrium (DUE). First, we discuss the properties of the DUE for passengers with desired departure time t_1^* and metro service fare τ_1. Moreover, t_1^a and t_1^b are the earliest and latest arrival times for passengers, respectively. We differentiate (5) with respect to t and set it to zero ($dc_1(t)/dt = 0$). Then, the arrival rate to the metro station when the boarding system achieves equilibrium is given by the following equation:

$$r_1(t) = \frac{\alpha}{\alpha - \beta} \cdot \omega \cdot s, \quad t \in \left(t_1^a, t_1^b\right). \tag{7}$$

The passengers who choose the earliest arrival time t_1^a will not meet queuing, and the passengers who choose the latest arrival time will get on the trains on time under the consideration of no late arrival. The travel costs of this group of passengers meet the following requirements:

$$\begin{aligned} c_1\left(t_1^a\right) &= \beta\left(t_1^* - t_1^a\right) + \tau_1, \\ c_1\left(t_1^b\right) &= \alpha\left(t_1^* - t_1^b\right) + \tau_1. \end{aligned} \tag{8}$$

The earliest and latest arrival times for this group of passengers are given by equations (10)–(11) by combining the condition of full-capacity operation during the boarding process $t_1^* - t_1^a = N_1/s$.

$$\begin{aligned} t_1^a &= t_1^* - \frac{N_1}{s}, \\ t_1^b &= t_1^* - \frac{\beta}{\alpha}\frac{N_1}{s}, \end{aligned} \tag{9}$$

$$T(t) = \frac{\beta}{\alpha - \beta}\left(t - t_1^a\right), \quad t \in \left(t_1^a, t_1^b\right). \tag{10}$$

Subsequently, we can easily obtain the individual travel cost for passengers on the metro service run with desired departure time t_1^* when the boarding system achieves UE state.

$$c_1(t) = \beta \cdot \frac{N_1}{s} + \tau_1. \tag{11}$$

Through a similar approach, the individual travel cost for passengers on the metro service run with desired departure time t_2^* is

$$c_2(t) = \beta \cdot \frac{N_2}{s} + \tau_2. \tag{12}$$

The benchmark pattern of the passengers has the same properties of the arrival time choice when the schedule gap of neighbor metro runs is $\Delta t = 0$ and the total demand satisfies $N = N_1$ or $N = N_2$, then different groups of passengers will achieve UE independently. Assume there are no fare differentials between two neighboring metro service runs in the benchmark pattern $\tau = \tau_1 = \tau_2$, then the total boarding-queue time (TBT) and total travel cost (TTC) are expressed as follows:

$$\begin{aligned} \mathrm{TBT} &= \frac{\beta}{\alpha} \cdot \frac{N^2}{4\omega s}, \quad N = N_1 \text{ or } N = N_2, \\ \mathrm{TTC} &= \beta \cdot \frac{N^2}{\omega s} + N\tau, \quad N = N_1 \text{ or } N = N_2. \end{aligned} \tag{13}$$

3. Biboarding Bottleneck Model with No-Fare Differentials

In this section, we discuss all possible congestion patterns in the boarding system when the fares of neighboring metro service runs are the same. In addition, the queuing time and boarding cost analysis are conducted to obtain the optimal schedule gap when the boarding system achieves UE. The total boarding cost for two groups of passengers on different metro service runs can be expressed as follows:

$$\mathrm{TBC} = \sum_{i=1,2} \left\{\alpha T_i(t) + \beta\left(t_i^* - t - T_i(t)\right) + \tau_i\right\}. \tag{14}$$

3.1. Fundamental Properties of the Biqueuing Model. Based on the principle of UE in the queuing bottleneck model, the passengers with different desired departure times have the same boarding cost and all passengers arriving at the metro station will board their desired runs. However, we provide propositions to explain the equilibrium properties in the mixed boarding system for further analysis.

Proposition 1. *With a given schedule gap of the metro service between rush hours and flat hours Δt and headway of metro s, the UE of the mixed boarding system cannot be achieved when $N_2 > \Delta t \cdot s$.*

Proof. A boarding queue forms during rush hour due to the limited capacity of the metro headway in the boarding process. Therefore, the maximum number of passengers arriving to the metro station and boarding the metro train runs successfully between time t_1^* and t_2^* is $\Delta t \cdot s$. All these passengers have a desired departure time t_2^* and board the corresponding metro runs. If $N_2 = \Delta t \cdot s$, the earliest arrival passenger who wants to take the metro run with a departure time t_2^* will meet the latest one who takes the previous metro run with a departure time t_1^* based on the property of the boarding pattern shown in Figure 2(b). Therefore, if the arrival passengers between the time t_1^* and t_2^* exceed maximum bottleneck capacity in number ($N_2 > \Delta t \cdot s$), there must be extra passengers with desired departure time t_2^* arriving at the station before the time t_1^* because they will miss their desired metro run if their arrival time is later than t_2^*. There will be a mixed queue composed of different types of passengers on the waiting platform. Suppose that the staggered passengers meet the mixed queue when they arrive to the metro station at time t, then the queuing cost for the

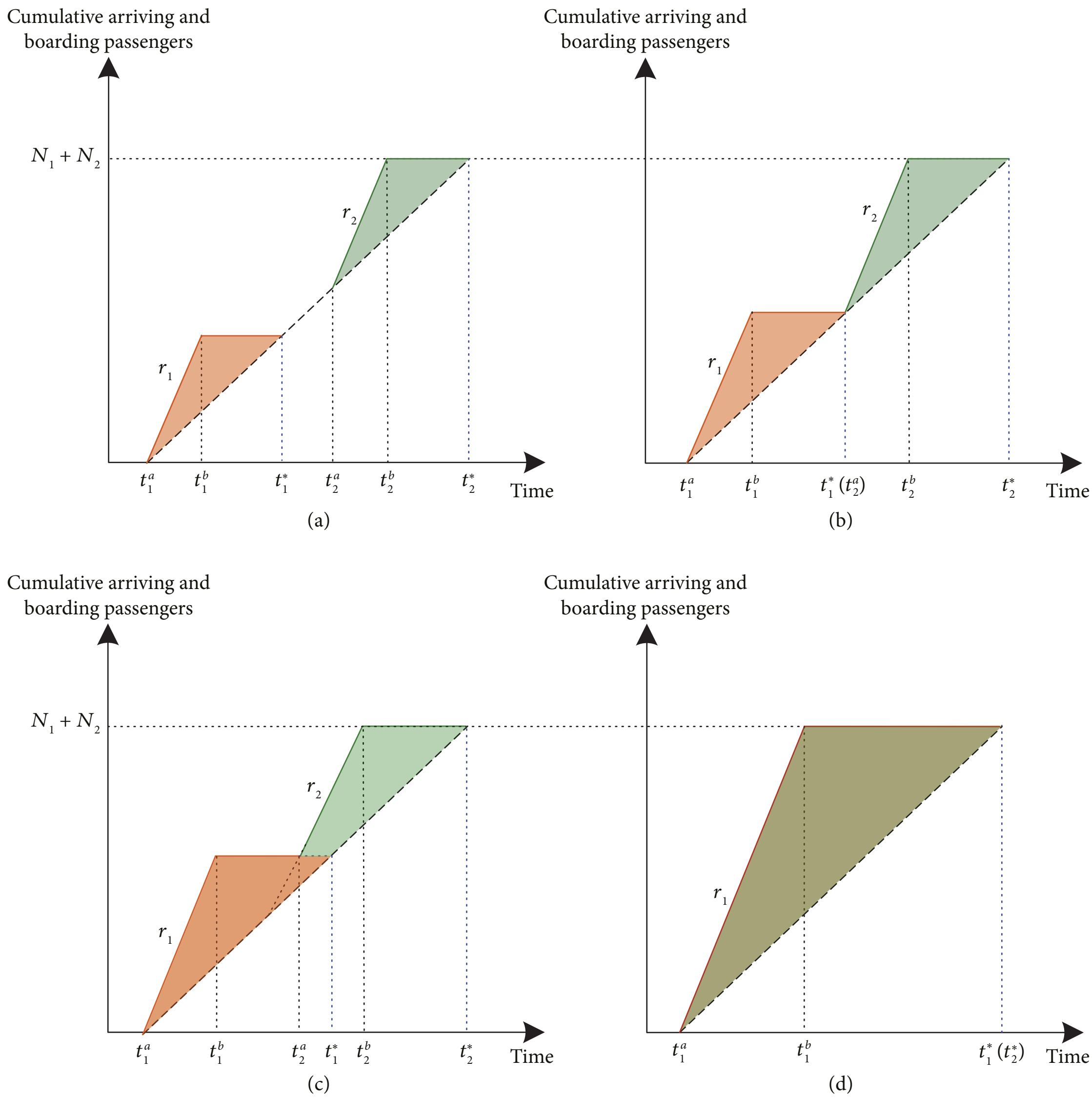

Figure 2: All equilibrium patterns of the boarding model with no-fare differentials. (a) Cumulative arriving and boarding passengers. (b) Cumulative arriving and boarding passengers. (c) Cumulative arriving and boarding passengers. (d) Cumulative arriving and boarding passengers.

staggered passengers who arrive at the station at time t can be expressed as follows:

$$c_1(t) = \alpha T(t) + \beta\left[t_1^* - t - T(t)\right] + \tau_1, \tag{15}$$

$$c_2(t) = \alpha T(t) + \beta\left[t_2^* - t - T(t)\right] + \tau_2. \tag{16}$$

With the condition of no fare differential ($\tau_1 = \tau_2$) and $\Delta t = t_2^* - t_1^* > 0$, the boarding cost of passengers who have the desired departure time t_2^* is higher than other passengers who want to depart at time t_1^* ($c_1(t) < c_2(t)$) when $N_2 > \Delta t \cdot s$. Therefore, the assumption of $N_2 > \Delta t \cdot s$ does not hold. □

Proposition 2. *With a definite schedule gap of metro service runs Δt and boarding bottleneck capacity s, the number of staggered passengers is equal ($N_1 = N_2$) if the boarding system with biservice runs achieves UE under the condition of $N_2 \leq \Delta t \cdot s$.*

Proof. According to Proposition 1, the mixed boarding system can achieve UE if the number of arrival passengers between the time t_1^* and t_2^* are less than the maximum bottleneck capacity ($N_2 \leq \Delta t \cdot s$). In addition, according to the condition of UE (equal cost for all commuters in the commuting system, $c_1(t) = c_2(t)$), we can get the result $N_1 = N_2$ when $N_2 \leq \Delta t \cdot s$ by combining the boarding cost formulas presented in (11) and (12). □

Theorem 1. *If the total travel demand of the metro biservice runs satisfies the condition $N \leq 2\Delta t \cdot s$, and the number of different passengers is equal ($N_1 = N_2$), the biboarding system achieves UE when the number of passengers with different desired times is equal ($N_1 = N_2$).*

Proof. According to Proposition 1 and Proposition 2, we know that the boarding system with staggered passengers cannot achieve UE when the number of passengers satisfies $N_2 > \Delta t \cdot s$. If $N_1 = N_2$, the boarding system of staggered passengers can achieve UE when $N_2 \leq \Delta t \cdot s$. Considering $N = N_1 + N_2$, if $N_1 = N_2$, the boarding system can achieve UE when $N \leq 2\Delta t \cdot \omega s$. □

Theorem 2. *If the total travel demand for biservice runs of the metro service satisfies the condition $N > 2\Delta t \cdot s$, the number of passengers who want to depart at time t_2^* is equal to $\Delta t \cdot \omega s$ ($N_2 = \Delta t \cdot \omega s$) and the number of passengers with another desired time t_1^* is $N - N_2$ ($N_1 = N - N_2$) when the commuting system achieves UE.*

Proof. See Appendix.

All four possible equilibrium boarding patterns with mixed passengers who have different desired departure times is presented in Figure 2 when the key time points (t_1^a, t_1^b, t_2^a, t_2^b) are derived. For instance, if the schedule gap of neighboring metro runs, $\Delta t = t_2^* - t_1^*$, is smaller (see in Figure 2(c)), some passengers on two neighboring metro runs will meet in the checking process, which means that they will mix and line up for checking and boarding between the time t_2^a and t_1^*. If $\Delta t = t_2^* - t_1^*$ is extremely large (much larger than $(N_1 + N_2)/s$), the checking and boarding of passengers with different desired departure times happen at different times, separating their boarding activity, as shown in Figure 2(a). However, the checking and boarding capacity of the metro station is wasted between the time t_1^* and t_2^a, at this time there are no passengers waiting for the security check, which reduces the operational efficiency of the boarding system during peak hours. The boarding pattern shown in Figure 2(b) is a critical one between the patterns in Figures 2(a) and 2(c). The difference is that two types of passengers are more connected (i.e., the earliest arrival passenger with desired departure time t_2^* joins the boarding queue behind the latest arrival passengers with desired departure time t_1^*) as the schedule gap Δt of neighboring metro runs becomes smaller. It follows that there is no wasted time in the checking and boarding process between the two types of passengers on adjacent metro runs. Therefore, it is very important to set an optimal schedule interval between two neighboring metro runs that will improve the operation efficiency of the boarding system and control the gathering of passengers during rush hours. □

3.2. Coordination of Optimal Schedule Gap with No-Fare Differentials. The coordination of the schedules of the metro service runs are investigated to reduce the total boarding queuing time and travel cost when there are no fare differentials. Assume $N^* = 2\Delta t \cdot s$ as the threshold of the travel demand in two neighboring metro service runs, then the queuing time and boarding cost under the condition of UE in staggered passengers will be analyzed in two patterns: when the total travel demand does not exceed the threshold of demand in two neighboring metro service runs $N \leq 2\Delta t \cdot s$ and when the total travel demand exceeds the corresponding threshold. $N > 2\Delta t \cdot s$

According to Proposition 2 and Theorem 1, for the first pattern ($N \leq 2\Delta t \cdot s$), the number of passengers with different desired departure times is equal ($N_1 = N_2$) when the biboarding system achieves UE. For the total boarding process, the boarding queuing length displays two equal peaks, whose value is half of the benchmark pattern, as shown in Figure 2(a).

TBT and TTC can be expressed by (17) and (18) by combining equations (11)–(13).

$$\text{TBT} = \frac{\beta}{\alpha} \cdot \frac{N^2}{4s}, \tag{17}$$

$$\text{TTC} = \beta \cdot \frac{N^2}{s} + N\tau, \tag{18}$$

where τ is the same for staggered passengers under the assumption that there are no fare differentials.

According to Proposition 2 and Theorem 1, if the total travel demand in two neighboring metro service runs satisfies the pattern $N > 2\Delta t \cdot \omega s$, the number of passengers who will take the next metro run is equal to $\Delta t \cdot s$ ($N_2 = \Delta t \cdot s$) when the biboarding system achieves UE. Subsequently, the number of passengers who will take the first metro is $N - N_2$ ($N_1 = N - N_2$). Under the condition of UE, TBT and TTC can be expressed as follows:

$$\text{TBT} = \frac{\beta}{\alpha} \cdot \frac{N_1^2}{s}, \tag{19}$$

$$\text{TTC} = \beta \cdot \frac{N_1^2 N_1 N_2}{s} + \left(\alpha T^f + \tau\right) \cdot N. \tag{20}$$

We can get the formulas of the TBT and TTC with different demand patterns by combining equations (18)–(21).

$$\text{TBT} = \begin{cases} \dfrac{\beta}{\alpha} \cdot \dfrac{N^2}{4s}, & N \leq 2 \cdot \Delta t \cdot s, \\ \dfrac{\beta}{\alpha} \cdot \dfrac{N_1^2}{s}, & N > 2 \cdot \Delta t \cdot s, \end{cases} \tag{21}$$

$$\text{TTC} = \begin{cases} \beta \cdot \dfrac{N^2}{s} + N\tau, & N \leq 2 \cdot \Delta t \cdot s, \\ \beta \cdot \dfrac{N_1^2 + N_1 N_2}{s} + N\tau, & N > 2 \cdot \Delta t \cdot s. \end{cases} \tag{22}$$

According to (21) and (22), the total queuing time in the boarding bottleneck model is determined by the total number of passengers and the boarding bottleneck capacity. The total queuing time of the biboarding bottleneck model is affected by the number of passengers that have the desired work start time t_1^* when the total number of passengers satisfies $N > 2 \cdot \Delta t \cdot s$. Conversely, the total queuing behavior does not matter for passengers who have the desired work start time t_1^*.

4. Biboarding Bottleneck Model with Fare Differentials

The fare differentials of the adjacent metro service run is set as $\Delta\tau$ in this section. In addition, the effect of the fare incentive (fare differentials) on the smoothing of the travel demand during peak hours and reduction of the boarding time of the biboarding system is analyzed. This section discusses the regulation effect of the incentive fare in different boarding patterns based on the conditions discussed previously.

4.1. The Effect of Fare Incentives in Less Demand Pattern (when $N \leq 2 \cdot \Delta t \cdot s$). Based on Section 3.1, if there are no fare differentials between staggered passengers ($\Delta\tau = 0$), the number of passengers on different service runs is equal ($N_1 = N_2$) when the queuing system achieves UE. In this case, the travel cost of two types of travelers is given by (11) and (12). The travel costs of two types of passengers with the same arrival time are different when the measure of fare differentials is implemented. Here, travelers will choose the service runs with the lowest fare to reduce their travel costs. The equilibrium proportion of staggered passengers in different metro runs changes compared with no-fare differentials. According to Theorem 1, the total travel demand of the biboarding system is necessary to satisfy the condition of $N \leq 2\Delta t \cdot s$ if the biboarding system can achieve UE. Combining (11) and (12), and $c_2(t) = c_1(t)$, $N = N_1 + N_2$ is given by the following equation:

$$\begin{aligned} N_1 &= \frac{N}{2} + \frac{s}{2\beta}\Delta\tau, \\ N_2 &= \frac{N}{2} - \frac{s}{2\beta}\Delta\tau. \end{aligned} \tag{23}$$

The number of staggered passengers who prefer to select the second metro satisfies $0 \leq N_2 \leq \Delta t \cdot s$ due to the boarding bottleneck limitation. We obtain the equilibrium condition of the biboarding system under the effect of fare differentials by including this condition in (23).

$$\beta\frac{N - 2\Delta t \cdot s}{s} \leq \Delta\tau \leq \beta\frac{N}{s}. \tag{24}$$

Therefore, the travel cost of the staggered travelers who depart from the station to their destination at time t_2^* is higher than others at time t_1^* when the travel demand does exceed the boarding threshold ($N \leq 2 \cdot \Delta t \cdot s$) and fare differentials between neighboring metro runs satisfy the condition $\beta N - 2\Delta t \cdot s/s \leq \Delta\tau \leq \beta N/s$. Therefore, some travelers change their choice of metro service from t_2^* to t_1^*. The boarding cost of travelers on the t_1^*-th metro service run increases with their number in this process of demand transfer. Moreover, the number of staggered passengers satisfies $N_1 = N/2 + s/2\beta\Delta\tau$ and $N_2 = N/2 - s/2\beta\Delta\tau$ when the new UE is achieved. The TBT can be derived as follows:

$$\text{TBT} = \frac{\beta}{\alpha}\frac{N^2}{4s} + \frac{s}{4\alpha\beta}(\Delta\tau)^2. \tag{25}$$

Similarly, considering the minimization of travel cost, all travelers prefer t_1^* as their desired departure time when the fare differentials satisfy the condition $\Delta\tau > \beta N/s$. In contrast, all commuters prefer t_2^* as their desired departure time when the fare differentials satisfy the condition $\Delta\tau < \beta N - 2\Delta t \cdot s/s$. The boarding queuing time in these two patterns can be expressed as follows:

$$\text{TBT} = \frac{\beta}{\alpha}\frac{N^2}{2s}. \tag{26}$$

Combining (25) and (26), the change of the total queuing time with different fare differentials can be expressed as follows:

$$\text{TBT} = \begin{cases} \dfrac{\beta}{\alpha}\dfrac{N^2}{2s}, & \Delta\tau > \beta\dfrac{N}{s} \text{ or } \Delta\tau < \beta\dfrac{N - 2\Delta t \cdot s}{s}, \\[2ex] \dfrac{\beta}{\alpha}\dfrac{N^2}{4s} + \dfrac{s}{4\alpha\beta}(\Delta\tau)^2, & \beta\dfrac{N - 2\Delta t \cdot s}{s} \leq \Delta\tau \leq \beta\dfrac{N}{s}. \end{cases} \tag{27}$$

4.2. Effect of Fare Incentives for a Higher Demand Pattern (when $N > 2 \cdot \Delta t \cdot s$). The relationship of passengers in different metro service runs satisfies the mixed equilibrium proportion ($N_2 = \Delta t \cdot \omega s$ and $N_1 = N - N_2$) if there are no fare differentials between different metro runs ($\Delta\tau = 0$) with high travel demand. If we introduce the measure of fare differentials to smoothen the travel demand between different metro runs, the biboarding system with fare differentials meets two possible patterns (patterns 1 and 2). In pattern 1, the biboarding system can still adjust the travel cost using endogenous variables (proportion of passengers with different desired departure times) to achieve a new UE within a certain controllable range of fare differentials. Conversely, in pattern 2, within an uncontrollable range of fare differentials, the system cannot adjust the travel cost using endogenous variables to achieve UE, leading to only one type of passenger in the metro network during peak hours (this pattern is likely the benchmark).

First, we analyzed and discussed the pattern when the moderating effect of fare differentials on staggered travelers can be controlled. Here, there must be $N_2 = \Delta t \cdot s$ when the biboarding system achieves UE. By combining the travel cost formula shown in equations (A.1), (A.2), and (A.7) in Appendix with $c_2(t) = c_1(t)$, $N_1 = N - \Delta t \cdot s$, we obtain the following:

$$\begin{aligned} t_2^a &= t_2^* - \frac{\beta}{\alpha}\frac{N - 2\Delta t \cdot s}{s} + \frac{1}{\alpha}\Delta\tau - \Delta t, \\ t_2^b &= t_2^* - \frac{\beta}{\alpha}\frac{N - \Delta t \cdot s}{s} + \frac{1}{\alpha}\Delta\tau. \end{aligned} \tag{28}$$

The controllable range of fare differentials can be expressed below in (29) by combining $t_1^b \le t_2^a \le t_1^*$ and equation (A-3) (i.e., $t_1^b = t_1^* - \beta/\alpha N - \Delta t \cdot s/s$).

$$-\beta\Delta t \le \Delta\tau \le \beta\frac{N - 2\Delta t \cdot s}{s}. \quad (29)$$

Therefore, the passengers on different metro service runs can achieve UE with the mixed proportion ($N_1 = N - \Delta t \cdot s$ and $N_2 = \Delta t \cdot s$) when the travel demand of two neighboring metro service runs is higher than the boarding threshold ($N > 2 \cdot \Delta t \cdot s$) and the fare differentials satisfy the condition $-\beta\Delta t \le \Delta\tau \le \beta N - 2\Delta t \cdot s/s$. The TBT is

$$\mathrm{TBT} = \frac{\beta}{\alpha}\frac{N^2 - 2(\Delta t \cdot s)^2}{2s} - \frac{(\Delta t \cdot s)^2}{\alpha}\Delta\tau. \quad (30)$$

The travel cost of staggered travelers with desired departure time t_2^* becomes less than other travelers with departure time t_1^* if the travel demand of two neighboring metro service runs exceeds the threshold level ($N > 2 \cdot \Delta t \cdot s$) and the fare differentials satisfy the condition $\Delta\tau < -\beta\Delta t$. The original equilibrium proportion of different travelers is broken if passengers prefer to choose t_2^* as their desired departure time, leading to the failure in the measure of fare differentials. Meanwhile, some travelers choose the desired departure time t_2^* arriving at the waiting room or platform before time t_1^* due to the boarding bottleneck and do not take the metro run that has vacant seats and will soon depart from the station at time t_1^*. Instead, they will wait until the next metro run that is less expensive, resulting in a wastage of transportation resources of the metro system and boarding congestion.

Similarly, the travel cost of passengers with the desired departure time t_2^* becomes more expensive than others who have the desired departure time t_1^* when the fare differentials of neighboring metro service runs satisfy $\Delta\tau > \beta N/s$. When passengers prefer to choose t_1^* as their desired departure time, the TBT of these two similar patterns can be expressed as follows:

$$\mathrm{TBT} = \frac{\beta}{\alpha}\frac{N^2}{2s}. \quad (31)$$

Finally, the travel cost of passengers on the t_2^*-th metro run is more expensive than others who take the t_1^*-th metro run when the fare differentials of neighboring metro service runs satisfy $\beta N - 2\Delta t \cdot s/s < \Delta\tau < \beta N/s$ and the total travel demand exceeds the threshold level ($N > 2 \cdot \Delta t \cdot s$). Therefore, travelers change their choice of metro service runs from t_2^* to t_1^*. Moreover, the boarding cost of travelers on the t_1^*-th metro service run increases with their number in this process of travel demand transfer. The number of staggered passengers satisfies $N_1 = N/2 + s/2\beta\Delta\tau$ and $N_2 = N/2 - s/2\beta\Delta\tau$ when the new UE is achieved. These patterns are shown in Figure 2(c). The TBT in this pattern is

$$\mathrm{TBT} = \frac{\beta}{\alpha}\frac{N^2}{4s} + \frac{s}{4\alpha\beta}(\Delta\tau)^2. \quad (32)$$

Combining (30), (31) and (32), the function of the TBT with fare differentials is expressed as follows:

$$\mathrm{TQT} = \begin{cases} \dfrac{\beta}{\alpha}\dfrac{N^2}{2s}, & \Delta\tau < -\beta\Delta t \text{ or } \Delta\tau > \beta\dfrac{N}{s}, \\[2ex] \dfrac{\beta}{\alpha}\dfrac{N^2 - 2(\Delta t \cdot s)^2}{2s} - \dfrac{(\Delta t \cdot s)^2}{\alpha}\Delta\tau, & -\beta\Delta t \le \Delta\tau \le \beta\dfrac{N - 2\Delta t \cdot s}{s}, \\[2ex] \dfrac{\beta}{\alpha}\dfrac{N^2}{4s} + \dfrac{s}{4\alpha\beta}(\Delta\tau)^2, & \beta\dfrac{N - 2\Delta t \cdot s}{s} \le \Delta\tau \le \beta\dfrac{N}{s}. \end{cases} \quad (33)$$

Based on the discussion above, with the fixed parameters, such as the unit cost of penalty β, the schedule gap Δt, and the capacity of the service s, the choice of arrival time at the station and metro service runs is not affected by the size of the total travel demand if the fare differentials satisfy the condition $\Delta\tau < -\beta\Delta t$ or $\Delta\tau > \beta N/\omega s$, and the adjustment effect of the incentive measure on the neighboring metro service demand by fare differentials is not effective. Meanwhile, for a determined travel demand N during a metro rush hour, if the fare differentials satisfy the condition $-\beta\Delta t \le \Delta\tau \le \beta N/\omega s$ when the travel demand exceeds the threshold level and if the fare differentials are $\Delta\tau = \beta N - 2\Delta t \cdot \omega s/\omega s$, the incentive measure of fare differentials reduces the boarding congestion. Here, the travelers in the checking process have the shortest queue length and the shortest queuing time. If $\Delta\tau = 0$, the boarding system has the shortest queue length when the fare differentials satisfy the condition $-\beta\Delta t \le \Delta\tau \le \beta N/\omega s$ when the travel demand is less than the threshold level. This means that the incentive measures of fare differentials have a negative effect on the regulation of the travel demand.

5. Numerical Analysis

The UE with the minimum disutility in the biboarding system when applying the measures of fare differentials is presented in this section. Various studies [5] conducted numerical analysis and verification for the related sensitivity properties of fare differentials to explore the influencing

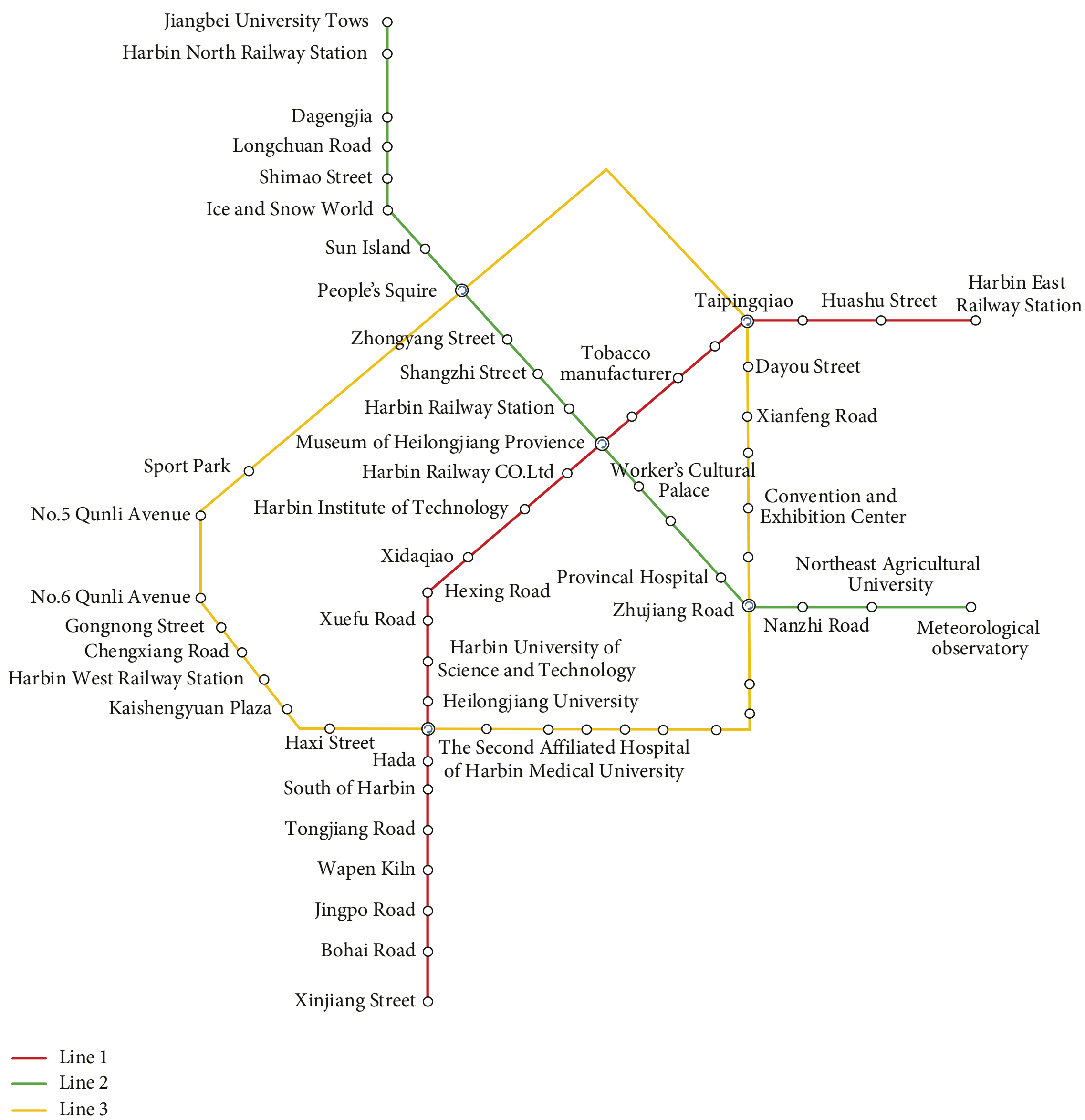

Figure 3: Map of the harbin metro system.

factors in the biboarding bottleneck model, providing theoretical guidance for improving future work.

In this study, we selected the Museum of Heilongjiang Province station as the study location, as shown in Figure 3. We also set s = 10 passenger/min, α = min, and β = . The relevant numerical simulation results are discussed as follows.

5.1. Relationship of Schedule Gap and Capacity $(\Delta t, s)$. Based on the previous sections, the number of commuters N_2 is affected by the schedule gap Δt and the total boarding capacity s, and the threshold of total travel demand is $N^* = 2\Delta t \cdot s$. The decision space of passengers between two neighboring metro service runs can be identified if the total travel demand is confirmed. Here, we list the three different cases (i.e., N = 600, N = 1200, and N = 1800), and Figure 3 shows the allocation detail of different groups of passengers by varying s and Δt. Figure 4 shows the relationship between the schedule gap and the boarding capacity when the demand of the neighboring metro service runs is fixed. Points A, B, and C represent three cases of passengers' demand in the metro station. Here, the red l_1, black l_2, and blue dotted lines l_3 shown in Figure 4 present the boundary choice patterns when $N_2 = \Delta t \cdot s$ for $N = 600$, $N = 1200$, and $N = 1800$, respectively. The upper part of the line indicates that the domain of staggered passengers (N_1, N_2) always satisfies $N_1 = N_2 = 600$ when the queuing system achieves UE, and we can see that it is not affected by Δt and s. For example, when the total travel demand is N = 1200, point A (50, 30) is at the upper part of line l_2 when the values of the schedule gap and total boarding capacity are determined. This means that the maximum number of passengers N_2 is 150 since it is higher than half of the total travel demand $(\Delta t \cdot s > 600)$. Therefore, after the passengers have made a

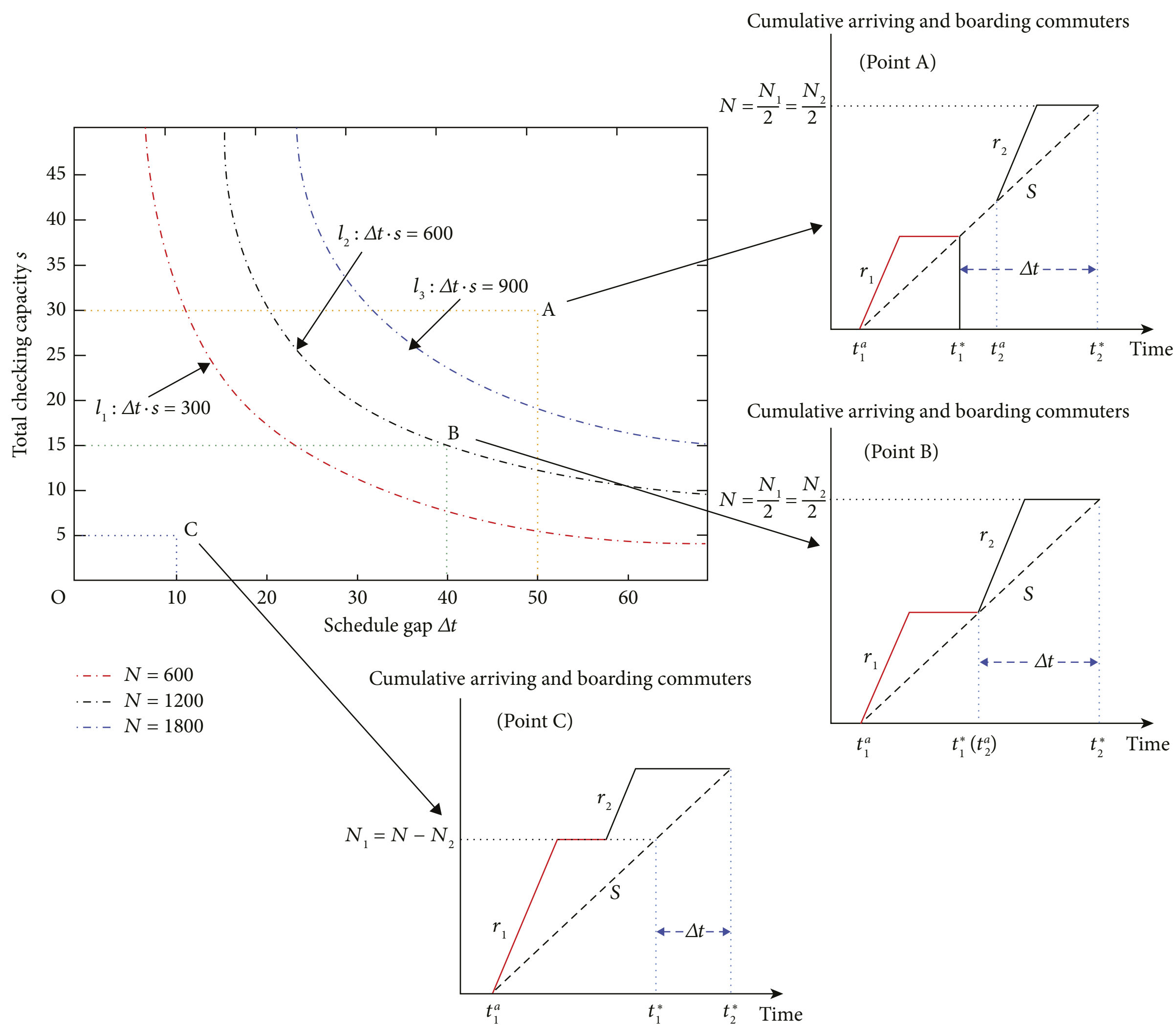

Figure 4: Decision space for PT shuttle choice among staggered passengers.

choice, the proportion of staggered passengers under the condition of UE is $N_1 = N_2 = 600$ (as shown in case 1 presented in Figure 2). In contrast, point C (10, 5) in the lower-left part of line l_2 indicates the case of $N > N^*$. Here, there are more passengers selecting the metro runs with an earlier schedule because the number of passengers who select the runs with desired departure time t_1^* will be decided by the domains of $(\Delta t, s)$. The number of passengers who select the metro runs with departure time t_2^* always satisfies $N_2 = \Delta t \cdot \omega s$ and $N_1 = N - N_2$. The maximum number of passengers N_2 is 50 when the domains of $(\Delta t, s)$ is point C and the number of staggered passengers is $N_1 = 1150$, as shown in point C in Figure 4. The boarding pattern is shown in case 3 in Figure 2. If the setting of Δt and s are distributed online l_2 $(\Delta t \cdot s = 600)$, the boarding queue can be described in case 2 in Figure 2. Meanwhile, the boarding system has the highest operation efficiency. Therefore, by comparing the different total travel demands, the probability of boarding in cases 1 (point A) and 2 (point B) becomes lower and higher, respectively, as the total travel demand increases.

5.2. Numerical Analysis on the Schedule Choice under the Influence of Boarding Bottleneck. Here, we analyze and discuss the impact of the different staggered schedule gaps between metro train clusters during rush and flat hours Δt and different fare differentials $\Delta\tau$ based on the different biboarding patterns. Based on the previously presented propositions and theorems, the threshold of the total travel demand is $N^* = 2\Delta t \cdot s$, which is defined by the black straight line A_1A_2 in Figure 4. In Figure 4, area A indicates that the boarding patterns of the total travel demand is less than the threshold $(N < 2\Delta t \cdot s)$, while area B shows that the total travel demand exceeds the threshold $(N < 2\Delta t \cdot s)$.

Table 1 shows the timetable (upstream) of metro runs during rush flat hours in the Museum of Heilongjiang Province station and Table 2 lists cases 1–6 and the settings of the schedule parameters of different metro runs. In addition, Figure 5 displays the sensitivity adjustment of different schedule gaps Δt on the strategy of metro runs selection. The red dotted lines P_1P_2 and P_2P_4 represent the optimal proportion of two staggered passengers when the

Table 1: The timetable (upstream) of metro runs during rush and flat hours in the Museum of Heilongjiang Province station.

Metro line	Timetable (upstream) of metro runs during rush hours	Timetable (upstream) of metro runs during flat hours
1	7:59, 8:03, 8:05, 8:08, 8:13, 8:16, 8:20, 8:25, 8:30, 8:36; 8:41	8:48, 8:55, 9:04, 9:13, 9:23, 9:32
2	8:02,8:04, 8:06,8:09, 8:15, 8:19, 8:22, 8:27, 8:33, 8:38; 8:44	8:50, 8:57, 9:08, 9:17, 9:28, 9:39

Table 2: The setting of schedule gap/fare differentials during rush and flat hours.

Case	Setting of schedule gap	Setting of fare differentials	Figure
1	$t_1^* = 8{:}\ 30, t_2^* = 9{:}\ 00$	$\tau_1 = 5, \tau_2 = 5$	Figure 5(a)
2	$t_1^* = 8{:}\ 00, t_2^* = 9{:}\ 00$	$\tau_1 = 5, \tau_2 = 5$	Figure 5(b)
3	$t_1^* = 8{:}\ 00, t_2^* = 10{:}\ 00$	$\tau_1 = 5, \tau_2 = 5$	Figure 5(c)
4	$t_1^* = 8{:}\ 00, t_2^* = 9{:}\ 00$	$\tau_1 = 6, \tau_2 = 3$	Figure 5(d)
5	$t_1^* = 8{:}\ 00, t_2^* = 9{:}\ 00$	$\tau_1 = 6, \tau_2 = 0$	Figure 5(e)
6	$t_1^* = 8{:}\ 00, t_2^* = 9{:}\ 00$	$\tau_1 = 6, \tau_2 = 2$	Figure 5(f)

system achieves UE in the patterns that the total travel demand is less or more than threshold N^*, as shown in Figures 5(a)–5(c), respectively. We take one setting of the two measures ($t_1^* = 8{:}\ 00, t_2^* = 9{:}\ 00;\ \tau_1 = 5, \tau_2 = 5$) as an example to discuss in this section, as shown in case 2 presented in Table 2 and Figure 5(b). Here, the threshold of demand is $N^* = \Delta t \cdot s = 1200$. The number of passengers who choose the metro runs with departure time t_1^* is equal to passengers who choose the next metro runs ($N_1 = N_2 = 400$, point Q_1) at area A of Figure 5(b) when the biboarding system reaches UE and the total travel demand is less than the threshold N^* ($N = 800$). This relationship is represented by the red dotted line P_1P_2 in Figure 4. Similarly, as shown in point Q_2 located at area B of Figure 5(b), if the total travel demand exceeds the threshold $N = 1400 > N^*$, the number of passengers who choose the metro runs with departure times t_2^* and t_1^* are $N_2 = 600$ and $N_1 = 800$, respectively, when the biboarding system reaches UE. Therefore, there must be some passengers who select the runs with departure time t_2^* that arrive at the station before time t_1^*, and will wait for the checking and boarding after all passengers of the previous run with a departure time t_1^*. This pattern is shown in Figure 2(c).

Meanwhile, the setting of parameters for different fare differentials shown in cases 4–6 listed in Table 2 indicates the sensitivity adjustment of the fare differentials gap $\Delta\tau$ on the strategy of the metro service runs selection. The optimal proportion of two staggered passengers under the adjustment of fare differentials is different from the patterns of no-fare differentials, as shown in Figures 5(d)–5(f). The point of UE in line A_1A_2 causes an offset, as shown in point P_2 in Figures 5(d)–5(f). Correspondingly, the red dotted lines P_1P_2 and P_2P_4 represent the optimal proportion of two staggered passengers when the system achieves UE when travel demand is less or more than threshold N^*, respectively. One setting of two measures ($t_1^* = 8{:}\ 00, t_2^* = 9{:}\ 00;\ \tau_1 = 0, \tau_2 = 10$) are chosen as an example for discussion, as shown in case 5 in Table 2 and Figure 5(e). Affected by fare differentials, the equilibrium proportion of staggered passengers changes when the threshold is $N^* = \Delta t \cdot s = 1200$ compared with patterns of no-fare differentials. The number of staggered passengers satisfies $N_1 = N/2 + \omega s/2\beta\Delta\tau$ and $N_2 = N/2 - \omega s/2\beta\Delta\tau$ when the travel demand is less than the threshold level ($N < 1200$), as shown in zone A in Figure 5(e), expressed as the red dotted line P_1P_2. In Figure 5(e), the black line A_1A_2 intersects at point P_3 (500,700) and is the optimal travel demand and proportion of staggered travelers, which also can be stated as—if the number of passengers N_1 and N_2 are 700 and 500, respectively, the fare incentive is the highest (the boarding pattern is consistent with Figure 2(b)). Moreover, the proportion of staggered passengers still satisfies $N_1 = N/2 + \omega s/2\beta\Delta\tau$ and $N_2 = N/2 - \omega s/2\beta\Delta\tau$ until $N_2 = 600$, as shown in P_2, if the total travel demand begins to exceed the threshold level. Subsequently, if the number of passengers N_2 remains constant and the number of passengers N_1 increases, and the mixed boarding queue is similar to the pattern shown in Figure 2(c). In Figure 5(f), only the fare of the later metro run is more expensive than the previous one ($\Delta\tau > 0$) compared with Figures 5(d)–5(e), and the measure of fare differentials reduces the boarding congestion in the station. However, the boarding congestion becomes worse if the fare differentials satisfy the condition of $\Delta\tau < 0$.

5.3. Numerical Analysis on Boarding Congestion and Incremental Revenue under the Influence of Fares Differentials. We set the schedule of the neighboring metro service runs as $t_1^* = 8{:}\ 00, t_2^* = 9{:}\ 00$ based on the definition of the threshold in travel demand $N^* = 2\Delta t \cdot s$ (in this study we get $N^* = 1200$) to explore the effect of different incentive measures of fare differentials for improving the boarding congestion model, and we analyzed its effect on the incremental revenue of the metro system.

Figure 6 shows the changing trend of the total queuing time of travelers with fare differentials under five patterns of travel demand ($N = 800$, $N = 1000$, $N = 1200$, $N = 1400$, and $N = 1600$). If the value of fare differentials satisfies $\Delta\tau < -30$ or $\Delta\tau > N/20$, all passengers choose the same metro service run and the total boarding time is constant (this pattern is shown in lines P_1P_2 and P_3P_4 in Figure 6 when the total travel demand is $N = 1200$). In contrast, if $-30 < \Delta\tau < N/20$, the changing trend of the queuing time of the passengers with different fare differentials can be divided

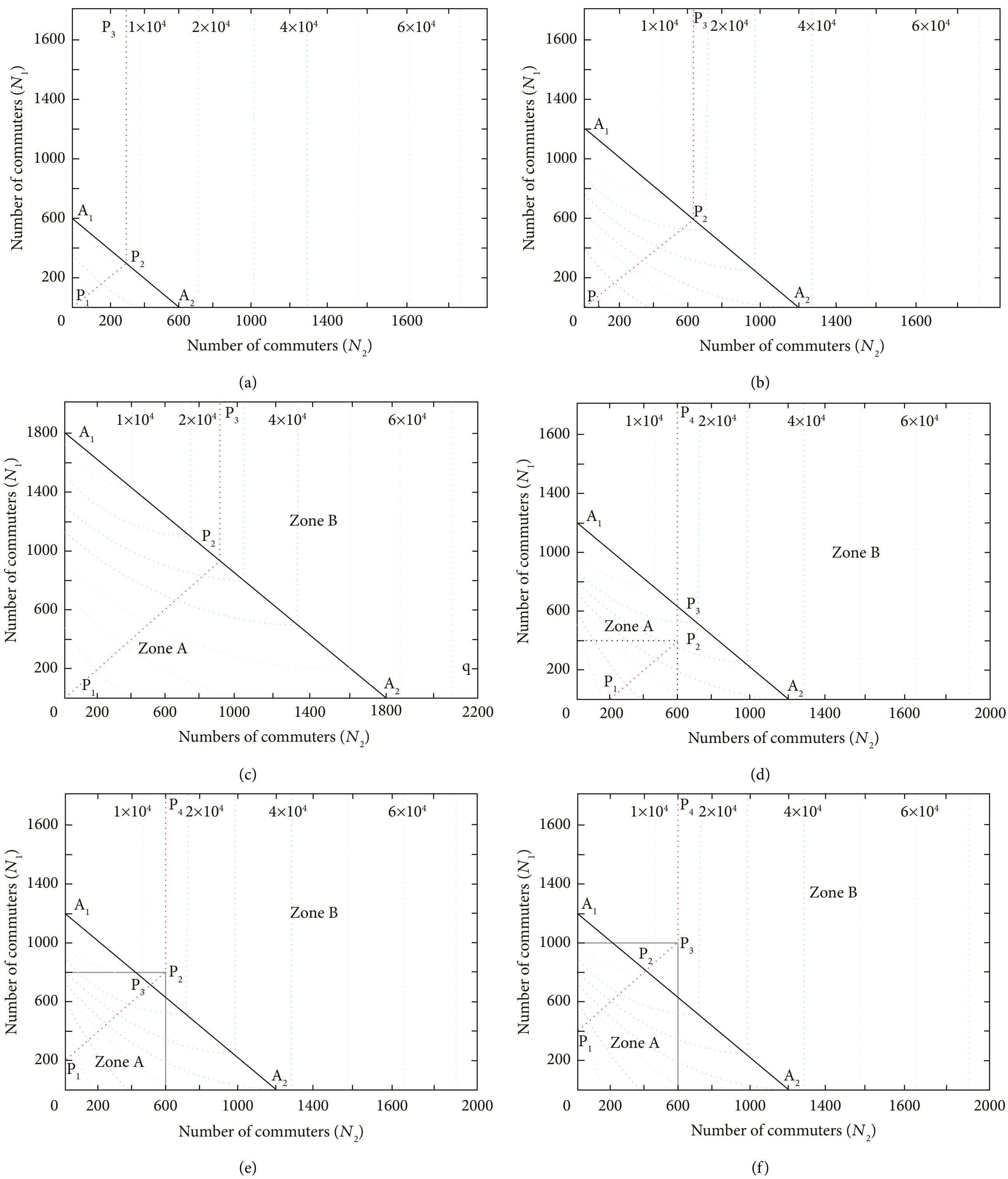

Figure 5: Equilibrium allocation detail between passengers with different desired departure times.

into two patterns, $N > N^*$ (e.g., $N = 1600$) and $N < N^*$ (e.g., $N = 800$). The total boarding queuing time decreases linearly first and then increases exponentially as the fare differentials increase when $N > N^*$ (e.g., $N = 1600$) if the fare differentials of the neighboring metro with service runs satisfy $\Delta\tau \in [-30, 80]$. At $\Delta\tau = 20$, the boarding system achieves the minimum total queuing time, as shown in point A_5 in Figure 6. At this time, the number of travelers who choose the metro runs with departure time t_2^* is $N_2 = \Delta t \cdot s = 600$, and the earliest passenger who arrives at the station at time t_1^* meets the latest passenger taking the previous metro train. Similarly, the minimum queuing time

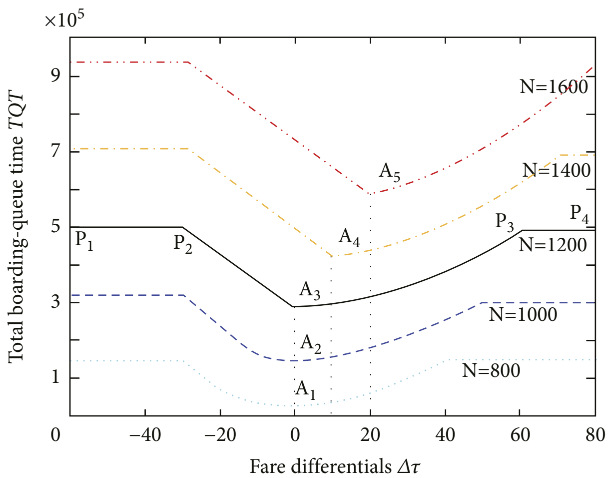

Figure 6: Effect of fare differentials on boarding congestion.

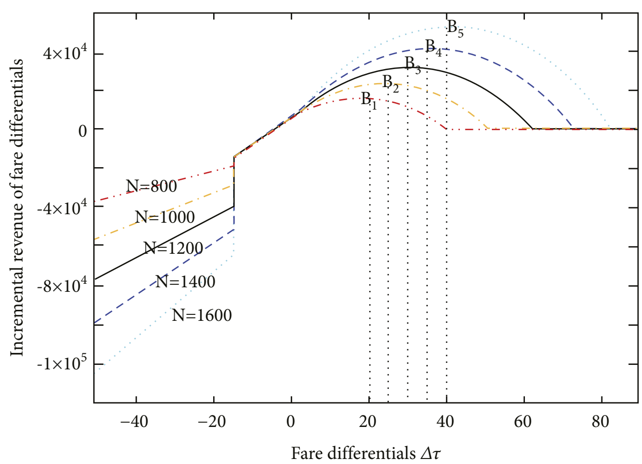

Figure 7: Effect of fare differentials on the total revenue of the metro system.

is obtained at point $\Delta\tau = 0$ when $N < N^*$ (e.g., $N = 800$), as shown in Figure 6 point A_1. Therefore, we concluded that the incentive measures (fare differentials) for smoothing travel demand is only effective when the demand is high. Conversely, unreasonable incentive measures will have a negative effect on the regulation of the metro system when the demand is low.

Figure 7 shows the changing trend of the total revenue of the travelers with fare differentials under five patterns of travel demand (i.e., $N = 800$, $N = 1000$, $N = 1200$, $N = 1400$ and $N = 1600$). First, compared with the no-fare differentials, the fare differentials can significantly increase the total revenue of the neighboring metro service runs when the fare of the later runs is higher than the one before. However, the measure of fare differentials negatively affects the total revenue when the fare of the later runs is less than the one before ($\Delta\tau < 0$). Second, the sensitivity of the travel behavior of the passengers for fare differentials decreases as the total travel demand increases (e.g., the optimal fare differentials are $\Delta\tau = 40$ and $\Delta\tau = 20$ when $N = 1600$ and $N = 800$, respectively). Finally, the optimal fare differentials should be $\Delta\tau = N/40$ (points B_1, B_2, B_3, B_4, and B_5).

6. Conclusion

This study proposes a biboarding bottleneck model that combines the measures of fare incentives for the boarding congestion problem in a metro station. On this basis, we also proposed a novel definition of the threshold to determine when to use the measures of fare differentials to help smooth the travel demand. The theoretical analysis and numerical simulation show that the measures of fare differentials can control crowd gathering in the metro station when the travel demand is high, reducing the boarding congestion of passengers in the station and increasing the total fare revenue. However, the measure of fare differentials gave bad results when the travel demand is less than the threshold and the fare of later runs is less than the previous one.

Although this study tried to consider all the relevant factors when modeling the boarding behavior in the metro station, the model proposed in this study cannot accurately reflect the actual pattern since we used a microeconomic model to perform a macroeconomic evaluation of management measures. In the future, the travel behavior and characteristics of heterogeneous metro passengers will be considered.

Appendix

UE Pattern When Travel Demand is Higher Than Threshold

According to Theorem 1, under the condition of UE in the system, the number of different passengers is $N_1 = N - N_2$, $N_2 = \Delta t \cdot s$ when travel demand is higher than the standard threshold ($N > 2 \cdot \Delta t \cdot s$). This means that the number of travelers with the desired departure time t_2^* is fixed when the neighboring metro schedule gap Δt is fixed in the case of $N > 2 \cdot \Delta t \cdot s$. Conversely, more travelers choose the t_1^*-th train run when $N > 2 \cdot \Delta t \cdot s$, as shown in Figure 2(c). If we know that:

$$c_2\left(t_2^a\right) = \alpha \cdot \left(t_1^* - t_2^a\right) + \beta \cdot \left[t_2^* - t_1^*\right] + \tau_2, \tag{A.1}$$

$$c_2\left(t_2^b\right) = \alpha \cdot \left(t_1^* - t_2^a\right) + \tau_2. \tag{A.2}$$

Then, we can calculate the earliest and latest arrival time of travelers on t_1^*-th train runs as follows:

$$t_1^a = t_1^* - \frac{N - \Delta t \cdot s}{s}, t_1^b = t_1^* - \frac{\beta}{\alpha}\frac{N - \Delta t \cdot s}{s}, \tag{A.3}$$

$$c_1(t) = \beta \cdot \frac{N_1}{s} + \tau_1. \tag{A.4}$$

, and by combining equations (A.1), (A.2), and (A.4), the earliest (latest) arrival time, total queuing time, and travel cost of travelers on t_2^*-th train runs can be expressed as follows:

$$t_2^a = t_2^* - \frac{N_2}{s} - \frac{\beta}{\alpha} \cdot \frac{N_1 - N_2}{s}, t_2^b = t_2^* - \frac{\beta}{\alpha}\frac{N_1}{s}, \tag{A.5}$$

$$T(t) = \frac{\beta}{\alpha - \beta} \cdot \frac{N_1 - N_2}{s}\left(t - t_2^a\right), \quad t \in \left(t_2^a, t_2^b\right), \tag{A.6}$$

$$c_2(t) = \beta \cdot \frac{N_1}{s} + \tau_2. \tag{A.7}$$

Data Availability

All the datasets in this study can be provided by the corresponding author to reviewers upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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