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Research Article

Resilient Transportation Network Design under Uncertain Link Capacity Using a Trilevel Optimization Model

Mohammad Rahdar,[1] **Lizhi Wang**,[2] **Jing Dong**,[3] **and Guiping Hu**[2,4]

[1]*Department of Engineering & Physics, St. Ambrose University, Davenport, IA 52803, USA*
[2]*Department of Industrial & Manufacturing Systems Engineering, Iowa State University, Ames, IA 50011, USA*
[3]*Department of Civil, Construction and Environmental Engineering, Iowa State University, Ames, IA 50011, USA*
[4]*Department of Sustainability, Golisano Institute for Sustainability, Rochester Institute of Technology, Rochester, NY, USA*

Correspondence should be addressed to Mohammad Rahdar; rahdar@sau.edu

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This study addresses uncertainty in a transportation network by proposing a trilevel optimization model, which improves resiliency against uncertain disruptions. The goal is to minimize the total travel time by designing a resilient transportation network under uncertain disruptions and deterministic origin-destination demands. The trilevel optimization model has three levels. The lower level determines the network flow, and the middle level assesses the network's resiliency by identifying the worst-case scenario disruptions that could lead to maximal travel time. The upper-level takes the system perspective to expand the existing transportation network to enhance resiliency. We also propose a formulation for the network flow problem to significantly reduce the number of variables and constraints. Two algorithms have been developed to solve the trilevel model. The results of solving the highway network in Iowa show that the trilevel optimization model improves the total travel time by an average of 41%.

1. Introduction

The transportation sector is among the critical infrastructure sectors in the United States because many other critical infrastructures such as emergency services, food and agriculture, healthcare and public health, and manufacturing depend on transportation networks to function properly [1]. The highway and motor carrier is a subsector of transportation systems and supports the mobility of people, goods, and services, which are essential for daily activities and emergency responses. However, road networks entail risk from natural and human-made events such as hurricanes, tsunamis, earthquakes, bridge collapse, and terrorist attacks. These hazards could result in significant efficiency reductions, direct damage to the physical infrastructure, or even negative impacts nationally and globally on the economy and social systems. Therefore, there is a need to improve reliability on components of interconnecting networks to guarantee safety and complete delivery in the presence of any unpredictable failures. Resilient network design ensures that the network functionality is at an acceptable level of service in the presence of all probabilistic failures.

The general mathematical model of the network design problem is a bilevel programming problem. The decisions related to investment improvements are made at the upper-level problem by traffic authorities in system's interest as a whole. On the other hand, the individual travelers decide where and how to travel at the lower-level problem. Some related pieces of research that proposed a bilevel programming model for network design are in [2–7]. Also, Karoonsoontawong and Waller [8] proposed a linear bilevel programming model for the network design problem. They developed a genetic algorithm, a simulated annealing, and a random search to solve the problem. Lin et al. [9] formulated a bilevel linear program for the network design problem and proposed a heuristic algorithm based on Dantzig–Wolf decomposition to solve it; the solution of the algorithm could be potentially local optimum. Hamid and Mehdi Sepehri [10] presented a single-level mixed-integer linear programming formulation for the bilevel network design problem. They also generated two valid inequalities to improve the efficiency of computation time. Khooban et al. [11] proposed a bilevel programming model for the network design problem. The upper-level problem of their model decides to expand capacity and determines signal settings at intersections; the lower-level was the user equilibrium assignment problem.

Network design problems can be classified based on the origin-destination demand, the decision-making level, and the design variables. These problems are usually divided into two modeling cases in terms of demand: deterministic and uncertain demands. When the demand is deterministic, it is assumed that the demand between each origin-destination pair is given. However when it is uncertain, the origin-destination trip matrices are taken as random variables. In addition, the network design problem involves making optimal decisions at three levels: strategic, tactical, and operational [12, 13]. The strategic level includes long-term decisions such as building new links or expanding existing routes. Tactical decisions can determine the orientation of a one-way road or the allocation of lanes. Finally, the operational level decisions are short-term, involving traffic flow control and scheduling problems [12]. Furthermore, network design problems can be classified into three different types based on the decision variables. The first class is the discrete network design problem which deals with adding a new lane or building a new road [14–16]. The second class is the continuous network design problem which makes decisions on capacity enhancement, signal setting at intersections, and road pricing [6, 17–19]. The third class involves both discrete and continuous design variables [7, 20, 21]. This paper studies the strategic decisions of link capacity expansions by adding new lanes to critical links when demand is deterministic. We propose a trilevel optimization model to identify the vulnerable links and decide on expansions.

Identifying critical links of a transportation network is a primary issue in the vulnerability analysis because the failure of these links significantly impacts the whole network. Some researchers [22, 23] assumed that the failure is a link or a group of links being fully disrupted and examined the effect of iteratively removing road links to calculate the network performance. As stated in [24], it ignores the potential combined effects of multiple links. For example, having two bridges on a river, one of them could be congested, and the traffic would shift to the other one. Thus, neither bridge may be considered a pivotal link individually, but the simultaneous disruption of both would make the system vulnerable. In addition, this approach can be computationally intensive [25]. Another approach to identify the critical links is preselecting potentially vulnerable links by calculating the stochastic traffic assignment. Knoop et al. [26] compared ten different criteria for selecting potentially vulnerable links in a network and concluded that none of these strategies accurately predicted the list of vulnerable links. Also, they stated that their combination did not accurately represent the full consequences of blocking a link. Therefore, these strategies are not good enough to properly identify the critical links in a road network.

This paper aims to propose a model to expand a transportation network given a limited budget, such that the effect of a disruption on the entire network is minimized. To build a resilient transportation network that faces uncertain disruptions, we work on the uncertainty of link capacities when origin-destination demands are constant and propose a new approach to perform network design. For example, crashes, stalled vehicles, or weather conditions can reduce the capacity of the road while the number of travelers and the distance for each link are fixed. Some of these travelers may choose to wait in a congested link, while others may select another route to reach the destination. Since reducing the link capacities degrades the performance of a transportation network and can delay or stop the movement, quantifying such impacts is critical to network design improvement.

The contributions of this paper are as follows: first, we developed a trilevel optimization model for the resilient network design problem. Second, we proposed a new formulation for the network flow problem (the third level of the trilevel model) to significantly reduce the number of variables and constraints. Third, we designed a heuristic algorithm for solving the trilevel optimization model to efficiently search for a worst-case scenario in the scenario space. We also conducted an experiment using the proposed model on a large transportation network for the state of Iowa.

The remainder of the paper is organized as follows: in Section 2, the problem formulation is discussed. Section 3 is devoted to algorithm development. Section 4 presents the case study and experimental results. Finally, the conclusion with a summary is reported in Section 5.

2. Model Formulation

2.1. Problem Statement. We address a traffic network design problem with uncertainty over available link capacity. The goal is to make optimal strategies to strengthen the network against future disruptions under limited resources. The model aims to identify critical links and increase their capacity under the given budget. The network design problem is a two-stage decision-making problem, and its objective function minimizes the travel time subject to the network construction budget and travel demand satisfaction. The link capacity expansion decisions are made at the first stage before the realization of disruption uncertainty. We focus on expanding the capacity of vulnerable links, not addition of a new link. At the second stage, flow variables are decided after observing the disruption. In the remainder of the section, we first give the deterministic version of the network design problem in Section 2.2 and then introduce the trilevel optimization model in Section 2.3.

2.2. The Network Flow Problem. Consider a directed network in which vehicles can travel on road segments. The travel time on a link is $t(f)$ when the flow rate of vehicles on the link is f. A popular link performance function that represents the relationship between resistance and volume of traffic has been developed by the Bureau of Public Roads (BPR). The BPR function has been used extensively to estimate the link travel time in a road network. We adopted the BPR function (1) as the travel time of a link in our model.

$$t(f) = \tau\left(1 + \alpha\left(\frac{f}{p}\right)^{\beta}\right), \tag{1}$$

where $t(f)$ is the average travel time for a vehicle on the link, τ denotes the free-flow travel time on the link, f is the link traffic flow, and p is the link's capacity. The coefficients α and β determine the shape of the function. Parameters α and β are classically 0.15 and 4, respectively. The BPR function is nonlinear, and it is almost flat for flows lower than the link capacity. However, the travel time increases significantly for higher flows.

There are generally two ways to formulate the traffic assignment problem: system optimum (SO) and user equilibrium (UE). The system optimum formulation minimizes the total travel time of all travelers. However, in the user equilibrium formulation, each driver intends to minimize its travel time independently. SO traffic assignment has been used in disruption scenarios in the literature. For example, Murray-Tuite [27] compared SO and UE on network resiliency under disruptions and found comparable performance between the two models. Angelelli et al. [28] developed an SO model with user constraints for traffic assignments. They bounded the ratio of the normal length of any path to the normal length of the shortest path for the same origin-destination pair. However, these bounds on the level of unfairness are in terms of normal lengths of paths and independent of the flow. The experienced unfairness in terms of the travel times on the restricted path set may be much higher than the specified level of normal unfairness, which is an a priori fixed quantity. He et al. [29] studied the robustness of multimodal freight transport networks against disruptions using both UE and SO models. Prashker and Bekhor [30] discussed the relation between the UE and the SO. They reported that the UE and SO solutions are close to each other in the low congested part of the graph. In this paper, we formulate the objective function as in SO to find the minimum total travel time to address important questions at the system level, such as identifying the most vulnerable links and enhancing network capacity to achieve an optimum social equilibrium.

The formulation of the network flow problem is represented in (2a)–(2f), and Table 1 includes the notation used in formulating the network flow model (2a)–(2f). The model's objective is to minimize the total travel time. The average individual travel time for a vehicle on link (i, j) is $t_{i,j}(f_{i,j})$ and can be calculated through equation (1). Total flow on link (i, j) is $f_{i,j}$, and the capacity of each link is $2000p_{i,j}$. It is assumed that the maximal flow rate for each lane is 2000 vehicles per hour under ideal conditions. The unit of link capacity $p_{i,j}$ is the number of lanes for link (i, j), so by multiplying 2000 by $p_{i,j}$, the unit is converted to the number of vehicles per hour. Constraint (2b) states that the total input flows to node s when their origin is r and destination is s equal to the travel demand from node r to node s. Constraint (2c) expresses that the total output flows from node r minus the total input flows to node r when their destination is s is equal to the travel demand from origin r to destination s. Constraint (2d) makes sure that the total input flows to node i are equal to the total output flows from node i when their origin is r and their destination is s. Constraint (2e) calculates the flow of each link, and constraint (2f) defines the domains of decision variables.

$$\min \sum_{(i,j)\in\mathscr{L}} t_{i,j}\left(f_{i,j}\right) f_{i,j}, \tag{2a}$$

$$\text{s.t.} \sum_{(j,s)\in\mathscr{L}} g_{j,s,r,s} = d_{r,s}, \quad \forall s \in \mathscr{S}, r \in \mathscr{R}(s), \tag{2b}$$

$$\sum_{(r,j)\in\mathscr{L}} g_{r,j,r,s} - \sum_{(j,r)\in\mathscr{L}} g_{j,r,r,s} = d_{r,s}, \quad \forall s \in \mathscr{S}, r \in \mathscr{R}(s), \tag{2c}$$

$$\sum_{(i,j)\in\mathscr{L}} g_{i,j,r,s} - \sum_{(j,i)\in\mathscr{L}} g_{j,i,r,s} = 0, \quad \forall s \in \mathscr{S},\ r \in \mathscr{R}(s),\ i \in \mathscr{N}\backslash\{s, \mathscr{R}(s)\}, \tag{2d}$$

$$f_{i,j} = \sum_{(r,s)\in\mathscr{D}} g_{i,j,r,s}, \quad \forall (i,j) \in \mathscr{L}, \tag{2e}$$

$$g_{i,j,r,s} \geq 0, \quad \forall (i,j) \in \mathscr{L}, (r,s) \in \mathscr{D}. \tag{2f}$$

One potential drawback of this formulation is that it generates a model with a huge number of variables and constraints. In this paper, we present an equivalent and more efficient model with fewer variables and constraints, which is presented in (4a)–(4f). In the new formulation, the definition of variable $g_{i,j,r,s}$ is changed to $g_{i,j,s}$, which is the total flow of link (i, j) from all origins $\mathscr{R}(s)$ to destination s.

$$g_{i,j,s} = \sum_{r\in\mathscr{R}(s)} g_{i,j,r,s}. \tag{3}$$

Therefore, the reformulation of the network flow problem is as follows in (4a)–(4f). Constraint (4b) states that the total input flows with destination s to node s should be equal to the total travel demand of node s. Similarly, constraint (4c) expresses that the total output flows of node r with destination s minus the total input flows to node r with destination s is equal to the travel demand from origin r to destination s. Constraint (4d) states that the output flows of node i with destination s equal to the input flows to node i with destination s in which there is no demand from i to s. Constraint (4e) calculates the flow of each link, and constraint (4f) defines the domains of decision variables.

$$\min \sum_{(i,j)\in\mathscr{L}} t_{i,j}\left(f_{i,j}\right) f_{i,j}, \tag{4a}$$

$$\text{s.t.} \sum_{(j,s)\in\mathscr{L}} g_{j,s,s} = \sum_{r\in\mathscr{R}(s)} d_{r,s}, \quad \forall s \in \mathscr{S}, \tag{4b}$$

$$\sum_{(r,j)\in\mathscr{L}} g_{r,j,s} - \sum_{(j,r)\in\mathscr{L}} g_{j,r,s} = d_{r,s}, \quad \forall s \in \mathscr{S},\ r \in \mathscr{R}(s), \tag{4c}$$

Table 1: Notation used in the network flow model.

Sets	
$\mathcal{N}$	Set of nodes
$\mathcal{L}$	Set of links
$\mathcal{S}$	Set of destination nodes
$\mathcal{R}(s)$	Set of origin nodes with destination $s \in \mathcal{S}$
$\mathcal{D}$	Set of origin-destination pairs for travel demands
Decision variables	
$g_{i,j,r,s}$	Flow of link $(i,j) \in \mathcal{L}$ from origin $r \in \mathcal{R}(s)$ to destination $s \in \mathcal{S}$
$f_{i,j}$	Total flow on link $(i,j) \in \mathcal{L}$
Parameters	
$\tau_{i,j}$	Free-flow travel time of link $(i,j) \in \mathcal{L}$
$p_{i,j}$	Number of lanes on link $(i,j) \in \mathcal{L}$
$d_{r,s}$	Travel demand from origin $r \in \mathcal{R}(s)$ to destination $s \in \mathcal{S}$

$$\sum_{(i,j)\in\mathcal{L}} g_{i,j,s} - \sum_{(j,i)\in\mathcal{L}} g_{j,i,s} = 0, \quad \forall s \in \mathcal{S},\ i \in \mathcal{N}\backslash\{s, \mathcal{R}(s)\}, \tag{4d}$$

$$f_{i,j} = \sum_{s\in\mathcal{S}} g_{i,j,s}, \quad \forall (i,j) \in \mathcal{L}, \tag{4e}$$

$$g_{i,j,s} \geq 0, \forall (i,j) \in \mathcal{L},\ s \in \mathcal{S}. \tag{4f}$$

Since the BPR function is nonlinear, we linearized it through defining a new variable $b_{i,j,k}$, which is the flow of link (i,j) using block k. We define a few blocks for each link. The variable $b_{i,j,1}$ is the flow through link (i,j) using block 1 capacity, that is, the flow within 100% capacity. Variable $b_{i,j,2}$ is the flow through link (i,j) using block 2 capacity, which is beyond 100% but within 200% capacity. Travel time is higher in this block; as k gets higher, the flow gets into a more costly capacity. The new objective function is $\sum_{(i,j)\in\mathcal{L}}\sum_k \gamma_k \tau_{i,j} b_{i,j,k}$, and new coefficients are $\gamma_k \tau_{i,j}$ which are travel times through link (i,j) using block k capacity. Parameter γ_k is a coefficient to adjust the cost of traveling in higher blocks. To determine the values of parameters γ_k, we defined breakpoints for the traffic flow as 100%, 200%, ...of the link capacity on BPR function and estimated travel time of one vehicle going through the link at these points. We determined γ_k parameters by minimizing the difference between the travel times obtained from the BPR function and our objective function at breakpoints. Values for parameter γ_k for $k = 1, 2, 3$ are 1, 5, and 32.8, respectively. Figure 1 shows the concept of block capacities for a 1-lane link when the free-flow travel time is 1 hour. Breakpoints are at 2000, 4000, ... vehicles per hour. For traffic flow under 2000 veh/hr, the average travel time for each vehicle is one hour, and it grows for higher flow rates.

For this linearization, we also need to modify constraint (4e) and add another constraint to the model. Constraint (5) is the modification of (4e) and relates two variables $b_{i,j,k}$ and $g_{i,j,s}$. It states that the total flow on link (i,j) in all blocks equals the total flow on this link to all destinations. New constraint (6) ensures that the flow on each block cannot exceed the link capacity.

$$\sum_k b_{i,j,k} = \sum_{s\in\mathcal{S}} g_{i,j,s}, \quad \forall (i,j) \in \mathcal{L}, \tag{5}$$

$$b_{i,j,k} \leq 2000 p_{i,j}, \quad \forall k, (i,j) \in \mathcal{L}. \tag{6}$$

2.3. Trilevel Optimization Model. Network design problems are usually formulated as a bilevel model. The decision variables of the upper-level are the link capacity expansions, and network flow is determined in the lower-level of the model. This type of model assumes that there is no disruption in the network. Relaxing the simplifying assumption of no disruption results in a two-stage decision-making problem, in which uncertain disruptions occur after making expansion decisions. Disruptions are uncertain variables in the problem, but we can estimate their lower and upper bounds. We propose a two-stage trilevel optimization model to make decisions on expansion, identify the disruptions, and determine the network flow. We assume that, after making decisions on the first stage (expansion decisions), uncertain disruptions will be observable, and thus, the second stage (network flow problem) becomes a deterministic model. To make expansion decisions in the first stage, we take a pessimistic view of uncertainty and anticipate the worst-case scenario for the second stage. By this formulation, we have three levels in the model, in which the first stage decisions are made in the upper-level, the worst-case scenario is identified in the middle-level given the first stage decision, and the second stage decisions are made in the lower-level under the worst-case scenario and given the first stage decision. The upper-level foresees the network flow under the worst-case scenario to determine the optimal network expansions. The trilevel optimization model is developed using the notation defined in Table 2.

To abstract the formulation of the trilevel model, we aggregate the decision variables of three levels into x, y, and z, respectively, and aggregate objective function coefficients into c. Using the notation of aggregated decision variables and parameters, we formulate the trilevel optimization model as follows.

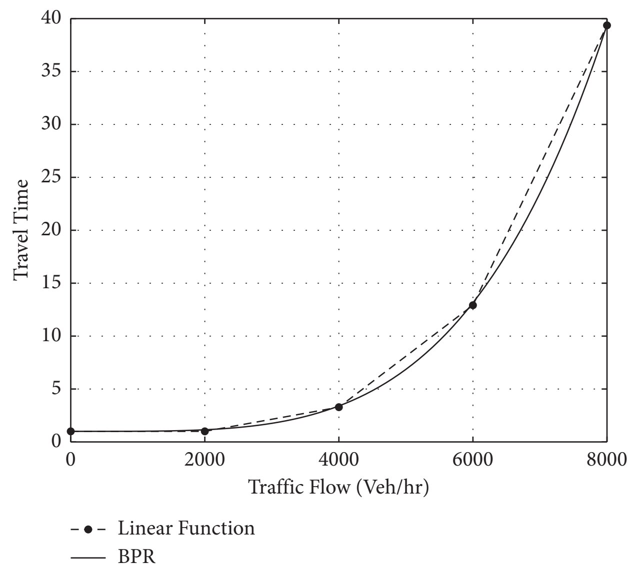


Figure 1: The Bureau of Public Roads (BPR) function and the linearized function, relating the travel time to the flow.

Table 2: Notation used in the trilevel model.

Sets	
$\mathcal{N}$	Set of nodes
$\mathcal{L}$	Set of links
$\mathcal{S}$	Set of destination nodes
$\mathcal{R}(s)$	Set of origin nodes with destination $s \in \mathcal{S}$
Decision variables	
Upper level: x	
$\pi_{i,j}$	Capacity expansion of link $(i, j) \in \mathcal{L}$
Middle level: y	
$q_{i,j}$	Capacity reduction of link $(i, j) \in \mathcal{L}$ because of disruption
Lower level: z	
$g_{i,j,s}$	Flow of link $(i, j) \in \mathcal{L}$ to destination $s \in \mathcal{S}$
$b_{i,j,k}$	Flow of link $(i, j) \in \mathcal{L}$ using block k capacity
Parameters	
γ_k	Coefficients of travel time for each block k
$\tau_{i,j}$	Free-flow travel time of link $(i, j) \in \mathcal{L}$
$a_{i,j}$	Capacity expansion cost of link $(i, j) \in \mathcal{L}$
$l^x_{i,j}$	Lower bound of capacity expansion for link $(i, j) \in \mathcal{L}$
$u^x_{i,j}$	Upper bound of capacity expansion for link $(i, j) \in \mathcal{L}$
B	Network expansion budget
Q	Network disruption upper bound
$l^y_{i,j}$	Lower bound of disruption for link $(i, j) \in \mathcal{L}$
$u^y_{i,j}$	Upper bound of disruption for link $(i, j) \in \mathcal{L}$
$d_{r,s}$	Travel demand from origin $r \in \mathcal{R}(s)$ to destination $s \in \mathcal{S}$
$p_{i,j}$	Initial number of lanes on link $(i, j) \in \mathcal{L}$

$$\min_{x\in\mathcal{X}}\left\{\max_{y\in\mathcal{Y}(x)}\left\{\min_{z\in\mathcal{Z}(x,y)} c^{\top}z\right\}\right\}. \tag{7}$$

Here, the lower level, $\min_{z\in\mathcal{Z}(x,y)} c^{\top}z$, solves a deterministic problem to minimize the total travel cost given the expansion decision, x, made at the upper level and the worst-case scenario of disruptions, y, identified by the middle level. Its objective function is $\sum_{(i,j)\in\mathcal{L}}\sum_k \gamma_k \tau_{i,j} b_{i,j,k}$, and the feasible set $\mathcal{Z}(x, y)$ is defined in (8). The compact form of (8) is represented in (9), where A_3, B_3, and C_3 are the constraints' coefficients in the feasible set of the lower-level problem. Parameter A_3 is the coefficient set for the upper-level variables ($\pi_{i,j}$); parameter B_3 implies the set of coefficients for the middle-level variables ($q_{i,j}$); and parameter C_3 indicates the coefficient set for the lower-level variables ($g_{i,j,s}$ and $b_{i,j,k}$).

$$\mathscr{Z}(x,y)=\left\{\begin{array}{c} z\colon \sum_{(j,s)\in\mathscr{L}} g_{j,s,s}=\sum_{r\in\mathscr{R}(s)} d_{r,s}\,\forall s\in\mathscr{S} \\ \sum_{(r,j)\in\mathscr{L}} g_{r,j,s}-\sum_{(j,r)\in\mathscr{L}} g_{j,r,s}=d_{r,s}\quad \forall s\in\mathscr{S},\, r\in\mathscr{R}(s) \\ \sum_{(i,j)\in\mathscr{L}} g_{i,j,s}-\sum_{(j,i)\in\mathscr{L}} g_{j,i,s}=0\quad \forall s\in\mathscr{S},\, i\in\mathscr{N}\backslash\{s,\mathscr{R}(s)\} \\ \sum_{k} b_{i,j,k}=\sum_{s\in\mathscr{S}} g_{i,j,s}\,\forall (i,j)\in\mathscr{L} \\ b_{i,j,k}\le 2000\left(p_{i,j}+\pi_{i,j}-q_{i,j}\right)\forall k,\,(i,j)\in\mathscr{L} \\ g_{i,j,s}\ge 0\quad \forall (i,j)\in\mathscr{L},\, s\in\mathscr{S} \end{array}\right\}, \tag{8}$$

$$A_3x+B_3y+C_3z\le b_3. \tag{9}$$

The middle level observes the expansion decision, x, made at the upper level and solves a bilevel optimization model, $\max_{y\in\mathscr{Y}(x)}\left\{\min_{z\in\mathscr{Z}(x,y)} c^\top z\right\}$, to identify the worst-case scenario of disruptions, anticipating the response of the lower level. The feasible set $\mathscr{Y}(x)$ is defined in (10), and its compact form is represented in (11), where B_2 is the coefficients of the middle-level variables, $(q_{i,j})$. The first constraint is bound on the total disruption in the network, and the second and third constraints set the lower and upper bounds for the disruption of each link (i,j) in the network.

$$\mathscr{Y}(x)=\left\{\begin{array}{c} y\colon \sum_{(i,j)\in\mathscr{L}} q_{i,j}\le Q \\ q_{i,j}\le u^{\mathrm{y}}_{i,j}\,\forall (i,j)\in\mathscr{L} \\ q_{i,j}\ge l^{\mathrm{y}}_{i,j}\,\forall (i,j)\in\mathscr{L} \end{array}\right\}, \tag{10}$$

$$B_2y\le b_2. \tag{11}$$

The upper level solves the trilevel optimization model (7), which minimizes the travel time, anticipating the response from the middle and lower levels. The feasible set $\mathscr{X}$ is defined in (12). Its compact form is represented in (13), where A_1 is the set coefficient for the upper-level variables $(\pi_{i,j})$. The first constraint is the budget limit on the total expansion in the network, and the second and third constraints set the lower and upper bounds for the expansion of each link (i,j) in the network.

$$\mathscr{X}=\left\{\begin{array}{c} x\colon \sum_{(i,j)\in\mathscr{L}} a_{i,j}\pi_{i,j}\le B \\ \pi_{i,j}\le u^{\mathrm{x}}_{i,j}\,\forall (i,j)\in\mathscr{L} \\ \pi_{i,j}\ge l^{\mathrm{x}}_{i,j}\,\forall (i,j)\in\mathscr{L} \\ \pi_{i,j}\in\mathbb{Z}\,\forall (i,j)\in\mathscr{L} \end{array}\right\}, \tag{12}$$

$$A_1x\le b_1. \tag{13}$$

3. Algorithm Design

The proposed trilevel model has three levels. The upper level of the trilevel model determines the capacity expansion. The middle level identifies the worst-case scenario for disruptions given the expansion decisions. The lower level determines traffic flow given the expansions and disruptions. This kind of problem is challenging to solve considering the three-level structure. However, several approaches are available in the literature to solve it, including heuristic and metaheuristic algorithms and decomposition methods. In this paper, we decompose the trilevel model into a master problem and a subproblem. To develop the master problem, first, we formulate the dual of the lower level problem. The compact form of the lower level problem is shown in (14a)–(14c) when expansions $(\widehat{x})$ and disruptions $(\widehat{y})$ are given.

$$\min c^\top z, \tag{14a}$$

$$\text{s.t. } C_3z\le b_3-A_3\widehat{x}-B_3\widehat{y}, \tag{14b}$$

$$z\ge 0. \tag{14c}$$

The dual of the lower-level problem is shown in (15a)–(15c), where λ is the dual variable of constraint (14b).

$$\max -\left(b_3-A_3\widehat{x}-B_3\widehat{y}\right)^\top\lambda, \tag{15a}$$

$$\text{s.t. } -C_3^\top\lambda\le c, \tag{15b}$$

$$\lambda\ge 0. \tag{15c}$$

The master problem $M(\widehat{y},\widehat{\lambda})$ is formulated in (16a)–(16c). It consists of the upper level problem constraints (16b) and a series of optimality cuts (16c) which are added to the master problem in each iteration. λ_w is the dual variable of constraint (14b) in iteration w. It has two decision variables: variable x which is the expansion decision and variable t_{M} which is the total travel time. Parameters $\widehat{\lambda}_w$ and $\widehat{y}_w$ are, respectively, the dual variable value of the lower level problem and the disruption amount, which are both estimated by solving the subproblem in iteration w.

$$\min_{t_{\mathrm{M}},x} t_{\mathrm{M}}, \tag{16a}$$

$$\text{s.t. } A_1x\le b_1, \tag{16b}$$

$$t_{\mathrm{M}}\ge -\left(b_3-A_3x-B_3\widehat{y}_w\right)^\top\widehat{\lambda}_w\forall w. \tag{16c}$$

The subproblem $S(\widehat{x})$ is represented in (17), and it is a bilevel optimization problem (the middle and lower levels) to assess the resiliency of the network and determine the flow. It has two variables: variable y, which is the disruption in the network, and z, which is the network flow. The parameter $\widehat{x}$ is the expansion decision made in the master problem.

$$\max_{y\in\mathscr{Y}(\widehat{x})}\left\{\min_{z\in\mathscr{Z}(\widehat{x},y)} c^{\top}z\right\}. \quad (17)$$

The idea is to iteratively solve the master problem $M(\widehat{y},\widehat{\lambda})$ and the subproblem $S(\widehat{x})$ to determine capacity expansions under the worst case of disruption. In the first iteration, we assume that there is no expansion for links and solve the subproblem (17) to assess the resiliency of the network and determine the network flow. Then, with the given network flow, we solve the master problem (16a)–(16c) to expand critical links. By having newly added link capacities, we solve the subproblem again to find the network flow under the worst-case disruption. The resulting bilevel model (17) either confirms the solution of the upper-level decision, so the algorithm terminates or yields a worst-case scenario that will be added as a new cut to the master problem in the next iteration.

Subproblem (17) is a bilevel programming problem that also needs an algorithm to be solved. Therefore, we also designed an iterative heuristic algorithm to solve the subproblem. It identifies the most critical links in the network and quantifies the negative impacts of capacity reductions of individual links. Uncertain disruptive road events such as traffic accidents, obstacles on the road, or adverse weather conditions reduce the capacity of a given link and increase the total travel time in the network. The algorithm of solving subproblem (17) reduces the capacity of links in the network up to the total disruption limit (Q) to find a worst-case scenario. It has two steps in each iteration: first, we find a worst-case scenario for disruption; second, we solve the lower-level problem to find the network flow under this scenario. With the network flow from step 2, we try to find a scenario in step 1 to disrupt the network more severely. It continues until there is no improvement in the objective function of the bilevel problem.

To start the bilevel problem algorithm with a good solution, we seek a considerable traffic disruption as an initial solution; then, the algorithm improves the initial solution. Therefore, in the first iteration of the algorithm of solving the subproblem, we try to disconnect (r,s) pairs with high demands as many as network disruption upper bound Q allows to obtain an initial disruption. To remove the minimum number of edges to disconnect two vertices in a graph, we can solve the max-flow min-cut problem. The max-flow min-cut theorem states that the maximum flow from an origin to a destination equals the sum of the edge weights that if they are removed, it will disconnect destination s from origin r. By using the Ford–Fulkerson algorithm [31] and given the link capacities, we can find the maximum flow between origin r and destination s and hence the residual network. Every edge of a residual graph has a residual capacity, which is basically the current capacity. They are equal to the original capacity of the edge minus current flow. When residual capacity is 0 for a link, it means there is no edge between two vertices. To disconnect (r,s), we need a cut that requires the origin r and the destination s to be in different subsets of the network. This cut includes edges going from the origin's side to the destination's side. To find all edges of the minimum cut, we need to obtain the residual graph by running the Ford–Fulkerson algorithm. Then, the set of nodes that are reachable from origin r in the residual network need to be found. All edges from a reachable node to a nonreachable node are minimum cut edges. By disrupting the minimum cut edges, origin r and destination s are disconnected. The algorithm to find the initial traffic disruption starts with disconnecting the (r,s) pairs with the highest demands and continues to separate them until it reaches the network disruption upper bound Q. After having this initial disruption, the lower-level problem is solved to find the network flow.

In the subsequent iterations with the new flow, we improve the initial solution by cutting more critical links and undoing the cuts of unimportant links. For this purpose, we employ two strategies to cut new lanes and undo the cut of the already disrupted links.

(i) Heuristic strategy 1: cut and undo the cuts based on flow of each link. It means that we cut five new lanes of the links with the largest flow and undo the cut for five already disrupted links with the smallest flow. The new disruption is $\widehat{y}_1$.

(ii) Heuristic strategy 2: cut and undo the cuts based on the dual variables for capacity constraints in the lower-level problem. Similar to strategy 1, cut five new lanes with the largest dual variable and undo the cut for five already disrupted links with the smallest dual variable. Return $\widehat{y}_2$ as the new disruption under this scenario.

Then, we solve the lower level problem with the given $\widehat{y}_1$ and $\widehat{y}_2$ and select the one with the largest total travel time as the disruption scenario of the current iteration. This procedure continues until there is no improvement in the objective function of problem (17). The steps of the algorithm are described in Algorithm 1. We solved the bilevel problem for different disruption limits and various values for the number of cuts and observed that five-lane cutting and undoing cuts outperform other parameter settings. If this number is too small, it could get stuck in a local optimum, and if it is too large, it forces the algorithm to undo the cuts of some critical links and cut some unimportant links.

We also introduce a greedy algorithm to expand the congested links greedily. In this method, we solve the bilevel programming problem (17) for different disruption limits to assess network resiliency. Then, the most congested links are expanded by one lane until the expansion budget allows. After expansion, the resiliency of the network is assessed again by solving the bilevel programming problem. The steps of the greedy method are represented in Algorithm 2.

4. Case Study: Transportation Network in Iowa

To demonstrate the model, we used the highway network in Iowa and simplified the network to contain 85 nodes and 332 links, as represented in Figure 2. The distance between two

(1) Inputs: $\mathcal{X}, \mathcal{Y}(x)$, and $\mathcal{Z}(x, y)$, $\quad \forall x \in \mathcal{X},\ y \in \mathcal{Y}$
(2) Initialize $\widehat{x} = 0$, $\zeta^L = -\infty$, $\zeta^U = +\infty$
(3) **while** $\zeta^L < \zeta^U$ **do**
Solve the subproblem (17) with the given $\widehat{x}$.
(4) Initialize $\widehat{\zeta} = -\infty$.
(5) Disconnect (r, s) pairs with high demands to estimate $\widehat{y}$.
(6) Solve the lower-level problem (14a)–(14c) with the given $\widehat{y}$ to get $(\widehat{z}, \widehat{\lambda})$.
(7) **while** there is an improvement **do**
(8) Update $\widehat{\zeta} \leftarrow \max\{\widehat{\zeta}, c^{\top}\widehat{z}\backslash\}$.
(9) Use strategy 1 to get $\widehat{y}_1$.
(10) Solve the lower level problem (14a)–(14c) with the given $\widehat{y}_1$ to get $(\widehat{z}_1, \widehat{\lambda}_1)$.
(11) Use strategy 2 to get $\widehat{y}_2$.
(12) Solve the lower level problem (14a)–(14c) with the given $\widehat{y}_2$ to get $(\widehat{z}_2, \widehat{\lambda}_2)$.
(13) **if** $c^{\top}\widehat{z}_1 \geq c^{\top}\widehat{z}_2$ **then**
(14) $\widehat{y} \leftarrow \widehat{y}_1$, $\widehat{z} \leftarrow \widehat{z}_1$, and $\widehat{\lambda} \leftarrow \widehat{\lambda}_1$
(15) **else**
(16) $\widehat{y} \leftarrow \widehat{y}_2$, $\widehat{z} \leftarrow \widehat{z}_2$, and $\widehat{\lambda} \leftarrow \widehat{\lambda}_2$
(17) **end if**
(18) **end while**
(19) Return $\widehat{y}$, $\widehat{z}$, and $\widehat{\lambda}$ as the solution to the subproblem (17).
(20) Update $\zeta^U \leftarrow \min\{\zeta^U, \widehat{\zeta}\}$.
Solve the master problem (16a)–(16c) with the given $\widehat{y}$ and $\widehat{\lambda}$.
(21) Add cut (16c) to the master problem (16a)–(16c) and solve it.
(22) Let $\widehat{x}$ and t_M denote an optimal solution to the master problem (16a)–(16c).
(23) Update $\zeta^L \leftarrow t_M$.
(24) **end while**
(25) Solve the subproblem (17) with the given $\widehat{x}$ to find $\widehat{y}$ and $\widehat{z}$.
(26) Return $x^* = \widehat{x}, y^* = \widehat{y}, z^* = \widehat{z}$, and $\zeta^* = c^{\top}\widehat{z}$

Algorithm 1: The trilevel model algorithm.

(1) Inputs: $\mathcal{Y}(x)$ and $\mathcal{Z}(x, y), \forall x \in \mathcal{X},\ y \in \mathcal{Y}$
(2) Solve the subproblem (17) with $\widehat{x} = 0$ as explained in Algorithm 1.
(3) Determine $\widehat{x}$ by expanding the most congested links by one lane until the expansion budget allows.
(4) Solve the subproblem (17) with the given $\widehat{x}$ as explained in Algorithm 1, to get $\widehat{y}$ and $\widehat{z}$.
(5) Return $x^* = \widehat{x}, y^* = \widehat{y}, z^* = \widehat{z}$, and $\zeta^* = c^{\top}\widehat{z}$

Algorithm 2: The greedy method.

locations is defined as the Euclidian distance in miles, and we used the gravity model to estimate the demand of each origin-destination pair. To apply the gravity model, we need friction and socioeconomic factors and the production/attraction matrix. The friction factor is calculated from Gamma function, $F_{ij} = a t_{ij}^{b} e^{c t_{ij}}$, where t_{ij} is the travel time between origin i and destination j, and it is estimated between all (r, s) pairs by using the shortest path algorithm. In our model, the trips between (r, s) pairs are home-based work (HBW); thus, the parameters of the Gamma function from [32] are $a = 28507$, $b = -0.02$, and $c = -0.123$. We also assumed that socioeconomic factor $K_{ij} = 1$ for all links. To estimate the production/attraction of each node, we used the annual average daily traffic (AADT) of the Iowa transportation network [33]. We applied a peak hour factor of 12% and assumed a 50%-50% split of two-directional flow. Therefore, we used 6% as the inflow and 6% as the outflow of the node. The production or the attraction of this node is 6% of AADT. If multiple links connect to one node, we calculate the total production/attraction by summing over all links. The formula of the gravity model for computing the number of trips between node i and node j is as follows:

$$d_{ij} = P_i \left(\frac{A_j F_{ij} K_{ij}}{\sum_k A_k F_{ik} K_{ik}} \right), \tag{18}$$

where d_{ij} is the number of trips (demands) from node i to node j, P_i is the number of trip productions in node i, A_j is the number of trip attractions in node k, F_{ij} is the friction factor relating the spatial separation between node i and node j, and K_{ij} is the socioeconomic factor between nodes i and j.

There are 85 nodes, 332 links, and 1215 origin-destination (r, s) pairs. We assume that the maximal flow rate for each lane is 2000 vehicles per hour under ideal conditions. Some links have one lane, and others have two lanes. The number of lanes for each link is based on the Iowa road network, and we considered real link capacities in the numerical example. The average speed is assumed to be 60

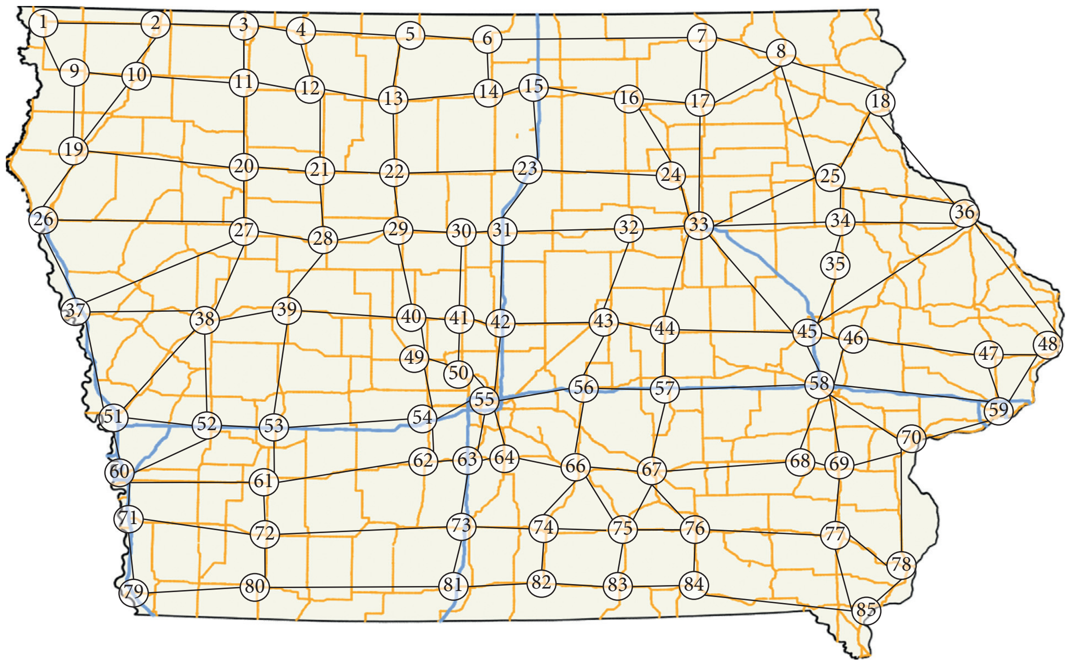


Figure 2: Simplified Iowa transportation network (the background map is a work of the U.S. federal government in the public domain: https://commons.wikimedia.org/wiki/File:Iowa_highways.svg).

miles per hour, so the travel time is one minute per mile for the first capacity block. The travel times for the next blocks are multiplied by the coefficients γ_k. In many studies, the capacity reduction is 100% of the link capacity, i.e., the link can be removed entirely from the network. However, complete closure does not accurately reflect the actual link capacity as a consequence of minor events. Also, the 85-node road network is an abstract representation of the statewide road network. A link in the abstract network represents not only the major road connecting the two nodes but also other local roads. Therefore, in this paper, we assume that each link keeps a capacity of 0.4 lanes after the maximal disruption. For example, the maximal disruption to 1-lane and 2-line links are, respectively, 0.6 and 1.6 lanes. We also assume that the capacity can be reduced by 0.1 lanes. For example, for a 1-lane link, disruption can be $\{0, 0.1, \ldots, 0.6\}$; if the disruption for a 1-lane link is 0.3 lanes, it means that the link capacity is reduced from 1 to 0.7 lanes, which is a reduction from 2000 to 1400 vehicles per hour. The expansion upper bound is one lane for all links. The expansion cost is estimated to be 1.5 million dollars per lane per mile, and the expansion budget is assumed to be 400 million dollars. The upper bound of total disruption in the network is different from 5 to 60 lanes. The demand of (r, s) pairs is obtained by applying the gravity model.

We conducted an experiment to test and compare the performances of the trilevel optimization model and an intuitive expansion strategy. First, we assumed there was no expansion and disruption in the system and estimated the total travel time in the network by solving the lower-level problem (14a)–(14c). Second, we solved the bilevel programming problem (middle and lower levels), assuming there was no expansion, but probabilistic disruptions could happen through the network to assess the network resiliency. Third, we improved the result of the bilevel model by applying intuitive expansions through Algorithm 2. Fourth, we ran the trilevel optimization model (Algorithm 1) to confront the worst-case scenario disruptions in the most resilient manner.

When there is no expansion and disruption, the total travel time is 7.38×10^4 hours, and the network flow is shown in Figure 3. We use different color codes and line styles to indicate the volume per capacity ratio on each link. The orange and red lines indicate highly congested links.

To show how our model can identify vulnerable links better than common criteria in the literature review, we assess resiliency using the bilevel programming model and two other criteria when there is no expansion. Congestion indicators can help find critical links for the performance of the road. There are several indicators in the literature to assess congestion based on the balance between supply and demand and trip time evaluation [34]. Among these congestion indicators, we adopt volume to capacity ratio (V/C) [35] as the first criterion because it is the most widely used measure to assess congestion of a link based on the impairment of the traffic flow capacity. The indicator V/C is a primary performance measure to estimate the level of congestion on a roadway by comparing roadway demand (volume) with roadway supply (capacity). Also, we consider the congestion index (CI) as the second criterion to find critical links to the network performance. The CI is the ratio between a trip time under congested conditions and free-flow conditions. A higher value for each criterion means that the predicted impact of blocking that link is more significant. After identifying critical links with either of the criteria, we disrupt the links with a higher value of each measure. Thus,

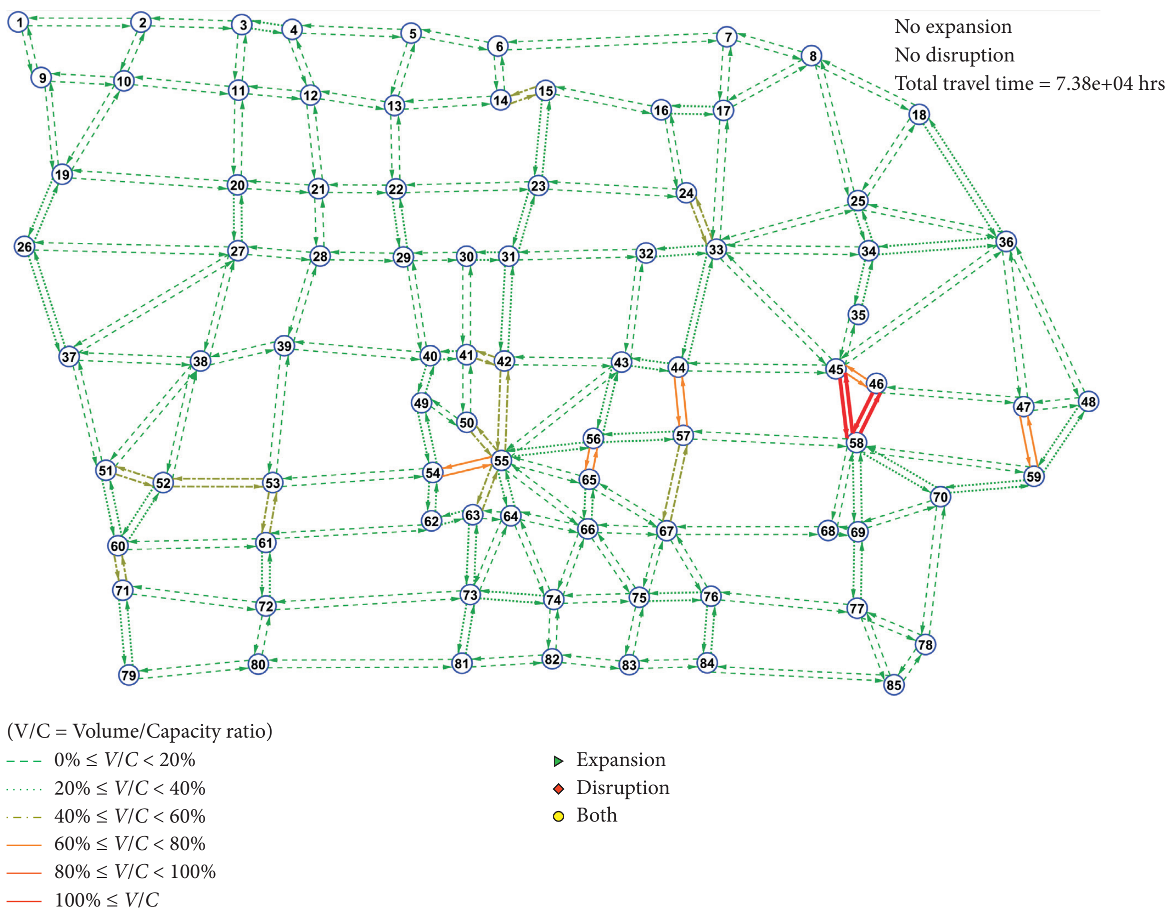


Figure 3: Iowa transportation network without expansion and without disruption.

the most congested links are blocked until the total network disruption limit allows. The total travel time can be obtained after blocking the critical links identified with different methods (criteria 1 and 2 and bilevel model).

There are 464 lanes in the network, and we set the total disruption limit to different values from 5 to 60 lanes. Figure 4 shows the total network travel time with various disruption limits when there is no expansion. The network travel time increases gradually as the number of disrupted lanes grows. A larger total travel time indicates the capability of an approach to identify more vulnerable links in the network. The results show that the bilevel programming model identifies the vulnerable links more accurately than the other two indicators.

Figure 5 presents the Iowa transportation network with disruption limits 15, 30, 45, and 60 lanes. The red rhombus shape on a link indicates that it has been disrupted. The disruption can be a coefficient of 0.1 up to 0.6 or 1.6 for a 1-lane or a 2-lane link, respectively.

In the next stage, we enhance the network resiliency in two ways: first, by solving the trilevel optimization model to determine what links need to be expanded (Algorithm 1); second, by adding lanes to the most congested links greedily (Algorithm 2); here, the most congested links are determined by solving the bilevel programming model. Since the proposed method to solve the subproblem (17) is a heuristic one, the solution to the trilevel optimization model is not optimal. However, the algorithm returns an optimal solution in case of finding the optimal solution for the subproblem.

Figure 6 compares the total travel time in the network from the greedy expansion and the trilevel optimization model against the results when there is no expansion. The trilevel model and the greedy method improve the total travel time in the network by an average of 41% and 21% for all disruption limits, respectively.

However, this improvement achieves with fewer miles expanded by the trilevel model. Table 3 gives the total expansion in miles for the greedy method and the trilevel model when network disruption upper bound is 5 to 60 lanes. The trilevel model can find critical links more effectively, so the total number of miles expanded in the trilevel model is by average 7% less than the greedy method over all cases.

Figure 7 shows the results of the greedy method and the trilevel model on the Iowa transportation network when the disruption limit is 30 and 60 lanes. The red rhombus shape on a link indicates that it has been disrupted. The green triangle on a link implies that the link has been expanded, but no disruption occurred. The yellow circle shows that the link has been both expanded and disrupted.

To test the results with a more realistic scenario, we also considered the flooding problem in Iowa and designed a lane closure scenario based on a previous road closure. In May

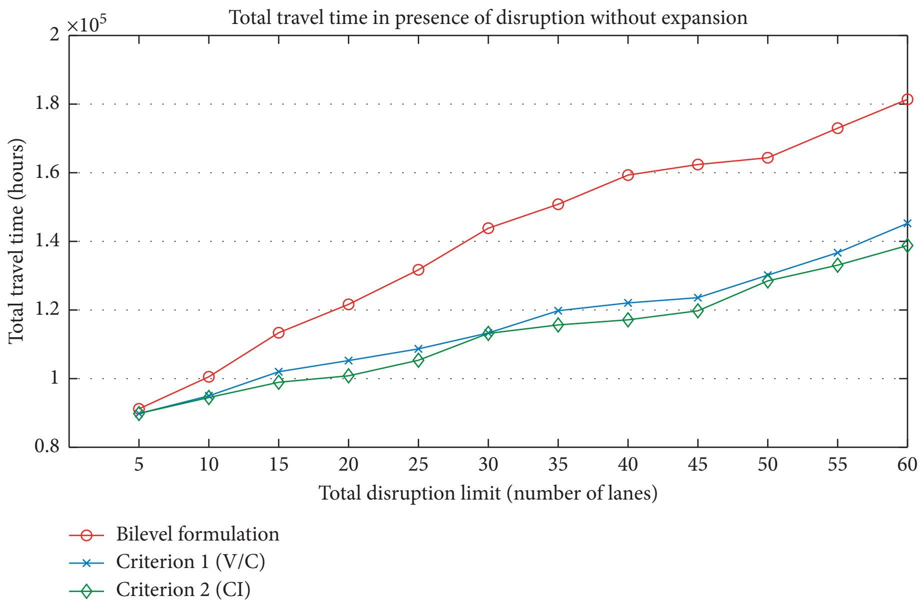
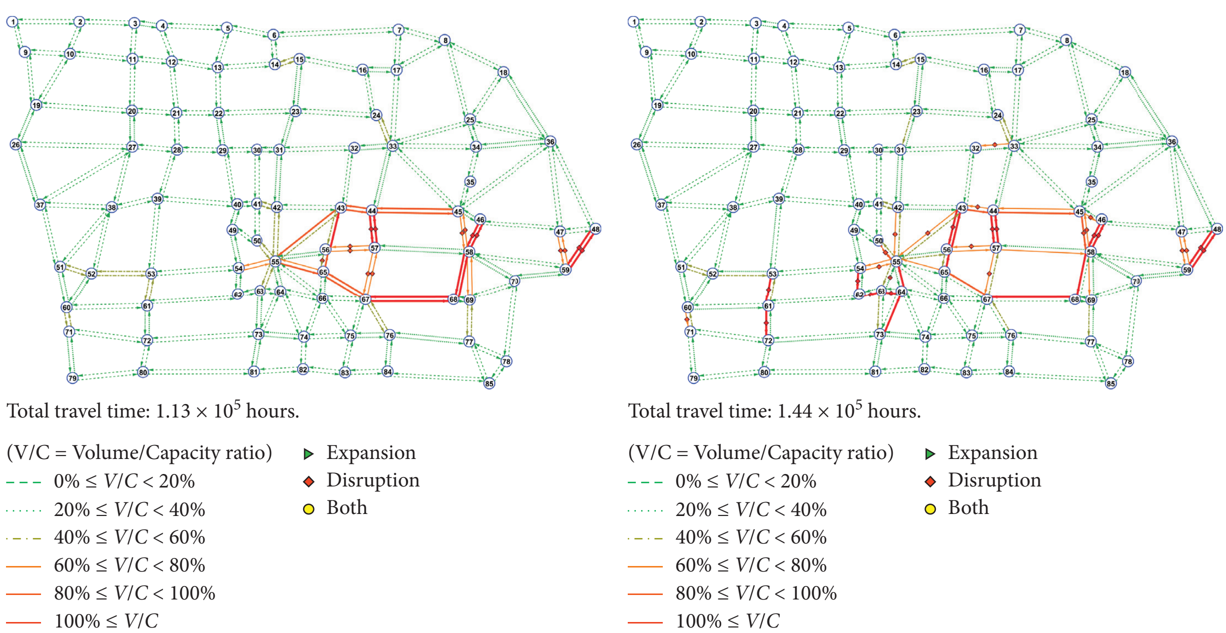


Figure 4: Total travel time after blocking critical links identified with different methods from 5 to 60 lanes without expansion.



(a) (b)

Figure 5: Continued.

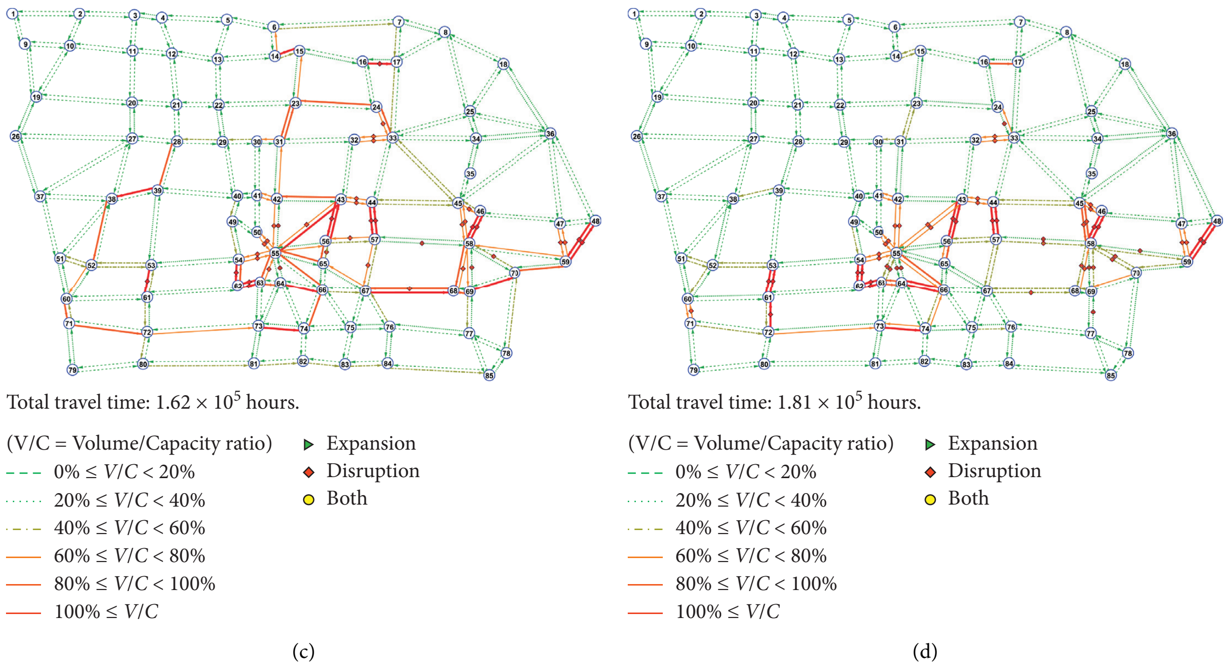


FIGURE 5: Iowa transportation network without expansion when disruption limit is 15, 30, 45, and 60 lanes. (a) Disruption: 15 lanes. Total travel time: 1.13×10^5 hours. (b) Disruption: 30 lanes. Total travel time: 1.44×10^5 hours. (c) Disruption: 45 lanes. Total travel time: 1.62×10^5 hours. (d) Disruption: 60 lanes. Total travel time: 1.81×10^5 hours.

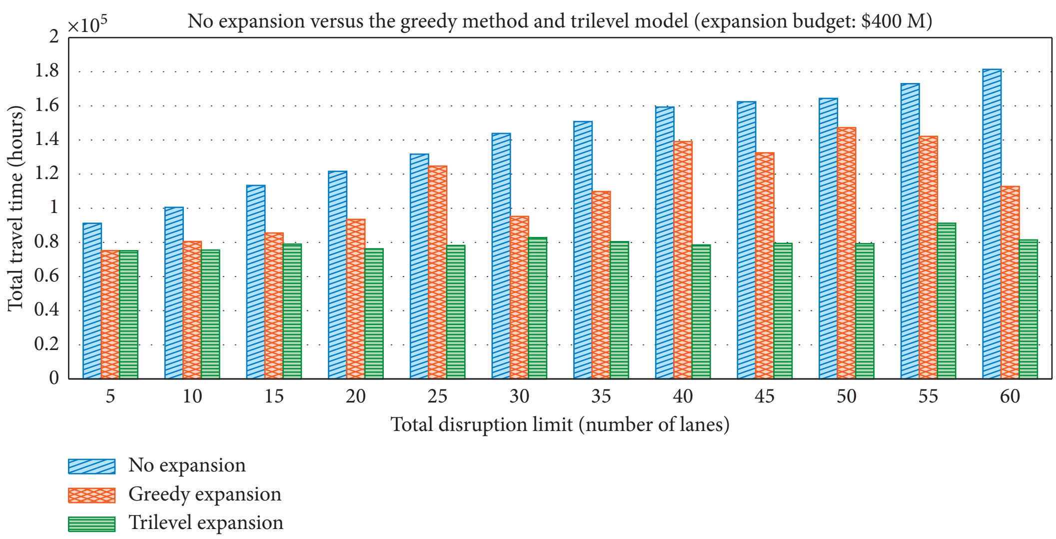


FIGURE 6: The comparison of total travel times obtained from greedy expansion and the trilevel optimization model against travel times when there is no expansion for different values of disruption limit from 5 to 60 lanes.

TABLE 3: The comparison of total expansion in miles between the greedy method and the trilevel optimization model when expansion budget is 400 million dollars.

Disruption limit Q (lane)	5	10	15	20	25	30	35	40	45	50	55	60
Greedy method (mile)	266	266	266	261	260	265	266	263	265	262	262	263
Trilevel model (mile)	102	222	249	263	261	263	260	262	264	263	266	266

2019, the Iowa Department of Transportation closed I-29 from Highway 34 near Glenwood to the Missouri state line [36]. Other roads were also closed across the state due to flooding, including Highway 169 near Adel, Highway 65 near Lucas, Highway 92 near Oskaloosa, Highway 21 near Belle Plaine, Highway 130 near Plainview, and Highway 67 in Davenport. Accordingly, we reduced the capacity of links (i, j) by $q_{i,j}$ lanes as reported in Table 4. As previously

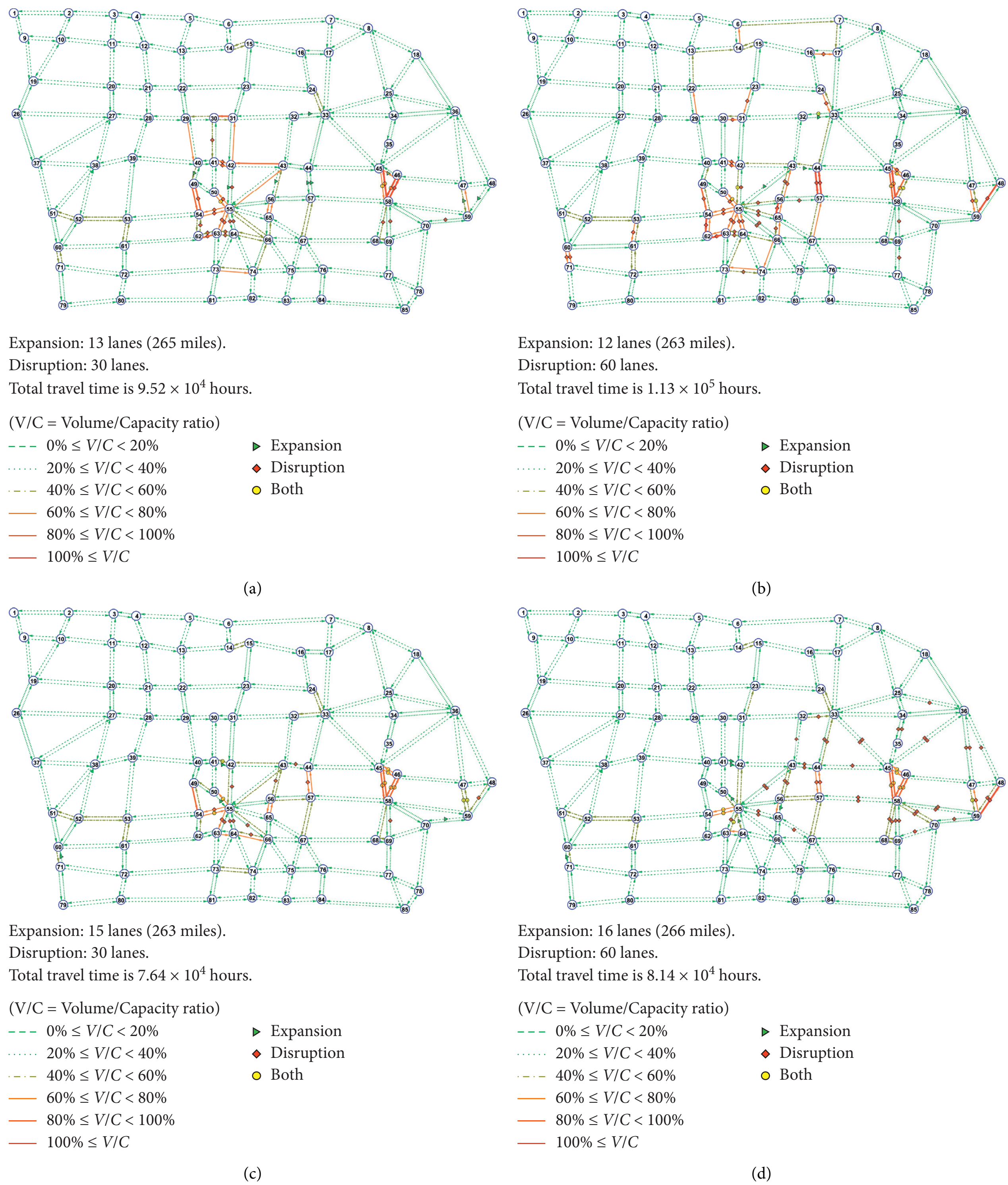


Figure 7: Iowa transportation network with expansions obtained from the greedy method and the trilevel model when disruption limit is 30 and 60 lanes. (a) Greedy method expansion: 13 lanes (265 miles). Disruption: 30 lanes. Total travel time is 9.52×10^4 hours. (b) Greedy method expansion: 12 lanes (263 miles). Disruption: 60 lanes. Total travel time is 1.13×10^5 hours. (c) Trilevel model expansion: 15 lanes (263 miles). Disruption: 30 lanes. Total travel time is 7.64×10^4 hours. (d) Trilevel model expansion: 16 lanes (266 miles). Disruption: 60 lanes. Total travel time is 8.14×10^4 hours.

mentioned, we consider capacity reduction instead of complete closure because the 85-node road network is an abstract representation of the real road network. Therefore, a link in the network represents the major road connecting the two nodes and also other local roads. For example, by closing Highway I-29, travelers might still be able to travel between

Table 4: Link disruptions based on the flooding incident in Iowa in May 2019.

Highway	Node i	Node j	$q_{i,j}$
I-29	60	71	1.6
I-29	71	60	1.6
I-29	71	79	1.6
I-29	79	71	1.6
169	49	54	0.6
169	54	49	0.6
65	64	74	0.6
65	74	64	0.6
65	74	82	0.6
65	82	74	0.6
92	66	67	0.6
92	67	66	0.6
21	44	57	0.4
21	57	44	0.4
130	58	59	0.8
130	59	58	0.8
67	59	70	0.8
67	70	59	0.8
67	69	70	0.6
67	70	69	0.6

Table 5: Comparison of total travel times with/without expansion and with/without flooding event in hours.

	Without flooding	With flooding	Difference
Without expansion	7.38×10^4	7.69×10^4	3116 hrs
With expansion	7.12×10^4	7.29×10^4	1691 hrs

the two nodes using other local roads. Therefore, the total capacity of the link is reduced, but the link is not removed.

Assuming there is no expansion, after reducing the capacity of the links affected by the Iowa flood in 2019 and solving the lower-level problem, the total travel time is 7.69×10^4 hours. However, the total travel time is 7.38×10^4 hours when there is no expansion and no disruption in the network. Therefore, the Iowa flood incident increased the travel time by 3116 hours. We calculated the total travel times again with and without the flood incident when roads were expanded using the trilevel model. The travel time after expansion with and without flooding event reduced to 7.29×10^4 and 7.12×10^4 hours, respectively. The travel time increase by flooding event is 1691 hours, which is almost 45% less than 3116 hours. Table 5 summarizes the results of testing the trilevel model with and without the Iowa flood scenario.

5. Conclusions

In this study, we propose a new approach to address uncertainty in a transportation network. The link capacities are uncertain parameters, and the origin-destination demands are constant. The objective is to design a resilient transportation network in the presence of disruption to minimize the total travel time. This study makes three contributions to the literature. First, we developed a trilevel optimization model for the resilient network design problem. The lower-level determines the network flow to minimize the total travel time; the middle-level assesses the resiliency of the network by identifying the worst-case scenario disruptions that could lead to a maximal cost to the transportation system, and the upper-level designs the optimal strategy to expand the existing transportation network so that it enhances the resiliency of the network. Second, we reformulated the network flow problem to reduce the number of variables and constraints significantly. Third, we designed a heuristic algorithm for solving the trilevel optimization model to efficiently enhance the resiliency of the network.

The results of solving the bilevel programming problem, assuming there is no expansion, show that reducing the link capacities due to probabilistic disruptions affects transportation network's performance and can delay or stop the movement. We improved the result of the bilevel model by applying two methods. The results show that the trilevel optimization model and the greedy expansion method improve the total travel time by an average of 41% and 21%, respectively.

This study is subject to several limitations that suggest future research directions. For example, the proposed model assumes the origin-destination demands are deterministic. Relaxing this assumption would require a more complicated model that reflects the uncertainty or time dependency over travel demands. Also, we formulated the problem as system optimum because our focus in this paper is on the higher system level and long-term planning to increase the network's capacity. Using a user equilibrium for the network flow model can be a possible direction for future work, although it would require new algorithms for bilevel and trilevel analysis to identify most vulnerable links and most effective capacity expansion strategies. Furthermore, the designed algorithm is heuristic; future research can be designing a more efficient algorithm or even developing an exact algorithm to find the optimal solution.

Data Availability

The data used to support the findings of this study are presented in the supplemental files. All other data used are contained within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Supplementary Materials

Supplementary 1: data_nodes file: coordinates (latitude and longitude) of all 85 nodes and production/attraction of each node to use in the gravity model. Supplementary 2: data_links file: all 332 links with their capacity (number of lanes). Supplementary Materials (*Supplementary Materials*)

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