


Research Article

Research on Direct Braking Force Estimation and Control Strategy Using Tire Inverse Model

Zhiguo Zhou ¹ and Xiaoning Zhu²

¹Zhejiang Institute of Communications, Hangzhou 311112, China

²Xi'an Aeronautics Computing Technique Research Institute, Aviation Industry Corporation of China, Xi'an 710065, China

Correspondence should be addressed to Zhiguo Zhou; zhouzg1107@126.com

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With the rapid development of computer control and vehicle intelligence technology, speed and safety of vehicles have been greatly improved, and the requirements for vehicle control performance are getting higher and higher. For the direct braking force control, in the process of deceleration, a fast braking response can be obtained, which improves the braking performance and vehicle safety. This paper concentrates on direct braking force estimation and control strategy using a tire inverse model based on the antilock braking system, and to solve the problem of the existing ABS system is mainly antilock braking function, no direct braking force control function. Taking magic formula model for reference inverse model, the critical parameters under different road surfaces are obtained according to experience data. Then, the desired slip ratio corresponding to braking force can be obtained via fast tire inverse model look-up table method. The tyre friction self-adjustment decision making is obtained using the tire inverse model method. A direct braking force antilock braking system (DBF-ABS) controller is built using the nonsingular fast terminal sliding mode method. The simulation results indicated that the control strategy has adaptability and stability to the change of road conditions.

1. Introduction

On behalf of adapting to the complex working conditions and enhancing the vehicle's safety and comfort, various types of automotive active electronic control systems are presented. The research on integrated vehicle dynamics control already became an urgent problem to be solved and has attracted extensive attention [1–3].

These research studies have improved vehicle performance to a certain extent but still have some problem to be solved. Some studies focus only on the design of the main circuit [4–6]. The calculated stable side forces and total yaw moments are applied without considering targeted production and allocation manners. The influence of tire dynamics is essentially treated as nominal parameters, such as the basic angular stiffness when the problem formulates. But there is an interaction between the non-linearity of tire characteristics and vehicle dynamics

[7–10]. However, these studies based on the main loop design can provide the maximum performance margins and theoretic insight, and the vehicle motion force generation process does not fully take the special interactions between tires and the road into account. It can lead to insufficient control accuracy or overly optimistic performance results. When need more tire force, for example, if the tires have been in big slip rate, applying large braking force will only make things worse. More importantly, the realization of tire forces is still a critical problem in relation to handling property [11–14].

This paper concentrates on direct braking force estimation and control strategy using tire inverse model based on the ABS. The main content of the rest section of the paper is as follows. Section 2 discusses the development and new features of vehicle integrated control and direct torque ABS control technology in recent years. Section 3 describes tyre friction self-adjustment decision-making

method and direct braking force DBF-ABS controller design. The results of simulation analysis are presented in Section 4. The conclusions and future related work is provided in Section 5.

2. Literature Review

ABS system has been developed since the early 20th century [15, 16]. At the end of 1970s, the great progress of digital electronic technology and large-scale integrated circuit laid the technical foundation of ABS. After the mid-1980s, the development of ABS paid more attention to its own cost performance ratio [17, 18]. The work during this period has increased the popularity of ABS. The ABS system is considered as the most important safety technical achievement since the adoption of safety belt in automobiles [19, 20].

With the improvement of vehicle speed and intelligent technology level, the related vehicle control technology based on ABS has also achieved new and rapid development. The EBD and ABS are integrated to form the automobile auxiliary integrated system using CAN bus [21]. Brake-by-wire control systems for intelligent vehicles are studied [22]. In the past decades, quite a few advanced intelligence, automatic control, and computer technologies have been widely used in ABS for smart vehicles, for example, distributed and self-adaptive vehicle speed estimation and control [23]; optimal slip rate is obtained and tracked based on the multiphase method [24, 25]. Besides, a nonlinear predictive control strategy was proposed [26, 27].

Especially with the improvement of vehicle integration, collaborative or optimal control has become a new research hotspot, such as, combined emergency braking, integrated vehicle chassis control [28–30], and self-learning adaptive control [31, 32]. In addition, with the development of modern computer and communication technology, some new technologies are applied to reduce traffic congestion and vehicle driving safety, such as advanced driving assistance system and autonomous driving [33–35]. In particular, the development of intelligent networked vehicle technology has further improved vehicle safety [36, 37]. Based on these previous studies, this paper concentrates on direct braking force estimation and control strategy using the tire inverse model based on the ABS to solve the direct braking force control problem.

3. Method

In this section, the control structure of the direct braking force self-adjustment decision making and control is designed. The desired direct braking force friction is estimated and tracked. Based on the estimated values of tyre friction, the desired slip ratio can be obtained, which is corresponded with the specific desired tyre friction, using the reverse look-up table method. Then, based on the tire braking force model, the terminal sliding mode method

ensures that the antilock braking system can achieve the desired slip rate to obtain direct braking.

3.1. Control Structure. The direct braking force estimation, self-adjusting decision, and control structure are shown in Figure 1. The details are as follows:

Step 1: the driver commands are received from the brake pedal system. The control inputs, namely, wheel control moments, are obtained by a servo loop to distribute force and torque to the four tire-road contact blocks.

Step 2: the direct braking force is estimated using direct braking force quick look-up table based on tire inverse model. The ideal tyre-road friction is obtained by direct braking force decision-making system.

Step 3: the control target error is calculated and direct braking target control force is obtained and assigned by direct braking force control system.

At last, simulations and results are analyzed based on the vehicle dynamic model, including tyre-road dynamic model, vehicle dynamic model, and braking force sensor model.

3.2. Vehicle Model. Tire-road friction has obvious nonlinear characteristics, which should be estimated. An attempt has been made to measure braking torque using force sensors mounted on caliper mounts. Assuming that braking torque can be obtained from sensors, then, tyre-road friction can be calculated [24].

In Figure 2, vehicle dynamics and brake model are built as follows:

$$\begin{aligned} \dot{u} &= \frac{\sum_{i=FL,FR,RL,RR} F_{fb,i}}{M}, \\ \dot{\omega}_i &= \frac{R_{b,i} F_{xb,i} - T_{b,i}}{J_{b,i}}, \end{aligned} \quad (1)$$

$$F_{xb,i} = \mu_i \cdot F_{Z,i},$$

$$F_{Z,i} = \frac{1}{4} Mg,$$

where M is the mass of vehicle, $F_{xb,i}$ is the friction force, ω_i is the angular speed, $J_{b,i}$ is the wheel inertia, $R_{b,i}$ is the radius of the vehicle wheel, $F_{fb,i}$ is the brake force measured by force transducer, $T_{b,i}$ is the brake torque, g is the acceleration of gravity, $F_{Z,i}$ is the vertical load of the wheel, and i is the front, rear, left, and right positions wheel.

3.3. Tyre Model. The tire model is derived from Magic formula. Magic formula is the universal semiempirical tire model [38]. The general form is as follows:

$$\mu_l = A_l \sin[B_l \arctan\{C_l \lambda - D_l (C_l \lambda - \arctan(C_l \lambda))\}], \quad (2)$$

where μ_l is the longitudinal friction coefficient, C_l is the stiffness factor of the tire, λ is the longitudinal slip of the

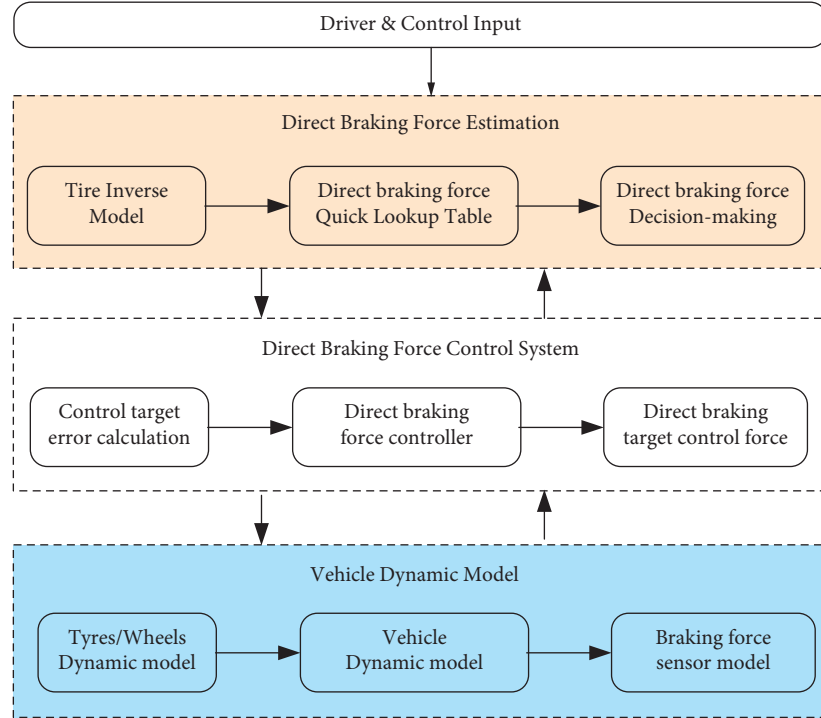


FIGURE 1: Schematic diagram of direct braking force estimation and control configuration.

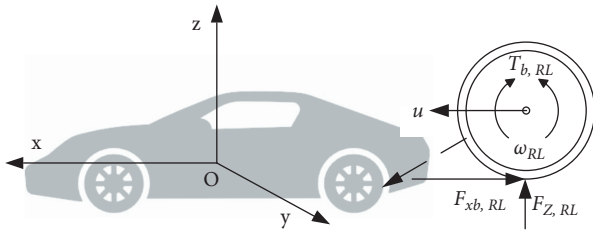


FIGURE 2: Vehicle braking dynamics model.

vehicle, A_l is the peak value, B_l is the shape factor, D_l is the curvature factor.

3.4. Direct Braking Force System Controller Design. In this section, a direct braking force controller based on ABS with terminal sliding mode control method is proposed, as shown in Figure 3.

After introducing the controller, the parameter uncertainty and the influence of external interference can be eliminated.

The following equation is the sliding surface designed in this paper:

$$S_{\text{DBE-ABS}} = \frac{d(F_{\text{Dir_Brak}} - F_{\text{Ref_Brak}})}{dt + \xi(F_{\text{Dir_Brak}} - F_{\text{Ref_Brak}}) + \zeta(F_{\text{Dir_Brak}} - F_{\text{Ref_Brak}})^{(a/b)}, \quad (3)$$

where $e \in R$; ξ, ζ are constants, and $\xi > 0, \zeta > 0$; a, b are positive odd integers. At the same time, $a < b < 2a$.

The dynamic adjustment process of sliding mode control consists of arrival stage and sliding control two stages. To make the switch manifold reachable, smooth, and fast convergence in a finite time, a “terminal attractor” is proposed to improve chatter less control while taking

full advantage of nonsingular fast terminal sliding mode control. This controller sliding surface design as follows:

$$\dot{S}_{\text{DBE-ABS}} = (-\varsigma s - \vartheta s^{(g/f)})(F_{\text{Dir_Brak}} - F_{\text{Ref_Brak}})^{(p-q)}, \quad (4)$$

where $\varsigma \in R^+$; $\vartheta \in R^+$; $m > 0$ are odd integers. $n > 0$ is odd integers. At the same time, $0 < g/f < 1$. And the direct braking force antilock braking control law is shown below:

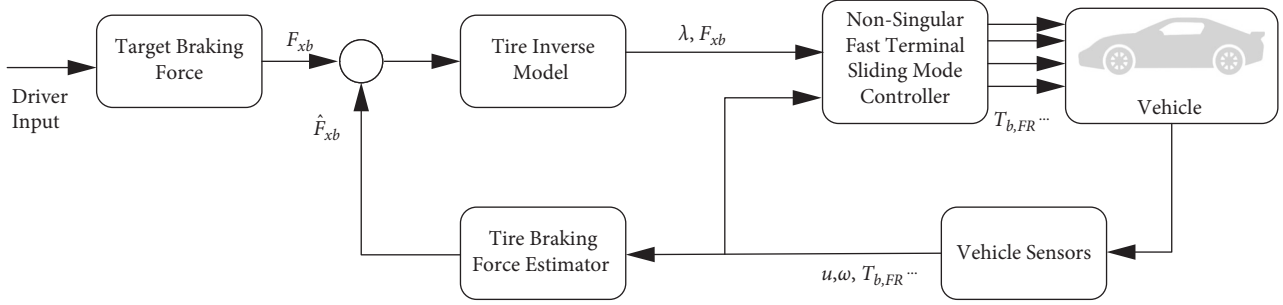


FIGURE 3: Schematic of direct braking force antilock braking system controller.

$$F_{Con_Brak} = \frac{u \cdot J}{r_b^2} \frac{q}{\zeta \cdot p} \left(-\ddot{F}_{Err_Brak} \cdot F_{Err_Brak}^{(q-p/q)} - \xi \cdot \dot{F}_{Err_Brak} \cdot F_{Err_Brak}^{(q-p/q)} + (\zeta s + \vartheta s^{(g/f)}) \right) - \frac{u}{r_b^2 \cdot J} \left(\frac{r_b^2 F_{Dir_Brak} + J(1-\lambda)(du/dt)}{u} - \dot{F}_{Inv_Ref_Brak} \right). \quad (5)$$

In the above equation, $F_{Err_Brak} = F_{Dir_Brak} - F_{Ref_Brak}$, since $0 < q - p/q$, the system eliminates the singularity problem and can converge to system equilibrium by tracking the sliding surface.

4. Simulations and Analysis

On behalf of proving the effectiveness of the direct braking friction self-adjusting decision and controller, simulation is carried out in this section. Firstly, the characteristics of tyre friction are presented by using three sets of different test points, analyzing the impact of these test points on vehicle speed control and vehicle braking distance. Then, the superiority of the nonsingular fast terminal sliding mode method-based DBF-ABS controller is compared with fast sliding mode control and Bang-Bang-based ones. Finally, the overall performance of friction self-tuning control in μ -split condition is achieved.

4.1. Parameter Set. Parameters required to build the simulation and analysis system are listed in Table 1.

The different road surface Magic formula empirical parameters can be obtained in Table 2. Based on the above information, A_L , B_L , C_L , and D_L can be constrained in the range of corresponding different roads [24].

4.2. Simulations of Quasilinear Braking Area Characteristics. The braking force between the tire and the ground has a characteristic that are transitioned from linear to nonlinear, including two areas quasilinear braking area and emergency braking nonlinear area, as shown in equation (2). In order to better estimate and control the direct braking force, it is necessary to analyze the interaction characteristics of these two regions. In the tyre friction self-adjustment decision

making, control sets in quasilinear braking area points $\lambda \in [0.01 \sim 0.10]$ have been selected 3 points, that is A, B, and C. And based on these points, the simulations are conducted. The results can be obtained in Figures 4 and 5. The control points of $(\hat{\lambda}_{xb}, \hat{F}_{xb})$ are within the area that $\lambda \in [0.01 \sim 0.10]$. And as is shown in Figure 5, little change is present in control slip rate, so there is a little effect on braking distance.

4.3. Simulations of Emergency Braking Area Characteristics. The control set points in emergency braking area, in $\lambda \in [0.10 \sim 0.20]$, have already selected 3 points, that is A, B, and C. Based on these points, the simulations are conducted. The results can be obtained in Figures 6 7.

The control points of $(\hat{\lambda}_{xb}, \hat{F}_{xb})$ are within the area that $\lambda \in [0.10 \sim 0.20]$. From Figures 6 and 7, although the step of the slip rate is the same as that of case 4.2, it has a greater impact on the braking distance. The result is significantly improved compared with Case 4.3, because the identification point information is more applicable to the nonlinear variation of tire-road friction. In conclusion, the sampling point λ contributes to more impact on the braking distance.

4.4. Simulations under μ -Split Condition of Different Road Surface. The braking force varies with different road environment. The scenarios of the vehicle running under μ -split condition of different road surfaces are chosen to verify the self-tuning and adaptive performances of proposed estimator and controller, as shown in Figure 8.

Set the constant reference friction force s_{ss} in this μ -split simulation. The road conditions change between the asphalt, dry road and the asphalt, wet road at 1.5s, as shown in Figures 9 and 10. In figures, the friction and vehicle acceleration have not changed drastically. As shown in Figure 9, although the friction force is same, the figure displays that the reference value at 1.0 seconds decreases from about 0.054 to 0.036 as the road conditions change. As shown in Figure 10, although the friction force is same, the vehicle

TABLE 1: Parameters used in simulations.

Notation	g acceleration of gravity	J_b wheel inertia	R_b wheel radius	M vehicle mass
Unit	m/s^2	$kg \cdot m^2$	m	kg
Value	9.8	12	0.25	1530

TABLE 2: Magic formula parameters of different roads.

Road	Snow	Cobblestone (wet)	Asphalt-wet	Cobblestone (dry)	Concrete (dry)	Asphalt (dry)
A_L	0.20	0.40	0.80	0.85	0.37	1.10
B_L	1.45	1.45	1.60	1.40	1.64	1.55
C_L	17.43	14.02	15.63	10.09	13.42	13.42
D_L	0.65	0.60	0.45	0.64	0.53	0.53

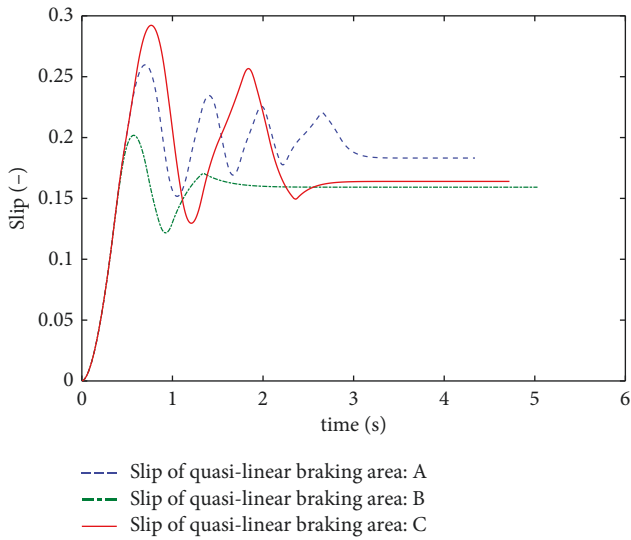


FIGURE 4: μ in quasilinear area.

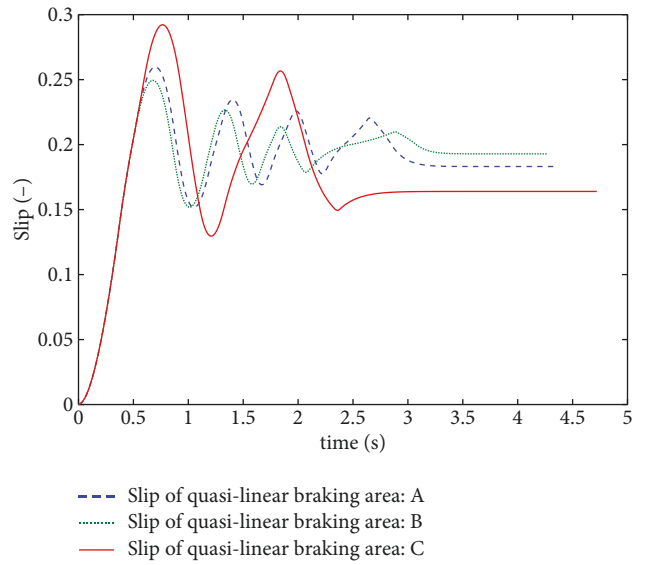


FIGURE 6: μ in emergency area.

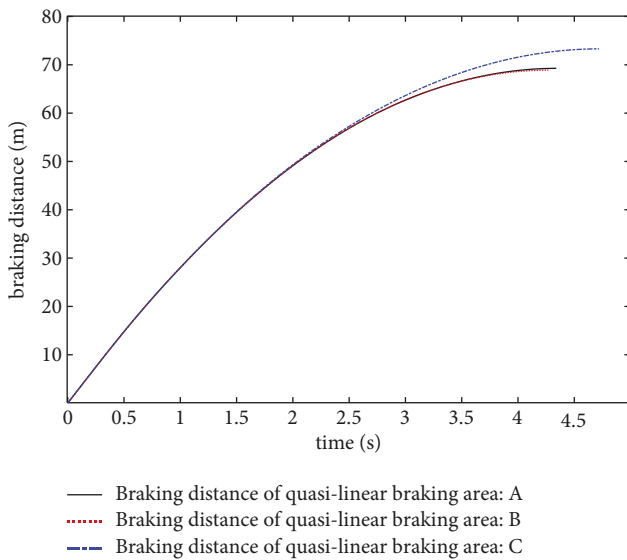


FIGURE 5: Braking distance in quasilinear area.

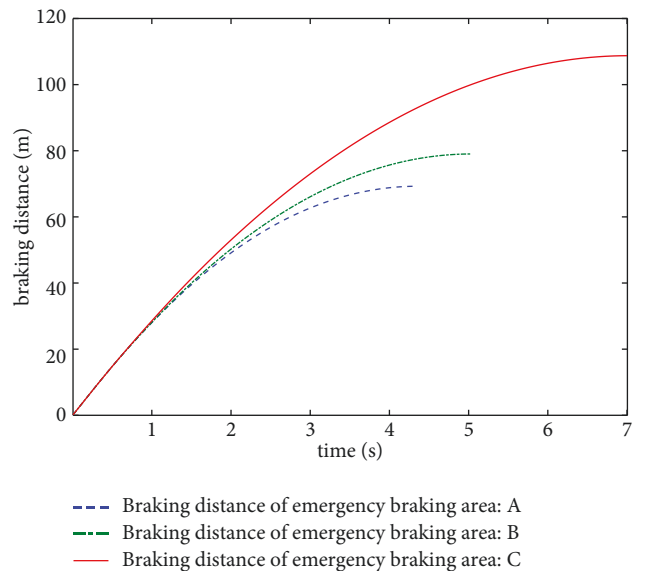


FIGURE 7: Braking distance in emergency area.



FIGURE 8: Scene of direct braking friction control under μ -split condition of different road surfaces.

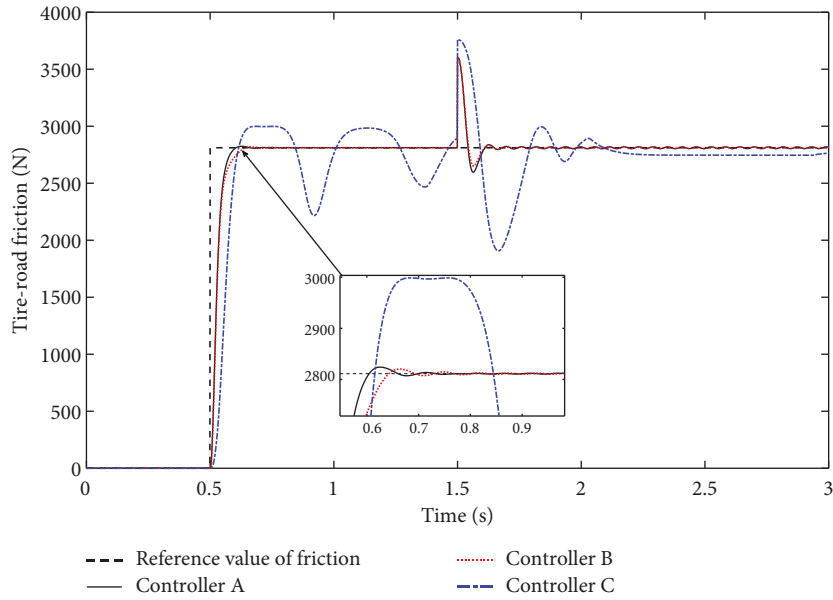


FIGURE 9: Reference and real value of friction.

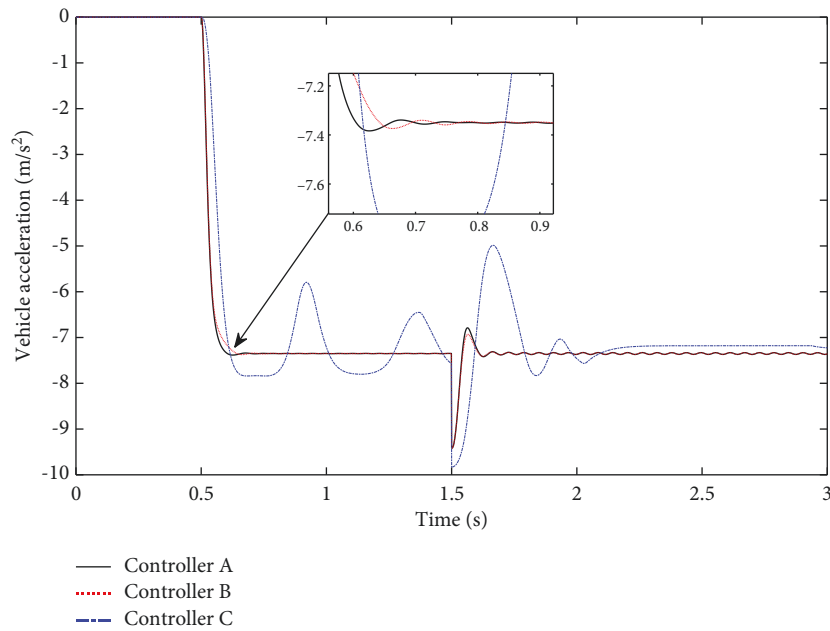


FIGURE 10: Vehicle acceleration.

maintains the same acceleration on different road surfaces. However, the figure displays that the acceleration value at 1.5 seconds decreases as the road conditions change. The results shows that the proposed self-tuning controller can estimate

and keep the tracking value consistent with the reference value under different road surfaces.

Furthermore, the proposed direct braking force controller (controller A) is compared with fast terminal sliding

mode controller (controller B) and Bang-Bang controller (controller C). From Figure 9, the controller A converges between reference and real value of friction faster than controllers B and C. In Figure 10, the results are similar for vehicle acceleration control. From the results, although the performance of the Bang-Bang controller is not as good as the other two, it is often used in engineering due to its simple structure and low requirements for the control processing unit. The figure shows that the proposed DBF-ABS controller can keep the tracking value consistent with better performance.

5. Conclusions and Future Work

Based on the existing ABS, a kind of direct braking force estimation and control strategy based on DBF-ABS control system was proposed to solve the problem of the existing ABS system which is mainly antilock braking function, no direct braking force control function. For the direct braking force control, in the process of deceleration, a fast braking response can be obtained, which improves the braking performance and vehicle safety. Firstly, taking magic formula model for reference inverse model, the critical parameters under different road surface are obtained according to experience data. Then, the desired slip ratio corresponding to braking force can be obtained via tire inverse model look-up table method. The tyre friction self-adjustment decision making is obtained using the tire inverse model method. A direct braking force antilock braking system (DBF-ABS) controller is built using the nonsingular fast terminal sliding mode algorithm. Finally, the simulations and analysis results show that the control method has adaptability and stability under different driving conditions.

Due to limited sensing equipment for direct braking force control data acquisition, future work will be focused on advanced sensing and data estimation. In addition, a wider range of dynamic adaptive direct braking torque control and matching with AEBS is also an interesting topic.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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