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Research Article

An Evaluation Method of Road Link Functionality Using Individual Trajectory Data and Weighted Network Analysis

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Road links within a city are hierarchical according to their structure and function. Upper-level road links, such as highways and arterials, are designed to maintain higher mobility and traffic flow, while lower-level road links should be more accessible. However, depending on the origin-destination demand pattern (O-D), drivers' route choice, land use, and urban infrastructure, the actual usage pattern of roads could be different from the designed intention. This difference ultimately puts a load on certain road links and causes traffic jams. In order to handle this issue, it is necessary to create an appropriate evaluation method for the functionality of road links in advance. The research suggests an evaluation method to examine the functionalities of the roadways by using real-world mobility data and weighted network analysis. In the study, the roles of links were defined and quantified by three network attributes, in-strength, out-strength, and betweenness centrality. Derived attributes were used to cluster links with similar travel patterns. Furthermore, the concept of link reliability was introduced to measure the reliance of the network on individual links. Those network indices make it possible to evaluate the functioning of roads based on people's travel patterns and to detect critical links that are irreplaceable and difficult to detour. This information can be used to determine the priorities of upcoming improvements and ultimately improve the efficiency of operation and maintenance of the road link networks.

1. Introduction

The hierarchy of the road is defined by their structural and systemic standards; upper-level roads are designed with better mobility, while lower-level roads require better accessibility than upper-level roads. However, road links often malfunction in the real world and do not perform as intended. It can cause traffic jams and inefficiencies in transportation operations. For example, urban highways or arterials are designed to have better mobility than accessibility. Although their design speed, speed limit, and signal system are tailored to fit their purpose, links that make up the arterials could have different uses of their own. Links that serve as origin and destination require higher accessibility, but in practice, O-D demand is high around the arterial, which can lead to traffic problems and inefficiencies such as parking space issues. In addition, links that are mainly used as connectors require higher mobility, but they also cause bottlenecks due to the small number of lanes or low capacity, even though the links are heavily traffic-intensive. In order to solve these problems, an appropriate evaluation method for the functionality of road links is required, and ultimately, it is necessary to change the design and function of the road links.

Meanwhile, the complex system theory, including the network theory and the graph theory, suggests that a system consisting of nodes and links can be analyzed using the framework of the networks. Based on these ideas, an evaluation method for the road link design and functionality of existing road links is presented in this study. This paper deals with road network and link attribute information, which can be analyzed in various ways using the complex system theory. Previous network analyses in transportation research have focused on examining network topology based on road and public transit networks [1–8]. These studies focused on deriving the underlying topological structures and network characteristics of transportation networks. Various network characteristics, such as degree and weight distributions [8], betweenness centrality [6], the robustness of the networks [7], capacity reliability [2], and route redundancy [3] are investigated based on the physical connections of the transportation networks.

Although plenty of studies examine topological characteristics of transportation networks, the detailed characteristics of human mobility that occurred on road networks cannot be observed. Therefore, to identify the roles and functions of the roadways, it is significant to analyze not only the physical characteristics of the road (e.g., number of lanes, width, length, speed limit) but also how travel occurs on the road. Previous studies have used origin and destination (O-D) information [9–11], but traffic flow and network analysis using only O-D data have limitations in identifying detailed functionalities of roads. How much traffic occurs and ends on the road is also an essential characteristic. However, considering that the most fundamental function of the road is connecting regions, it is necessary to analyze individual trajectory data of vehicles and derive the network connectivity of the roads.

In the end, the paper evaluates the performance and roles of the road links by using real-world mobility flow data that contains detailed trajectories of vehicles. We derived key attributes of road links and clustered road links with four different functionalities. In Section 2, link attributes are defined and interpreted based on network theory. In Section 3, links are clustered based on their attributes and analyzed to determine the reliability of the links. Conclusions and potential applications drawn from the results are discussed in Section 4. In this section we finally suggest ways to improve transportation operations with the results.

2. Deriving Network Properties

2.1. Background Theory. There have been attempts to interpret and explain complex social phenomena as simple networks with nodes and their connections. Including human mobility, real-world networks such as social networks [12], internet [13], protein interactions [14, 15], metabolic [16], and brain networks [17] were interpreted through the network science. Since transportation and mobility networks also have similar structures, they can be represented as complex networks.

Attributes of nodes can be defined by a few parameters such as degree, strength, closeness, betweenness, etc. The degree of a node represents the number of edges connected to it. On the other hands, the strength quantifies the sum of weights assigned to a node (Please see ta. 1).

The strength of a node in a graph is defined as the increase in the number of connected components in the graph upon removal of the node. In other words, strength can be defined as the sum of weights of outbound links or the sum of weights of inbound links (Please see Figure 1(b)). In a directed graph, strength (s) of a node has two parts: in-strength and out-strength determined by the direction of the traffic as described in Eq. (1). That is, in-strength and out-strength of a vertex i with an adjacent vertex j represent the sum of weights (w) of in-coming and out-going traffic, respectively:

$$s_i^{\text{in}} = \sum_{j \neq i} w_{ij} s_i^{\text{out}} = \sum_{j \neq i} w_{ij} \tag{1}$$

The betweenness centrality (BC) of each vertex is defined as the number of shortest paths that pass through the vertex [18]. That is, the BC measures the extent to which a vertex is on the path between other vertices, and it is measured as follows:

$$BC_i = \sum_{h \neq i \neq j} \frac{\sigma_i^{hj}}{\sigma^{hj}} \tag{2}$$

where σ^{hj} is the total number of shortest paths from vertex h to vertex j, and σ_i^{hj} is the number of those paths that pass through vertex i when the vertex i is not an endpoint. A vertex with a high BC can have significant influence within the network because it controls the traffic passed between other vertices. A node with a high BC plays an important role in the network, because if the node is removed or fails to work, it will cause a serious failure of the entire network [19]. Also, it means that high BC nodes are those where information flow is more likely to occur. Practically, removing high BC nodes can hinder information or transportation flow within the network as large portion of shortest paths are disconnected.

We use this concept of betweenness centrality as a tool to determine the functionality of road links. In this paper, betweenness centrality is defined differently from the conventional definition as the number of vehicles that pass through each node, since all trips in this study are assumed to be generated along the most cost-efficient and time-saving routes. The purpose of this study is to determine the role of each road link when traffic flow occurs, and the relative importance among road links in the network is not compared. Therefore, it can be a more appropriate indicator to express the node's characteristic as the absolute size of the traffic volume, rather than as a ratio, as in the existing BC definition. In this study, the BC of the road link in the weighted road network ($\widehat{BC_i}$) is newly defined as Eq. (3).

$$\widehat{BC_i} = \sum_{h \neq i \neq j} \sigma_i^{hj} \tag{3}$$

Meanwhile, this study applies a different network topology analysis method where we substitute nodes for 'road links'. The idea of this inverse approach is based on Space Syntax theory by [20], and there have been numerous studies from this original idea. The axial map by Hillier represents the whole length of a street (i.e., axial line) as a node in a network that is linked to the other streets that cross it.

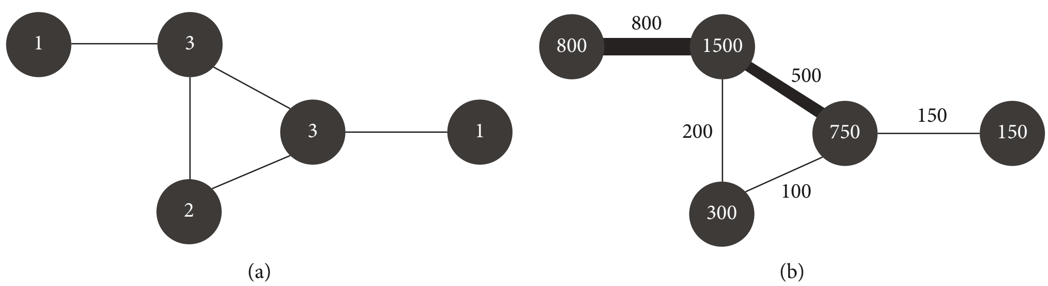

Figure 1: Schematic diagrams of (a) node degree, and (b) node strength in network theory.

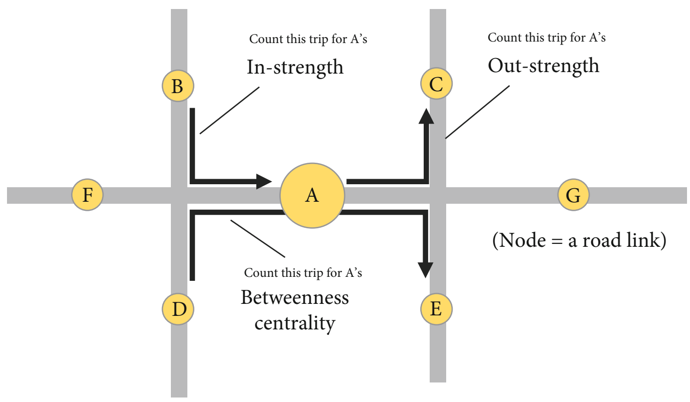

Figure 2: Concepts of a node, in-strength, out-strength, and betweenness centrality in a road network.

This allowed follow-up studies to overcome the limitations of conventional transport models where a street system is described as a network of nodes at street intersections that are linked by segments of streets between these intersections [21]. In most cases of network theory, nodes play an important role while links merely connect the nodes. In road network analysis, however, various attributes are assigned to the links rather than the nodes [22]. Thus, in this paper, network attributes are assigned to the links as if they were nodes of the typical network theory, as shown in Figure 2.

Figure 2 shows how the link attributes are defined in this study. Navigation data does not include all travel route information of all vehicles, and the penetration rate of the probe vehicles is also not known. All vehicles that pass through a node are defined as 'load' in this study, and it is the sum of in-strength, out-strength, and BC. That is, one link has a value for each of the three attributes, and the function of the link is defined by combining these characteristics. In addition, the load value, which is the sum of all three attributes, is used to measure link reliability, which indicates the reliance of the network on individual links.

2.2. Network Properties

2.2.1. Data and Site Description. In this research, road links within Seoul in South Korea are utilized to construct a transportation network. Road network data provided by Intelligent Transport Systems Standard Node Link (ITSSNL), including 101,378 links nationally and 7,695 links within Seoul, are used. These links are 3,242 km in length and 0.4 km long on average. The road network data is updated once a month, and anyone can download it online in the form of GIS data at the website (ITS Standard Node Link in South Korea, https://www.its.go.kr/nodelink/nodelinkRef).

Individual trajectory data from the car navigation system, which Korea Transport Institute (KOTI) collected for many years, are used to examine travel patterns and link characteristics. This data, known as View-T data, can be accessed through the webpage-based transportation database provided by KOTI (View-T Data from Korea Transport Institute, https://viewt.ktdb.go.kr/cong/map/page.do), and it is an SQLite file with an '.db' filename extension. Each route contains sequential information on the links traveled along the route. From the sequential data, we can infer where the traffic demands are generated and which links the drivers chose to travel in between. It also contains the number of times a route data (i.e. path with sequence) was collected and the number of links in the route. The vehicle's trajectory data contains one year travel route data in the Seoul metropolitan area, and the total number of trips is 783,793. The data structure is described in Figure 3.

2.2.2. Link Attributes. Figure 4 shows link attributes of out-strength, in-strength, and BC in the Seoul road network. In-strength and out-strength (Figure 4(c) and Figure 4(d)) exhibit trips generated and concluded in the links. The higher value of strength signifies that the areas adjacent to the link cause more traffic demands. It is worth noting that upper-level road links, such as urban highways shown in Figure 4(a), have relatively low in-strength and out-

TripID	Path Sequence	TPCnt	TLinkCnt
1	100001, 1000037, 1000043, 1000042, 1000039, 1000020, 1000017, 1000049, 1000054, 1000050, 1096663, 0, 1000034	2	13
2	1000003, 1000055, 1000059, 1000116, 1000098, 1000096, 1000084, 1000086, 1000089, 1000173, 1000177, 1000179, 1000339, 1000336, 1000426, 1000438, 1000395, 1000392, 1082007, 1000406, 1000417, 0, 1082041	4	23
3	1000008, 1097050, 1000103, 1000107, 1000112, 1000109, 1000098, 1000096, 1000084, 1000086, 1000089, 1000173, 1000177, 1000179, 1000339, 1000336	2	16
4	1000008, 1097050, 1000103, 1000107, 1000112, 1000109, 1000073, 1091667, 1000288, 1000297, 1000294, 1000302, 1000309, 1000278, 1000274, 1000314, 1000318, 1000326, 1000324, 1000322, 1000311, 1091684, 1000515, 1000513	1	24
…	…	…	…
(example)			

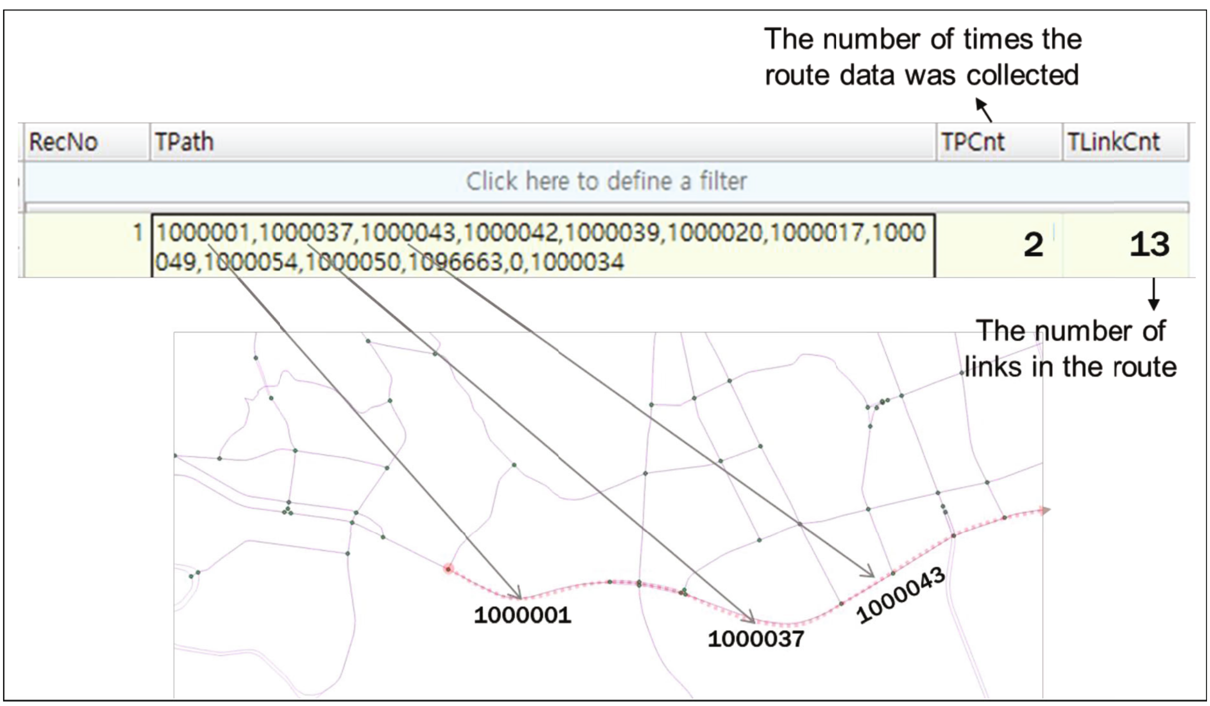

Figure 3: Data scheme of individual trajectory data (View-T data).

strength. In addition, in-strength and out-strength show similar spatial distributions, and local road links around the city center and small towns in the city tend to have higher values of strength than those around arterial road links.

On the other hand, road links with high BC are on major urban highways such as riverside expressways and highways connecting other cities. BC does not show similar spatial distribution to that of in-strength and out-strength. BC distribution shown in Figure 4(e) indicates the number of through traffic in road links of Seoul. The high BC links that appear along the urban highways shown in Figure 4(a) have a lot of through traffic and do not correspond to the origin or destination spot. However, it should be noted that there may be links with high BC values even though they are not the upper-level road links such as urban highways. These links are sections where the mobility function needs to be improved due to high traffic volume. That is, even though it is designed as a lower-level road link, it is a section that can easily cause traffic congestion due to the large amount of traffic volume in the city center area (Please see Figure 4(b)). In Chapter 3, we investigate how to find these malfunctioning sections and improve the vulnerable road links.

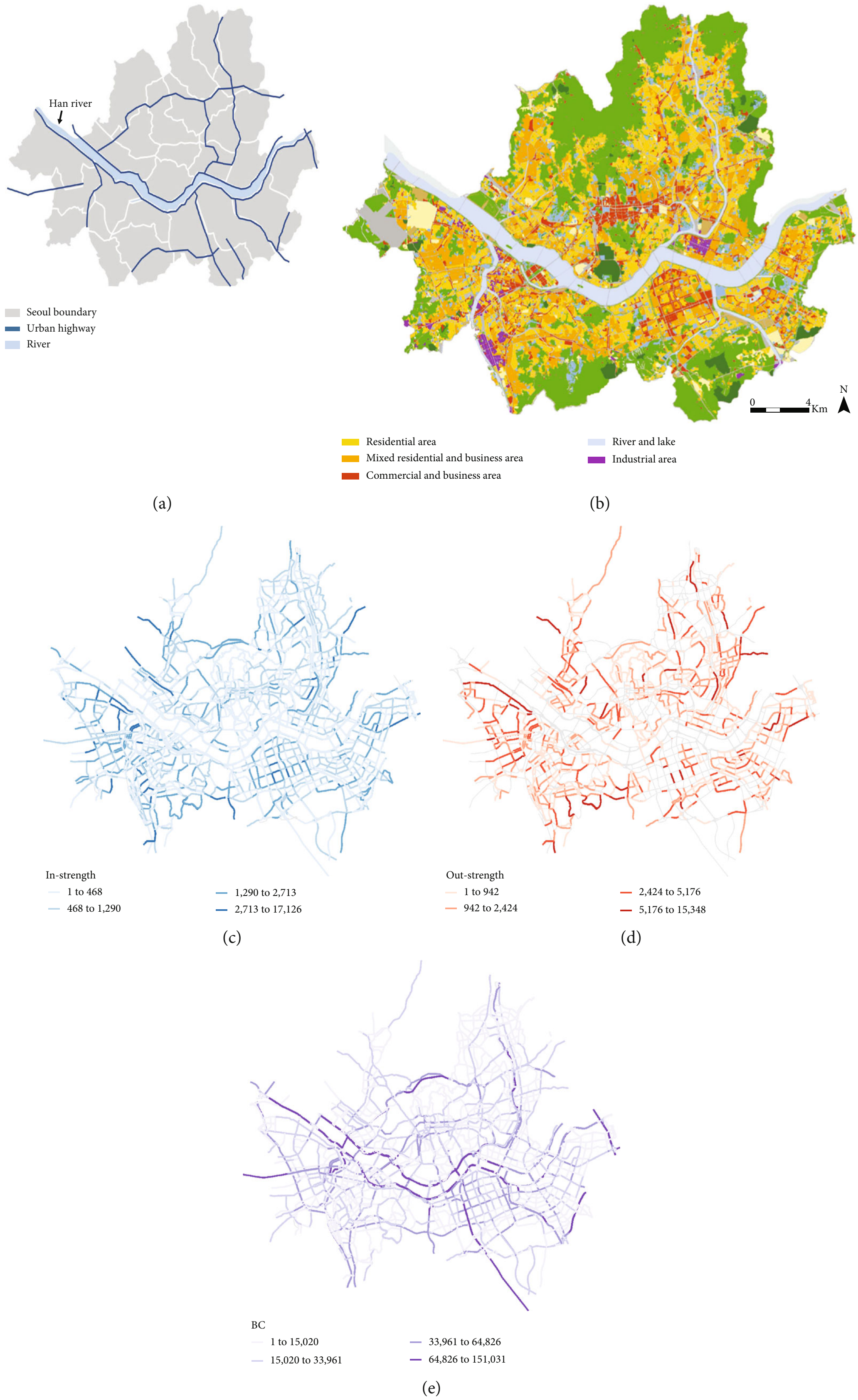

Figure 4: Link attributes in Seoul road network; (a) urban highways in Seoul, (b) land use map of Seoul (The Seoul Research Data Service, https://data.si.re.kr/node/45), (c) in-strength, (d) out-strength, (e) betweenness centrality of road links in Seoul.

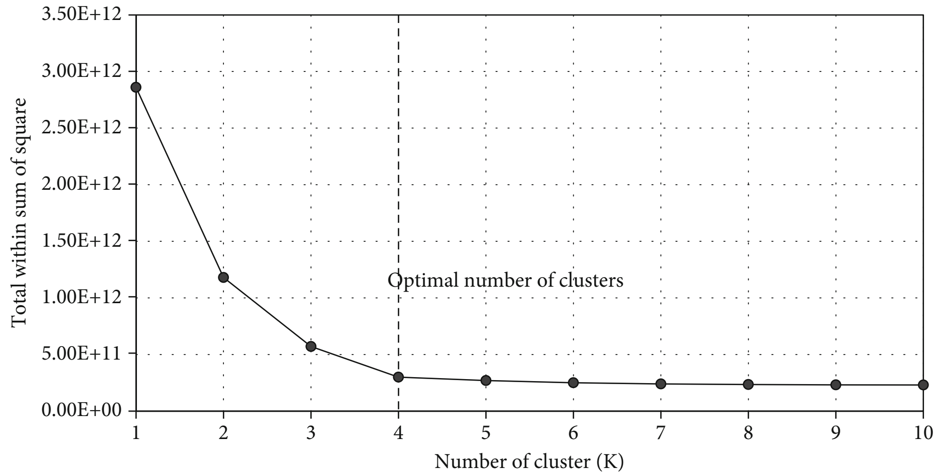

Figure 5: The optimal number of clusters with PAM clustering method.

Table 1: Results of link clustering.

Group	In-strength	BC	Out-strength	# of link	Total length
1	121	6238	130	3250	1,035 km
2	1224	19514	1434	1507	1,183 km
3	208	24262	68	2069	615 km
4	203	61720	43	869	409 km

3. Link Functionality

3.1. Link Clustering. To define the functionality of the links, the partitioning around medoids (PAM) clustering method was used to divide them into different functionality groups using three previously defined attributes of the links: out-strength, in-strength, and BC. In this study, PAM was used instead of the K-means clustering method to address its limitation: its inability to integrated categorical values and its susceptibility to outliers. The PAM produces more robust results than K-means does as PAM minimizes the distances between the non-medoid objects and the medoid [23]. The optimal number of clusters was defined using the Elbow method, as shown in Figure 5. It shows that the total within the sum of squares (WSS) improves marginally after four clusters; hence the optimal number of clusters is determined as four.

Table 1 shows the results of clustering road links according to their attributes. These results confirm that values of in-strength, out-strength, and BC vary depending on the usage and the hierarchy of the link. The four groups derived as a result of clustering have a representative value of a point located at the center of each cluster (Please see Figure 6). These four functionality groups are defined primarily by their BC. and secondarily by in-strength and out-strength. The correlation coefficient between in-strength and out-strength of all links within the network resulted in a positive value of 0.39, but BC showed little correlations with in-strength and out-strength: 0.08 and -0.01, respectively. Here, we need to note the different link functionalities between in-strength and out-strength although there are explicit distinctions between strengths and BC. The difference between in-strength and out-strength usually depends on time of day and land use. That is, if the strength of a road link in a residential area is calculated over time, the out-strength is likely to be high in the morning peak, the in-strength is likely to be high in the evening peak, and vice versa. However, rather than the differences in node characteristics that appear differently over time, we are interested in dealing with the characteristics of roads and their role in the network that appear over a long period of time. Therefore, this study estimates the node characteristics over the entire period of data collection.

Group 1, which is shown a grey circle symbol in Figure 6, possibly represents minor local roads that have low in-strength, out-strength, and BC. Low values in all attributes imply that these links are minor and local roads with small traffic demands. Road links corresponding to Group 1 accounted for the most, with 3,250 out of 7,695 road links, accounting for 42% of the total. The left part of Figure 7 shows the geographic visualization of clusters in the Seoul road network. In the case of minor local roads corresponding to Group 1, they are located in outlying areas outside the city center or are distributed in locations away from major residential/commercial areas.

Group 2 is represented by the blue triangle mark in Figure 6. This group includes the links that have very high in-strength and out-strength but they have around average BC value. These links are frequently used as origin and destinations and are a major cause of traffic demands. These links are likely adjacent to residential/commercial areas with high accessibility. As shown in Figure 7, most of the road links in residential/commercial areas in Seoul are classified as Group 2, and the length of the road links belonging to Group 2 is the longest among the four groups. That is, most areas requiring high accessibility are clustered as Group 2.

Group 3 is represented by the orange square mark in Figure 6. It has links that have low in-strength, low out-strength, and relatively high BC. Group 3 represents lower-

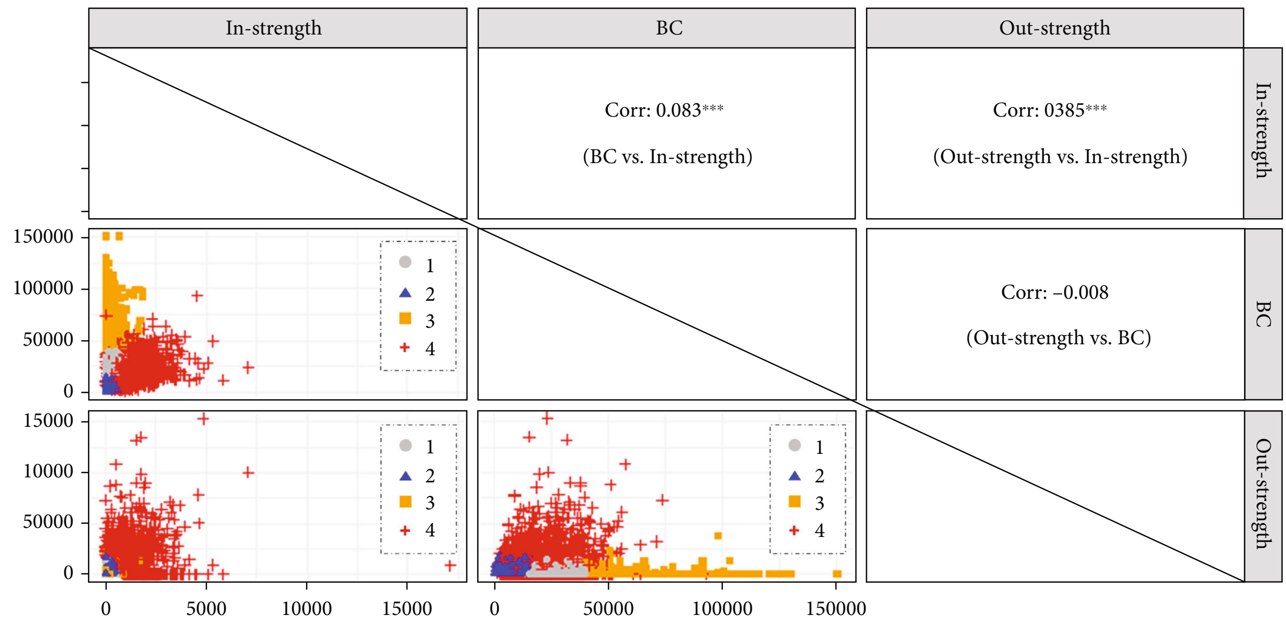

Figure 6: Clustered links and correlation among the three attributes.

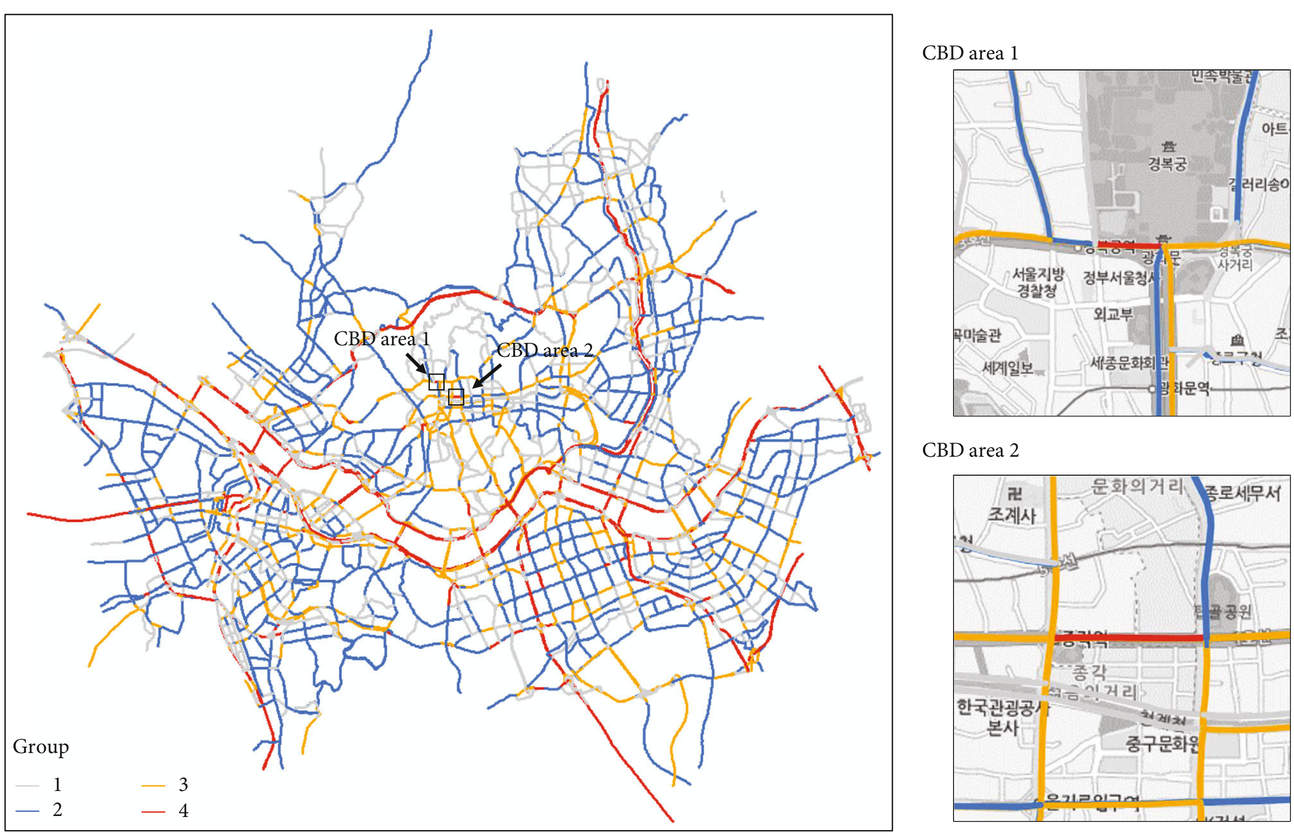

Figure 7: Spatially visualized result of clustering in Seoul City and examples of link malfunction.

level highways that are not used as much as major expressways. Its relatively high BC requires these links to have higher mobility than their accessibility. Most links that are included in this group are normal arterial roads near the central business district (CBD) area, but the number of links and road length is much smaller and shorter than other road links belonging to Group 1 and Group 2.

Group 4, denoted by the red plus symbol, are major highways that are not used as either origin or destination and have very high BC. It can be expected that Group 4

FIGURE 8: lr_i of the first quartile load links in Seoul.

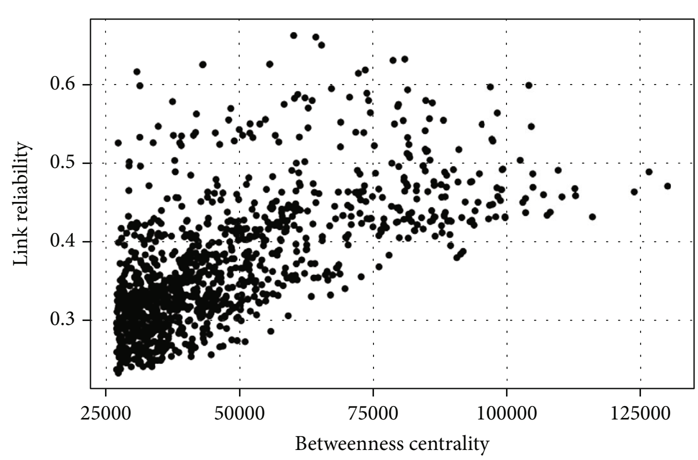

FIGURE 9: Relationship between lr_i and BC.

consists of most highways and major arterials with high through traffic volume. Its extremely high BC implies that these are major highways with multiple lanes and high mobility, but traffic congestion can be caused by these links if some links of this group do not work properly. It is worth noting that there are links corresponding to Group 4 within the CBD area, as shown in the enlarged view of Figure 7. This implies that these links exceed the design limit and are likely to cause a bottleneck due to links' malfunction. To this end, if usage patterns of the roads can be analyzed and visualized as this study suggests, it allows us to detect any malfunctions in which the use of links exceeds or is far below the design limits.

3.2. Link Reliability. This section examines the reliability of the links. Reliability in the road transportation system is defined as susceptibility to incidents that can result in significant reductions in road network serviceability [24].

Previous research focused on predicting the change in performance level due to the changed road geometry caused by a disabled link [4, 25, 26]. The focus of this research is to define link reliability based on the usage pattern of the road using real-world travel route data. Link reliability can be derived by the following formula:

$$lr_i = \frac{\widehat{BC_i}}{T_{od_i}} \quad (4)$$

where lr_i is reliability of link *i*, od_i means all O-D pairs that pass link *i*. BC_i is the number of all vehicle trajectories that pass link *i*, corresponding to the BC of the link. T_{od_i} is the number of all trajectories generated from identical O-D pairs that pass through the link *i*. Thus, higher link reliability suggests that there is a lack of alternative routes to this link and implies that the trip network relies on the performance of the link.

Eq. (3) is applied to links in the Seoul roadway network, and the link reliability map is shown in Figure 8. Links with 25% highest loads were chosen because alternative routes are less significant when there is not enough traffic flow through a link, and then the load is too small. Urban highways like riverside expressways showed high link reliability, and arterials and local roads showed relatively low link reliability. This suggests that arterials have many alternative routes, but highways and expressways lack alternative routes, and people choose not to detour.

It is possible to detect links that lack alternative routes and have funneled traffic flow by comprehensively looking at the network characteristics of links and link reliability. Figure 9 is a scatter plot with the *x*-axis as link reliability and the *y*-axis as BC. These two variables have a high correlation coefficient of 0.66. This result tells us that links with high through traffic inevitably have high link reliability. We are most interested in the links with high values in both BC and link reliability. When both numbers are high, the through traffic is dense in the link without an alternative route. Therefore, these links with high values in both BC and link reliability should be prioritized for any upgrades or investments of road links.

4. Conclusion

This research suggested an evaluation method to examine the functionalities of the current road network links. Weighted network analysis using travel route data was performed for the evaluation, and the methodologies and the measures were developed to indicate the functionalities and characteristics of road links.

In this research, three network attributes were derived from the individual trajectory data, and the link characteristics were determined using the attributes. The derived attributes are in-strength, out-strength, and betweenness centrality (BC), representing different characteristics of the links. In this study, the in-strength and the out-strength measure the amount of inbound and outbound traffic, respectively, and the betweenness centrality is defined as the number of vehicles that pass through certain links. The concept of each attribute was slightly modified to apply individual data-based measurements to the road link network.

Also, the roles of links within a network were quantified in this research. The derived attributes were used to cluster links with similar usage patterns and formed four functionality groups and this result was verified by the PAM clustering analysis. Each group showed that they were roughly defined by the hierarchy level, but some links had usage patterns that contrast their design purpose. Although in-strength and out-strength appear differently depending on the time, detailed comparison analysis by time was not covered in this study due to the limitations of the data used. The analysis of road link functionality over time can be addressed in subsequent studies with another dataset.

Meanwhile, the concept of road link reliability was also newly introduced to measure the reliance of the network on each link. With results drawn from this research, it is possible to evaluate the functionality and importance of each link in a road network. This information is able to be used to determine the priorities of upcoming improvements and ultimately improve the efficiency of operation and maintenance of the road network. More specifically, the proposed link functionalities and reliability could be a criterion for prioritizing the physical improvement (e.g., extension, geometrical structure improvement) of congested roads. Moreover, the indices derived in the study can also be directly used when planning the underground road network, which has recently received significant attention. In addition, in the case of South Korea, the area of the Seoul Metropolitan area is gradually expanding, and the traffic congestion problem caused by megaregions is also a big issue worldwide. The result that the link reliability is high on the road connecting Seoul and subregions outside Seoul suggests that the road infrastructure that connects the urban cores and the suburban areas is insufficient.

Data Availability

In this research, road links within Seoul in South Korea are utilized to construct a transportation network. Road network data provided by Intelligent Transport Systems Standard Node Link (ITSSNL), including 101,378 links nationally and 7,695 links within Seoul, are used. These links are 3,242 km in length and 0.4 km long on average. The road network data is updated once a month, and anyone can download it online in the form of GIS data at the website (https://www.its.go.kr/nodelink/nodelinkRef). Individual trajectory data from the car navigation system, which Korea Transport Institute (KOTI) collected for many years, are used to examine travel patterns and link characteristics. This data, known as View-T data, can be accessed through the webpage-based transportation database provided by KOTI (https://viewt.ktdb.go.kr/cong/map/page.do), and it is an SQLite file with an '.db' filename extension.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] W. Cao, X. Feng, J. Jia, and H. Zhang, "Characterizing the Structure of the Railway Network in China: A Complex Weighted Network Approach," *Journal of Advanced Transportation*, vol. 2019, Article ID 3928260, 10 pages, 2019.

[2] A. Chen, H. Yang, H. K. Lo, and W. H. Tang, "A capacity related reliability for transportation networks," *Journal of Advanced Transportation*, vol. 33, no. 2, 200 pages, 1999.

[3] W. Jing, X. Xu, and Y. Pu, "Route Redundancy-Based Network Topology Measure of Metro Networks," *Journal of Advanced Transportation*, vol. 2019, Article ID 4576961, 12 pages, 2019.

[4] V. Latora and M. Marchiori, "Is the Boston subway a small-world network?," *Physica A: Statistical Mechanics and its Applications*, vol. 314, no. 1-4, pp. 109–113, 2002.

[5] S. Lämmer, B. Gehlsen, and D. Helbing, "Scaling laws in the spatial structure of urban road networks," *Physica A: Statistical Mechanics and its Applications*, vol. 363, no. 1, pp. 89–95, 2006.

[6] W. Liu, X. Li, T. Liu, and B. Liu, "Approximating Betweenness Centrality to Identify Key Nodes in a Weighted Urban Complex Transportation Network," *Journal of Advanced Transportation*, vol. 2019, Article ID 9024745, 8 pages, 2019.

[7] G. Ren, "Robustness Analysis of Air Route Network Based on Topology Potential and Relative Entropy Methods," *Journal of Advanced Transportation*, vol. 2021, Article ID 5527423, 11 pages, 2021.

[8] M. Zanin, X. Sun, and S. Wandelt, "Studying the Topology of Transportation Systems through Complex Networks: Handle with Care," *Journal of Advanced Transportation*, vol. 2018, Article ID 3156137, 17 pages, 2018.

[9] A. Chen, P. Kasikitwiwat, and C. Yang, "Alternate capacity reliability measures for transportation networks," *Journal of Advanced Transportation*, vol. 47, no. 1, 104 pages, 2013.

[10] P. M. Murray-Tuite and H. S. Mahmassani, "Methodology for determining vulnerable links in a transportation network," *Transportation Research Record*, vol. 1882, no. 1, pp. 88–96, 2004.

[11] Y. Yang, J. Liu, P. Shang, X. Xu, and X. Chen, "Dynamic Origin-Destination Matrix Estimation Based on Urban Rail Transit AFC Data: Deep Optimization Framework with Forward Passing and Backpropagation Techniques," *Journal of Advanced Transportation*, vol. 2020, Article ID 8846715, 16 pages, 2020.

[12] R. Albert and A. L. Barabási, "Statistical mechanics of complex networks," *Reviews of Modern Physics*, vol. 74, no. 1, pp. 47–97, 2002.

[13] R. Albert, H. Jeong, and A.-L. Barabási, "Diameter of the World-Wide Web," *Nature*, vol. 401, no. 6749, pp. 130-131, 1999.

[14] R. Albert, "Scale-free networks in cell biology," *Journal of Cell Science*, vol. 118, no. 21, pp. 4947–4957, 2005.

[15] H. Jeong, S. P. Mason, A.-L. Barabási, and Z. N. Oltvai, "Lethality and centrality in protein networks," *Nature*, vol. 411, no. 6833, pp. 41-42, 2001.

[16] H. Jeong, B. Tombor, R. Albert, Z. N. Oltvai, and A.-L. Barabási, "The large-scale organization of metabolic networks," *Nature*, vol. 407, no. 6804, pp. 651–654, 2000.

[17] M. Rubinov and O. Sporns, "Complex network measures of brain connectivity: uses and interpretations," *NeuroImage*, vol. 52, no. 3, pp. 1059–1069, 2010.

[18] M. Barthelemy, "Betweenness centrality in large complex networks," *The European Physical Journal B*, vol. 38, no. 2, pp. 163–168, 2004.

[19] M. Bellingeri, D. Bevacqua, F. Scotognella, R. Alfieri, and D. Cassi, "A comparative analysis of link removal strategies in real complex weighted networks," *Scientific Reports*, vol. 10, no. 1, pp. 1–15, 2020.

[20] B. Hillier, A. Leaman, P. Stansall, and M. Bedford, "Space syntax," *Environment and Planning B: Planning and design*, vol. 3, no. 2, pp. 147–185, 1976.

[21] C. Yamu, A. Van Nes, and C. Garau, "Bill Hillier's legacy: space syntax—a synopsis of basic concepts, measures, and empirical application," *Sustainability*, vol. 13, no. 6, p. 3394, 2021.

[22] X. Zhang, E. Miller-Hooks, and K. Denny, "Assessing the role of network topology in transportation network resilience," *Journal of Transport Geography*, vol. 46, pp. 35–45, 2015.

[23] A. P. Reynolds, G. Richards, B. de la Iglesia, and V. J. Rayward-Smith, "Clustering rules: a comparison of partitioning and hierarchical clustering algorithms," *Journal of Mathematical Modelling and Algorithms*, vol. 5, no. 4, pp. 475–504, 2006.

[24] A. J. Nicholson and Z. P. Du, "Improving network reliability: a framework," *Proceedings of 17th Australian Road Research Board Conference*, vol. 17, no. 6, pp. 1–17, 1994.

[25] L. Dall'Asta, A. Barrat, M. Barthélemy, and A. Vespignani, "Vulnerability of weighted networks," *Journal of Statistical Mechanics: Theory and Experiment*, vol. 2006, no. 4, article P04006, 2006.

[26] V. L. Knoop, M. Snelder, H. J. van Zuylen, and S. P. Hoogendoorn, "Link-level vulnerability indicators for real-world networks," *Transportation Research Part A: Policy and Practice*, vol. 46, no. 5, pp. 843–854, 2012.