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Research Article

Reliability Assessment Model and Simulation of Journal Bearing of Railway Freight Cars Based on Bayesian Method under Small Sample Sizes

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Generally, the box bearing of railway freight cars has no bearing sample failure data at the end of the time-terminated reliability test. However, it is expensive and has high service reliability requirements. Given a small sample size and zero-failure data, the traditional failure probability calculation formula based on a large sample size and the reliability modeling technique cannot easily assess the reliability of rolling bearings accurately. Considering the applicability of the bearing of railway freight cars, this study integrated the prior information of samples and the simulation test information according to Bayes statistical theory, deduced the mathematical model of cumulative failure probability under failure-free data, calculated the distribution parameters using the least square method, and established the reliability estimation model of rolling bearings on the basis of Weibull distribution. The failure-free simulation data of rolling bearings were produced according to the Monte Carlo simulation, and the reliability of the journal bearing of railway freight cars was simulated and assessed by three methods. Simulation results demonstrate that the proposed reliable Bayes multilayer estimation method could not only meet the design requirements of the ISO 281 rolling bearing standards on that basis of the failure-free data and small sample size of the time-terminated simulation, but also assess the reliability of the rolling bearing of railway freight cars.

1. Introduction

As the key bearing component, the tapered journal roller bearing of railway freight cars bears the bogie and car body vibration loads during service and the random flexural-torsional loads during contact with the wheel track. Rolling contact fatigue is the major damage mode. The journal bearing may cause temperature increase in the axle and can even cut the axis and cause train derailment once it fails [1]. Railway freight traffic is developing toward large power and heavy loads, implying the increasing requirements on the reliability and service time of rolling bearings. The reliability of journal bearings directly determines the reliability of railway freight cars. Hence, the accurate estimation of the service time of rolling bearing is particularly important [2]. Number/time-terminated fatigue life tests are practical methods of obtaining the reliability assessment data of bearings. Within the regulated time, if failures occurred, then the reliability can be estimated using the classical probability statistical method and a big sample size [3]. However, if no failure occurred, traditional methods, like maximum likelihood estimation and optimal linear unbiased estimation, cannot obtain accurate assessment results [4]. The journal bearing of railway freight cars is expensive and has high reliability, therefore being unsuitable for the long-period and high-cost number-terminated and complete failure test. It usually chooses the small-sample-sized time-terminated test. The establishment of a reliability model and the estimation of the service time of journal bearings with a small sample size and failure-free data are the weak links of the study on the reliability of the rolling bearing of railway freight cars and must be urgently addressed.

Increasing the size of the sample information and using the traditional bootstrap method, maximum likelihood estimation [5], and other reliability modeling methods are effective ways of increasing the reliability assessment accuracy under a small sample size. However, the calculation of failure probability has some defects because these methods ignore the prior information of samples, and the reliability assessment cannot meet the requirements. The Bayesian method has attracted significant attention because it can make full use of all sample information and increase the reliability assessment accuracy [6]. Currently, the bearing reliability under a small sample size has been widely applied. However, the assessment of the reliable service time of the rolling bearing of railway freight cars lacks relevant studies. For this reason, prior information and the time-terminated simulation data based on the Monte Carlo method were integrated first using beta distribution as the prior distribution, thereby obtaining the cumulative failure probability (CFP) formula on the basis of the Bayesian method. The Weibull distribution of unknown parameters was estimated according to the least square method, and the reliability estimation model of the journal bearing of railway freight cars was established using a small sample size. Moreover, the simulation assessment of reliability was carried out according to the European standards. The research results provide references for the reliability assessment of the journal bearing of railway freight cars using a small sample size and failure-free data.

2. Literature Review

As the core part of a rotating machine, rolling bearing plays an important role in protecting the reliability and safety of transportation. With respect to the reliability and service time problems of bearing under service, scientific researchers have conducted numerous intensive studies. Warda and Chudzik [7] studied the influences of radial cylindrical roller bearing misalignment on contact fatigue life through the finite element method. Prakash [8] proposed two Bayes hierarchical models; one model uses life data and the other uses structural health monitoring data to construct the degradation model and assess the reliability of rolling bearings. Sakaguchi and Harada [5] discussed the contact fatigue life of angular contact ball bearing under practical working conditions through modeling using the ADAMS software. Leturion et al. [9] established a multibody model of rolling bearing and generated synthesis data that cover the degradation related with the bearing surface; some output variables are used as covariables of the proportional risk model, which estimates the reliability of bearings through training. Nelias et al. [6] studied the influences of the upsetting moment and bearing clearance on the service time of bearings by establishing the statics analysis model of bearing. According to the three-parameter Weibull distribution simulation, Ferreira et al. [10] discussed the statistical propagation law of the fatigue life of bearings by using the reliability data of the railway car axle's ball bearing. To estimate the residual service time of ball bearings, Sotrisno et al. [11] developed three algorithms for predicting the reliable service time of bearings according to 17 ball bearing test data, including 6 bearing data for algorithm training and 11 bearing data for testing.

Bearings have several failure modes. Rolling contact fatigue is one of the major failure modes. Recently, many scholars have built various fatigue life calculation models of bearings according to different stress life standards. Espejel and Gabelli [12] built a bearing prediction model that separates the surface rolling contact and rollaway fatigues, considering various normal operation conditions of bearings, including load, lubrication, and shape parameters. Ying et al. [13] built a reliability model of rolling bearing according to vibration signals, which were collected through a simulation test and analyzed. To increase the accuracy of fatigue life calculation during rolling bearing simulation, Kabus et al. [14] simulated the rollaway contact according to high-precision elastic space theory and proposed a new quasistatic time-domain tapered roller bearing model with multiple degrees of freedom and zero friction. Yakout et al. [15] corrected the traditional Lundberg–Palmgren rolling bearing life formula. According to the corrected formula and dynamic characteristics of rolling bearing, the fatigue life of bearings was predicted through the vibration modal analysis method. Morales and Gabelli [16] studied the dynamic sliding effect of Hertz contact and its influences on the fatigue life of rolling bearings from three perspectives by using tribological modeling.

Fatigue life test, which consumes considerable labor forces and materials, is a simple method of obtaining bearing reliability data. However, the bearing does not usually fail at the end of a time-terminated test. Studies on the reliability problem without failure data have attracted increasing interest from the academic and engineering circles. Given that the service time of bearings follows the Weibull distribution [17], Li et al. [18] proposed a reliability analysis method without failure data, which assesses the reliability of rolling bearings by expanding the optimal confidence interval. Meanwhile, the influences of the bearing grouping mode and the shape parameters of the model on reliability assessment results were analyzed. Xia [19] proposed a gray bootstrap method of reliability estimation under incomplete bearing information and estimated the life probability distribution using the defined empirical failure probability function. When the life probability distribution is unknown and failure-free data exist, this method can assess the reliability of rolling bearings. Considering the relatively high predicted values of bearing life under extreme stress, Gupta [20] found that correcting the critical subsurface shearing stress related with the contact fatigue of rolling bearings could significantly improve the reliable life of rolling bearings. To ensure the safety of reliable assessment, Nguyen [21] applied the three-parameter Weibull distribution model and predicted the life probability related with the fatigue of rolling bearings under any reliable conditions.

The above analysis indicates that the reliability test of bearings can select a small sample capacity, which has not failed yet. How to analyze and assess small-sample-sized parameters is the core problem of reliability estimation. The famous Bayes theory can handle small-sample-sized parameters. Given that integrating sample and test information can also shorten the bearing test time, the Bayesian method has been widely used in the small-sample-sized reliability assessment of bearings. On the basis of the Bayesian method, Kwon [22] proposed the reliability verification method without failure samples for samples with bearings following Weibull and logarithmic normal distributions. Considering the changes in reverse Weibull reliability, Pandya and Jadav [23] proposed a variable-point model and achieved the Bayes estimation of unknown variable points using the asymmetric loss function. Shimizu [24] viewed rolling bearing as a system formed by the series connection of the inner ring, the outer ring, and rollers and believed that the contact fatigue life observes the three-parameter Weibull distribution. On this basis, the three-parameter Weibull distribution bearing life prediction model based on contact fatigue was constructed. Kotzalas [25] discussed the fatigue life statistical distribution in the high reliability region of tapered roller bearings, calculated the fatigue life of rolling bearings, which follows the two-parameter Weibull distribution, and carried out a reliability verification experiment on the test data. Most recently, it is worth noting that Bayesian methods have also been leveraged to address the limited data in training deep learning-based models. This helps avoid overfitting issues and shows its advantages to enhance the trustworthiness of the deep learning-based approaches [26, 27]. Generally, these methods can be adopted to estimate various reliability parameters in case of failure data in reliability test. However, with the extension of product service time and the improvement of reliability, a large number of nonfailure data have appeared in the timed truncation life test of rolling bearings. The traditional statistical methods, such as simple Bayesian method and Wellbull distribution method, have been unable to meet the needs of reliability evaluation.

The journal bearing of railway freight cars experiences complicated random service loads, interweaving dynamic loads, and the coexistence of the multifailure mechanism and failure modes. A long test time is required to obtain the failure data of bearings. The reliable life of bearings is usually analyzed and assessed using a small sample size and the time-terminated test. The traditional failure probability calculation formula based on a large sample size and the reliability modeling method cannot meet the reliability assessment requirements. Therefore, the tapered journal double-row roller bearing of railway freight cars was selected as the research object. To assess the reliability through the effective use of the small sample sized data of the time-terminated test, the priori and test information of the simulation bearing samples were integrated first using Bayes statistical theory, and the mathematical model of the CFP of journal bearing was deduced. Next, the Weibull distribution parameters were calculated using the least squares method, and a reliability estimation model of rolling bearing was constructed. Meanwhile, the failure-free simulation data of bearings were obtained using the Monte Carlo method, and the numerical simulation of the reliability assessment of bearings was carried out using different methods. According to the simulation results, the proposed Bayes multilayer estimation method of bearing reliability follows the ISO 281 design standards of rolling bearings and can estimate reliability of journal bearing of railway freight cars.

The remainder of this paper is organized as follows. Section 3 discusses the integration of the prior and test information using the multilayer Bayes estimation method and the construction of a mathematical model of CFP. The undetermined parameters of the two-parameter Weibull distribution are solved according to the least square method, and a mathematical model of the reliability assessment of the journal bearing of railway freight cars was constructed. Section 4 presents the generation of random numbers that conform to Weibull distribution according to the Monte Carlo method, thus getting the failure-free simulation data of bearing. Three methods were used in the reliability simulation tests, and the simulation results were compared. Section 5 summarizes the conclusions.

3. Materials and Methods

3.1. Multilayer Bayes Model of CFD

3.1.1. Simplified Model of Failure Probability. If n_i is the number of samples, t_i is the sampling time, and $(t_i, n_i)(i = 1, 2, \ldots, m)$ are the failure-free test data, in which n products are involved in the time-terminated test, including r products that fail before end time t and $(n-r)$ products that do not fail at t. The time that product r fails is denoted as t_r, and the time that the first product fails after t is recorded as t_{r+1}. If the rth and $(r+1)$th-order statistics of uniformly distributed samples are $F(t_r)$ and $F(t_{r+1})$, respectively, then their mathematical expectations are given as $E\,[F\,(t_r)] = r/(n+1)$ and $E\,[F\,(t_{r+1})] = (r+1)/(n+1)$. Given that the form estimated value is slightly lower than the latter one, compromised term $(r+0.5)/(n+1)$ is used for the estimation results of $F\,(t_r)$.

The failure-free data is indeed a special situation of the failure data problem; that is, no failure product should exist before end time t or $1/2\,(n+1)$ can be estimated when $r=0$. Hence, the classical calculation formula of failure probability $\widehat{p}_i$ is given as $\widehat{p}_i = 1/2\,(s_i+1)$.

3.1.2. Empirical Bayes (E-Bayes) Model of Failure Probability. If λ is the maximum failure probability, $\widehat{p}_{i-1}$ is estimation of last failure probability, and p_i distributes uniformly on the interval of $[\widehat{p}_{i-1}\lambda]$. When it meets the condition $\widehat{p}_{i-1} < p_i < \lambda$, the prior distribution function of $\pi\,(p_i)$ is then deduced as $\pi\,(p_i) = 1/(\lambda - \widehat{p}_{i-1})$; otherwise, $\pi\,(p_i) = 0$.

To obtain the posterior distribution of failure probability p_i, the conjugate prior distribution of p_i is beta distribution, and its mean and variance are given as follows:

$$\begin{cases} E_b(p_i) = \dfrac{a_i}{(a_i + b_i)}, \\ D_b(p_i) = \dfrac{a_i b_i}{(a_i + b_i)^2 (a_i + b_i + 1)}. \end{cases} \tag{1}$$

Here a_i and b_i are hyperparametres. Since there exists the formula $B(a_i, b_i) = \int_0^1 x^{a_i - 1}(1-x)^{b_i - 1} dx$, the conjugate prior distribution function $\pi((p_i|a_i), b_i)$ can be deduced as

$$\pi((p_i|a_i), b_i) = \frac{1}{B(a_i, b_i)} p_i^{a_i - 1} (1 - p_i)^{b_i - 1}. \tag{2}$$

The empirical Bayes estimation p_i can be derived, which is called E-Bayes estimated value $\widehat{p}_i$ with $\widehat{p}_i = \int \widehat{p}_i(b)\pi(b)db$, and the corresponding E-Bayes estimated value is obtained as below.

$$\widehat{p}_i = \frac{1}{c-1} \ln \frac{s_i + c + 1}{s_i + 2}. \tag{3}$$

Multilayer Bayes Model of CFD. Define the function $\Gamma(x) = \int_0^1 t^{x-1} e^{-t} dt$; then the posterior distribution function of $h(p_i|s_i)$ can be derived from the Bayes theorem; that is,

$$h(p_i|s_i) = \frac{\Gamma(a_i)\Gamma(s_i + b_i)}{\Gamma(s_i + a_i + b_i)} p_i^{a_i - 1} (1 - p_i)^{s_i + b_i - 1}. \tag{4}$$

According to the point estimation of the Bayesian method, the mean $E[h(p_i|s_i)]$ of the posterior distribution is used as the Bayes estimated value of p_i (i.e., $\widehat{p}_i = a_i/(s_i + a_i + b_i)$).

The probability density curve on the interval of [0, 1] is shown in Figure 1. When it meets the condition $a \le 1$ and $b > 1$, the probability density of the beta distribution is monotone decreasing, which is consistent with the prior information that the probability of relatively high p_i is low, and the probability of relatively low p_i is high. For calculation convenience, let $a = 1$ and $1 < b < u$, where u is determined by the failure-free data information; then, the multilayer prior distribution function of $f(p_1|u)$ can be obtained as

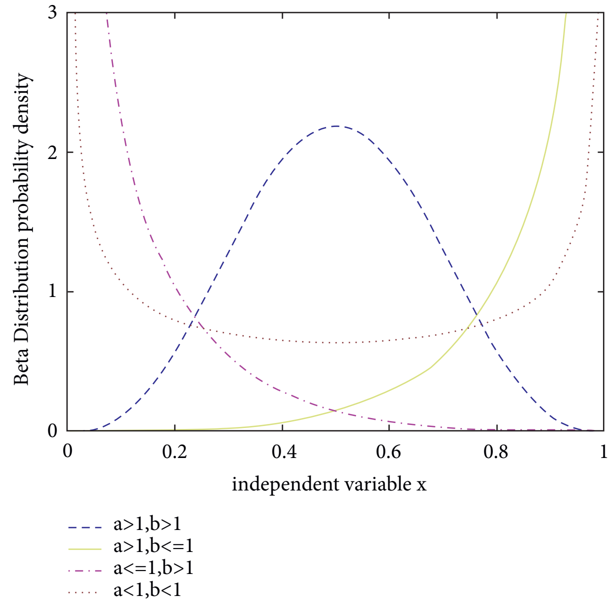

Figure 1: Probability density functional curve of beta distribution.

$$f(p_1|u) = \int_1^u \frac{(1-p_1)^{b-1}}{B(1,b)} \cdot \frac{1}{u-1} db. \tag{5}$$

Given that Bayes point estimation is the estimation of minimum risk, the Bayes point estimation based on the loss function $L(\widehat{p}_i, p) = (\widehat{p}_i - p)^2$ is the posterior mean. Finally, the Bayes estimation model can be deduced as

$$\widehat{p}_i = \widehat{p}_{i-1} + \frac{(1 - \widehat{p}_{i-1})[(1 + s_i)\ln s_i + u + 1/s_i + 2 - s_i \ln s_i + u/s_i + 1]}{[u - 1 - s_i \ln s_i + u/s_i + 1]}. \tag{6}$$

Obviously, the previous probability $(\widehat{p}_{i-1})$ is used when $\widehat{p}_i$ is calculated, which makes full use of the sample information by repeatedly using the data. Hence, the Bayes multilayer estimation model can make accurate reliability estimation.

3.2. Reliability Estimation Model. For Weibull distribution function $F(t) = 1 - \exp[-(t_i/\eta)^\beta]$, Bayes estimated value $\widehat{p}_i$ is used to replace p_i, with $\widehat{p}_i = 1 - \exp[-(t_i/\eta)^\beta]$. Let $y_i = \ln\ \ln(1 - p_i)^{-1}, x_i = \ln t_i, a = -\beta \ln \eta, b = \beta$; then $\widehat{p}_i$ can be transformed into the linear equation as, $y_i = a + bx_i$ $(i = 1, 2, \ldots, m)$.

The nonlinear least squares method (NLSM) is very suitable for solving the problem of parameter estimation of nonlinear functions. NLSM transforms the nonlinear problem into a linear problem through a specific transformation method. After the linear function estimate is obtained, it is transformed into the nonlinear function estimate according to transformation relation. The values of a and b are estimated according to the least square method as follows: $\widehat{a} = (B - A^2)/(D - AC)$ and $\widehat{b} = exp[(BC - AD)/(B - A^2)]$, where $A = \sum_{i=1}^m w_i y_i$, $B = \sum_{i=1}^m w_i y_i^2$, $C = \sum_{i=1}^m w_i x_i$, and $D = \sum_{i=1}^m w_i x_i y_i$. w_i is the weighted coefficient and has two forms. In the design based on end time,

$w_i = t_i / \sum_{j=1}^{m} t_j$. In the design based on the total test time, $w_i = n_i t_i / \sum_{j=1}^{m} n_j t_j$. They are called the Bayes weighted coefficient method 1 (BWM1) and the Bayes weighted coefficient method 2 (BWM2).

The point estimation of Weibull distribution at any time can be calculated as follows.

$$\widehat{\beta} = \widehat{b}, \widehat{\eta} = \exp\left(\frac{-\widehat{a}}{\widehat{b}}\right). \tag{7}$$

Finally, the reliability calculation model is constructed as follows.

$$\widehat{R}(t) = \exp\left[-\left(\frac{t_i}{\widehat{\eta}}\right)^{\widehat{\beta}}\right]. \tag{8}$$

4. Results and Discussion

Tapered roller bearing is characterized by compact structure, small radial size, high bearing capacity, and low operation temperature and has been widely used in the journal bearing of railway freight cars. Figure 2 shows that the journal bearing of freight cars at the left and right of the axle bears not only radial loads but also axial loads during service, interference fit exists between the inner ring of the bearing and the axle, and clearance fit exists between the outer ring and the journal box, serving as the support.

The reliability test on the rolling bearing is shown in Figure 3. Considering the limitations of the testing machine, one set of test bearing and two sets of accompanying test bearings are selected during the test. Three sets of bearings are installed on the test shaft, with the test bearing in the middle and the accompanying test bearing on both sides. The rear of the testing machine is a radial loading cylinder, and the left side is connected to the transmission system. The test fixture is installed in the testing machine, and the comparative test method is used in the test process. The test is divided into the number-terminated full-life and time-terminated tests. To obtain complete test data, the number-terminated test requires the failures of all samples and reliable assessment results. However, it is disadvantageous when the test period is long and implies huge costs. The terminated life test only lasted to failure of partial samples, and the life assessment results are calculated by collecting some test data. Considering the high cost, high reliability requirement, long test period, and high consumption of bearings of railway freight cars, a small sample size and the time-terminated simulation test are used in the reliability life assessment to save time and decrease the cost without influencing the life reliability assessment of bearings.

Supposing n tapered roller bearing samples were selected randomly in the time-terminated test and divided into m groups, then each group has n_i ($i = 1, 2, ..., m$) samples. The terminated time of each group is t_i, which satisfies following condition: $t_1 < t_2 < \cdots < t_m$. Let s_i be the total number of nonfailed bearing samples before t_i. In other words, s_i samples that have not exited the test at t_i: $s_i = n_i + n_{i+1} + \cdots + n_m$. Supposing none of the bearing samples failed after finishing the test, then the failure-free test data of bearings is (t_i, n_i).

Currently, most of the rolling bearings used in railway freight cars are manufactured by SKF (Sweden) and FAG-INA (Germany). The service time of bearings conforms to the two-parameter Weibull distribution [10, 25]. According to the requirements of European Standards EN12082, the reliability of the journal bearing of railway freight cars shall be checked according to ISO 281 standards [28].

During the time-terminated life simulation test, the samples are supposedly chosen freshly for each terminated test. If the bearing samples do not fail at the first terminated test, they are used in the second time-terminated test. This iteration continues until all the samples fail. The vertical and radial loads applied onto the double-row tapered roller bearing of heavy-duty freight cars were $p_v = 204.78$ kN and $P_H = 77.4$ kN, respectively. Railway freight cars move at rated speed under rated loads, and the life of the rolling bearing is no shorter than the travel distance of 60×10^6 km [28]. In this study, the rated speed of the heavy-duty freight cars was 80 km/h, which corresponds to the rotating speed of the journal bearing of 505 rpm. If the service time of the bearing exceeds 8000 h, it can meet the reliability design requirements.

According to recommended value of ISO 281, $\beta = 1.5$ and $\eta = 40000$ are the initial values of the Weibull distribution. The probability density model of rolling bearing is expressed as follows.

$$F(t) = \frac{1.5t^{0.5}}{40000^{1.5}} \exp\left[-\left(\frac{t}{40000}\right)^{1.5}\right]. \tag{9}$$

The point estimated values of β and η are calculated through the maximum likelihood estimation of the Weibull distribution parameters: $\widehat{\beta} = 1.4898$ and $\widehat{\eta} = 39832$. Hence, the mathematical model of reliability estimation is expressed as follows.

$$\widehat{R}(t) = \exp\left[-\left(\frac{t}{39832}\right)^{1.4898}\right]. \tag{10}$$

The estimation curves of reliability and failure probability, which are calculated and plotted using the CEM, BWM1, and BWM2, are shown in Figure 4. Obviously, the reliability estimated by BWM2 is the most ideal.

With $\widehat{\beta}$ and $\widehat{\eta}$, a group of Weibull distribution random numbers are generated using the Monte Carlo method and sorted in ascending order. Ten data are placed in one group, and 30 bearing samples are divided into 10 groups for 10 time-terminated tests. The end time of each time (unit: hour) is set as t_i. The test is terminated when the Monte Carlo simulation continued to the 10th time and the test time of the bearing is 9,079 h without failure data. According to the previous repair cycle requirement that the travel distance of railway freight cars should be no shorter than 60×10^6 km [28], the end time of this simulation is determined to be 9,079 h.

The simulation test data of the time-terminated test are listed in Table 1, where i is the termination order, n_i is the number of bearing samples, and s_i is the number of nonfailed

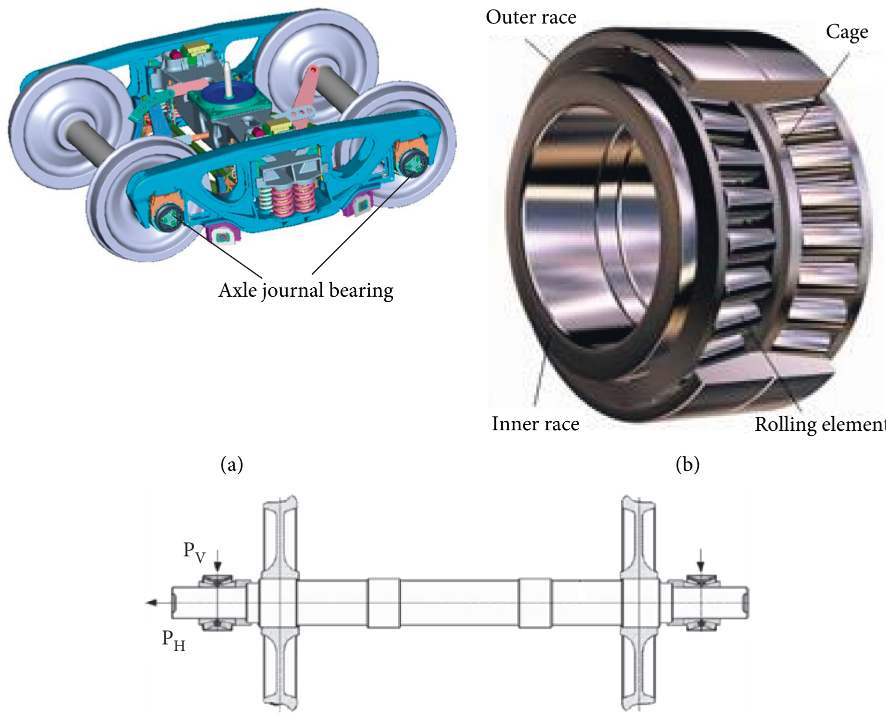

Figure 2: Structure of axle journal bearing for railway freight wagon. (a) Car bogie of railway freight. (b) Double-row tapered roller bearing. (c) Force analysis of axle journal bearing.

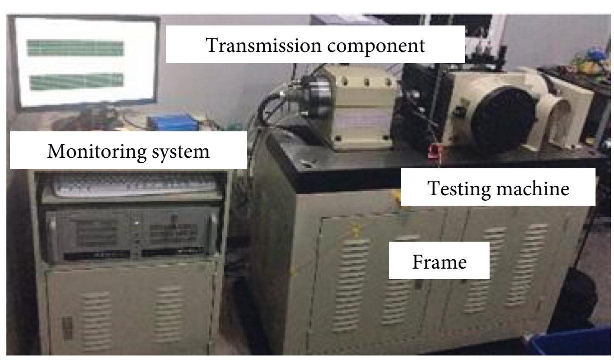

Figure 3: Reliability test bench of axle journal bearing.

bearing samples. The operation time and driving distance of railway freight cars under different numbers of bearing samples in the terminated simulation test are shown in Figure 5.

Supposing the ultimate reliability of the journal bearing of railway freight cars when traveling 60×10^6 km is 0.9975, that is, the maximum failure probability does not exceed 0.0025. The relation curves between the CFP, which are obtained in the four terminated simulation tests, and upper limit parameter u are shown in Figure 6. The satisfying curve is derived according to Figure 6. Finally, the value of u in the equation (5) is determined to be 1000.

After determining the parameters in Table 2, the estimated value $(\widehat{p}_i)$ of $\widehat{p}_i$ is obtained using the CEM, BWM1, and BWM2. The estimated values of the Weibull parameters and the calculated results of reliability using the CEM, BWM1, and BWM2 are listed in Table 3.

Tables 2 and 3 indicate that the estimated results of the CEM differ significantly from the expectation, while the estimated results of BWM1 and BWM2 are relatively good. The Bayes weighted methods are chosen. When the reliability is approximately 0.97, the reliability life of the bearing is calculated to be 8,190 h, and the corresponding traveling distance is 65.5×10^6 km. The reliability assessment results conform to the design requirements of the ISO 281 standards.

The Weibull parameters, which are calculated using failure-free data and failure probability, and the relative errors (REs) are listed in Table 4. According to the above calculation formulas, BWM1 only considers the end time of

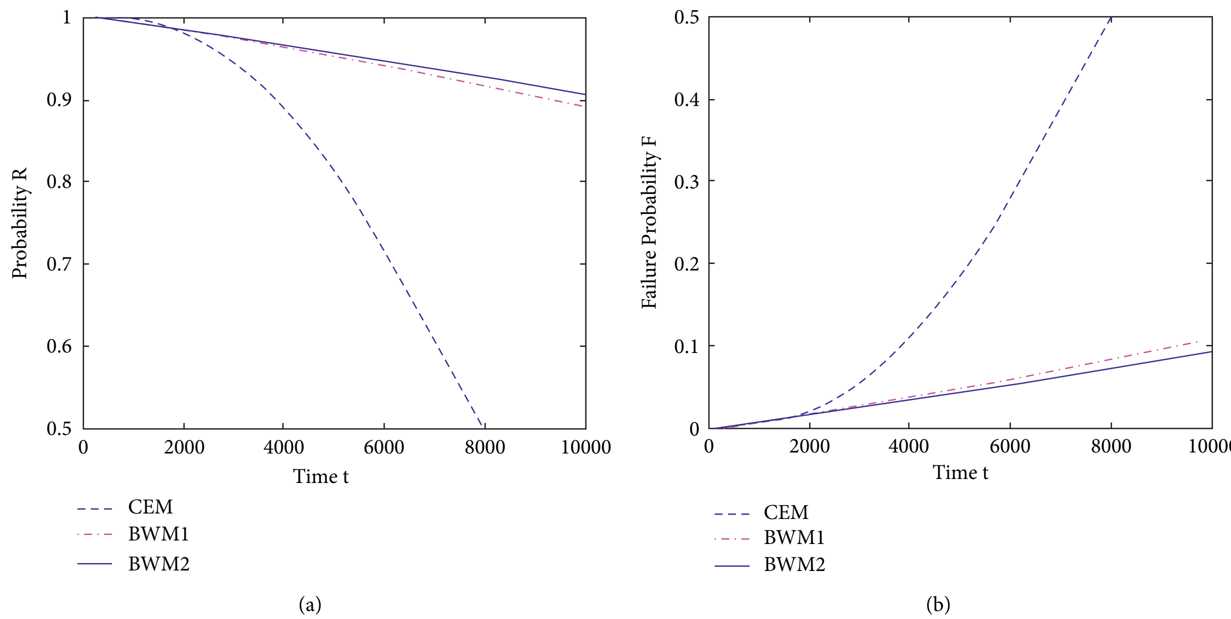

Figure 4: Reliability and failure probability curves of the axle journal bearing for railway wagon. (a) Reliability curves. (b) Failure probability curves.

Table 1: Time truncated data.

i	t_i	n_i	s_i
1	981	7	30
2	1885	6	23
3	2768	5	17
4	3683	3	12
5	4583	2	9
6	5491	2	7
7	6378	1	5
8	7205	1	4
9	8190	1	3
10	9079	1	2

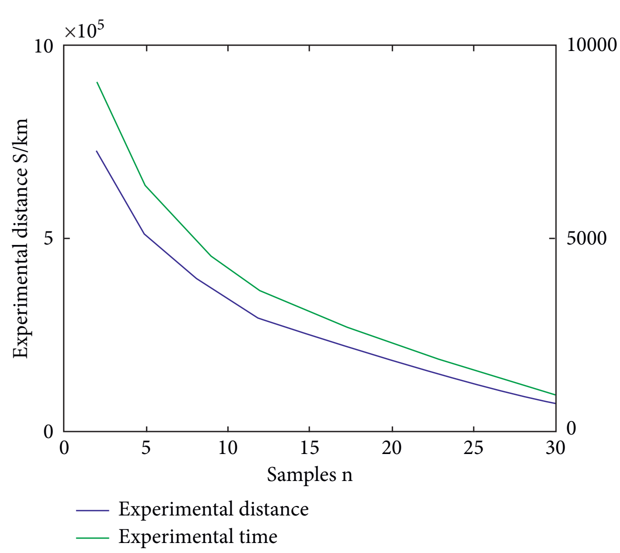

Figure 5: Relationship between sample number, test distance, and test time.

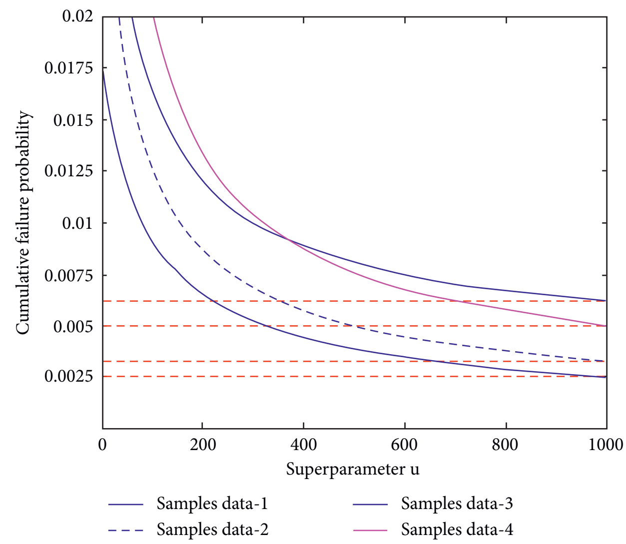

FIGURE 6: Relationship diagram between failure probability and parameter (u).

TABLE 2: CFP estimation $\times 10^{-2}$.

CFP	CEM	BWM1	BWM2
$\widehat{p}_1$	0.89	0.24	0.24
$\widehat{p}_2$	1.09	0.49	0.49
$\widehat{p}_3$	1.35	0.76	0.76
$\widehat{p}_4$	1.72	1.05	1.05
$\widehat{p}_5$	2.27	1.36	1.36
$\widehat{p}_6$	3.13	1.7	1.7
$\widehat{p}_7$	4.55	2.06	2.06
$\widehat{p}_8$	7.14	4.47	4.47
$\widehat{p}_9$	12.5	2.93	2.93
$\widehat{p}_{10}$	25	3.46	3.46

TABLE 3: Parameter and reliability estimation.

Parameter	CEM	BWM1	BWM2
β	1.8455	1.4187	1.3389
η	28706	32621	36962
R (981)	0.9980	0.9979	0.9978
R (1885)	0.9935	0.9953	0.9952
R (2768)	0.9867	0.9924	0.9923
R (3683)	0.9776	0.9892	0.9892
R (4583)	0.9667	0.9858	0.9859
R (5491)	0.9539	0.9823	0.9825
R (6378)	0.9396	0.9787	0.9791
R (7205)	0.9250	0.9752	0.9758
R (8190)	0.9059	0.9709	0.9715
R (9079)	0.8874	0.9640	0.9649

TABLE 4: Comparison of calculation errors.

Method	β	η	RE of β	RE of η
CEM	1.8455	28706	0.2303	0.2823
BWM1	1.4187	32621	0.0542	0.1844
BWM2	1.3389	36962	0.1074	0.0085

the samples, whereas BWM2 comprehensively considers the influences of the end time and the sample size on the Weibull distribution parameters. Therefore, BWM1 shows a higher calculation error than BWM2, while BWM1 presents a relatively high calculation accuracy.

Table 4 shows that the Weibull parameter error based on the multilayer Bayesian methods is significantly lower than that based on the CEM because the multilayer Bayesian methods make full use of the prior information and simulation data of bearing samples. Furthermore, the estimation results of the Bayesian methods are more accurate than those of the CEM, which uses few information. Hence, the multilayer Bayesian methods are more applicable to the reliability modeling and reliability life assessment of the rolling bearings of railway freight cars on failure-free data.

5. Conclusions

Journal bearing of railway freight cars is expensive and has high reliability requirements. The failure data of bearings takes quite a long time to obtain. The reliable life of journal bearing is usually estimated through the small-sample-sized, time-terminated test. The traditional calculation formula of failure probability and the reliability modeling method based on a big sample size cannot easily assess the reliable life of bearings accurately. Thus, beta distribution is used as the a priori distribution. The prior information of the samples and the simulation test data are integrated according to Bayes statistical theory through which the mathematical model of CFP on failure-free data is deduced. The bearing reliability assessment model based on Weibull distribution is constructed by combining the least squares method. Moreover, the failure-free simulation data of bearings are simulated according to the Monte Carlo method. A numerical simulation study on reliable life assessment is carried out using different methods. Meanwhile, the simulation results are compared. The following conclusions could be drawn:

(1) The failure-free simulation data of rolling bearing, which is obtained from the Monte Carlo method, can expand the parameter ranges based on multilayer Bayes estimation, increasing the estimation accuracy of model parameters in the numerical simulation test.

(2) The multilayer Bayes reliability assessment model is compatible with the sample data and the distribution functional information. The model information size is significantly higher than that of the traditional method. In particular, the reliability assessment accuracy of Bayes weighted methods is far higher than that of the traditional probability statistical method.

(3) During implementation of the small-sample-sized, time-terminated test of the journal bearing of railway freight cars, the estimation results of the bearing reliability model, which is built according to multilayer Bayesian method, and the Weibull distribution conform to the design requirements of the ISO 281 standards. This model can accurately assess the reliability life of journal bearings.

In this study, the failure-free data (t_i, n_i) were obtained from the simulation results of the Monte Carlo method. Determining the end time (t_i) of the failure-free data and the number of bearing samples (n_i) in the reliability life test will be discussed intensively in future studies. Additionally, the proposed method is based on failure-free data and inapplicable only to the assessment of the reliability life of bearings once failure data is produced in the reliability test. Therefore, a more reasonable reliability assessment method must be developed.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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