Research Article
Locating Urban Consolidation Centers under Shipper Rationality

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An urban consolidation center (UCC) is a logistics facility used to combine deliveries of multiple shippers for reducing freight traffic in dense urban areas. Despite their benefits, such as reducing less-than-truckload deliveries, former applications of UUCs have faced backlash from shippers that were forced to use them even though they did not experience any cost savings. Such failures are often due to an unsuitable location of the UCC and the additional costs of facility operation and maintenance that are unfairly divided between the shippers. This paper introduces shipper rationality to measure the willingness of shippers to use an optimally located UCC. We use a continuum approximation approach to derive the routing costs of the shippers as closed-form expressions. We present three objective functions depending on the preference of the regulatory agency that chooses the location of the UCC, allowing rational shippers to join only if they experience cost savings from the UCC. We discuss influential factors in the optimal location of a UCC, including the customer density of each shipper, the establishment cost of the UCC, and the location of each shipper’s fulfillment center.

1. Introduction

Urban consolidation centers (UCCs) are logistics facilities commonly located in suburban or industrial areas and used for consolidating shipments into fewer trucks with higher load factors. Benefits of UCCs range from social to environmental advantages, such as noise reduction [1], lower greenhouse gas emissions [2], and shorter total traveled distances. Despite their potential benefits, UCCs have not always been successful in practice. In the past three decades, up to 150 UCC projects were initiated in Europe, of which very few have survived [3]. An example, the Maastricht Consolidation Center of the Netherlands in 1989 failed due to the strong opposition from the local business community, as major retailers saw participation obligation as a loss of competitive edge in their logistics arrangements [4]. The Stuttgart Consolidation Center of Germany in 1993 failed because the transportation cost savings were considerably lower than the handling costs, and the environmental benefits were partially offset by operating costs [5, 6]. The Zurich Consolidation Center of Switzerland was unsuccessful because few retailers participated, and transportation savings were minimal. In many of these pilots, government subsidies were stopped after a while, forcing participants to leave the program as they were unwilling to pay the full costs of the facility [7].

A main contributing factor to the failure of UCCs is the reluctance of shippers to participate in such programs due to the additional costs. Hence, any UCC feasibility study, pilot, or implementation project must consider shipper rationality, which captures the financial incentive of shipper to join and remain in the program. Shipper rationality implies that a shipper uses the UCC only if its new logistics costs (when using the UCC) are lower than its original costs. This paper’s objective is (i) to develop a framework to assess shipper participation in UCC programs under shipper rationality; (ii) to investigate the optimal location of a UCC on a corridor connecting a distribution center to an urban area where recipients are located; and (iii) to assess the impacts of the UCC on various stakeholders, such as local governments and shippers, using a multiobjective optimization approach.
Shipper rationality (shipper rationality in other context can include decisions regarding manufacturing, supply main management, and marketing) in the presence of a UCC can depend on many parameters, including the location of the UCC, the number of other participating shippers, the cost of the facility, the number of delivery stops of each shipper, the type of commodity, and time constraints. Among such parameters, the location of the UCC is important because (i) it impacts all the rational shippers and influences whether they would use the facility or not; (ii) there is more flexibility in changing the location of the UCC compared to changing the location of the shippers, because a UCC is typically constructed in the presence of firms and not prior to their establishment; and (iii) UCCs have a fixed cost that can depend on how close they are to the service region. Hence, one major focus of this study is to find the optimal location of a UCC under user rationality.

The UCC location problem requires knowledge of shippers’ customer locations and the demands to compose vehicle routing schemes. However, construction and the use of a UCC is a strategic decision. Thus, it is impractical to consider very detailed routing aspects due to the difficulty of accurately predicting data. An alternative approach for modelling routing is the continuous approach method, which advocates the use of aggregate data instead of detailed inputs, thus smoothing minor dynamic and stochastic variations, which are less critical in strategic planning [8, 9]. Data aggregation also has other benefits, such as enhanced tractability for larger-size problems and provision of closed-form solutions that are appropriate for gaining managerial insights. The scientific contribution of this paper is the development of a UCC location model using a continuous approximation model with aggregate shipper data.

The remainder of this paper is organized as follows. Section 2 provides a literature review. Section 3 presents the proposed model for capturing user rationality incorporating continuous approximation models. Section 4 identifies the three objective functions and also presents a sensitivity analysis. Section 5 provides the conclusions of the study.

2. Background

Hub location problems are generally treated as a many-to-many distribution issue with transhipments, as there are many shippers involved and each delivering to many customers [10–12]. The UCC problem is a special case of the hub location problem; it can be viewed as a collection of several one-to-many distribution configurations (with or without transshipment), where each one-to-many configuration belongs to one shipper that decides whether to use the UCC (with transshipment) or not (without transshipment). This small change in distribution composition, as illustrated in Section 3, helps us consider shipper rationality. Moreover, the UCC problem is distinguished from previous research on the hub location problem in that the UCC is generally located along a corridor oriented towards an urban area where deliveries have to be performed (see Figure 1). This feature of the UCC implies that the location of a UCC is a one-dimensional distance that can be measured using any reference point along the corridor. In conventional hub-location problems, however, the hub(s) can be located anywhere from a set of discrete candid locations [12] or anywhere along a Euclidean grid [13].

Shipper rationality is acknowledged to be an influential factor in the logistics of UCCs. Daganzo [14] implicitly considers shipper rationality, ensuring that a truck will only use a UCC if $d_m + D \leq D_m$, where $d_m$ is the distance from a distribution center $m$ to the UCC, $D$ is the distance from the UCC to a single delivery point, and $D_m$ is the direct distance from the distribution center $m$ to the single delivery point. Clearly, a shipper only uses a UCC if there are potential cost savings. Shipper rationality can also be compared to passenger rationality (in the context of public transportation) when deciding whether to use a park-and-ride facility. Holguin et al. [15] capture passenger rationality by indicating that a passenger uses a park-and-ride facility if and only if $g_{ij}^{PR} \geq g_{ij}^{PR}$, where $g_{ij}^{PR}$ is the cost of traveling from $i$ to $j$ using the auto mode and $g_{ij}^{PR}$ is the cost of traveling from $i$ to $j$ using a park-and-ride facility located at $p$. The formulations presented in this paper build on such rationality concepts.

The UCC problem in combinatorial format is NP-hard because the vehicle routing problem is embedded in it [16]. We use the continuum approximations of Newell and Daganzo [17, 18] to estimate routing costs. Jabali et al. [8] presented a continuous approximation model for the fleet composition problem by partitioning a service region into zones, where each zone is served by one vehicle. A particularly similar methodology is used in this paper to find the required number of vehicles with respect to their load factors. Detailed explanations are provided in Section 3.

3. A Continuous Approximation Model

The UCC problem is composed of a densely populated service area where delivery points are located and one or more corridors connect shipper distribution centers to the service area (Figure 2). Without loss of generality, we assume there is only one corridor leading to the service area with a total distance of $B$. Considering that each shipper accesses its nearest corridor, one can model more than one corridor leading to the service area, each with specific properties. A shipper $i$’s location on the corridor is specified by $x_i$, which is measured from the start of the corridor. A potential consolidation center’s location is specified by $x_c$ and the point of the corridor’s end is specified by $x_e$. The service area is assumed to have an area of $S$ where a total of $D_i$ delivery points of shipper $i$ are randomly distributed. This amounts to a density of $\sigma_i(= D_i/S)$ deliveries for shipper $i$. 

![Figure 1: Vehicle routing in a zone with width \(w\).](image-url)
To model the UCC problem using continuous approximation, we consider a circular trapezoid service area with parallel equi-travel-time contours (Figure 3). Equi-travel-time contours [18] imply that the travel time is the same when traveling from \( x_e \) to any two points in the service area that have the same direct distance from \( x_e \). The circular trapezoid has three variables that define its shape. These are \( r_1 \), \( r_2 \), and \( \theta \), which are determined so that the area of the trapezoid in Figure 3 is the same as the area of the service area in Figure 2 (S). Assuming that \( \theta \) and \( r_1 \) are constant and selected to best represent the geometry of the service area, we have

\[
r_2 = \sqrt{\frac{2S}{\theta}} + r_1^2.
\]

(1)

\[
r = r_2 - r_1.
\]

(2)

The service area, as presented in Figure 3, is partitioned into smaller trapezoid zones. Each zone is assigned to one vehicle and has the dimensions \( w \) and \( l \). As discussed by Newell and Daganzo [18], the circular trapezoids are approximated by rectangles presented in Figure 1. As an approximation for the vehicle routing distance in each zone, Daganzo [17] presents a strip strategy (heuristic) that divides each zone into two rectangles of length \( w/2 \) where the top half of the customers are visited on the way out and the bottom half of the customers are visited on the way in. Newell and Daganzo [18] then reinterpret formulations so that the total distance is composed of a line-haul distance and a local distance. Line-haul is the distance between the distribution center and the center of the zone. Local distance is the distance from the zone center to the far end of the zone and back. The other half of the local traverse distance is accounted for in the line-haul distance. The average local distance per point in the zone, as presented by Newell and Daganzo [18], is

\[
\frac{w}{6} + \frac{1}{w \sigma}
\]

(3)

Equating the derivative of (2) to zero gives the optimal width \( w^* \) (hereafter referred to as \( w \)), which is

\[
w = 2\sqrt{\frac{3}{2\sigma}}
\]

(4)

Furthermore, the total local distance in every zone is obtained from the product of the interpoint distance traveled within the rectangle \( \sqrt{2/3\sigma} \) and the number of delivery points in that zone \( (w\sigma) \):

\[
w\sigma \sqrt{\frac{2}{3\sigma}}
\]

(5)

3.1. Problem Setting. The following are the assumptions made in the approximate model:

(i) The service area topology consists of small zones that are partitioned together to make a circular trapezoid shape.

(ii) Delivery point density is constant through the service area for every shipper.

(iii) Multiple shippers may have different delivery point densities but the same type of commodity.

(iv) Only one type of vehicle with a given capacity is used in the problem for better exposition. This assumption is prudent in cases where the firms are homogenous, which makes it reasonable for them to use the same vehicle type. With heterogeneous firms, however, this assumption may not be logical, as heterogeneous firms could use different vehicle types. This assumption can be relaxed at the cost of tractability.

(v) The vehicle can serve a limited number of deliveries, and each delivery point requires one unit of capacity from the vehicle.

(vi) Handling and ordering costs are a constant value in the formulations, thus adding no insight to the model. This cost can be assumed to be part of the consolidation center operating cost. The formulation will slightly change if handling costs are a function of the order size, in which case the constant holding cost assumption would have to be relaxed.

The notations used throughout this paper are defined below (Table 1).

In the case where shipper \( i \) uses a consolidation center, the subscript of \( t_{ij} \) and \( g_{ij} \) denotes shipper \( i \) and the superscript denotes all the shippers who bundle their items with shipper \( i \). Therefore, \( g_{ij}^k \) is the total cost for shipper \( i \) when shipper \( i \) bundles its items with shipper \( j \). The superscripts of \( t \) and \( g \) can have more than two elements,
where the number of elements represents the number of shippers who bundle their items together.

3.2. Continuous Model without Time Constraints. In this section, we consider the scenario with no time constraints. Four different cases are examined. In Case 1 and Case 2, we present total delivery costs (i.e., transportation and fixed vehicle costs) when one shipper uses and does not use a consolidation center, respectively. In Case 3 and Case 4, we present total delivery costs when multiple shippers use and do not use consolidation centers. In computing the costs, we ignore holding costs (renting cost and stationary inventory cost) because in daily deliveries there is a specific number of deliveries that have to be performed every day. Holding costs, therefore, are constant whether a consolidation center is used or not. We can also ignore pipeline inventory costs for daily deliveries when there is a deadline before which deliveries have to be completed. Instead of delivery deadlines, however, we assume a work shift hour limit for drivers and assume that this constraint is always binding when compared to the delivery deadline time. The four presented cases are distinguished according to the number of shippers (one or multiple) and the presence of a consolidation center. We now present the four cases.

3.2.1. Case 1: One Shipper without Consolidation. When one shipper completes its deliveries without any consolidation, the total cost is composed of line-haul and local transportation costs. Line-haul travel time for each vehicle consists of traveling from the distribution center to the end of the corridor \( (x_{ab}/v_1) \), from the end of the corridor to the start of the service area \( (r_1/v_2) \), and from the start of the service area to the center of each zone. To account for the latter, we acknowledge that there are \( n_w \) strips that stretch in the radial direction; that is, there are \( n_w \) zones in the transverse direction, where every strip is the collection of a set of adjacent zones in the radial direction. \( n_w \) is calculated as \( n_w = r/\theta_w \), where \( w \) is obtained from equation 4. In each strip, the line-haul travel time from the start of the service area to the center of zone \( j \) is \( (j - (1/2))/v_2 \). This cost, however, has to be summed up over all zones in the strip, which gives \( \sum_{j=1}^{n_w} [(j - (1/2))/v_2] \), where \( n_w \) is the number of zones along the radial direction (i.e., number of zones in each strip) and is \( r/l \). Given that there are \( N_i \) vehicles, the total line-haul travel time is (Congestion effects of general population are assumed to be included in the speed parameters (i.e., \( v_2 \)).

\[
t_{lh} = 2 \left( N_i \left( \frac{x_{ab}}{v_1} + \frac{r_1}{v_2} \right) + n_w \frac{\sum_{j=1}^{n_w} [(j - (1/2))/v_2]}{v_2} \right),
\]

where the term in the bracket is multiplied by 2 to account for the return trip and \( x_{ab} \) is the distance from shipper \( i \)'s distribution center to the end of the corridor. In (5), \( N_i \) is the total number of vehicles (i.e., the product of the number of zones along the radial direction \( n_r \) and the number of zones along the transverse direction \( n_w \)).

\[
N_i = n_r n_w
\]
By setting $n_l$ to $r/l$, (7) can be rewritten as
\[ r^2 \frac{2}{J} \]

Equation (8) requires $n_l$ to be an integer. This requirement, however, can be relaxed because $r$ is relatively larger than $l$ and a small change in $l$ results in a large change in $r/l$. Moreover, as $r$ itself is an approximation of the geographic shape of the service region, one can slightly modify it to ensure the integer nature of some variables. Newell and Daganzo [18] explain this tactic in partitioning a service region for a variant of the vehicle routing problem. Inputting (7) and (9) into (6), and considering that $nl$ can be relaxed because $r$ is relatively larger than $l$, equation (6) can be rewritten as
\[ t_{li}^{lh} = 2n_w \frac{r}{J} \left( \frac{x_{ic} + r_1 + r}{v_1} + \frac{r}{2v_2} \right). \]

We now move on to local travel time. The local travel time for every zone is the product of the number of deliveries in that zone and the transversedistance per customer, which is $\sqrt{2/3}$. Thus, the total local travel time for shipper $i$ is
\[ t_{li}^{hc} = 2n_w \frac{r}{J} \left[ \frac{wl}{v_2} \sigma \frac{r}{\sqrt{2/3}} \right], \]
where the term outside the brackets is the total number of vehicles and the term $wl \sigma$ inside the brackets is the total number of deliveries in each zone. For a better exposition, we have omitted the service time at each delivery point. By simplifying the sum of (9) and (10), we have
\[ t_i = \left[ 2n_w \frac{r}{J} \left( \frac{x_{ic}}{v_1} + \frac{r_1 + r}{v_2} \right) \right] + \left[ \frac{r^2 \theta \sqrt{2\sigma}}{v_2} \right]. \]

Equation (11) (second term; also shown in (10)) shows that the length of the zones does not impact the total local travel time. This is intuitive because no matter what zone width and length are used, leading to a different number of vehicles, the vehicles have to visit all delivery points. The total local travel time does, however, increase with $\sigma$, which means higher delivery densities require longer local travel times. Equation (11) also shows that the total line-haul travel time (first term in (11)) is only a function of the zone length $l$, where $t_i$ decreases with $l$. This means that in cases where there are no hours-of-service constraints, the shipper selects the largest possible value of $l$, which is
\[ l^* = \min \left( \frac{c_{max}}{u \sigma}, r \right). \]

For the case of small items where vehicle capacity is not reached, one can say $l^* = r$ and $n_l = 1$. For bigger items where vehicle capacity is reached, we have $l^* = c_{max}/u \sigma$ and $n_l = rw \sigma/c_{max}$. Given (11), we define the total cost of delivery for shipper $i$ as
\[ g_i = rt_i + pn_w n_l, \]
where the first term is the monetary value of travel time $t_i$ and the second term is the total fixed cost associated with each vehicle. $r$ and $p$ in equation (13) are the unit costs for each unit of travel time and each dispatched vehicle, respectively. Minimizing the total cost of equation (13) still gives the optimal $l$ values of equation (13).

### 3.2.2. Case II: One Shipper with Consolidation

In Case II, we consider the situation where one shipper uses a consolidation center. The travel time for this case is composed of two parts. The first part (also known as the first leg) is the total travel time from the distribution center to the consolidation center, and the second part (aka, second leg) is the total travel time from the consolidation center to all delivery points. The latter is somewhat similar to the travel time of Case I with the exception that the distribution center is now located at the consolidation center. The total travel time of Case II is
\[ t_i^l = \left[ 2N_i^l x_{ic} \right] + \left[ 2n_w \frac{r}{J} \left( \frac{x_{cc}}{v_1} + \frac{r_1 + r}{v_2} \right) + \frac{r^2 \theta \sqrt{2\sigma}}{v_2} \right], \]
where the first term is the total cost of the first leg of the trip. We assume that vehicles are filled to capacity in the first leg of the trip. This is a reasonable assumption since any fill volume lower than the vehicle capacity, in the case of no hours-of-service constraints, leads to more required vehicles and higher costs. The total number of required vehicles for the first leg is
\[ N_i^l = \frac{\sigma S}{c_{max}}, \]
where each of the $N_i^l$ vehicles in (14) has to travel from the distribution center to the consolidation center a distance of $x_{ic}$. The same distance has to be traveled from the consolidation center back to the distribution center. This explains why the first term in (14) is multiplied by 2. The second term of (14) is the total travel time from the consolidation center to all delivery points. The total cost for shipper $i$ when using the consolidation center is equal to
\[ g_i = rt_i^l + \rho \left( N_i^l + N_i^2 \right), \]
where $N_i^2$ is the total number of vehicles for the second leg of the trip and is equal to $(n_w r/l)$. According to (14), and similar to (11), shipper $i$ has to select the largest allowable $l$ to minimize its costs. Therefore, for Case II, similar to Case I, the $l$ value is obtained from equation (13).

We now investigate whether it is beneficial for one shipper to use a consolidation center alone. This is an economical action if the cost of using the consolidation center is lower than the cost of direct delivery. The difference between the two costs is
\[ g_i - g_i = \tau \left( 2N_i^{x_e}v_{x_j} + 2n_{w_i}(x_{i} - x_{i}) \right) + \rho N_i \],

where the first term is the extra cost of travel time for the first leg of the shipment, the second term is the difference in the cost of line haul travel time, and the third term is the extra fixed vehicle cost for the first leg of the shipment. Since the vehicles on the first leg of the trip are full, \( N_i \) can also be written as \( n_x n_y \). This simplifies equation (17) to \( \rho N_i \), which means that there is always an extra cost of shipment when a consolidation center is used and vehicles are full. In summary, Case II is analogous to changing shipper \( i \)’s distribution center location from \( x_i \) to \( x_e \) and paying for the additional fixed vehicle cost \( \rho N_i \).

3.2.3. Case III: Multiple Shippers without Consolidation. In Case III, there are multiple shippers that do not use the consolidation center. Therefore, the total cost for each shipper is as presented in equation 14.

3.2.4. Case IV: Multiple Shippers with Consolidation. In Case IV, multiple shippers use a consolidation center. We present the situation where two shippers bundle their freight at the consolidation center, which can be extended to any number of shippers. When two shippers \( i \) and \( j \) decide to consolidate their freight, the average delivery density becomes \( \sigma_i + \sigma_j \). This leads to the following total travel time:

\[ t_{ij}^* = \left[ 2N_i^{x_e}v_{x_j} + \frac{2N_i^{x_e}v_{x_j}}{c_{\text{max}}} \left( x_{i} + r_{1} + \frac{r}{2v_{2}} \right) \right] + \left[ \frac{\sqrt{2}}{3} \frac{N_i^{x_e}v_{x_j}}{v_{2}} \left( \sigma_i + \sigma_j \right) \right], \]

where the first term is the total travel time from the consolidation center to the distribution center and back \((2x_{i}/v_{x_j})\) when there are \( N_i \) vehicles (similar to equation (14)). The second term is the line-haul travel time to the zones and back. The third term is the local travel time. Both terms are obtained from (11). The first term is dependent on \( \sigma_i \) (due to (15)) whereas the second and third terms have \( \sigma_i + \sigma_j \). This is because the leg of the delivery from the distribution center to the consolidation center is the shipper’s responsibility and is composed solely of its items. The second two terms, however, involve both shippers.

Although equation (18) is the total travel time, some modifications have to be applied before computing the total cost for each shipper. The second and third terms of equation (18) are the total travel time incurred by both shippers. This cost has to be divided between the two with a reasonable ratio. Assuming that shipper \( i \) pays only for his/her share of the cost, the last two terms in equation (18) are multiplied by \( \sigma_i/\sigma_i + \sigma_j \) for shipper \( i \). This ensures that a shipper with more customers pays a higher proportion of the total cost. Any other ratio may be used for cost allocation. One alternative would be to assume that cost allocation is itself a variable, which would lead to a game theory model with shipper coalition formation.

The total travel time cost for shipper \( i \) is

\[ t_{ij} = \tau \left( 2N_i^{x_e}v_{x_j} + \frac{2N_i^{x_e}v_{x_j}}{c_{\text{max}}} \left( x_{i} + r_{1} + \frac{r}{2v_{2}} \right) \right) \]

\[ + \tau \left[ \frac{\sqrt{2}}{3} \left( \frac{\sigma_i}{\sigma_i + \sigma_j} \right) r \left( \frac{\sigma_i}{\sigma_i + \sigma_j} \right) \right], \]

where setting \( \sigma_j \) to zero gives (15), which is travel time cost of shipper \( i \) when using a consolidation center on its own. The total fixed vehicle cost for this case is

\[ \rho \left( N_i + \frac{\sigma_i}{\sigma_i + \sigma_j} \right), \]

where the first term in the brackets is the fixed vehicle cost of delivery from the distribution center to the consolidation center, which is incurred only by shipper \( i \), and the second term is the fixed vehicle cost of delivery, divided between shippers \( i \) and \( j \), from the consolidation center to the delivery points. The total delivery cost for Case IV is therefore equal to the sum of equations (19) and (20) and is denoted by \( g_i \). Rational shippers will only agree to consolidate their items if the total consolidation cost \( g_i \) is lower than the cost of no consolidation \( g_i \). By setting \( g_i \geq g_i \) and simplifying the equations, we have

\[ \tau \left( \frac{\sqrt{2}}{3} \sqrt{\sigma_i} \geq \tau \left[ \frac{\sqrt{2}}{3} \frac{\sigma_i}{\sigma_i + \sigma_j} + \rho \frac{\sigma_i}{c_{\text{max}}} \right] \right). \]

where the left-hand term and the first term on the right-side are the local travel time costs of shipper \( i \) with and without consolidation, respectively. The second term on the right-side is the fixed vehicle cost in the second leg of the deliveries from the consolidation center to the customers. Equation (21) implies that in cases where there are no time constraints, and vehicles are filled to capacity, the line-haul travel time costs do not change, but the local travel times do change with respect to the given customer densities. There is also an additional fixed vehicle cost from the consolidation center to the customers. Equation (21) will not hold in many practical cases when the fixed cost of a vehicle is larger than the monetary value of time \( (r \ll \rho) \). We conclude from this section that when vehicles of a shipper are filled to capacity, it is never economical to consolidate with any other shipper. This conclusion is analogous to Daganzo [11].

3.3. Continuous Model with Time Constraints. We now consider the scenario with time constraints. We apply time constraints in terms of allowable work shift hours of the drivers. We present four cases similar to Section 3.2.

3.3.1. Case I: One Shipper without Consolidation. We assume that the lengths of all the zones are equal. This leads to longer travel times for drivers who have to travel to the furthest zones in the service area. Although this may seem
unfair, drivers in practice change shifts on a weekly or monthly basis so that no one is mistreated. Each day, however, the travel time for every driver has to be smaller than the allowable work shift hours. This constraint is formulated as

\[
2 \left( \frac{x_{ic}}{v_1} + \frac{r_1}{v_2} \right) + 2 \left( \frac{r - l/2}{v_2} \right) + \frac{r^2 \theta}{v_2} \sqrt{v_2} \leq T. \tag{23}
\]

The first term on the left-hand side of equation (22) is the travel time from the distribution center to the start of the service area and back. The second term is the travel time from the start of the service area to the center of the last zone in each strip. The third term is local travel time in one zone. Simplifying (23) gives

\[
l \leq \min \left( \frac{T - 2 \left( \frac{x_{ic}}{v_1} + \frac{r_1}{v_2} \right)}{v_2}, \frac{c_{\text{max}}}{w \sigma} \right). \tag{24}
\]

Given \( l \), the total cost is

\[
g_i = \frac{\tau}{2} \left( \frac{2n_w}{r} + \frac{r_1}{v_1} + \frac{r_1}{v_2} \right) + \frac{r^2 \theta}{v_2} \sqrt{v_2} \frac{2 \sigma}{3} + \rho \left[ n_w \frac{r}{l} \right], \tag{25}
\]

where the first term in the bracket is the travel time cost and the second term is the fixed vehicle cost. Equation (24) is the same as (13) except for the input \( l \) value.

3.3.2. Case II: One Shipper with Consolidation. In Case II, there are two different sets of drivers. Those who drive the vehicles from the distribution center to the consolidation center (first leg) and those who drive the vehicles from the consolidation center to the delivery points (second leg). We assume that vehicles are filled to capacity in the first leg of the shipment. In other words, we assume that the work shift constraint is not binding in the first leg of the trip. Given the assumptions, the total vehicle cost for Case II is

\[
g_i^j = \frac{\tau}{2} \left( \frac{2n_w}{r} + \frac{r_1}{v_1} + \frac{r_1}{v_2} \right) + \frac{r^2 \theta}{v_2} \sqrt{v_2} \frac{2 \sigma}{3} + \rho \left[ n_w \frac{r}{l} \right], \tag{26}
\]

where the first and second brackets present the total travel time and fixed vehicle cost from the consolidation center to the delivery points, respectively. The third term is the total travel time and fixed vehicle cost from the distribution center to the consolidation center. \( l \) in (25) is

\[
l \leq \min \left( \frac{T - 2 \left( \frac{x_{ic}}{v_1} + \frac{r_1}{v_2} \right)}{v_2}, \frac{c_{\text{max}}}{w \sigma} \right). \tag{27}
\]

where \( l \) is smaller than the value presented in equation (24). Shippers use the consolidation center only if their new cost (equation (26)) is lower than their original cost (equation (25)). Hence, the difference between the cost of using a consolidation center and not using one (when \( l = \left( \frac{T - 2 \left( \frac{x_{ic}}{v_1} + \frac{r_1}{v_2} \right)}{v_2} \right) \)) is

\[
g_i^j - g_i = y_k \left[ \sqrt{\theta \left( \frac{y_0 - \left( \frac{x_{ic}}{v_1} \right)}{y_1 + \frac{x_{ic}}{v_1}} \right)} - \frac{y_0}{y_1} \right] + \left[ \sigma \left( r_2 x_{ic} + y_3 \right) \right], \tag{28}
\]

where

\[
y_0 = \frac{x_{ic}}{v_1} + \frac{r_1}{v_2} + \frac{\rho}{2 \tau},
\]

\[
y_1 = \left[ T - 2 \frac{x_{ic}}{v_1} - 2 \frac{r_1}{v_2} - 2 \frac{r}{2v_2} \right] v_2,
\]

\[
y_2 = \frac{2 \tau r^2 \rho}{v_1 c_{\text{max}}},
\]

\[
y_3 = \frac{\theta r^2 \rho}{c_{\text{max}}},
\]

\[
y_4 = \rho \frac{r^2 \theta}{v_2},
\]

are constants. The first bracket in equation (27) presents the difference in the line haul and local travel time costs of Cases I and II, and the second term presents the extra travel time cost and fixed vehicle cost of travel from the distribution center to the consolidation center. While the first term is always negative, the second term is always positive. Equation (27) shows that increasing \( \sigma \) decreases the possibility of a shipper to relocate its distribution center closer to the service area. That is, for high \( \sigma \) values, the extra cost of transporting commodities from \( x_i \) to \( x_c \) (second term of (27)) is high, thus overcoming the “potential benefit” of using the UCC. Moreover, for higher \( \sigma \) values, the first term of (27) (“potential benefit”) grows more slowly than the second term.

Given a fixed \( \sigma \) and variable consolidation center location \( x_{ic} \), (27) can have zero, two, or, in very specific cases, one root in the positive \( x \)-direction. If there are no roots, the equation is always positive, and it is never economical to use a consolidation center. This is a common case since \( y_3 \) is a large number that incorporates the fixed cost of vehicles. If there are two roots, there then exists a range of \( x_{ic} \) (between the two roots) where using the consolidation center is economical.

3.3.3. Case III: Multiple Shippers without Consolidation. Similar to Case III in section 3.2, the total cost of each shipper is calculated from equation (25).
3.3.4. Case IV: Multiple Shippers with Consolidation. The total cost when shippers \( i \) and \( j \) bundle their items is

\[
g_i^{ij} = \frac{\sigma_i}{\sigma_i + \sigma_j} \left[ \frac{r^2 \theta}{l} \left( \frac{2(\sigma_i + \sigma_j)}{\sigma_i + \sigma_j + \frac{r_1}{v_1} + \frac{r}{2v_2}} \right) \right] + \rho \frac{\sigma_i}{\sigma_i + \sigma_j} \left[ \frac{r^2 \theta}{l} \left( \frac{\sigma_i + \sigma_j}{6} \right) \right] + \frac{r^2 \theta \sigma_i}{\epsilon_{\text{max}}} \left( \frac{2x_{\text{ic}}}{v_1} + \rho \right),
\]

where the first term (in brackets) is the line haul and local travel time cost for the second leg of the trip, the second term in brackets is the fixed vehicle cost for the second leg of the trip, and the third term is the fixed vehicle cost and travel time cost for the first leg of the trip. The costs related to the second leg of the trip (first and second term) are divided between the shippers, whereas the first leg cost (third term) is incurred only by shipper \( i \). The length of the zones in equation (29) can be obtained from equation (27). One major assumption of equation (29) is that shippers \( i \) and \( j \) do not share any customers. That is, \( \sigma_i \) and \( \sigma_j \) are obtained from two separate sets of customers, which are mutually exclusive. We relax this assumption in Section 4.

Assuming that all shippers are rational, shipper \( i \) would only choose to bundle its items with shipper \( j \) if the new cost (29) is lower than the original cost (25). Therefore, we have

\[
g_i^{ij} - g_i = y_4 \sqrt{\frac{\sigma_i}{\sigma_i + \sigma_j}} \left( \frac{y_0 - \left( x_{\text{ic}}/y_1 \right)}{y_1 + x_{\text{ic}} (2v_2/v_1)} \right) \left( \frac{y_0}{y_1} \right) + \frac{y_4}{v_2} \sqrt{\sigma_i} \left( \frac{\sigma_i}{\sigma_i + \sigma_j} \right) \left( y_2 x_{\text{ic}} + y_3 \right),
\]

where the first and the second terms present the difference in the line haul and local travel time cost, respectively. The third term is the fixed vehicle and travel time cost of the first leg of the trip from the distribution center to the consolidation center. The first and second terms of equation (30) are always negative (or zero) and the third term is always positive. The following can be concluded from equation (30):

1. Shipper \( i \) only considers shipper \( j \)'s delivery point density when deciding whether consolidation with shipper \( j \) is a beneficial move. Therefore, the location of shipper \( j \)'s distribution center is irrelevant to shipper \( i \)'s decision.

2. Increasing \( \sigma_i \) makes equation (30) more negative and increases the savings of shipper \( i \). That is, every shipper would be better off consolidating items with other shippers with higher customer density.

3. Setting \( \sigma_j \) to zero in equation (30) would imply that shipper \( i \) is consolidating its own items. Hence, the second term becomes 0, and equation (30) turns into equation (28).

4. The location of the consolidation center also impacts shipper \( i \)'s decision.

5. Similar to Case II of Section 3.3, if equation (30) has two roots, there is a range for the location of the consolidation center which would benefit shipper \( i \).

These five features, although pertinent to urban consolidation, are similar to the conclusions of Daganzo [11] on hub locations. This highlights the consistency between urban consolidation and higher-level hub consolidation.

4. Urban Consolidation Center Location with Two Shippers

In this section, we use the equations from Section 3 to find the optimal location of a consolidation center under shipper rationality (i.e., a shipper will use a consolidation center if its costs can be reduced). The optimal location is dependent on what objective function we pursue. In Section 4.1, we define three different objective functions that have benefits and drawbacks for shipper and for residents of the service region. In Section 4.2, an example problem is presented, and sensitivity analysis is provided in Section 4.3.

4.1. Objective Functions. In addition to the costs explained in Section 3, we define an investment cost for the UCC, which includes the cost of acquiring land. It is reasonable to assume that this cost is highest when closer to the service area (i.e., end of the corridor at \( x_c \)) and decreases when located farther away. Similar assumptions can be found for locating Park-and-Ride facilities.

We define the land cost of the UCC (denoted by \( f \)) as

\[
f = F \left( 1 - \frac{x_{\text{ac}}}{B} \right)^{3},
\]

where \( F \) and \( \alpha \) are constants. \( f \) is divided between the shippers with respect to the ratio of their customer densities. For instance, given shippers \( i \) and \( j \), shipper \( i \)'s land cost would be \( f \sigma_i / (\sigma_i + \sigma_j) \).

Given (32), we define three objectives:

1. MaxiMax: maximize the maximum of the total savings of all shippers

2. MaxiMin: maximize the minimum of the total savings of all shippers
(3) MinVKT: minimize total Vehicle Kilometers Traveled (VKT) in the service area

The first two objective functions are likely to be adopted by the shippers themselves, whereas the third is likely to be imposed by a third-party authority. The three objectives are depicted in Figure 4, illustrating two curves pertaining to the total savings of the two shippers (i.e., $[(g_i - g_j^i) - f(\sigma_i/\sigma_i + \sigma_j)]$ and $[(g_i - g_j^i) - f(\sigma_i/\sigma_i + \sigma_j)]$). As is depicted, under the MaxiMin objective function, both shippers obtain the same value of savings (i.e., "a"), whereas, under the MaxiMax objective function, shipper $i$ receives a savings of "a*" monetary units and shipper $j$ receives a savings of "a**" monetary units. The MinVKT objective function in Figure 4 occurs at a location closest to the boundary of the service region where both shippers acquire a positive value of savings. Since the MinVKT is used as a measure of maximum community welfare, it is likely that the third-party authority does not consider shipper rationality and enforces all shippers to use the UCC, thus leading to negative savings for some shippers. For the first two objectives where the UCC is initiated by the shippers themselves, the savings for each shipper has to be a positive value. Hence, one constraint needs to be added for each shipper to ensure the nonnegativity of savings. The feasibility of a solution depends on whether or not the nonnegativity constraints are respected.

As the UCC gets closer to the service area, the second leg vehicles, dispatched from the UCC to the service region, obtain a higher truck load because they are granted more routing time. From some point on (denoted by $x_{ce}^*$) when the vehicles reach their capacity, locating a consolidation center closer to the service region (i.e., $x_{ce} \leq x_{ce}^*$) does not affect the routing configuration because in all such cases the vehicles are dispatched full. Figure 5 shows that the MinVKT objective function can be obtained when the UCC is within some range. Under MinVKT, similar to MaxiMax, the shippers would have different saving values (i.e., "b" and "b**"). Intuitively, and under general conditions, locating the UCC farther from the service region (i.e., right at $x_{ce}^*$) would be better, as shippers could benefit through positive savings. This leads to pareto-optimality between the MinVKT objective and the other two objectives.

Pareto-optimality is closely tied to shipper’s rationality. The decision by a third-party authority to construct a UCC should consider whether shippers would benefit from the UCC or not. As indicated earlier in the introduction, there are many failure cases of UCCs due to a lack of acknowledgement of shipper rationality. Hence, a well-devised UCC should consider both the MinVKT and one of the first two objectives. This decision can be made through the study of pareto-optimality between the objective functions.

4.2. Example Problem with Two Shippers. In this section, we present an example problem with two shippers. The parameters of the problem have been set up similar to Jabali et al. [8]. We model a corridor and a service region where $B = 150$ km, $r_i = 35$ km, $r_j = 70$ km, $\theta = (\pi/3)$, $v_i = 70$ (km/h), $v_j = 35$ (km/h), and $S = 5000$ km$^2$. The two shippers $i$ and $j$ are originally positioned so that $x_{ce} = 145$ and $x_{ce} = 150$, unless stated otherwise. Customer densities for both shippers are set to 0.9, unless mentioned otherwise. We set $\tau_i = 65$, $\tau_j = 25$, $\rho_i = 20\tau_i$, and $\rho_j = 20\tau_j$. The maximum land cost of the UCC is set to $F = 6 \times 10^5$ monetary units and $\alpha = 1$. Vehicles (trucks) have a capacity of 250 units.

Figure 6 presents the savings curves of the two shippers and the optimal location of the UCC under the three objective functions. Under MinVKT, the UCC is positioned so that vehicles are fully dispatched, and the shippers have positive savings. Figure 7 illustrates the respective routing zones of shipper $i$ under (a) no consolidation center, (b) the MaxiMin objective function, (c) the MaxiMax objective function, and (d) the MinVKT objective function. As depicted, $n_w$ (number of routing zones in the transverse direction) is smaller for the case of no consolidation (Figure 7(a)) but larger for the other three cases (Figures 7(b)–7(d)). Therefore, using a consolidation center generally increases $n_w$ but decreases $n_l$ depending on the chosen objective function.
4.3. Shipper’s Distribution Center Location, Customer Density, and Vehicle Capacity. We vary the location of shipper $j$’s distribution center ($x_{je}$) from 150 to 0. As illustrated in Figure 8, $x_{je} = 135$ is the minimum required distance in order to justify constructing a UCC. That is, shipper $i$ can consolidate with shipper $j$ (with a specified $\sigma_j$) only if $x_{je} \geq 135$. As was elaborated in Section 3, changing shipper $j$’s location does not impact the savings curve of shipper $i$; it does, however, impact the optimal location of the UCC and whether constructing one is beneficial at all.

Figure 9 presents the savings curve of the two shippers with respect to the customer density of shipper $j$ when increased from 0.3 to 4. As illustrated, the MaxiMin objective function and the MaxiMax objective function (for shipper $j$) have asymptotic features with respect to shipper $j$’s customer density (due to equation (30)) and converge to $2.9 \times 10^5$. This is the maximum value of savings that shipper $i$ can obtain. Furthermore, as shipper $j$’s customer density increases, shipper $j$ incurs a higher UCC land cost ($\lim_{\sigma_j \rightarrow \infty} f_{\sigma_j / \sigma_i + \sigma_j} = f$) and shipper $i$ uses the consolidation center for free ($\lim_{\sigma_j \rightarrow \infty} f_{\sigma_j / \sigma_i + \sigma_j} = 0$).

4.4. Shared Customers between Shippers. One assumption made in Section 3 is that shippers do not share customers; that is, $\sigma_i$ and $\sigma_j$ are obtained from two separate sets of customers, which are mutually exclusive. We relax this assumption by introducing $\omega_i$ and $\omega_j$ as the shared customer
Figure 8: Savings curves of the two shippers when $x_{ie} = 135$ km.

Figure 9: Savings curve with respect to customer density of shipper $j$.

Figure 10: MaxiMin objective function with respect to $\omega_i$. 
ratio of shipper $i$ and $j$. For instance, a $\omega_i$ value of 0.5 indicates that half of shipper $i$’s customers are also shipper $j$’s customers. Hence, we have

$$S_i \omega_i = S_j \omega_j,$$

(33)

where the left-hand side is the total number of shipper $i$’s customers that are shared with shipper $j$, and the right-hand side is the total number of shipper $j$’s customers that are shared with shipper $i$. Equation (32) can be simplified to $\sigma_i \omega_i = \sigma_j \omega_j$. In such cases, the total customer density changes from $\sigma_j + \sigma_i$ (which was assumed in Section 3) to $$(1 - \omega_j) \sigma_j + (1 - \omega_i) \sigma_i + \sigma_i \omega_i,$$

(34)

where the first (second) term represents customer $j$’s ($i$’s) exclusive customers, and the third term represents the customers shared between both shippers. Substituting equation (31) into equation (32) gives

$$\sigma' = \sigma_i \left(1 - \omega_i + \frac{\omega_i}{\omega_j}\right).$$

(35)

By substituting equation (32) into (29), the new cost of shipper $i$ when consolidating with shipper $j$ (i.e., $g_i^j$) is

$$g_i^j = r \frac{\alpha_i}{1 + \alpha_i} \left[r^2 \theta + \frac{2 \alpha_i}{3} \frac{v_2}{v_1} + \frac{r_1}{v_2} \right] + \left(1 - \omega_i \right) \theta \sigma_i \left[\frac{2 \tau x_i}{v_1} + \rho \right],$$

(36)

where $l$ is still obtained from equation (27). Figure 10 presents the total amount the maximin objective function increases with respect to $\omega_i$ when $\omega_i = \omega_j$ and $\sigma_i = \sigma_j = 0.9$. This means that shipper $i$’s savings would increase if there are more customers shared between the two shippers, leading to a lower total number of stops in the service region.

5. Conclusions

Locating UCCs is of strategic importance to shippers and local governments. Shippers can, in certain cases, create coalitions to reduce their logistics costs, and local governments can enhance traffic conditions in urban areas by reducing total freight vehicle trips. Despite their benefits, there are many instances of failed urban consolidation centers. One major underlying factor in their failure is lack of shipper participation, as shippers are unwilling to use the facility if they accrue any additional logistics costs. This study investigates the optimal location of a UCC under shipper rationality to consider the financial incentive of each shipper. Results show that shippers would only consolidate when drivers are constrained by delivery time and vehicles are not full. The proposed framework can be used for strategic decision-making when considering at a high level whether the deployment of a UCC would be beneficial.

Sensitivity analysis of the closed-form equations leads to managerial insights. For instance, the model shows that given a fixed UCC location, each shipper considers other shippers’ customer densities as the only influential factor in deciding whether to consolidate. Moreover, each shipper benefits from consolidating with shippers of high customer density to cut down costs. Analysis of the UCC location shows that there exists a stretch of the corridor where user rationality is satisfied. Construction of the UCC anywhere beyond this range could lead to failure.

The proposed framework of this study can be extended in future research. For any number of shippers and for each combination of shippers, the cost savings curves can be used to find the optimal location of the UCC. There are also some assumptions in this study that could be relaxed for future research. By taking heterogeneous commodities into account, the results could change, as shippers would be interested in other shippers’ commodity type as well as customer density. Moreover, relaxing the assumption of unlimited UCC capacity introduces competition (for UCC capacity) and collision (between shippers) to the model. Finally, consideration of a more realistic rectangular grid instead of a radial grid can enhance the accuracy of the results.

Data Availability

The data used and analyzed in this study are available in this paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

References


