Stability Analysis of Heterogeneous Traffic Flow with Connected and Automated Vehicles: Joint Consideration of Communication Failures and Driver Takeover

1. Introduction

In recent years, intelligence agents with connection characteristics have attracted much attention, especially in the field of connected and automated vehicles (CAVs). CAVs can take rational dynamics automatically by perceiving information with intervehicle communications [1, 2], including vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication. However, fully mature CAVs, which meet Level 5 in the Society of Automotive Engineers (SAE) J3016, are not available because of the current limitations on communication technologies [3]. Currently, vehicular communication has been introduced into multiple auxiliary driving systems to form the primary connected and automated vehicles, in which the cooperative adaptive cruise control (CACC) system is the most representative and has widely been applied in commercial vehicles [4, 5].

CACC vehicles take significant advantages of accurate perception and timely reaction over the traditional human-driven vehicles (HDVs) by introducing intervehicle communication [6]. Past works have proved the beneficial effects of the CACC system on improving traffic capacity, ensuring traffic safety, and reducing traffic emissions [7–10]. Yet, this automatic driving technology is also accompanied by some unexpected safety risks under sudden communication failures caused by all kinds of equipment failures and malicious cyberattacks [11, 12]. Taking malicious cyberattacks as an example, they can result in two categories of communication failures, covering bogus messages and transmission delay [13]. The former makes vehicles receive bogus messages, such as overestimated or underestimated...
headway and speed, from the preceding vehicles. The latter makes message packets delayed in delivery by constructing a jammed communication channel. Also, there are other communication failures, such as message interruption [14], which can be recognized as transmission delay with an infinite delay time.

Considering that the stability of CACC vehicles will be greatly disturbed by communication failures, different methods are proposed to avoid the potential collision risks caused by anomalous dynamics, in which the representative ones are active forward collision warning (FCW) and drivers’ takeovers [15, 16]. The FCW will remind the drivers to take over the vehicle in time once it detects a serious speed fluctuation or potential collision risk, and then, anomalous vehicle dynamics can be effectively avoided with the correct perception of drivers who are not directly affected by communication failures [17]. Therefore, it appears that CACC vehicles have opportunities to turn to the HDVs in case of communication failures, making the traffic flow with CACC vehicles have characteristics of heterogeneity. Besides, recent studies point out that it will take great efforts for the market penetration rate (MPR) of autonomous vehicles to grow to a considerable proportion [18], so the heterogeneous traffic flow mixed with CACC vehicles and HDVs is bound to exist for a long time. Drivers’ delayed reaction in HDVs is also a potential risk for speed oscillations and traffic stability. These points all indicate that it is better to analyze the impact of communication failures on traffic stability in heterogeneous traffic flow.

In view of the mentioned topic, this study aims at analyzing the stability of heterogeneous traffic flow with CACC vehicles and HDVs by considering communication failures and driver takeover together. Partial CACC vehicles request drivers’ takeover when they have anomalous vehicle dynamics, and then, CACC vehicles turn to HDVs. The stability of different traffic flow configurations was firstly analyzed with the possibility of communication failures and the proportion of drivers’ takeovers. Then, numerical experiments were simulated to validate the stability of heterogeneous traffic flow under several typical speed-oscillation cases. This study helps to distinguish the most critical cases of traffic instability with communication failures and gives reasonable suggestions for heterogeneous traffic flow to keep stability.

The remainder of this study is organized as follows. Section 2 makes a literature review on stability analysis of heterogeneous traffic flow. Section 3 introduces the methodology of traffic flow configuration and car-following models. Section 4 makes the theoretical analysis of traffic stability, and Section 5 takes the numerical experiments to validate the theoretical results. Then, Section 6 conducts a discussion on traffic capacity. Finally, Section 7 concludes this study with the main findings and future directions.

2. Literature Review

Relevant research on the stability of heterogeneous traffic flow with CAVs has attracted much attention in recent years. The initial approach is to introduce the CAVs into the traffic flow of HDVs and evaluate the impact of CAVs on traffic stability [19, 20], where some common variables such as the MPR and desired headway of CAVs are widely discussed. Afterwards, some novel factors are considered in mixed-flow scenarios with a deepening understanding of intervehicle communication, leading to more realistic and richer results in the stability analysis. Consequently, this section does a literature review in accordance with the classifications of discussed factors in relevant research, including CAVs/drivers’ properties, traffic operation, and communication failures.

2.1. CAVs/Drivers’ Properties. In mixed traffic, some inherent properties of CAVs and HDVs will affect the traffic stability and have widely been considered in existing studies to make the research scenarios more realistic. The discussed properties of CAVs concentrate on communication delays and information flow topology (IFT), and those of HDVs concentrate on perception-reaction time (PRT) and drivers’ attitudes.

Communication ability is a key characteristic of CAVs, inevitably accompanied by communication delay and closely related to the IFTs. In terms of communication delay, Zhang et al. [21] incorporated it explicitly into CACC control design and analyzed three stability properties: local stability, string stability, and traffic flow stability. Jin and Orosz [22] designed an optimal control for connected vehicle systems with communication delay based on the linear quadratic regulation. Ruan et al. [23] jointly considered platoon organization and multiple time delays in the stability analysis of heterogeneous traffic flow. The results showed that time delays have a slightly negative impact on linear stability and demonstrated the necessity of introducing communication delays. Similar conclusions can be found in Jia and Ngoduy [24], Qin and Orosz [25], and Tian et al. [26]. In terms of IFTs, the predecessor following (PF) topology is most widely studied and regraded as an initial topology for intervehicle communication. Connected vehicles can only receive information from their immediately preceding vehicles under the PF topology, and traffic stability cannot be ensured when intervehicle communication is interrupted [14, 27]. To solve this problem, multi-anticipations were gradually considered in previous research, with which connected vehicles can receive information from multiple preceding vehicles [24, 28]. The representative IFTs with multi-anticipations include two predecessors following (TPF), two predecessor-leader following (TPLF), and multiple predecessor-leader following (MPLF) [29]. Ruan et al. [30] constructed the stability criteria for CAV-HDV mixed flow with multiple IFTs and proved that multi-anticipations could efficiently improve linear stability.

Unlike automated vehicles controlled by the central processing unit, HDVs controlled by drivers have perception delays and need a certain time to take action in an emergency. Consequently, the PRT is a common factor considered for HDVs in past research; for example, Yao et al. [31] introduced multiple reaction times to the speed of the preceding vehicle, the speed difference, and the space.
Because CAVs follow their preceding vehicles closely with V2V communication instead of visual perception like HDVs, the mixed flow with CAVs has great differences from the homogeneous flow of HDVs in traffic operation. In this field, researchers consider some special traffic operation characteristics of the mixed flow and discuss the included vehicle types together. Specifically, traffic flow mixed with CAVs and HDVs is most commonly studied and serves as a basic scenario for stability analysis. Related research aims at evaluating the beneficial effects of CAVs on traffic stability and discussing the influence of traffic flow characteristics; for example, Yao et al. [19] presented that the unstable interval gradually decreases with the increase in the MPR of CAVs and the mixed flow always keeps stable when the MPR of CAVs is greater than 47%. The similar conclusion can also be found in the research of Chang et al. [23]. Furthermore, existing research realized that the information interaction would be interrupted between CAVs and HDVs, especially for the PP topology, and introduced vehicle degradation. The most common scene is that CACC vehicle degrades into an ACC vehicle when it follows an HDV, which can be seen in Qin et al. [35], Wang et al. [10], Yao et al. [34], Yu et al. [14], and Zhou et al. [36]. Their results showed that vehicle degradation could cause serious damage to traffic stability.

Besides, platoon organization is a significant strategy for the traffic flow of CAVs and has been stressed in past research. First, platoon organization leads to new vehicle types, such as string-leader and string-follower, in the traffic flow. Xiao et al. [8] suggested that the string-leader should take a larger interstring gap for the separation between strings than the intrastring gap taken by the string-followers. Existing results showed that platoon organization could contribute to the stable traffic flow from the perspective of reducing speed oscillation [8, 37]. After that, novel traffic analysis methods are gradually developed for the traffic flow with platoons of fixed length. Jia and Ngoduy [24] defined the criterion for platoon stability for full-CAVs platoons with the consideration of intervehicle communication. Liu et al. [38] proposed a model for head-to-tail string stability in mixed platoons, in which a lack of string stability for HDVs is allowed as it can be suitably compensated by CAVs sparsely inserted in the platoon.

2.3. Communication Failures. Communication security has attracted increasing concern in the connected environment. However, current communication network is vulnerable and easily damaged by malicious attacks and unexpected equipment failures. Then, communication failures will potentially appear this way and cause traffic instability. Wang et al. [13] divided the potential communication failures into three categories: bogus messages, communication delay, and collusion attacks. Both bogus messages and collusion attacks make CAVs receive underestimated or overestimated information of speed and position from preceding vehicles. The difference between them is that bogus messages only affect a single vehicle while collusion attacks act on multiple vehicles simultaneously. Besides, as one of the more serious communication failures, communication interruption was studied together with vehicle degradation by Yu et al. [14]. Earlier research mainly evaluated speed oscillation and traffic safety under communication failures. The results showed bogus messages on position and communication interruption have a significantly negative impact on traffic operation and cause serious stability-related and safety-related problems [39].

Recently, communication failures were gradually taken into consideration in the stability analysis of traffic flow with CAVs, and researchers contributed to proposing advanced controls to keep stability under communication failures. Specifically, Pirani et al. [40] presented a comprehensive study on the impact of IFTs on cyber-security and then revealed how the IFTs affect the ability to reject communication disturbances. Khattak et al. [41] emulated three types of cyberattacks, including message falsification, dedicated denial of service, and spoofing attacks, in the traffic flow with CAV platoons. The results revealed CAV platoons are unstable under these cyberattacks, and the worst case is represented by the message falsification attack. Zhai and Wu [42] introduced cyberattacks and additional delay time into a connected vehicle environment and then proposed a continuous delay feedback control for improving the stability condition. Relevant research can also be found in Dong et al. [43] and Kashyap et al. [44].

2.4. Literature Summary. Based on the above literature review, relevant research on the stability analysis of heterogeneous traffic flow with CAVs or CACC vehicles is summarized in Table 1.

According to the literature summary, mixed traffic with CAVs/CACC vehicles and HDVs accounts for the majority, with discussions of penetration rate and desired headway. In recent years, ACC vehicles, automated vehicles (AVs), and connected vehicles (CVs) are gradually introduced into the mixed traffic, accompanied with traffic operations of platoon organization and vehicle degradation. Table 1 also shows that although the stability analysis of heterogeneous traffic
flow has been widely studied in past works, few works like this study jointly considered communication failures and their coping strategy: driver takeover in the stability analysis. Because driver takeover will cause the vehicle conversion from CACC vehicles to HDVs, these two vehicle types are most affected by communication failures and driver takeover. Thus, the mixed flow only with CACC vehicles and HDVs is chosen in this study to make the research scenario clearer and can further study the impact of communication failures and driver takeover on traffic stability. After that, multiple factors such as CACC penetration rate, driver takeover probability, communication delay time, and information deviation degree are discussed to make this study more realistic.

### Table 1: Literature summary on the stability analysis of heterogeneous traffic flow.

<table>
<thead>
<tr>
<th>Publication</th>
<th>Vehicles</th>
<th>Research object</th>
<th>Analysis factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chang et al. [21]</td>
<td>CAV, HDV</td>
<td>Platoon organization</td>
<td>CAV penetration rate, Desired headway, Maximum platoon size</td>
</tr>
<tr>
<td>Chen et al. [34]</td>
<td>CACC, HDV</td>
<td>Platoon organization</td>
<td>CACC penetration rate</td>
</tr>
<tr>
<td>Hajdu et al. [45]</td>
<td>CAV, HDV</td>
<td>Communication delay</td>
<td>Delay time, Maximum uncertainty</td>
</tr>
<tr>
<td>Jia et al. [46]</td>
<td>CAV, HDV</td>
<td>Platoon organization</td>
<td>HDV penetration rate, Communication topology</td>
</tr>
<tr>
<td>Li et al. [28]</td>
<td>CAV, HDV</td>
<td>Platoon organization</td>
<td>Platoon size</td>
</tr>
<tr>
<td>Qin et al. [35]</td>
<td>CAV, ACC, CACC</td>
<td>Vehicle degradation</td>
<td>CACC penetration rate</td>
</tr>
<tr>
<td>Qin and Li [47]</td>
<td>CACC, HDV</td>
<td>Platoon organization</td>
<td>CACC penetration rate</td>
</tr>
<tr>
<td>Qin and Orosz [25]</td>
<td>CAV, HDV</td>
<td>Platoon organization</td>
<td>Penetration rate, Delay time</td>
</tr>
<tr>
<td>Rahman et al. [48]</td>
<td>CAV, HDV</td>
<td>Platoon organization</td>
<td>Penetration rate, Drivers’ compliance, Platoon policies</td>
</tr>
<tr>
<td>Rahman et al. [49]</td>
<td>CAV, CV, AV, HDV</td>
<td>Multi-anticipation</td>
<td>Penetration rate</td>
</tr>
<tr>
<td>Ruan et al. [23]</td>
<td>CAV, HDV</td>
<td>Platoon organization</td>
<td>CAV penetration rate</td>
</tr>
<tr>
<td>Ruan et al. [30]</td>
<td>CAV, HDV</td>
<td>Communication topology</td>
<td>Desired headway, Disturbance degree</td>
</tr>
<tr>
<td>Wang et al. [10]</td>
<td>CACC, ACC, HDV</td>
<td>Vehicle degradation</td>
<td>CACC penetration rate</td>
</tr>
<tr>
<td>Yao et al. [19]</td>
<td>CAV, HDV</td>
<td>Car-following behaviors</td>
<td>CAV penetration rate</td>
</tr>
<tr>
<td>Yao et al. [31]</td>
<td>CAV, HDV</td>
<td>Vehicle degradation</td>
<td>CAV penetration rate</td>
</tr>
<tr>
<td>This study</td>
<td>CAV</td>
<td>Communication failures</td>
<td>CACC penetration rate</td>
</tr>
<tr>
<td></td>
<td>HDV</td>
<td>Driver takeover</td>
<td>Driver takeover probability, Communication delay time</td>
</tr>
</tbody>
</table>

3. Methodology

#### 3.1. Traffic Flow Configuration

This study takes the homogeneous traffic flow of CACC vehicles as the initial case, in which each CACC vehicle updates dynamics with the messages, including location and speed, from the immediately preceding vehicle, as Figure 1(a) shows. This connectivity structure comes from the field test where the empirical models have been derived [50, 51]. Then, communication failures appear in partial CACC vehicles due to the possible equipment failures and malicious cyberattacks. CACC vehicles will receive bogus messages or delay messages when suffering from communication failures, leading to anomalous dynamics and traffic instability.
Because drivers have different degrees of trust in automatic driving technology, partial drivers will choose to take over the vehicle actively out of the distrust of automatic driving technology or be remained to take over the vehicles by the FCW. Consequently, partial CACC vehicles will turn to the HDVs, which run with drivers’ visual perception and are not directly affected by communication failures, as Figure 1(b) shows. As a result, the homogeneous traffic flow of CACC vehicles turns to the heterogeneous traffic flow mixed with CACC vehicles and HDVs under communication failures, and CACC vehicles can be further divided into vehicles with communication failures (denoted as CF) and normal vehicles without communication failures (denoted as C).

We assumed that the proportion of CACC vehicles suffering from communication failures is \( p \) and the proportion of driver takeover under communication failures is \( q \). \( N \) denotes the total number of vehicles in the heterogeneous traffic flow, and numbers of C, CF, and HDV can be formulated as \( N \cdot (1 - p) \), \( N \cdot p \cdot (1 - q) \), and \( N \cdot p \cdot q \), respectively.

3.2. Car-Following Models. Car-following models are essential for analyzing vehicle dynamics in the traffic flow. In this study, the realistic CACC model proposed by the California PATH program, which has been calibrated and validated by real experiments [50, 51], was used to simulate CACC vehicles. Furthermore, the Intelligent Driver Model (IDM) [52] was chosen to simulate the HDVs because it involves fewer parameters and variables with clear actual physical meaning and is easily calibrated and optimized with the field dataset. Furthermore, the IDM can output vehicle acceleration with the acceleration constraints for comfort and safety-critical consideration [53], making the control of vehicles have better transition performance with the actuator system than controls based on the Gipps model [54] and optimal velocity model [55].

It is worth noting that both CACC vehicles and HDVs adopt the car-following models in discrete time in this study, considering that current communication standard IEEE 802.11p stipulates the electric control unit (ECU) to send or receive information with a fixed time interval because the design of wireless access in vehicular environment (WAVE) shall allow both single- and multireceiver units and the communication channel cannot be used simultaneously [56]. Although HDVs are not affected by the communication interval and are continuous-time mechanical systems, their driving models also take a discrete-time form in this study to be consistent with the CACC model and convenient for analyzing the dynamics of different vehicles in the mixed flow.

3.2.1. General CACC Car-Following Model. The general CACC car-following model proposed by the PATH is written as

\[
v_n(t + \Delta t) = v_n(t) + k_p e_n(t) + k_d \dot{e}_n(t),
\]

where \( \Delta t \) is the time step, with a value of 0.01 s; \( s_n(t) = x_{n-1}(t) - x_n(t) \) is the headway distance between the vehicle \( n \) and its preceding vehicle \( (n - 1) \); \( s_0 \) is the safe headway distance at a standstill; \( L_{veh} \) is the length of vehicle, and \( t_{hw} \) is the desired time gap for CACC vehicles; \( e_n(t) \) is the spacing error of the vehicle \( n \), and \( \dot{e}_n(t) \) denotes the derivative of spacing error; and \( k_p, k_d \) are the model coefficients.
Take the derivative of Equation (1) and substitute Equation (2) into (1), and the linear acceleration function of car-following dynamics for CACC vehicles is obtained as

\[
v'_n(t + \Delta t) = f^C[v_n(t), s_n(t), \Delta v_n(t)]
\]

\[
= \frac{-k_p (s_{n+1}(t) + k_s s_n(t) + k_d \Delta v_n(t) - k_p (s_n + L_{veh}))}{(k_d s_{n+1} + \Delta t)}
\]

(3)

where \(\Delta v_n(t) = (v_{n-1}(t) - v_n(t))\) is the speed difference between vehicle \(n\) and preceding vehicle \((n-1)\).

3.2.2. CACC Car-Following Model with Communication Failures. Normal CACC vehicles receive correct messages of location and speed from their preceding vehicle. However, the messages propagated from the preceding vehicle will be at fault when CACC vehicles are affected by communication failure. As mentioned in the Introduction section, the fault data received by CACC vehicles mainly include the following two types:

(1) Bogus messages: Communication failures that result in underestimated or overestimated messages of location and speed propagated from the preceding vehicles [13].

(2) Transmission delay: Communication failures that result in an additional delay to the messages of location and speed, except for the general communication delay.

To get the car-following model for CACC vehicles with communication failures, Equation (3) can be modified as

\[
v'_n(t + \Delta t) = f^{CF}[v_n(t), \tilde{x}_n(t), \Delta \tilde{v}_n(t)]
\]

\[
= \frac{-k_p (s_{n+1}(t) + k_s s_n(t) + k_d \Delta \tilde{v}_n(t) - k_p (s_n + L_{veh}))}{(k_d s_{n+1} + \Delta t)}
\]

(4)

where \(\tilde{x}_n, \Delta \tilde{v}_n\), and \(\tilde{t}\) denote underestimated/overestimated headway, underestimated/overestimated speed difference, and transmission delay caused by communication failures, respectively.

3.2.3. Intelligent Driver Model. The intelligent driver model (IDM) for simulating the vehicle dynamics of HDVs is formulated as the following equations:

\[
v'_n(t + \Delta t) = f^{HDV}[v_n(t), s_n(t), \Delta v_n(t)]
\]

\[
= a \left(1 - \frac{v_n(t)}{v_0} \right) ^4 \left(1 - \frac{v_n(t) - s_n(t) - L_{veh}}{s_n(t) - L_{veh}} \right) ^2.
\]

(5)

\[
s^*(v_n(t), \Delta v_n(t)) = \left(s_0 + T v_n(t) - \frac{v_n(t) \Delta v_n(t)}{2 \sqrt{ab}}\right).
\]

(6)

where \(\Delta t\) is time step, consistent with that of CACC model; \(a\) is the maximum acceleration, and \(b\) is the desired acceleration; and \(v_0\) represents the desired speed, and \(T\) is the safety headway.

Noted that vehicle dynamics of HDVs are updated with the drivers’ visual perception, which is not affected by communication failures. Consequently, the input data of the IDM should be actual data but not fault data. In this way, the anomalous dynamics of CACC vehicles caused by communication failures can be corrected by drivers’ takeover.

3.2.4. Parameter Setting for Car-Following Models. In this study, the parameters in car-following models are determined by taking the empirical values validated in the real experiments, as shown in Table 2.

4. Stability Analysis

4.1. Definition of Stability. Traffic stability represents the ability of traffic flow against small perturbations to equilibrium state. When traffic flow is at an equilibrium state, we assume that all vehicles travel at an equilibrium speed \(V\) and there is no speed difference between any two adjacent vehicles. The acceleration of vehicles remains 0, and all vehicles remain an equilibrium headway \(S\). Then, small perturbations to speed and headway are introduced to traffic flow, denoted as \(\mu_n(t) = (v_n(t) - V)\) and \(\gamma_n(t) = (s_n(t) - S)\), respectively.

Then, Taylor expansions are used to obtain the linearization of car-following models of normal CACC vehicles, CACC vehicles with communication failures, and HDVs near the equilibrium point. The linearization models are shown in the following equations, respectively:

\[
\frac{d\mu_n}{dt} = L^C(\mu_n, \gamma_n, \Delta \mu_n) = f^C_{\mu_n \mu_n} + f^C_{\gamma_n \mu_n} + f^C_{\gamma_n \gamma_n} \Delta \mu_n,
\]

(7a)

\[
\frac{d\gamma_n}{dt} = L^C(\mu_n, \gamma_n, \Delta \mu_n) = f^C_{\gamma_n \mu_n} + f^C_{\gamma_n \gamma_n} \Delta \mu_n.
\]

(7b)

\[
\frac{d\Delta \mu_n}{dt} = L^{HDV}(\mu_n, \gamma_n, \Delta \mu_n) = f^{HDV}_{\mu_n} \mu_n + f^{HDV}_{\gamma_n} \gamma_n + f^{HDV}_{\Delta \mu_n} \Delta \mu_n.
\]

(7c)

where \(f^k, f^k_{\mu_n}, f^k_{\gamma_n}, f^k_{\Delta \mu_n}\) represent the partial derivatives of car-following models of \(k \in \{C, CF, HDV\}\), for speed, headway, and speed difference in an equilibrium state, respectively.

Furthermore, we substitute \(\mu_n(t) = U e^{\lambda t} \mu_n^0\) and \(\gamma_n(t) = Y e^{\lambda t} \gamma_n^0\) into Equation (7) and jointly consider the correlation \(\frac{d\mu_n}{dt} = (\mu_n^0 - \mu_n)\). The formulations are shown as follows:
Table 2: Parameter setting for car-following models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameters</th>
<th>Values</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car-following model</td>
<td>$k_p$</td>
<td>0.45 s$^{-1}$</td>
<td>Milanés and Shladover [50]</td>
</tr>
<tr>
<td>for CACC vehicles</td>
<td>$k_d$</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_{bw}$</td>
<td>0.60 s</td>
<td>Milanés et al. [51]</td>
</tr>
<tr>
<td></td>
<td>$L_{veh}$</td>
<td>5 m</td>
<td></td>
</tr>
<tr>
<td>Intelligently Driver Model</td>
<td>$T$</td>
<td>1.5 m</td>
<td>Treiber et al. [52]</td>
</tr>
<tr>
<td></td>
<td>$a$</td>
<td>1 m/s$^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$b$</td>
<td>2 m/s$^2$</td>
<td>Kesting et al. [5]</td>
</tr>
<tr>
<td></td>
<td>$s_0$</td>
<td>2 m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$v_0$</td>
<td>33.3 m/s</td>
<td></td>
</tr>
</tbody>
</table>

\[
\lambda^2 = \lambda \left[ f_{n,v}^C + f_{n,s}^C \left( 1 - e^{-i\omega} \right) \right] - f_{n,s}^C \left( 1 - e^{-i\omega} \right), \quad (8a)
\]

\[
\lambda^2 = \lambda \left[ f_{n,v}^{CF} + f_{n,s}^{CF} \left( e^{-i\omega} - 1 \right) e^{-\frac{i}{\alpha t}} \right] - f_{n,s}^{CF} \left( 1 - e^{-i\omega} \right), \quad (8b)
\]

\[
\lambda^2 = \lambda \left[ f_{n,v}^{HDV} + f_{n,s}^{HDV} \left( 1 - e^{-i\omega} \right) \right] - f_{n,s}^{HDV} \left( 1 - e^{-i\omega} \right). \quad (8c)
\]

Substitute $\lambda = (i\lambda_1 + \lambda_2 \omega^2)$ into Equation (8), and keep the 2-power polynomial. The parameters $\lambda_1$ and $\lambda_2$ can be represented as the following equations, respectively:

\[
\lambda_1 = \frac{f_{n,v}^C}{f_{n,s}^C} \lambda_2 = \frac{f_{n,v}^C}{f_{n,s}^C} - \frac{f_{n,-1}^C}{f_{n,v}^C} - \frac{\lambda_1^2}{f_{n,v}^C}, \quad (9a)
\]

\[
\lambda_1 = \frac{f_{n,v}^{CF}}{f_{n,s}^{CF}} \lambda_2 = -\frac{1}{f_{n,v}^{CF}} \lambda_1^2 - \frac{f_{n,s}^{CF}}{f_{n,v}^{CF}} + \frac{f_{n,s}^{CF}}{f_{n,v}^{CF}} \left( \frac{1}{2} + \lambda_1 \right), \quad (9b)
\]

\[
\lambda_1 = \frac{f_{n,v}^{HDV}}{f_{n,s}^{HDV}} \lambda_2 = \frac{f_{n,v}^{HDV}}{f_{n,s}^{HDV}} - \frac{f_{n,s}^{HDV}}{f_{n,v}^{HDV}} - \frac{\lambda_1^2}{f_{n,v}^{HDV}}. \quad (9c)
\]

Substitute the formulation of $\lambda_1$ into the formulation of $\lambda_2$, and we can get the stability criteria for three types of vehicles, which is given as the following equations, respectively:

\[
F_C = \frac{\left( f_{n,v}^C \right)^2}{2} - f_{n,s}^C - f_{n,s}^C f_{n,v}^C, \quad (10a)
\]

\[
F_{CF} = \frac{\left( f_{n,v}^{CF} \right)^2}{2} - f_{n,s}^{CF} - f_{n,s}^{CF} f_{n,v}^{CF}, \quad (10b)
\]

\[
F_{HDV} = \frac{\left( f_{n,v}^{HDV} \right)^2}{2} - f_{n,s}^{HDV} - f_{n,s}^{HDV} f_{n,v}^{HDV}, \quad (10c)
\]

4.2. Stability Analysis of Homogenous Traffic Flow. This section analyzes the stability of homogeneous traffic flow. Firstly, the partial derivatives for car-following models of CACC vehicles are calculated to understand the stability of the homogeneous traffic flow of CACC vehicles. It is noted that the partial derivatives are not affected by communication failures, neither Bogus messages nor transmission delay, as shown in the following equations:

\[
f_{n,v}^C = \frac{-k_p t_{hw}}{(k_u t_{hw} + \Delta t)} f_{n,v}^C, \quad (11a)
\]

\[
f_{n,s}^{CF} = \frac{k_p}{(k_u t_{hw} + \Delta t)} f_{n,s}^{CF}, \quad (11b)
\]

\[
f_{n,v}^{HDV} = \frac{k_d}{(k_u t_{hw} + \Delta t)} f_{n,v}^{HDV}. \quad (11c)
\]

By substituting Equation (11) into Formula (10a) and Formula (10b), we get the stability criterion for the homogeneous traffic flow of normal CACC vehicles and CACC vehicles with communication failures in the following formulas, respectively:
Figure 4: Stability values for heterogeneous traffic flow under different scenarios.
According to the stability criterion, the stability values for normal CACC vehicles are determined by the car-following parameters. However, the stability values for CACC vehicles with communication failures are significantly affected by the transmission delay $\Delta t$ and the desired time gap $thw$. It is noted that not all communication failures have an impact on traffic stability and the communication failure of bogus messages is excluded. Then, we analyzed the impact of transmission delay and desired time gap on stability values in Figure 2, in which the black line represents the borderline between stable and unstable conditions, with a stability value of 0.

Figure 2 shows that the stability value decreases as the transmission delay increases and the desired time gap decreases. It indicates that a large transmission delay will affect traffic stability because it makes the received information deviate from the actual values. Besides, a small desired time gap shortens the headway distance from preceding vehicles and vehicles have not enough time to make a reaction to sudden speed change, leading to a more serious speed fluctuation.

Besides, HDVs will be introduced into the traffic flow by drivers’ takeovers, so stability analysis for homogeneous traffic flow of HDVs is also essential. The partial derivatives for the IDM can be found in past works, such as Yao et al. [19], Chang et al. [20], and Chen et al. [34]. By substituting them into Equation (10c), we get the stability criterion for homogeneous traffic flow of HDVs:

$$F_{\text{HDV}} = 0.5 \left( \frac{4aV^3}{v_0^4} + \frac{2aT - 2aT(V/v_0)^4}{(s_0 + TV)} \right)^2$$

$$+ \sqrt{ab} \left( \frac{V(1 - (V/v_0)^4)}{s_0 + TV} \right) \left( \frac{4aV^3}{v_0^4} + \frac{2aT - 2aT(V/v_0)^4}{(s_0 + TV)} \right) > 0.$$  \hspace{1cm} (14)

It shows the stability value for homogeneous traffic flow of HDVs is determined by the equilibrium speed $V$. The relationship between stability value and equilibrium speed is shown in Figure 3, in which the traffic flow of HDVs is unstable in the speed range between 0.57 m/s to 21.48 m/s and keeps stable under other equilibrium speeds.

4.3. Stability Analysis of Heterogeneous Traffic Flow. The homogeneous traffic flow will turn into heterogeneous traffic flow when partial CACC vehicles are taken over by drivers under communication failures. Consequently, the heterogeneous traffic flow is composed of normal CACC vehicles, CACC vehicles with communication failures, and HDVs.

In 2009, Ward [57] derived a general stability criterion of heterogeneous traffic flow, as follows:

$$\sum_n \left( \left( \frac{f_{nx}}{2} - f_{nx} - f_{nx}f_{nx} \prod_m f_{mx} \right)^2 \right) > 0.$$  \hspace{1cm} (15)

It indicates that the stability value of heterogeneous traffic flow is determined by the proportion of different vehicles. Then, we substitute the vehicle numbers into Equation (15) and rewrite the stability condition of heterogeneous traffic flow, as follows:
Figure 6: Continued.
As suggested by the above analysis, the stability values of heterogeneous traffic flow are determined by the equilibrium speed $V$ and some key parameters $p,q,t$. It indicates that not all communication failures will affect traffic stability. Although bogus messages will certainly disrupt vehicle dynamics, it has no impact on the stability of traffic flow. Then, we analyze the stability values of different cases by adjusting the influencing factors, as shown in Figure 4.

Figure 4 shows that the equilibrium speed and configuration parameters have a significant impact on the stability values of heterogeneous traffic flow. Firstly, the unstable range expands as more CACC vehicles suffer from communication failures, especially under the equilibrium speed between 0.57 m/s and 21.48 m/s. It indicates that communication failures would seriously affect traffic stability. Secondly, increased transmission delay also worsens traffic stability as it makes the received messages greatly deviate from the actual messages and induces vehicles to take an anomalous acceleration or deceleration. Thirdly, a large proportion of drivers’ takeovers harms traffic stability. It is because the drivers’ takeover introduces the HDVs, which are inherently unstable under the speed range between 0.57 m/s and 21.48 m/s, into the heterogeneous traffic flow. The number of unstable vehicles increases as drivers are more likely to take over vehicles under communication failures; hence, traffic flow turns to instability.

Furthermore, some extreme cases are also worth studying. There are no HDVs in the heterogeneous traffic flow when $q$ is equal to 0 and there are no CACC vehicles with communication failures when $q$ is equal to 1. In these cases, the heterogeneous traffic flow only consists of two types of vehicles. From Equation (13) and Equation (14), the stability of CACC vehicles with communication failures is affected by the transmission delay and that of HDVs is affected by the equilibrium speed. Besides, the proportion of communication failures is a common factor influencing the traffic stability. Thus, stability values in cases of $q = 0$ and $q = 1$ are discussed with their respective influencing factors in Figure 5.

Figure 5(a) shows that the instability area of $q = 0$ mainly concentrates in the lower right corner where both the transmission delay and proportion of communication failures are large. This conclusion is consistent with the discussion in Figure 4. Besides, when CACC vehicles with
Figure 7: Continued.
Communication failures are completely taken over by drivers, the stability condition is worse than traffic flow with some vehicles remaining in the CACC mode even if under communication failures. It further indicates that the driver’s takeover is not an ideal method for reducing the impact of communication failures on traffic stability because it increases the number of unstable HDVs in the heterogeneous traffic flow.

5. Numerical Analysis

This section proposes a series of numerical simulations to verify the proposed model in Equation (16) and demonstrate its major properties. Because the theoretical model shows that the transmission delay is an influencing factor of traffic stability and bogus messages could disrupt the vehicle’s dynamics, we take the transmission delay and bogus messages as communication failures and analyze their impact on traffic stability in multiple cases.

5.1. Startup and Braking Analysis. We carry out the same simulation as that in past work [58] to analyze the vehicle dynamics under a traffic signal and examine certain properties of the proposed model. A platoon, denoted as 10 vehicles following a leading vehicle, is simulated to analyze the stability in the cases of startup and braking. Firstly, the traffic signal is red and all vehicles are stationary at the initial moment. Afterward, the signal changes to green and the leading vehicle starts at 0 s with a constant acceleration of 1 m/s² until the speed reaches 15 m/s and then keeps uniform speed until the 60 s. Next, the leading vehicle decelerates at $-1 \text{ m/s}^2$ until it stops before another red signal.

The dynamics of the following 10 vehicles are analyzed in the cases of startup and braking, respectively, as shown in Figures 6 and 7. When CACC vehicles suffer from communication failures, they can turn to HDVs or keep driving with communication failures. Thus, three types of vehicles are included in the traffic flow, including normal CACC vehicles (denoted as red solid lines), CACC vehicles with a transmission delay of 5 s (denoted as blue solid lines), and HDVs (denoted as blue dotted lines). We set the proportion of drivers’ takeover under communication failures is 0.5, $q = 0.5$, and then, we make stability analysis by discussing the proportion of CACC vehicles suffering from communication failures.

Figure 7: Vehicle dynamics with transmission delay during braking case. (a) $p = 0$. (b) $p = 0.2$. (c) $p = 0.4$. (d) $p = 0.6$. (e) $p = 0.8$. 
Figure 8: Vehicle dynamics under communication failures for incidents analysis. (a) $p = 0, q = 0$. (b) $p = 0.3, q = 0$. (c) $p = 0.3, q = 0.6$. (d) $p = 0.6, q = 0$. (e) $p = 0.6, q = 0.6$. (f) $p = 0.9, q = 0$. (g) $p = 0.9, q = 0.6$. 

<table>
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<tr>
<th>Time (s)</th>
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<th>Speed (m/s)</th>
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Figure 9: Continued.
Figure 9: Space-time evolution of the headway under communication failures. (a) $p = 0.1, q = 0.2$. (b) $p = 0.1, q = 0.8$. (c) $p = 0.3, q = 0.2$. (d) $p = 0.3, q = 0.8$. (e) $p = 0.5, q = 0.2$. (f) $p = 0.5, q = 0.8$. (g) $p = 0.7, q = 0.2$. (h) $p = 0.7, q = 0.8$. (i) $p = 0.9, q = 0.2$. (j) $p = 0.9, q = 0.8$.

Figure 10: Traffic capacity with multiple penetration rates under transmission delay. (a) No HDVs. (b) 25% HDVs. (c) 50% HDVs. (d) 75% HDVs.
Figure 6 shows that transmission delay has a significant impact on stability. Firstly, CACC vehicles update the speed untimely with a transmission delay, resulting in an expanded speed difference between vehicles. Besides, HDVs introduced by drivers’ takeovers have more serious fluctuations in vehicle dynamics. It is because the large time gap makes HDVs insensitive to speed change and unable to follow preceding vehicles closely. Otherwise, the large reaction time hinders the HDVs from adjusting vehicle dynamics untimely, leading to a longer duration of traffic instability.

Furthermore, traffic instability is worsened with an increased proportion of communication failures, which is reflected by the expanded amplitude of speed fluctuation and the extended duration for speed to return to stability. These points are consistent with the theoretical results, validating that the proposed models for the stability analysis of heterogeneous traffic flow are reliable.

The vehicle’s dynamics are also seriously affected by transmission delay in the case of braking. Just similar to the conclusions of the startup case, the speed difference between vehicles is expanded by the transmission delay and HDVs are insensitive to speed change with a large perception-reaction time, resulting in a long-duration traffic instability. Besides, the fluctuations in speed and deceleration become more serious with an increased proportion of communication failures. This conclusion shows that communication failures have a negative impact on traffic stability in the cases of both startup and braking.
5.2. Incidents Analysis. This section simulates the case in which the platoon slows down due to an incident. We take a platoon of 100 vehicles with an initial speed of 15 m/s, then, the leading vehicle is assumed to decelerate with a deceleration of \(-0.5\) m/s\(^2\) at 10 s and lasts for 2 s, and then, the leading vehicle keeps a uniform speed of 14 m/s until the end of the simulation. Traffic stability is analyzed with speed fluctuations under different proportions of transmission delay and drivers’ takeovers, as shown in Figure 8, in which normal CACC vehicles (denoted as red lines) are probably affected by transmission delay of 5 s, and then turn to the HDVs (denoted as black lines) or keep with the transmission delay (denoted as blue lines).

From Figure 8, vehicles decelerate in turn by following their preceding vehicles. There is an obvious speed fluctuation before each vehicle returns to the uniform speed; that is, speed increases first and then decreases. CACC vehicles can follow the speed change of their preceding vehicles timely when there are no communication failures. In this case, traffic flow can return to stability rapidly, and the whole process of speed fluctuations only lasts for about 70 s, as shown in Figure 8(a).

Besides, Figure 8 also shows that traffic stability is determined by the proportion of communication failures and drivers’ takeover. Both duration and amplitude of speed fluctuation expand as the proportion of transmission delay or drivers’ takeover increases. It shows that the transmission delay undermines traffic stability, and speed fluctuations are transmitted and amplified to the rear of traffic flow. Especially, HDVs introduced by drivers’ takeover have a greatly negative impact on traffic stability, even compared with CACC vehicles with

\[\tau = 0.0 \text{ s, } \alpha = 0.3 \]
\[\tau = 0.2 \text{ s, } \alpha = 0.7 \]
\[\tau = 0.4 \text{ s, } \alpha = 0.7 \]
\[\tau = 0.6 \text{ s, } \alpha = 0.7 \]

\[\tau = 0.0 \text{ s, } \alpha = 0.3 \]
\[\tau = 0.2 \text{ s, } \alpha = 0.7 \]
\[\tau = 0.4 \text{ s, } \alpha = 0.7 \]
\[\tau = 0.6 \text{ s, } \alpha = 0.7 \]

**Figure 12:** Traffic capacity with multiple transmission delay. (a) No HDVs. (b) 25% HDVs. (c) 50% HDVs. (d) 75% HDVs.
5.3. Density Wave. This section analyzes the impact of small disturbances on traffic stability. A platoon of 100 vehicles is simulated, and a transmission delay of 5 s is introduced as communication failures. Besides, a small disturbance of 0.1 m on location is put on the leading vehicle and vehicle dynamics between 1,000 s and 1,150 s are recorded. The periodic boundary is selected and the length of vehicles is set to 5 m. Figure 9 shows the typical traffic patterns after a sufficiently long time, and $H$ represents the ratio of actual headway to desired headway for each vehicle in the platoon.

Figure 9 shows that traffic flow can keep stability when the proportion of communication failures is at a low level, whereas small disturbances will be amplified and density waves appear when the proportion of communication failures and drivers’ takeover is both more than 0.5. Furthermore, it shows that the HDVs have a more negative impact on traffic stability, compared to CACC vehicles with communication failures. This point can be indicated by Figure 9(g), in which density waves are not obvious with a low proportion of drivers’ takeover, even if the proportion of communication failures has reached 0.7. By comparison, density waves appear significantly with a larger proportion of drivers’ takeovers in Figure 9(h).

6. Discussion

In this section, the impact of communication failures on traffic capacity is discussed. We simulate a 10-km straight...
segment with one lane and a cyclic boundary. Each simulation takes 110,000 steps with the first 100,000 steps as the preheating process. The results of the last 10,000 steps are recorded, and their average is calculated. Besides, the types and location of vehicles are determined randomly; hence, each simulation is repeated 300 times to eliminate contingency. Finally, the reported result is the average of these 300 runs.

In the simulation, transmission delay is taken into consideration as it can affect the stability of traffic flow. Besides, bogus messages on headway and speed can affect the vehicle’s dynamics even if they have no impact on traffic stability. We only consider the bogus messages on headway in the simulation as they have a more serious impact on vehicle dynamics than those on speed, as suggested by recent works [39].

### 6.1. Discussion on Penetration Rate

We take a transmission delay of 0.5 s and discuss traffic capacity with multiple penetration rates, as shown in Figure 10, in which the penetration rate of CACC vehicles with transmission delay to all CACC vehicles is denoted as $\alpha$.

Figure 10 represents that the traffic capacity declines as the penetration rate of CACC vehicles with transmission delay increases, indicating transmission delay has a negative impact on traffic capacity. Besides, the optimum density, denoted as the density corresponding to traffic capacity, becomes smaller under a larger penetration rate of CACC vehicles with transmission delay. It is because transmission delay makes vehicles unable to follow the speed change of preceding vehicles and expands the headway. Thus, both the optimum density and traffic capacity are reduced with transmission delay.

It also indicates that CACC vehicles perform better than HDVs in improving traffic capacity, even if suffering from transmission delay. Traffic capacity declines as the penetration rate of HDVs increases and the traffic capacity of mixed flow is larger than that of homogeneous traffic flow of HDVs in all cases. It can be explained by the large headway taken by HDVs due to their long perception-reaction time. Thus, optimum density reduces compared with those of mixed traffic flow.

Furthermore, we introduce bogus messages on headway to CACC vehicles and make the perceived headway 3 m larger than actual headway. The impact of bogus messages on traffic capacity is analyzed with multiple penetration rates, as shown in Figure 11, in which the penetration rate of CACC vehicles with bogus messages to all CACC vehicles is denoted as $\alpha$.

Figure 11 shows that bogus messages have little impact on traffic capacity. Traffic capacity keeps the same, but the optimum density increases as the penetration rate of CACC vehicles with bogus messages increases, especially for cases with high density. It is because bogus messages make CACC vehicles overestimate the headway and speed up to shorten it, causing the actual headway to be less than desired headway. Besides, traffic capacity declines as the proportion of HDVs increases, which is consistent with the conclusions in Figure 10.

### 6.2. Discussion on Transmission Delay

This section simulates different degrees of transmission delay and analyzes their impact on traffic capacity, as shown in Figure 12, where $\tau$ denotes the delay time.

Figure 12 shows that both traffic capacity and optimum density decline as the transmission delay increases. It indicates a large delay time leads to an expanded headway, further reducing the density and traffic capacity. Besides, transmission delay only has a serious impact on CACC vehicles but has little impact on HDVs. Traffic capacity keeps stable with different delay times when the proportion of HDVs is at a high level. Figure 12 also shows that HDVs and a high penetration rate of CACC vehicles with transmission delay harm traffic capacity, just like the conclusions in Figures 10 and 11.

### 6.3. Discussion on Bogus Messages

We simulate different bogus messages on headway and analyze their impact on traffic capacity, as shown in Figure 13, in which $d$ denotes the deviation between perceived headway and actual headway.

Figure 13 represents that traffic capacity increases when the perceived headway is larger than the actual headway, but decreases when the perceived headway is smaller than the actual headway. Traffic capacity with bogus messages on headway depends on whether the headway perceived by CACC vehicles is underestimated or overestimated. Vehicles will slow down to extend the underestimated headway or speed up to shorten the overestimated headway. Thus, the actual headway deviates from the desired headway, resulting in a changed traffic capacity. It indicates that traffic capacity is more seriously affected as the deviation between bogus messages and actual values deepens. Otherwise, the impact of bogus messages on traffic capacity is weakened with the increased proportion of HDVs, just the same as the conclusions in Figure 12.

### 7. Conclusion

This study analyzes the stability of heterogeneous traffic flow with communication failures. Two types of communication failures, including bogus messages and transmission delay, are formulated to analyze their impact on traffic flow at the theoretical level. Then, some methods like drivers’ takeover are introduced to react to communication failures and CACC vehicles have opportunities to turn to HDVs. Consequently, the heterogeneous traffic flow consists of normal CACC vehicles, CACC vehicles with communication failures, and HDVs. Finally, a series of numerical analyses, including startup and braking analysis, incidents analysis, and density waves, are made to verify the theoretical models and demonstrate their major properties.

The main conclusions of this study are summarized as follows:

1. Transmission delay has a negative impact on the traffic stability of CACC vehicles. By comparison, bogus messages on headway or speed have little impact on traffic stability of CACC vehicles, even if they can affect vehicle dynamics;
(2) Traffic stability deteriorates gradually as the proportions of communication failures and drivers' takeover increase, indicating that the driver's takeover is not an ideal solution for reducing the impact of communication failures as HDVs have worse stability than CACC vehicles;

(3) Increasing strength and penetration rate of communication failures will exacerbate fluctuations in speed, acceleration, and headway and extend the durations under cases of startup and braking, incidents, and density wave.

(4) Transmission delay will reduce the traffic capacity, and this impact will aggravate as the strength and penetration rate of transmission delay increase. Yet, the impact of bogus messages on traffic capacity depends on whether the headway perceived by CACC vehicles is underestimated or overestimated. Underestimated headway reduces traffic capacity, while overestimated headway increases traffic capacity.

Stability for heterogeneous traffic flow with communication failures has been analyzed theoretically, whose reliability is verified by a series of numerical simulations in this study. Besides, some directions can be further improved based on known conclusions. First, this study takes the PF topology and CACC vehicles receive information from their immediately preceding vehicle. Multi-anticipation is expected to be studied in the stability analysis of heterogeneous traffic flow. Second, the mixed traffic with CACC vehicles and HDVs is adopted in this study to analyze the impact of communication failures and driver takeover on stability. More vehicle types, such as ACC vehicles, AVs, and CVs, will be introduced to the mixed traffic, and the corresponding stability criteria will be developed in future research.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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