Research Article

Bus Predictive-Control Method considering the Impact of Traffic Lights

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Amid the COVID-19 pandemic, many travelers have switched from public transit to other modes. How to maintain the stability and service quality of the bus system under regular pandemic prevention and control, so as to maintain the attractiveness of the bus, is an important research direction. Predicting operation states and adopting appropriate control measures for running buses are effective means of improving the bus system’s schedule reliability and service quality. Focusing on the impacts of intersection traffic lights on the link’s travel time durations, we establish a probabilistic prediction model for bus headways, classifying the bus headways into three states: bunching, stable, and big gap states. Based on the prediction of bus headways, the most suitable control strategy is selected by the proposed method from the plan set, such as holding control, speed-adjusting control, and stop-skipping control to minimize the bus headway deviation. Simulation experiments were employed to verify the effectiveness of the proposed method. Compared with the no-control situation, the expected headway variation, average passenger waiting time, and bus bunching frequency for 100 simulations by the proposed method are reduced by 77.73%, 41.66%, and 87.11%, respectively. Compared with some control methods without prediction, the proposed method is more robust, maintains good control performance, and reduces bus bunching despite significant variations in environmental parameters. In addition, the model still performs well when considering the execution errors of bus drivers.

1. Introduction

The worldwide pandemic of COVID-19 has had a great impact on the world economy and production as well as people’s lifestyle [1]. At the beginning of the pandemic, countries imposed strict restrictions on traffic, resulting in a sharp drop in bus ridership [2–4]. On the other hand, failing to provide safe travel conditions resulted into solo driving [5, 6]. In the middle of the development of the pandemic, after some measures [7, 8], such as reducing bus capacity, wearing masks, and frequent disinfection, were implemented, the pandemic was brought under control effectively, people regained confidence in the safety of buses, and bus ridership began to recover. At present, many people have been vaccinated and the pandemic is fully under control and has entered the stage of normalized prevention and control. In China, people use the health code and the travel code for nationwide information sharing. Green of the health code means that the person is healthy, and he/she is allowed to enter public places and take public transportation. This method can guarantee the daily travel of healthy people, as long as they have a green health code. By requiring passengers to display the health code and measure body temperature when boarding the bus, bus travel has basically resumed as before. But these actions will lead to longer boarding time durations, which can lead to longer bus dwell time durations at stops and an increased risk of bus bunching. Therefore, ensuring that the stable operation of buses under regular pandemic prevention and control is important, this article aims to study...
the reliability of the bus system in the later stage of the pandemic.

More passengers gather at a bus stop while bus arrivals lagged due to traffic congestion or intersection traffic lights, which induces extra passenger boarding time and exacerbates the delay of this bus. In turn, the following bus with fewer passengers served may depart from the bus stop earlier before the scheduled time [9]. Two or more buses may arrive at the same bus stop at almost the same time, resulting in bus bunching (or bus cluster) [10–12], which reduces the operation reliability and service quality of the bus system [13]. Therefore, it is necessary to predict the likelihood of bus bunching so that effective control measures can be implemented to distribute buses more evenly along the bus route and alleviate bus bunching.

Establishing a predictive-control framework is a common idea for solving the bus bunching problem [14]. Researchers have developed various prediction models to identify the bus bunching phenomenon [15, 16]. Some fit bus link travel time durations with historical data by regression analysis or machine learning [17, 18]. The others combine real-time data with historical data to achieve real-time predictions [19, 20]. Existing studies often use a random term to represent the effect of intersection traffic lights and other disturbances such as traffic congestion, driver operational variability, and unforeseen circumstances on bus link travel time durations [21]. All of the existing research studies do not consider bus delays at intersections adequately. Traffic signal control is a common means of traffic flow control in the urban transportation system. It breaks off the continuous flow for meeting traffic demand in different directions, eliminating intersection conflicts, and has a massive impact on bus link travel time durations, so it should not be overlooked. To improve prediction accuracy, we consider the influence of intersection traffic lights and establish a headway probability prediction model of buses to calculate the bus headway probabilities and identify bus operation states.

Over the past few decades, researchers have proposed and validated a variety of practical measures to address bus bunching [9], such as stop-skipping strategies [22], limited-boarding strategies [23], static or dynamic holding strategies [24–28], speed-adjusting strategies [29], transit signal priority strategies [30], bus substitution strategies [31, 32], and combined strategies [33, 34]. Holding strategies can effectively alleviate bus bunching, but they often lead to confusion and dissatisfaction among passengers on board and increase bus operating time durations and costs [35]. Although easy to implement, stop-skipping strategies are prone to a decline in passenger satisfaction. They are usually implemented when there are significant delays on the bus, no passengers needing to get off the bus, and few passengers waiting at the stop. In contrast, speed-adjusting strategies can avoid these problems and achieve similar control effects as holding and stop-skipping strategies. Studies have shown the effectiveness and practicality of speed-adjusting strategies under different traffic disturbances [29]. However, the speed-adjusting strategies are limited by the road environment and traffic conditions. When there are many disturbances in the link’s flow environment, the speed-adjusting control may not be able to restore the stability of bus headways. Therefore, to enhance the control effect and improve the robustness of the control strategy, this paper proposes a combined control method in which speed-adjusting control is adopted prior to holding control and stop-skipping control.

This paper aims to determine the optimal control parameters and implement real-time control to resolve the bus bunching by calculating the headway probabilities and then selecting a suitable control strategy based on the results calculated. The paper is structured as follows: Section 2 introduces the probability prediction method of bus headways. In Section 3, a combined control method is presented based on the prediction of the bus headway state. Section 4 conducts simulation experiments and results analysis. Some conclusions and suggestions for future research are presented in Section 5.

2. Probability Prediction Method of the Bus Headway

On the same bus route with the total bus stop J, buses will follow the departure schedule to run away from the starting station. Buses can be considered to operate independently of each other, i.e., will not depart immediately after arriving at the bus stop J. Still, they will follow the departure schedule of stop J [29]. So we assume that the bus route is a one-way traffic corridor with J bus stops (Figure 1). Bus stops divide the bus route into J − 1 links, and the endpoints of the link zj,j+1 are the stop j and the stop j+1, respectively. Depending on whether links contain intersection r, (r = 1, 2, . . . , R), the set K of links of a bus route can be divided into two sets K′ and K′′, satisfying K = K′ ∪ K′′ and K′ ∩ K′′ = ∅. zj,j+1 ∈ K′ means that the link zj,j+1 contains intersections and zj,j+1 ∈ K′′ means that the link zj,j+1 does not contain intersections.

The link’s predictive-control model [29] is extended in the paper. We consider that the bus arriving at a bus stop is the beginning of the predictive-control cycle. When the bus arrives at a bus stop, the state of its headway at the next stop is predicted and a combined control will be implemented based on the prediction results. When the controlled bus arrives at the next stop, a new round of predictive control is started and so on.

The innovative prediction method that considers the influence of intersection traffic lights is shown in Figure 2. The prediction process is divided into three steps: bus departure time prediction, bus arrival time prediction, and bus headway probability prediction. The bus departure time prediction takes into account the impact of the passenger arrival rate on the bus dwell time at the stop, and the bus arrival time prediction fully considers the impact of the traffic lights on the link travel time. At the same time, the headway probability prediction divides the bus headways into three states, bunching, stable, and big gap states, and then calculates the probabilities of the three states, respectively. The steps of the prediction process are as follows.
Firstly, the arrival time of the bus $i$ at the stop $j$ is obtained from GPS data, and then, the number of boarding passengers $B_{ij}$ at the stop $j$ and its probability $P(B_{ij})$ are predicted based on the average passenger arrival rate and the average number of alighting passengers at the stop $j$ and the departure time $d_{ij}$ and its probability $P(d_{ij})$ of the bus $i$ at the stop $j$ are calculated based on these predicted results. Secondly, all possible travel time instances of the bus $i$ on the link $z_{ij,1}$ affected by the intersection traffic lights and all possible arrival time instances $a_{ij}(k)$, $(k = 1, 2, \ldots, n)$ of bus $i$ at stop $j + 1$ and their probability $P(a_{ij}(k))$, $(k = 1, 2, \ldots, n)$ can be calculated based on the discretized average link speeds $v_k \in (v_1, v_2, \ldots, v_n)$. Thirdly, the probability distribution of different headways, $h_{ij}^*$, between bus $i$ and the preceding bus at stop $j + 1$ can be predicted based on the expected departure time instances and arrival time instances, and the probability of bus headway $h_{ij}^*$ in the bunching state $P(s_{ij}^1)$, in the stable state $P(m_{ij}^1)$, and in the big gap state $P(l_{ij}^1)$ are calculated, respectively. The final output of the prediction model is the probability of the three states of bus headway $h_{ij}^*$ of the bus $i$ at the bus stop $j$.

2.1. Predicting Bus Departure Time Instances. The bus dwell time at bus stops without control can generally be divided into two parts. The first part is the fixed time $c$, i.e., the door opening and closing time of buses after arriving at bus stops. The second part is the passenger’s boarding and alighting time, related to the number of passengers to be served at the stop [36]. Assume that the average passenger boarding time is presented by $b_1$ sec per person, the symbol $b_2$ sec per person is used as the average passenger alighting time, and the bus passenger capacity is described as $N$ people per vehicle. Considering the impact of COVID-19, passengers need to show a health QR (Quick Response) code to the bus driver and measure their body temperature when boarding the bus. These operations will prolong the average boarding time of passengers. $b_1$ is usually 2–3 seconds per person, but if the impact of the pandemic is considered, it will be 5–10 seconds per person. In addition, in order to maintain the social distance between passengers, the bus capacity is limited to 50% and the value of $N$ is 60–80 in normal time, which is 30–40 during the pandemic. If the arrival time of the bus $i$ at the stop $j$ is illustrated by $a_{ij}$, then the dwell time $s_{ij}$ and the departure time $d_{ij}$ without control can be obtained as follows:

$$s_{ij} = c + \max \{b_1 \times B_{ij}, b_2 \times A_{ij}\}, \quad (1)$$

$$d_{ij} = a_{ij} + s_{ij}, \quad (2)$$

where $A_{ij}$ represents the number of alighting passengers and $B_{ij}$ is the number of boarding passengers.

If the number of onboard passengers of bus $i$ is $Q_{ij}$ when it arrives at stop $j$ and the onboard passengers get off at the stop $j$ according to a specific rate, the number of alighting passengers at the stop $j$ for the bus $i$ can be calculated as follows:

$$A_{ij} = [Q_{ij} \times \alpha_j], \quad (3)$$

where $\alpha_j$ represents the passenger alighting rate.

Assuming that the passenger arrival pattern conforms to the Poisson distribution, the average passenger arrival rate is $\mu$ person per minute. Based on the passenger actual arrival
rate of $\mu_j^i$, and the bus headway $h_j^i$, the number of boarding passengers and its probability can be calculated as follows:

$$B'_j = \min \left( \frac{\mu_j^i h_j^i}{60} + M_j^{-1}, N - Q_j^i + A'_j \right). \quad (4)$$

$$\mathcal{P}(B'_j) = \mathcal{P}(\mu_j^i = k) = \frac{k^k}{k!} e^{-\mu}, \quad (5)$$

where $M_j^{-1}$ represents the rejected passengers by the bus $i - 1$ at the stop $j$ and $M_j^{-1} = \max(0, \mu_j^i h_j^i - N + Q_j^i - A_j^{i-1})$.

The number of onboard passengers of the bus $i$ when it leaves the stop $j$ is calculated based on the number of boarding and alighting passengers, which is equal to the number of onboard passengers of the bus $i$ when it arrives at the stop $j + 1$.

$$Q_{j+1}^i = Q_j^i - A_j^i + B'_j. \quad (6)$$

The probability of the bus departure time $d_i^j$ is equal to the probability that the number of boarding passengers is $B'_j$, which can be obtained as follows:

$$\mathcal{P}(d_j^i) = \mathcal{P}(B'_j), \quad (7)$$

where the departure time probability $\mathcal{P}(d_i^j)$ of each bus at the first stop is 1 for all.

Considering buses departing at the first stop with a fixed headway $H$, the departure time of each bus at the first stop satisfies the following relationship:

$$d_1^i = d_1^i + i \times H, \quad (8)$$

where $d_1^i = 0$.

2.2. Predicting Bus Arrival Time Instances. Without interference, the bus speed conforms to the normal distribution; then, the probability density function of the bus speed under idealistic conditions is

$$\mathcal{P}(v) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(v-\bar{V})^2/2\sigma^2}, \quad (9)$$

where $\sigma$ represents the standard deviation of bus speeds and $\bar{V}$ represents the mean of bus speeds. Considering that other vehicles will affect buses and the speed limits of the general urban roads, we assume that the actual speed of the bus is within the interval $[V_{\min}, V_{\max}]$ and then the probability density of the interval $(-\infty, V_{\min})$ and $[V_{\max}, +\infty)$ can be superimposed on the interval $[V_{\min}, V_{\max}]$, so the actual probability of the bus speeds is obtained as follows:

$$\mathcal{P}(v) = \left\{ \begin{array}{ll}
0, & v \in (-\infty, V_{\min}) \cup (V_{\max}, +\infty), \\
\mathcal{P}(v) + \int_{V_{\min}}^{V_{\max}} \frac{p(m)\,dm}{\mathcal{P}(m)\,dm} \left[ \int_{-\infty}^{V_{\min}} \mathcal{P}(m)\,dm + \int_{V_{\max}}^{+\infty} \mathcal{P}(m)\,dm \right], & v \in [V_{\min}, V_{\max}],
\end{array} \right. \quad (10)$$

The departure time of the bus $i$ at the stop $j$ is $d_j^i$, and the location of the stop $j$ is $x_j$. If the link $z_{j+1}$ contains $n'$ intersections, these intersections are indexed as $r_k, k = 1, 2, \ldots, n'$, their locations are $x_{r_k}, x_{r_{k+1}}, \ldots, x_{r_{n'}}$, the number of vehicles queuing at intersection $r$ is $Z_r$, the average length of the vehicles is $u$ meters, and the average headway distance is $y$ meters, and the distance between the bus and the intersection $r_k$ when it stops is $x_{r_k} - (u + y)Z_{r_k}$.

The number of vehicles queuing at intersection $r_k$ at the departure time $d_j^i$ for the bus $i$ at the stop $j$ plus the travel time $x_{r_k} - (u + y)Z_{r_k} - x_j / v_{j+1}$ and the delay $\sum_{m=1}^{k-1} v_{j+1}(r_m)$ of the link $z_{j+1}$, which satisfies the following relationship:

$$d_j^i + y' \sum_{m=1}^{k-1} v_{j+1}(r_m) + \frac{x_{r_k} - (u + y)Z_{r_k} - x_j}{v_{j+1}} = C_{r_k} \times \tau + \omega. \quad (13)$$

$$y' = \left\{ \begin{array}{ll}
1, & n' > 1 \text{ and } k = 2, 3, \ldots, n', \\
0, & n' = 1 \text{ and } k = 1,
\end{array} \right. \quad (14)$$

where $C_{r_k}$ represents the signal circle length of the intersection $r_k$, $\tau$ is the natural number, $\omega$ is numeric in the
interval \([0, C_{r_k}]\), and \(e_{j,j+1}^i(r_k)\) represents the delay of the bus \(i\) at the intersection \(r_k\) on the link \(z_{j,j+1}\). The intersection traffic lights considered in this paper have fixed circle length, and the valid time duration is included in the red-light phase, with the green-light phase as the starting phase and \(\beta_{r_k}\) as the green time ratio. Then, \(\omega \leq C_{r_k} \beta_{r_k}\) represents that the bus \(i\) arrives at the intersection \(r_k\) at the time when the traffic light is green; otherwise, it is red.

However, the impact of intersection traffic lights on the bus link travel time cannot be ignored. There are two situations. The first is the bus arrives at the stop line when the traffic light is green. The bus can directly pass the intersection without stopping. The intersection delay is 0, and the probability of bus link travel time is the same as formula (12). The second situation is that the bus arrives at the intersection in the red-light period, needs to stop, and wait. The intersection delay depends on the vehicle queue length and the red-light time in the current cycle of the signal. If the number of queuing vehicles is 0, the intersection delay is the remaining red-light time. If the number of queuing vehicles is 0, the intersection delay is the remaining red-light time when the bus stops plus the red-light time caused by waiting for the vehicle ahead to cross the intersection. Therefore, the delay at the intersection \(r_k\) is calculated as follows:

\[
\begin{align*}
\Delta (t_{j,j+1}) & = \begin{cases} f(t_{j,j+1}), & \omega \leq C_{r_k} \beta_{r_k}, \\
\frac{f(t_{j,j+1}) + f(t_{j,j+1} - e_{j,j+1}^i)}{\beta_{r_k}}, & \omega > C_{r_k} \beta_{r_k} \end{cases} \\
\end{align*}
\]

where \(t_{j,j+1}^i \leq (L_{j,j+1} C_{r_k} (\beta_{r_k} + \tau) - d_j - \gamma^j e_{j,j+1}^i)/x_{r_k} - x_j - (u + y)Z_{r_k}^j + e_{j,j+1}^i\) is derived from \(\omega \leq C_{r_k} \beta_{r_k}\) of formula (15).

Based on the departure time \(d_j^i\) and the possible bus link travel time \(t_{j,j+1}^i\), the possible arrival time \(a_{j+1}^j\) of the bus \(i\) at the stop \(j + 1\) and its probability \(P(a_{j+1}^j)\) can be predicted and calculated as follows:

\[
\begin{align*}
a_{j+1}^j & = d_j^i + t_{j,j+1}^i, \\
P(a_{j+1}^j) & = \bar{f}(a_{j+1}^j - d_j^i) = \bar{f}(t_{j,j+1}^i).
\end{align*}
\]

2.3. Predicting Probabilities of Different Bus Headways.

The bus headway at each stop can be expressed as the difference between the bus departure time of itself and its preceding bus at the same stop [22], which can be calculated as follows:

\[
h_j^i = d_j^i - d_j^{i-1}.
\]

According to formula (7) and formula (20), we can calculate the probability of bus headways as follows:

\[
P(h_j^i) = P(d_j^i) \times P(a_{j+1}^j) = P(B_j) \times \bar{f}(t_{j,j+1}^i).
\]

The bus departure interval is \(H\) seconds. In this paper, the headway deviation coefficients \(\psi_1\) and \(\psi_2\) are set in the range of \(0 < \psi_1 < 1 < \psi_2\), \(\psi_1\) is the minimum stable headway coefficient, which is generally taken as 0.6–0.8 [13], and \(\psi_2\) is the maximum stable headway coefficient, which can be set according to the bus route and bus departure schedule. We define that the headways in the interval \([-\infty, \psi_1 H]\) indicate the bus will bunch, in the interval \([\psi_1 H, \psi_2 H]\) indicate the bus will cause a big gap, and in the interval \([\psi_2 H, \infty]\) indicate the bus is in the stable state. The cumulative probabilities of bus headways for these three intervals can be calculated as follows:

\[
\begin{align*}
P_i^{h_j^i} & = P(h_j^i < \psi_1 H), \\
P_m^{h_j^i} & = P(\psi_1 H < h_j^i < \psi_2 H), \\
P_i^{h_j^i} & = P(h_j^i > \psi_2 H),
\end{align*}
\]

where \(P_i^{h_j^i}\) represents the bunching probability, \(P_m^{h_j^i}\) represents the stability probability, and \(P_i^{h_j^i}\) represents the big gap probability.
2.4. Probabilistic Analysis of Bus Headways in the Bunching State. To investigate the impacts of the maximum speed limit $V_{\text{max}}$ and the average passenger arrival rate $\mu$ on the bus headway probability in the bunching state, a bus route with 18 bus stops is established for simulation. The bunching probability and the variance of the bunching probability at each stop are calculated on different parameters. In Figure 3, it is found that the bunching probability of each stop on the bus route varies and the maximum speed limit has a negligible impact on the variance of bunching probability. In contrast, the average passenger arrival rate has an enormous impact, as shown in Figure 4. With the increase in the passenger arrival rate, the variance of bunching probability tends to be stable. In addition, the cumulative bunching probability of stops along the route decreases as the maximum speed limit increases. At the same time, no clear pattern is shown for the different average passenger arrival rates.

3. Combined Control Method

This section will propose a real-time bus control method based on the headway probability prediction, which is a two-layer decision process, as shown in Figure 5.

The upper layer is the control strategy selection layer. Only the speed-adjusting control is used when the bus headway at the next stop is predicted to be in a stable state. In the bunching state, the holding control and the speed-adjusting control are combined to use. In the big gap state, the stop-skipping control and the speed-adjusting control are combined.

The lower layer is the control parameter optimization layer. The optimal holding time or adjusted speed and whether to skip the next stop should be determined according to the objective function at this layer.

3.1. Combined Control Objective Function. To make buses more evenly distribute on the bus route and avoid bus bunching, we set the deviation reduction of bus headway from the bus departure interval as the control objective and the objective function is as follows:

$$C_j = |\hat{h}_{j+1} - H_i|.$$  

3.1.1. Stable State. If the condition $P_{m_{j,j+1}} \geq \max(P_{s_{j,j+1}}, P_{l_{j,j+1}})$ is satisfied, the speed-adjusting control is implemented and the driver is informed of the recommended speed in the following link. The adjustable range of speeds is assumed as interval $[VA, VB]$. The bus’ optional travel speed $\tilde{v}_{j,j+1}$ on the following link satisfies inequation $VA \leq \tilde{v}_{j,j+1} \leq VB$ and $\tilde{v}_{j,j+1} \in (v_1, v_2, \ldots, v_n)$. The value of the objective function for the corresponding case of the possible speed $\tilde{v}_{j,j+1}$ is calculated, and the speed $\tilde{v}_{j,j+1}$ with the smallest objective function value $C_j$ is selected as the proposed driving speed $\tilde{v}_{j,j+1}$ for the following link.

3.1.2. Bunching State. If the condition $P_{s_{j,j+1}} > \max(P_{m_{j,j+1}}, P_{l_{j,j+1}})$ is satisfied, a bus bunching warning is issued to the bus driver and the driver is informed of the holding time for the current stop and the recommended speed for the following link. Assume that the holding time is in the range of interval $[0, W]$. All possible holding time instances $\tilde{w}_j$ at bus stops can be obtained in steps. The objective function values $C_j$ are calculated for all possible holding time instances $\tilde{w}_j$ and optional travel speed $\tilde{v}_{j,j+1}$. The control parameters with the smallest $C_j$ are selected as the optimal holding time $\tilde{w}_j$ and the proposed driving speed $\tilde{v}_{j,j+1}$ for the following link.

3.1.3. Big Gap State. If the condition $P_{l_{j,j+1}} > \max(P_{s_{j,j+1}}, P_{m_{j,j+1}})$ is satisfied, a big gap warning is issued to the driver and the driver is informed of the recommended driving speed for the following link and whether the next stop needs to skip. Firstly, the proposed speed $\tilde{v}_{j,j+1}$ and the link travel time $\tilde{t}_{j,j+1}$ of the following link are determined by minimizing the objective function, and the bus headway $\hat{h}_{j+1}$ after implementing the speed-adjusting control is calculated. If the inequation $\tilde{h}_{j+1} \geq \varphi_2 H$ is satisfied, that is, the bus
headway at the next stop, which is being controlled, is still in a big gap state, then the bus skips the next stop and the bus dwell time of the next stop is 0. Otherwise, the bus will dwell at the next stop, so passengers can board and alight the bus at the next stop.

3.2. System State Update. Figure 6 illustrates the predictive-control principle. The prediction module outputs the cumulative probabilities of the three states of bus headway, and the control module outputs the control parameters based on the prediction results and uses them to update the state of the bus. The updated states are then fed into the prediction module, and a new prediction and control cycle begins.

The control module will output the holding time $\bar{u}_j$, the driving speed $\bar{v}_{j,j+1}$, and whether to skip the next station. According to these instructions, the departure time $\tilde{d}_j$ at the stop $j$, the arrival time at the stop $j + 1$, and the headway at the stop $j + 1$ of the bus $i$ are updated as follows:

$$\tilde{d}_j = a_j^i + \bar{s}_j + \bar{u}_j. \quad (27)$$

$$\bar{a}_{j+1}^i = \tilde{d}_j + \bar{v}_{j,j+1}. \quad (28)$$

$$\tilde{h}_{j+1} = \bar{a}_{j+1}^i - \bar{a}_{j+1}^{i-1}. \quad (29)$$

3.3. Evaluation Indicators

3.3.1. Coefficient of Headway Variation. The coefficient of headway variation (CHV) is selected as the main evaluation index of the control performances, which can compare the variety of bus headways in different mean cases.

$$\text{CHV} = \frac{S_h}{D_h}; \quad (30)$$

where $S_h$ represents the standard deviation of bus headways and $D_h$ represents the mean of bus headways.
3.3.2. Average Passenger Waiting Time. Passenger waiting time can reflect the impact of the control strategy on passengers’ service qualification and is divided into two parts: the waiting time of newly arrived passengers and the waiting time of passengers rejected by the previous bus [38]. The total passenger waiting time $PWT$ can be calculated as follows:

$$PWT = \frac{1}{2} \sum_{j=1}^{I} \sum_{i=1}^{L} (\mu_j^i h_j^i) + \sum_{j=1}^{I} \sum_{i=1}^{L} (M_j^i-1 h_j^i).$$

(31)

The average passenger waiting time $\overline{PWT}$ is the total passenger waiting time divided by the sum of the boarding passengers and the passengers rejected by the previous bus.

$$\overline{PWT} = \frac{PWT}{\sum_{j=1}^{I} \sum_{i=1}^{L} (B_j^i + M_j^i)} = \frac{1}{2} \sum_{j=1}^{I} \sum_{i=1}^{L} (\mu_j^i h_j^i + 2M_j^i-1 h_j^i)$$

(32)

3.3.3. Total Number of Passengers Served. The number of passengers served by buses during the operation period is an essential indicator of the bus company’s operating revenue, so we count the total number of passengers served, $TNP$, as the third evaluation indicator.

$$TNP = \sum_{j=1}^{I} \sum_{i=1}^{L} B_j^i.$$

(33)

3.3.4. Average Running Time. The average running time $\overline{ART}$ is used to evaluate the impact of control strategies on the operating costs of bus companies.

$$\overline{ART} = \sum_{j=1}^{I} \frac{(d_j^i - d_i^j)}{J}.$$

(34)

3.3.5. Bus Bunching Frequency. Reducing bus bunching frequency, $BBF$, is the primary goal of the predictive-control method, so this article counts the bus bunching frequency to evaluate the effectiveness of control strategies as follows:

$$BBF = \sum_{j=1}^{I} \sum_{i=1}^{L} bb f_j^i,$$

(35)

where $\bar{h}_j^i < \mu, H$ represents that the bus headway remains in the bunching state after control; then, $bb f_j^i = 1$. Otherwise, $bb f_j^i = 0$.

4. Simulation Experiments

The previous sections describe the principle of the predictive-control method, and this section will implement simulation experiments to verify the effectiveness of the predictive-control method proposed in this paper. Meanwhile, the proposed method is compared with the holding control and the speed-adjusting control to analyze the control performance.

4.1. Simulation Environment and Parameters. A bus route with 18 bus stops and 8 intersections is constructed. The information about the links and intersections is shown in Tables 1 and 2.

The signal cycle length is 120 seconds, the ratio of green time to cycle length is 0.6, and the starting phase is the green-light phase. In the simulation experiment, the average speed is $V = 6m/s$, the variance of speed is $\sigma^2 = 30$, the adjustable speed range is $[4.74, 7.14]m/s$, the maximum holding time is $W = 60s$, and the departure interval between buses is $H = 300s$. The average boarding time is $b_1 = 5$ second per person, and the average alighting time $b_2 = 1$ second per person. The actual passenger arrival rate is randomly generated, with a desirable range of $[0, 6]$ people per minute, and the average passenger arrival rate for all stations is $\mu = 3$ people per minute. The passenger alighting rate at the origination stop is set to 0, at the destination stop is set to 1, and at stops between them is set to a random number of $[0, 1]$. The headway deviation coefficients $\varphi_1$ and $\varphi_2$ are taken as...
and 2, respectively, and the passenger capacity of buses is \( N = 40 \) people.

### 4.2. Numerical Experiments

#### 4.2.1. Prediction Based on Links

Based on the arrival and departure information of the previous bus, the arrival time of the current bus, and the number of passengers on board, the headway of the current bus at the next bus stop can be predicted (Figure 7).

Assume that the arrival time of the bus \( m \) at the stop \( k \) is \( a_k^m = 473 \) s, the number of passengers in the bus \( m \), \( Q_k^m \), is 20, and the passenger alighting rate at the stop \( a_k \) is 0.2. According to formula (3), the alighting passenger number of the bus \( m \) at the stop \( k \) is \( A_k^m = Q_k^m \times a_k = 20 \times 0.2 = 4 \). The arrival time of the bus \( m - 1 \) at the stop \( k \) is \( a_{k-1}^m = 153 \) s. According to formula (21), the headway of the bus \( m \) at the stop \( k \) is \( h_k^m = a_k^m - a_{k-1}^m = 473 - 153 = 320 \) s. If the actual arrival rate of passengers is \( \mu_k^m \) people per minute, the number of passengers rejected by the bus \( m - 1 \) at the stop \( k \) is \( M_{k-1}^m = 0 \). According to formula (4) and formula (5), the number of boarding passengers of the bus \( m \) at the stop \( k \) and its probability can be calculated as \( B_k^m = 10 \) people and \( P(B_k^m) = 0.0842 \). According to formula (6), the number of passengers on board when the bus \( m \) arrives at the stop \( k + 1 \) is \( Q_k^{m+1} = Q_k^m + B_k^m \) and \( h_k^m = 26 \) people. According to formula (1) and formula (2), we can calculate the dwell time and the departure time of the bus \( m \) at the stop \( k \) as \( s_k^m = 25 \) s and \( d_k^m = a_k^m + s_k^m = 498 \) s, respectively. According to formula (7), the probability of the bus departure time can be calculated as \( P(d_k^m) = 0.0842 \).

<table>
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<tr>
<th>Link</th>
<th>Origin stops</th>
<th>Destination stops</th>
<th>Origin location</th>
<th>Destination location</th>
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<td>600</td>
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<td>1100</td>
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<td>600</td>
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<tr>
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</tr>
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<td>2500</td>
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<td>600</td>
</tr>
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<td>( z_{6,7} )</td>
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</tr>
<tr>
<td>4</td>
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<td>( z_{7,8} )</td>
<td>300</td>
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<td>7</td>
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<td>( z_{15,16} )</td>
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<td>200</td>
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<tr>
<td>8</td>
<td>10000</td>
<td>( z_{17,18} )</td>
<td>300</td>
<td>300</td>
</tr>
</tbody>
</table>

Figure 7: Individual prediction process.
Suppose the length of the link \( z_{k+1} \) is 600 m, containing an intersection that is 300 m from the link origin. According to formulas (9)–(20), the link travel time \( t_{k+1} \), the possible arrival time instances of the bus \( m \) at the stop \( k + 1 \), and the probabilities \( P(a_{k+1}) \) can be calculated for all possible speeds. If the arrival time of the bus \( m - 1 \) at the stop \( k + 1 \) is \( a_{k+1} = 313 \) s, according to formulas (21) and (22), the bus headway \( h_{m,k}^{\text{opt}} \) and its probability \( P(h_{m,k}^{\text{opt}}) \) can be obtained, as shown in Figure 8.

According to formulas (23)–(25), the probabilities that the bus headway is in bunching, stable, and big gap states are calculated as \( P_{sm}^m = 0.0046 \), \( P_{sm}^m = 0.0725 \), and \( P_{sm}^m = 0.0071 \), respectively.

### 4.2.2. Combined Control

Comparing the value of the probabilities \( P_{sm}^m, P_{sm}^m, \) and \( P_{sm}^m \), it is found that the probability \( P_{sm}^m \) is in the stable state at the stop \( k + 1 \) is the largest, so the speed-adjusting control is selected.

Calculating the bus headway \( h_{m} \) and the objection \( C_{j} \) (Figure 9) for all possible adjusted speeds \( v_{j} \) and \( V_A \leq v_{j} \leq V_B \), it can be found that when the adjusted speed takes the value of 5.58 m/s, the bus headway is 292.5269 s, while the control objective function value is the smallest, 7.4731. Therefore, the proposed speed of the following link for the bus \( m \) is chosen as \( v_{m,k+1}^{\text{opt}} = 5.58 \) m/s, and the holding time at the stop \( k \) is \( \Delta h_{k+1} = 0 \). According to formulas (27)–(29), we update the departure time at the stop \( k \), the arrival time at the stop \( k + 1 \), and the bus headway for bus \( m \) at the stop \( k + 1 \) after control:

\[
\tilde{a}_{k+1}^{j} = \tilde{a}_{k}^{m} + \tilde{s}_{k}^{m} + \tilde{w}_{k}^{m} = 498, \quad \tilde{a}_{j+1}^{j} = \tilde{a}_{j}^{j} + \tilde{r}_{j+1}^{j} = 605.5269 \text{ s}, \text{ and } h_{j+1}^{j} = \tilde{a}_{j+1}^{j} - \tilde{a}_{k+1}^{j} = 292.5269 \text{ s}.
\]

### 4.3. Other Control Strategies

#### 4.3.1. Holding Control

If the bus headway \( h_{j}^{j} \) of the target bus is less than the departure interval \( H \), then the target bus is controlled at the stop with the holding time \( \min (\Delta h_{j}^{j}, W) \). Otherwise, no control is implemented.

#### 4.3.2. Speed-Adjusting Control

The speed can be adjusted in the range of \( [V_A, V_B] \). The deviation of bus headway and the departure interval after the implementation of each adjusted speed is calculated, and the speed with the slightest variation of bus headway, \( \Delta h_{j}^{j} \), is selected as the optimal adjusted speed.

### 4.4. Simulation Results and Discussion

Operational data from 20 buses were selected for analysis. Bus trajectories with no control (NC), holding control (HC), speed-adjusting control (SC), and the proposed combined control (CC) for the same parameter settings are presented in Figure 10.

Figure 10(a) shows that buses without control are subject to severe bunching. Still, the amount of bunching can be significantly reduced and the stability of the headways can be improved by implementing control, whether the holding control, the speed-adjusting control, or the combined control, as shown in Figures 10(b)–10(d). However, bus bunching and a big gap occur in both HC and SC, both of which reduce the amount of bunching to some extent compared to the no-control case, but did not maintain a stable state of bus headways for the whole operation period. Figure 10(b) shows that when bus headways are greater than the bus departure interval, HC cannot maintain the stability of the bus system. Figure 10(c) indicates that SC cannot avoid bus bunching completely when the speed of the preceding bus varies widely. Bus bunching occurs in both HC and SC. But, CC under the same conditions can predict the bus headway state and its probability and then control the bus to avoid bunching, as shown in Figure 10(d). Therefore, the CC based on headway probability prediction is not only effective in reducing bus bunching and restoring headway stability, but it is also more effective and robust than the HC and the SC. The predictive-control method maintains good performance under both normal conditions and high disturbance conditions.
The distribution of bus headways of the four cases is illustrated in Figure 11. The results show that the headway distribution in the no-control case is most dispersed, with a large proportion of headways in the bunching and the big gap states, which indicates that the headways vary dramatically and are extremely unstable. In contrast, the headway distribution in the other three cases is more concentrated, with more headways in the stable state, which indicates that the headway stability can be effectively improved by controlling the bus. HC and SC effectively control the bus headway at the front stops on the bus route, and the bus headway can be maintained in a stable state. However, as the control errors accumulate, the headways at the rear stops in the bunching and the big gap states increase significantly, indicating that the performance of HC and SC will gradually weaken. In addition, HC cannot take the bus to catch up with the previous bus when there is a big gap between them, and the bus headway cannot restore stability in such cases. SC is more effective than the holding control, but it cannot avoid bus bunching entirely due to the adjustable speed range limitations. In contrast, CC can effectively respond to various situations and keep the bus headways in the stable state, avoiding the bus bunching phenomenon in most cases.

Figure 12 shows the mean, median, and variance of the bus headways for the four cases, and it can be found that the mean and median of the bus headway for the four cases are relatively close. However, the variances of bus headways have a significant difference. The bus headway variance in NC is the largest, which improved to a more considerable extent after control. The bus headway variance and the 1.5 IQR in CC are small, which indicates that the distribution of bus headways in CC is the most concentrated and stable.

In order to quantify and analyze the effects of different control strategies on bus headways, passengers, operating costs, and bus bunching under various scenarios, the expected values of the performance indicators for 100 simulations are calculated as shown in Table 3. The statistical results show that CC can effectively reduce the coefficient of headway variation (CHV), the average passenger waiting time (PWT), the total number of passengers served (TPN), the average running time (ART), and the bus bunching frequency (BBF) with 77.73%, 41.66%, 3.25%, 8.23%, and 87.11% improvement, respectively, and has better performance than the HC and the SC. SC has the smallest average running time (ART), indicating that SC has the best operating cost savings performance.
In addition, the total holding time and the variation of the adjusted speed were calculated separately, as shown in Table 4. The results show that CC saves 52.05% of the total holding time compared with HC, effectively reducing the operating time. The variation of the adjusted speed for both SC and CC are relatively small, and the difference is negligible. The variation of the adjusted speed for CC is slightly lower than that of SC, which indicates that it is less complicated for bus drivers to operate under CC and more beneficial for bus drivers to implement control commands.

The previous simulation experiments have verified the validity and robustness of the predictive-control model during the pandemic on a theoretical level. However, in the actual operation process, the bus driver may not be able to fully execute the control instructions due to the external environment or personal operations, resulting in execution errors. In order to explore the influence of the execution error on the predictive-control model, an execution error coefficient $e_p$ is set in the next experiment, that is, the optimal adjusted speed and the optimal holding time are multiplied by $(1 + e_p)$ as the actual values and the bus system state is updated according to the actual adjusted speed and holding time. For the command of stop-skipping, it is assumed that the bus driver can execute it completely, and the bus trajectories under the no-control (NC) situation and the predictive-control situation with different execution errors ($-80\%$, $0$, and $80\%$) are shown in Figure 13.

It can be found that although the execution error reaches $-80\%$ and $80\%$, the predictive-control model still improves the instability of bus operation and reduces the bus bunching greatly. When the execution error $e_p$ takes a negative value, that is, the actual running speed and holding time are smaller than the optimal speed and holding time, the performance of
the model is worse than when $e_p$ is greater than 0, indicating that the predictive-control model is more sensitive to the negative value of the execution error.

Figure 14 shows the improvement of the evaluation indicators under different execution errors compared to the no-control situation (NC). It can be found that the improvement of BBF and CHV is relatively stable when the execution error changes, maintaining more than 90% and 60%, respectively. However, when $e_p$ is negative, PWT, TPN, and ART show a relatively large reduction, indicating that the negative execution error will have a greater impact on passengers.
5. Conclusion

This paper proposes a combined control method based on headway probability prediction to solve the bus bunching problem. The main findings of this paper are as follows:

1. Compared to the no-control case, the proposed method effectively avoids the bus bunching while considering the impact of the COVID-19 pandemic, improving the coefficient of headway variation (CHV), the average passenger waiting time (PWT), the total number of passengers served (TPN), the average running time (ART), and the bus bunching frequency (BBF) by 77.73%, 41.66%, 3.25%, 8.23%, and 87.11%, respectively.

2. The proposed method is robust with a wider range of adaptations and better performance than holding control and speed-adjusting control. It can effectively reduce the total holding time compared to holding control, saving 52.05% of bus operating time. Compared to speed-adjusting control, the variance of the adjusted speed is lower, which facilitates the bus driver to implement control commands.

3. The proposed method shows good robustness to the bus driver’s execution errors, and the improvement of bus bunching frequency (BBF) can be maintained above 90%.

However, this study treats the departure interval as a constant value. The proposed model implements real-time control based on the predicted results and does not avoid bus bunching at the planning level. Therefore, future research needs to consider the factor of departure interval and pay attention to the phenomenon of less passenger demand, increased passenger boarding time, and reduced bus capacity during the pandemic, improving the bus system by adjusting the departure interval and implementing real-time prediction control to stabilize the bus system and meet the passenger demands.

Notations

- $J$: Number of bus stops
- $j$: Index of bus stop $j$
- $I$: Number of buses
- $i$: Index of bus $i$
- $R$: Number of intersections
- $r$: Index of intersection $r$
- $x_j$: Location of bus stop $j$
- $x_r$: Location of intersection $r$
- $H$: Departure interval
- $z_{j+1}$: Index of link
- $L_{j+1}$: Length of link $z_{j+1}$
- $V_{\text{min}}$: Minimum average speed of links
- $V_{\text{max}}$: Maximum average speed of links
- $d_i$: Arrival time of bus $i$ at bus stop $j$
- $d_i$: Departure time of bus $i$ at bus stop $j$
- $s_i$: Dwell time of bus $i$ at bus stop $j$
- $v_i$: Speed of bus $i$ on the link $z_{j+1}$
- $t_i$: Travel time of bus $i$ on the link $z_{j+1}$
- $e_i$: Delay of bus $i$ on the link $z_{j+1}$
- $h_i$: Headway of bus $i$ at bus stop $j$
- $c$: Time of opening door and closing door
- $N$: Bus capacity
- $Q_i$: Passenger number of bus $i$ at bus stop $j$
- $\alpha_i$: Passenger alighting rate at bus stop $j$
- $A_i$: Passenger alighting number
- $B_i$: Passenger boarding number
- $M_i$: Passenger number rejected by bus $i$
- $\mu_i$: The passenger arrival rate of bus $i$ at bus stop $j$
- $\mu$: Average passenger arrival rate
- $\omega$: Holding time of bus $i$ at bus stop $j$
- $P_b$: Probability of bus bunching
- $P_m$: Probability of bus in a stable state
- $P_l$: Probability of bus in a big gap
- $C_i$: Control objective of bus $i$ at bus stop $j$
- $V_A$: Maximum adjusted speed
- $V_B$: Minimum adjusted speed
- $W$: Maximum holding time
- CHV: Coefficient of headway variation
- PWT: Average passenger waiting time
- TNP: Total number of passengers
- ART: Average running time per bus
- BBF: Bus bunching frequency.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.
Acknowledgments

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References


