

## Research Article

# Multiobjective Approach to the Transit Network Design Problem with Variable Demand considering Transit Equity

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Received 24 January 2022; Revised 21 June 2022; Accepted 7 July 2022; Published 7 October 2022

Academic Editor: Michela Le Pira

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Existing research on the transit network design problem has tended to focus on minimizing the various costs for both transit operators and users. However, to implement an appropriate and effective transit network in urban environments, it is important not to overly simplify the intrinsically complex nature of real-life network designs. In particular, the minimization of variance in transit service levels typically employed in existing methods can be significantly improved by incorporating a transit equity component. This paper adopts a multiobjective approach that considers system efficiency, user inconvenience, and transit equity without the use of weights in order to design a more realistic and efficient transit network. In particular, the multiobjective Nondominated Sorting Genetic Algorithm-II and the neighborhood local search method are employed in a logit-based mode-choice model in order to incorporate the variable transit demand arising from the private vehicle traffic volume. A toy test network and a real-life network from the city of Goyang, Republic of Korea, are used to verify the effectiveness of the proposed model. The model finds a set of solutions that improve transit equity with minimal losses of other objectives when compared to existing approaches, which produce a significant variance in the level of service, mainly due to the spatially condensed and overlapping distribution of their transit networks. In addition, the relationship between the three objective functions and their resulting patterns in response to key influential factors are also analyzed to verify the robustness of the proposed method in response to changing future conditions.

## 1. Introduction

Traditionally, the transit network problem (TNP) consists of five stages: (1) network design, (2) frequency setting, (3) timetable development, (4) vehicle scheduling, and (5) crew scheduling. The first two stages are known to strongly influence the total cost due to their higher weights [1]. Most previous studies have dealt with the transit network design problem (TNDP) by formulating objective functions for the monetary costs associated with the operating costs for the operator and the travel costs for the users. These models can be improved by considering additional factors such as user

demand, congestion, environmental pollution, the required number of transfers for passengers, and differences in regional characteristics. However, converting these factors into monetary costs is not straightforward, and often they counteract each other or require trade-offs. As such, a single objective approach has major drawbacks when trying to incorporate these additional factors, thus necessitating the use of a multiobjective approach [2].

This paper develops a multiobjective approach that considers transit equity along with system efficiency and user inconvenience. Transit equity is another crucial factor that has been overlooked. When maximizing the efficiency

of a transit network for operators and users as a whole, some transit users located at less dense areas can be neglected because busier and more profitable regions are prioritized. Therefore, it is essential to consider transit equity in the design of a new transit network so that passengers in all regions receive a more acceptable level of transit service when compared to the existing network [3].

It has been noted that to evaluate the service level and the competitiveness of the transit system, the modal split between private vehicles and the transit system should be considered, leading to variation in transit demand. Under variable transit demand, it is inappropriate to use the total or average passenger travel time to measure the efficiency of the system because this can lead to lower transit demand or to the avoidance of transit connections that require longer travel times. In this study, system efficiency is measured as a combination of fare revenue maximization, which is equivalent to transit ridership, and operator cost minimization.

This paper proposes a methodology for solving the transit network design and frequency setting problem (TNDFSP) that considers multiple components, including system efficiency, user convenience in terms of unmet demand, and transit equity related to regional differences in transit competitiveness, using a multiobjective approach for the design of a public transit network. Nondominated Sorting Genetic Algorithm-II (NSGA-II) is combined with a neighborhood local search process to efficiently find multiple Pareto optimal solutions.

The rest of this paper is organized as follows. Section 2 presents a literature review, while Section 3 formulates the multiple objective functions for transit equity and other factors, thus extending existing single-objective-based methods. Section 4 describes the proposed optimization method. Sections 5 and 6 presents the results obtained from testing our proposed method with a toy network and a real-life network and performs sensitivity analyses for various scenarios. Section 7 concludes this paper by suggesting future research directions.

## 2. Literature Review

Most research on transit network design has focused on maximizing profit [4–6], minimizing total travel time and passenger transfers [7, 8], minimizing operating and user costs [1, 9–31] minimizing external costs (e.g., subsidies or vehicle operating costs) [32–35], minimizing air pollutants such as greenhouse gases [36–38] and maximizing equity as an indicator of the regional public transportation service level [2, 3, 39–43]. There have been more variations in the methods prioritizing users than those focused on operators. For example, some researchers have introduced the concept of service coverage to represent transit demand and attempted to maximize it while minimizing the number of passenger transfers [44–47]. Various studies have also employed interesting approaches to deal with unmet demand as a function of the number of passenger transfers and have attempted to minimize their total travel time [8, 48–50]. Nikolić and Teodorović [51] sought to minimize unmet

demand, total travel time, and total passenger transfers, while Camporeale et al. [2] minimized unmet demand, operator costs, and user costs. Szeto and Jiang [52] pointed out that, even though the total travel time for passengers is indeed important, minimizing the number of passenger transfers can improve the passenger-related performance of a transit network; in particular, an increase in transfers can reduce network performance while also increasing passenger travel times. Generally, passenger transfers increase the travel time. Thus passengers tend to avoid or minimize them [32, 49], though a certain number of transfers can improve network performance.

Some studies on transit networks have considered two or more objective functions. Early research focused on prioritizing the objectives or using weighted sums. Recent multiobjective approaches employing evolutionary algorithms have attempted to find multiple Pareto optimal solutions that consist of nondominated solution sets. Most of these have considered user and operator costs as objective functions under various constraints [13, 14, 18, 19, 21–23, 53]. According to Pternea et al. [32], transit demand is dependent on the transit network and transit frequency, meaning that variable demand should be employed. Some studies have considered variable transit demand for a single objective function that combines weighted objective functions [33, 54, 55]. However, there have been very few studies in which variable demand is considered in the multiobjective TNDFSP without weighted factors [53].

There have been some attempts to include transit equity in the TNDFSP. For example, Camporeale et al. [39] sought to achieve a homogeneous transit network distribution for transit users, while Fan and Machemehl [3] introduced the concept of spatial equity as a limiting parameter that guaranteed that additional travel times for users remained below a certain threshold for their proposed network in comparison to the existing network. Ferguson et al. [40] also considered equitable access to basic amenities such as employment, supermarkets, and medical services and the minimization of regional differences in order to incorporate transit equity into the transit frequency-setting problem (TFSP). Camporeale et al. [2, 41] considered horizontal and vertical transit equity from the perspective of transit service supplies for selected regions, while Jiang [42] incorporated transit equity by analyzing the difference between user costs before and after modifications to a transit network. Kim et al. [43] utilized the travel time ratio between transit and private cars to define transit equity and identified the regions with the lowest equity as targets for improvement. However, to the best of the authors' knowledge, no previous study has employed an appropriate multiobjective approach to incorporate transit equity into the TNDFSP. It is challenging to quantify transit equity so that it can be directly compared with operating or user costs. Therefore, it is necessary to independently consider system efficiency, user inconvenience, and transit equity to find a solution set that satisfies all of the objective functions in a quantifiable and balanced manner and that does not depend on subjective and qualitative opinions or experiences.

### 3. Mathematical Formulation

In this paper, three main components are considered in the TNDFSP: system efficiency, user inconvenience, and equity. A description of each of these is provided in Table 1.

In the fixed transit demand problem, it is natural to consider the total travel time of transit passengers when assessing passenger-side benefits. However, for variable demand in which the transit system competes against private vehicles, the total passenger travel time can be minimized if the transit demand is reduced by lowering the service quality of the transit system. To avoid this undesirable situation, it is better to maximize the transit demand than the total passenger travel time. A rise in transit demand suggests that the transit service quality, including the travel time, is higher than that of private vehicles for individual origin–destination pairs. In the fixed fare system, the transit demand is equal to the revenue or fare income for the transit operator, which can be easily integrated into operator profit. The net profit of the operator, which is calculated as the difference between the fare revenue and operator costs, can also represent the efficiency of a transit system aiming to be competitive against private vehicles and having low operating costs. The combination of fare revenue and operator costs also helps to reduce the number of objective functions and thus simplifies the problem.

An objective function for the system efficiency is formulated using total demand ( $d_{ij}^t$ ) and transit fare ( $\beta$ ) to calculate total revenue and subtract expenses, which are a function of the bus operating costs ( $o$ ), the fleet size ( $f s^k$ ) for each route, and the route length ( $l^k$ ). When designing a transit route, the route length and route frequency (i.e., the fleet dispatching rate) are determined to accommodate demand by introducing long-distance routes or ensuring an appropriate level of service (LOS).

Transit users are assumed to perceive two or more transfers as inconvenient, and this is considered to be unmet demand. Stern [56] conducted a survey with transit users and reported that more than half of the survey participants were willing to transfer once without perceiving it as an inconvenience. Zhao et al. [49] also found that, for any given origin and destination pair, two or more transfers should be avoided for effective transit operations. This has been verified by data from the Republic of Korea (hereafter Korea) provided by the Korea Transportation Safety Authority (Table 2). According to transit smart-card data from the Greater Seoul Area, for one weekday in May 2017, 75% of transit users traveled with no transfer, 21% transferred once, 3% transferred twice, and 1% transferred three or more times. In other words, 96% of transit users traveled with no or one transfer. The objective function for the user inconvenience is calculated as the total transit demand ( $d_{ij}^t$ ) minus direct demand ( $d_{ij}^0$ ) and single-transfer demand ( $d_{ij}^1$ ).

Ferguson et al. [40] defined modal equity as the minimization of the standard deviation of the accessibility measures. In the variable demand problem employed in this study, transit accessibility can be represented by the travel time advantage over private vehicles because more

passengers will use the transit system when the transit travel time to the destination is competitive in relation to the travel time for private vehicles. In addition, the private vehicle travel time, which can be used as the reference travel time, better represents the spatial gap between the origin and destination than the direct distance in the transportation network. Thus, the gap between the transit and reference travel time is equivalent to the accessibility of the transit system. The objective function for transit equity is formulated as the standard deviation of transit accessibility, which is the difference in travel time between private vehicles and the transit system (equation (3)).

Table 3 presents the notations used in this paper.

$$\text{Object1 } \text{Max} \beta \sum_{ij} d_{ij}^t - o \sum_k f s^k * l^k, \quad (1)$$

$$\text{Object 2 } \text{Min} \sum_{ij} d_{ij}^t - d_{ij}^0 - d_{ij}^1, \quad (2)$$

$$\text{Object3 } \text{Min} \sqrt{\frac{\sum_{ij} (E_{ij} - \bar{E})^2}{nz^2}}, \quad (3)$$

$$\text{s.t. } f_{\min} \leq f^k \leq f_{\max} \forall k \in K, \quad (4)$$

$$f s^k = \frac{2t_r^k}{h^k} \forall k \in K, \quad (5)$$

$$l^k \leq l_{\max} \forall k \in K, \quad (6)$$

$$NL_{\min} \leq NL \leq NL_{\max}, \quad (7)$$

$$ns_{\min}^k \leq ns^k \leq ns_{\max}^k \forall k \in K, \quad (8)$$

$$E_{ij} = \frac{\min t_{ij}^t - \min t_{ij}^a}{\min t_{ij}^a} \begin{cases} i f \min t_{ij}^t - \min t_{ij}^a < 0, & E_{ij} = 1, \\ \text{otherwise,} & 0, \end{cases} \quad (9)$$

$$\sum_{ij} d_{ij}^t = \sum_{ij} d_{ij}^0 + \sum_{ij} d_{ij}^1 + \sum_{ij} d_{ij}^{um}, \quad (10)$$

$$t_{ij}^t = t_{ij}^{wt} + t_{ij}^{\text{in veh}} + t_{ij}^{\text{access}} + t_{ij}^{\text{egress}} + t_{ij}^{\text{transfer}}. \quad (11)$$

Equations (4)–(11) are the constraints for the object functions. Equation (4) limits the service frequency of a route, equation (5) relates the route frequency to the fleet size for a route, and equation (6) limits the maximum route length. Equations (7) and (8) represent the number of lines in the network and the number of bus stops for a route, while equation (9) is an index that measures transit accessibility. In the ideal case where the transit travel time for a particular origin–destination pair is shorter than that for a private car, the index will have a value of 1. In equation (10), total transit demand consists of direct demand, single-transfer demand, and two-or-more-

TABLE 1: Object function components.

Component	Description
System efficiency	Fare revenue (proxy for transit demand) minus operator costs, equivalent to net profit for the operator
User inconvenience	Unmet demand (more than the maximum number of transfers)
Equity	Deviation of transit accessibility

TABLE 2: Number of transfers per day (weekdays).

Number of transfers	Trip (%)
None	16,273,347 (75.0%)
One	4,568,921 (21.1%)
Two or more	854,405 (3.9%)
Total	21,696,673 (100%)

Source: Korea Transportation Safety Authority, <https://www.kotsa.or.kr/>.

TABLE 3: Problem notation.

<i>Sets</i>	
$K$	Set of lines in the transit network
$N$	Set of nodes $n$
<i>Parameters</i>	
$o$	Operating cost (\$/km)
$\beta$	Fare (\$)
$\alpha_1, \alpha_2$	Parameters of utility function
<i>Function</i>	
$U_c$	Utility of mode $C$
$P(C)$	Probability of choice for mode $C$
$C_c$	Coefficient of utility function
<i>Variables</i>	
$h^k$	Headway for line $k$
$f^k$	Frequency for line $k$
$f_{\min}$	Minimum frequency
$f_{\max}$	Maximum frequency
$f_s^k$	Fleet size for line $k$
$l^k$	Length of line $k$
$l_{\max}$	Maximum length of a line
$d_{ij}^t$	Transit demand from $i$ to $j$
$d_{ij}^{t_0}$	Transit demand from $i$ to $j$ without transfer
$d_{ij}^{t_1}$	Transit demand from $i$ to $j$ with one transfer
$d_{ij}^{t_n}$	Transit demand from $i$ to $j$ with two or more transfers
$E_{ij}$	Accessibility from $i$ and $j$
$nz$	Number of zones
$U_{ijc}$	Utility of mode $C$ from $i$ to $j$
$TL_{ijc}$	Total travel time from $i$ to $j$ using mode $C$
$TC_{ijc}$	Total travel cost from $i$ to $j$ using mode $C$
$2t_r^k$	Round trip travel time for line $k$
$t_{ij}^t$	Transit travel time from $i$ to $j$
$t_{ij}^d$	Passenger car travel time from $i$ to $j$
$t_{ij}^{wt}$	Passenger waiting time from $i$ to $j$
$t_{ij}^{\text{in-veh}}$	Passenger in-vehicle time from $i$ to $j$
$t_{ij}^{\text{access}}$	Passenger access time from $i$ to $j$
$t_{ij}^{\text{egress}}$	Passenger egress time from $i$ to $j$
$t_{ij}^{\text{transfer}}$	Passenger transfer time from $i$ to $j$
$NL$	Number of lines in the network
$NL_{\max}$	Maximum number of lines in the network
$NL_{\min}$	Minimum number of lines in the network
$ns^k$	Number of stations on line $k$
$ns_{\max}^k$	Maximum number of stations on line $k$
$ns_{\min}^k$	Minimum number of stations on line $k$
$w$	Number of modes
$D_c$	Dummy variable

transfer demand, the latter of which is taken to be unmet demand. The travel time for public transit consists of in-vehicle, waiting, access, egress, and transfer time (11)

This paper adopts the convex combination method [57, 58] for the user-equilibrium (UE) traffic assignment for private cars and the optimal strategy assignment model [59] for transit assignment. To capture varying mode-choice behavior that is dependent on established transit networks, equation (12) determines mode-choice probabilities based on a logit model that uses the travel times and costs associated with the trip assignment.

$$P(C) = \frac{\exp(U_C)}{\sum_w \exp(U_w)} \quad (12)$$

$$U_{ijc} = \alpha_1 * TT_{ijc} + \alpha_2 * TC_{ijc} + D_c + C_C.$$

## 4. Methodology

TNDFSP is a well-known NP-hard problem due to its vast solution search space and combination of various decisions for optimization [50]. Numerous algorithms have been applied to multiobjective transit network design problems, such as genetic algorithms [12, 18, 21–24, 37, 54], simulated annealing [46, 54], tabu search [16, 17, 55], ant colony optimization [9, 15], the artificial fish swarm algorithm [28], and particle swarm optimization [29]. NSGA-II [60] is known to be a very effective algorithm for solving multiobjectives [26, 53] and has been used to find optimal solutions for cases where one objective function is not dominant over the others while still satisfying the constraint conditions.

NSGA-II computes the fitness values for the solutions using fast nondominated sorting and a crowding distance approach. The calculated ranking of these solutions can be differentiated using a dominance rule, although they belong to the same front, unlike the existing method [61].

An optimal solution that is not dominated by other solutions is selected as a candidate solution because it is impossible to optimize multiple objectives at the same time. Assuming that all objective functions  $Z_a(x)$ ,  $\forall a \in A$  need to be minimized, given a feasible solution  $X = \{x_1, x_2, x_3, \dots, x_b\}$ ,  $x_1$  dominates  $x_2$  when the following two conditions are satisfied ( $x_1 \succ x_2$ ):

- (1) If all objective functions  $Z_q(x_1) \leq Z_q(x_2)$ ,  $\forall q \in Q$
- (2) If  $Z_q(x_1) < Z_q(x_2)$ ,  $\forall q \in Q$  for at least one objective function.

A Pareto optimal solution is not dominated by other solutions in the solution set; this nondominant set is referred to as a Pareto optimal set or Pareto optimal solutions [62].

Crowding distance is used to calculate the density of two adjacent solutions to determine fitness values within the same rank and to produce a diversity of solutions. Because the crowding distance is a Euclidean distance between two solutions, if the scale of an objective function differs from others, the influence of that objective function will be biased. To address this scaling issue, a normalization method

employed in previous studies is adopted here (equation (13)), [60, 63]:

$$v_m(x) = \frac{v_m(x) - v_{\min}}{v_{\max} - v_{\min}}, \quad (13)$$

where  $v_m(x)$  is the normalized  $m^{\text{th}}$  value

The transit network optimization is performed as shown in Figure 1.

**4.1. Network Encoding and Network Creation.** The transit network is expressed as an individual in the algorithm, and in this study, the encoding scheme has been designed to simultaneously consider the transit routes and frequency. Unlike Chai [26], where stations and lines are encoded separately, this method can perform crossovers or mutations without modifying the line information. As shown in Figure 2, the border (the red line) separating each bus route and the bus stops for a route are sequentially listed, followed by the frequency of that route. Our proposed approach generates a transit network according to a process that considers the characteristics associated with public transit, with the maximum route length and number of routes limited by the network size. The network creation process consists of the following steps:

- (i) Step 1: Determine the number of bus lines ( $NL$ ).
- (ii) Step 2: Determine the number of stations for line  $k$  ( $ns_{\min}^k < ns^k < ns_{\max}^k$ ).
- (iii) Step 3: Check the maximum line length constraint. If the line satisfies the constraint, then go to the next step. If not, go back to Step 2.
- (iv) Step 4: Check the minimum and maximum station constraints. If the line satisfies the constraints, then go to the next step. If not, go back to Step 2.
- (v) Step 5: Randomly set the frequency of bus line  $k$  within a given range ( $f_{\min} < f^k < f_{\max}$ ).
- (vi) Step 6: Repeat Steps 2 to 5 until all bus lines have been created.

**4.2. Crossover and Mutation Operator.** Transit network crossovers and mutations are employed to find the optimal solutions. In particular, the crossovers consist of two types: line and station crossovers. In a line crossover, part of the network is exchanged based on a random point for the transit networks  $K1$  and  $K2$  (Figure 3). If the number of routes does not meet the compatibility conditions, the new transit network is removed as a solution candidate. As shown in Figure 4, a station crossover selects two bus routes on the same transit network. If the two bus lines share a station, an exchange is performed based on that station (Figure 4(a)); otherwise, it is based on a random station (Figure 4(b)). Similar to the line crossover process, after the crossover, if a new route does not satisfy the line length or number of stations constraint, it is excluded. Mutations are only applied to lines, with a random route selected within a transit network (Figure 5). Existing research suggests a crossover rate of 0.8–0.9 and a mutation rate of 0.05–0.1.

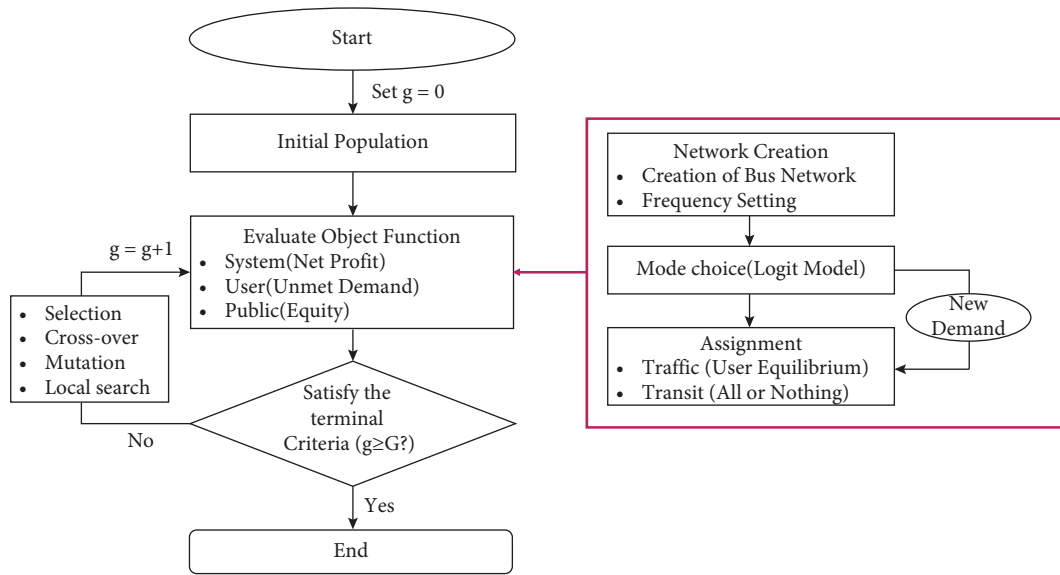


FIGURE 1: Overview of transit network optimization.

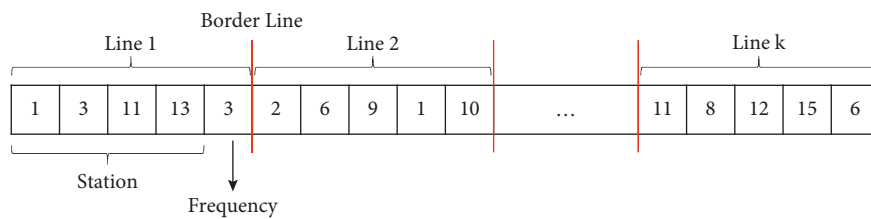


FIGURE 2: Network encoding.

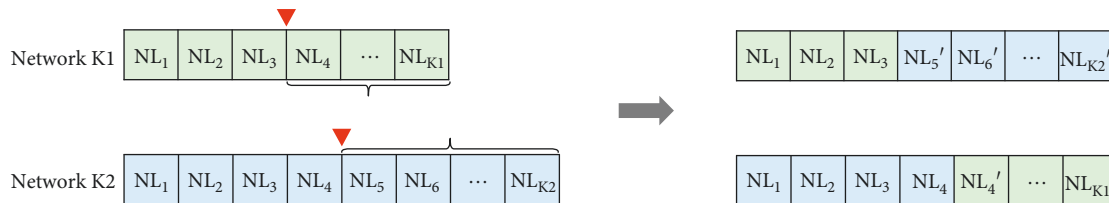


FIGURE 3: Line crossover.

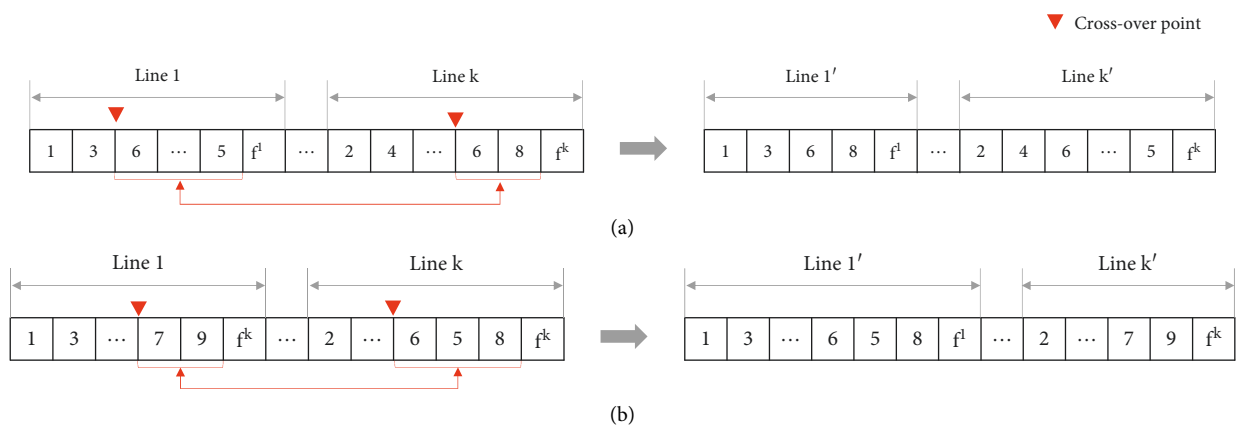


FIGURE 4: Station crossover. (a) With. (b) Without the same station.



FIGURE 5: Line mutation.

This paper arbitrarily chooses 0.8 as the crossover rate and 0.1 as the mutation rate.

**4.3. Local Search.** Neighborhood local search is combined with the NSGA-II in this study. For an individual generated from the crossover and mutation procedures, a local search technique is employed to improve the efficiency of the solution search. All of the generated offspring that share identical bus stops are determined. Parts of the routes are then switched, followed by checking whether the newly formed individuals are compatible with the limiting conditions associated with the maximum route length and the minimum number of bus stops for a route. The new individual then replaces the two individuals in the previous step if their fitness value is higher (Figure 6).

## 5. Numerical Results

**5.1. Toy Network.** An ideal grid-form network consisting of 16 zones, 65 nodes, 160 links, and 128 connectors is constructed (Figure 7). Every node is a candidate for a bus stop, and all links are 2 km long and have an average speed of 60 km/h. The total demand is 80,000 trips/day, the demand for the CBD area is 30% higher than in other regions, and the demand for each area is uniformly distributed. The proposed optimization method is coded and run using EMME4/API software. The parameters for the numerical experiments are set as follows: crossover probability of 0.8, mutation probability of 0.1, population size of 50, and 50 iterations.

Optimal solutions are found by simultaneously considering objective functions related to the system, users, and the public (Figure 8). Figure 8(a) shows that the final population performs generally better than the initial population. In particular, the Pareto solution set from which a final population can be chosen exhibits significant improvements with respect to all three objective functions.

The Pareto surface illustrated in Figure 8(a) is an interpolated smooth surface that clearly shows a Pareto optimal rather than connecting the actual points. The data points in Figure 8(a) are projected against each objective function to generate two-dimensional graphs, and the associated Pareto optimal solutions are presented in Figures 8(b)–8(d), revealing hidden relationships between the three objective functions. Because fluctuations in one objective function can influence other objective functions differently, these relationships can be utilized as guidelines for the design of transit networks for specific purposes.

Table 4 presents the correlation coefficients from Spearman correlation analysis for the associated variables.

As shown in Figure 9(a), the system efficiency tends to increase as the passenger volume per route length increases. Figure 9(b) shows the relationship between average route

length and unmet demand for origin-destination pairs. The unmet demand tends to decrease as the transit route length increases because there are more locations that can be reached without transferring. Figure 9(c) presents the changes in transit equity according to the total daily operating length, defined as the product of route length and route frequency. It is found that as the total daily operating length increases, the transit equity improves. Because transit services are provided over a geographically larger area, regional differences in accessibility to these services tend to shrink and transit equity consequently improves.

Unlike most previous studies that have considered system efficiency and unmet demand, this research also accounts for transit equity in the design of an optimal transit network. Table 5 compares the results with and without transit equity. When transit equity is not considered, the system efficiency is higher, and the percentage of unmet demand in relation to total demand is lower. However, both the number of routes and the total route length are lower, indicating that each route tends to be shorter, which in turn is indicative of a very limited transit network. When system efficiency and unmet demand are optimized, and transit equity is not considered, transit demand decreases due to the dense and overlapping spatial distribution of the routes (Figures 10(b) and 10(d), respectively). In addition, when transit equity is not considered, the Pareto solution set for each objective function does show any improvement (except for transit equity), yet the transit services are not distributed evenly throughout the network and tend to be denser in regions with higher demand. When transit equity is considered, system efficiency and unmet demand are slightly worse, but transit equity has improved overall. In addition, services are distributed in various regions compared to when transit equity is not considered (Figures 10(a) and 10(c)).

The suggested algorithms with neighborhood local search are compared with nonlocal search cases in terms of the evolution of the average objective function for each iteration (Figure 11). It is confirmed that all objective functions improve with the use of local search. Figure 12 depicts the Pareto optimal of the final population of two cases, with the solutions found with the local search mostly dominating the solutions found without it.

**5.2. Large Network Analysis.** The proposed method is applied to an existing transit network to verify its applicability in real life. The network is in the city of Goyang, Gyeonggi Province, Korea (Figure 13). This city included 3,828 nodes, 39 zones, 8,963 links, and a population of about 1.04 million in 2017. The network data (road and transit) and origin-destination demand (traffic and transit) are collected from the Korean Transportation Database (KTDB). The existing transit network consists of 68 bus lines.

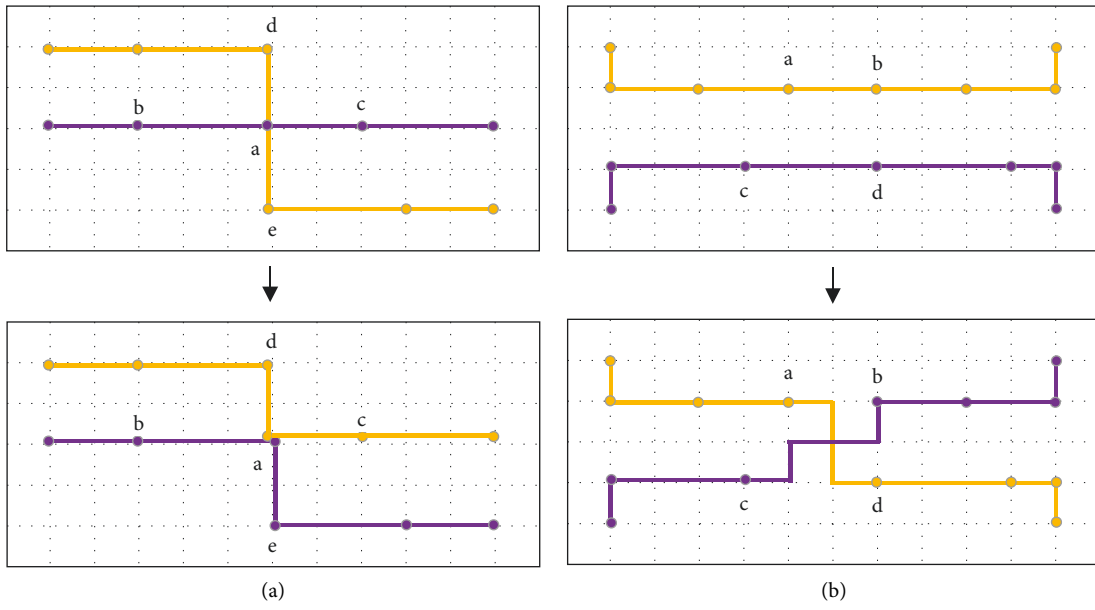


FIGURE 6: Illustration of the local search process.

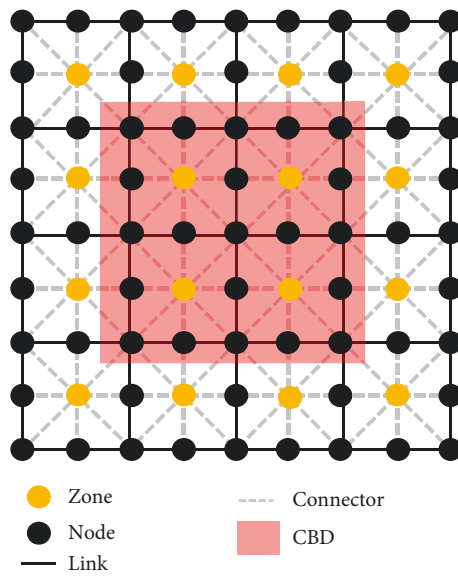
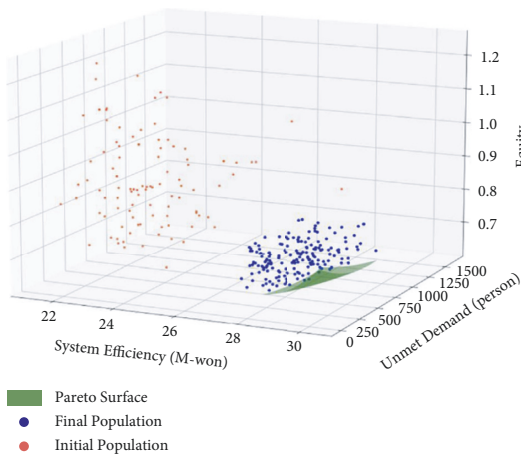
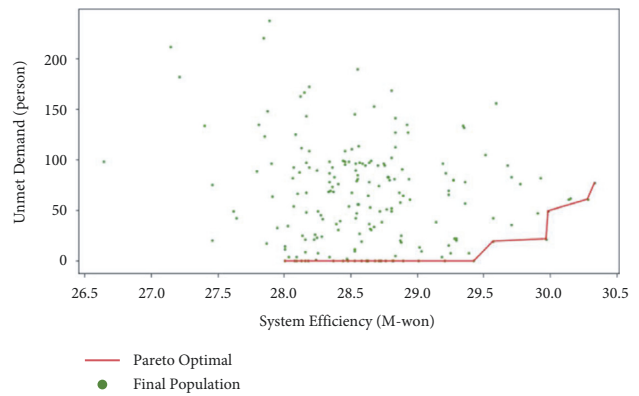


FIGURE 7: Structure of the toy network.



(a)



(b)

FIGURE 8: Continued.



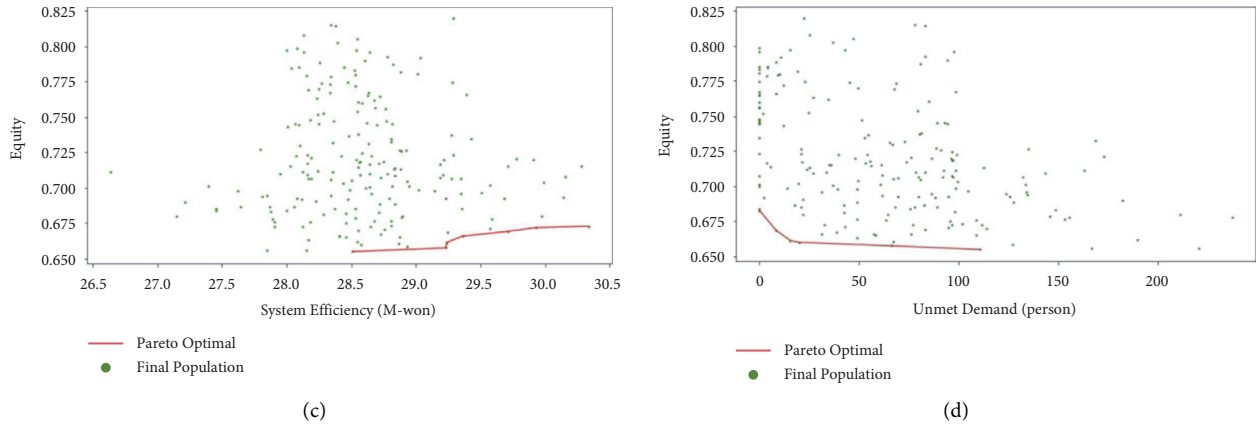


FIGURE 8: Pareto surface and Pareto optimal. (a) Pareto surface between the three objectives. (b) Pareto optimal between system efficiency and equity. (c) Pareto optimal between system efficiency and unmet demand. (d) Pareto optimal between unmet demand and equity.

TABLE 4: Results of spearman correlation analysis.

	Modal split (transit, %)	Passenger volume per km	Daily total operating Length (km)	Average line length (km)	Total length (km)	Total fleet	No. Lines
System efficiency	-0.446	0.898	-0.849	-0.806	-0.795	-0.630	0.043
Unmet demand	0.116	0.207	-0.114	-0.396	-0.277	0.135	0.345
Equity	-0.804	0.772	-0.819	-0.532	-0.685	-0.779	-0.387

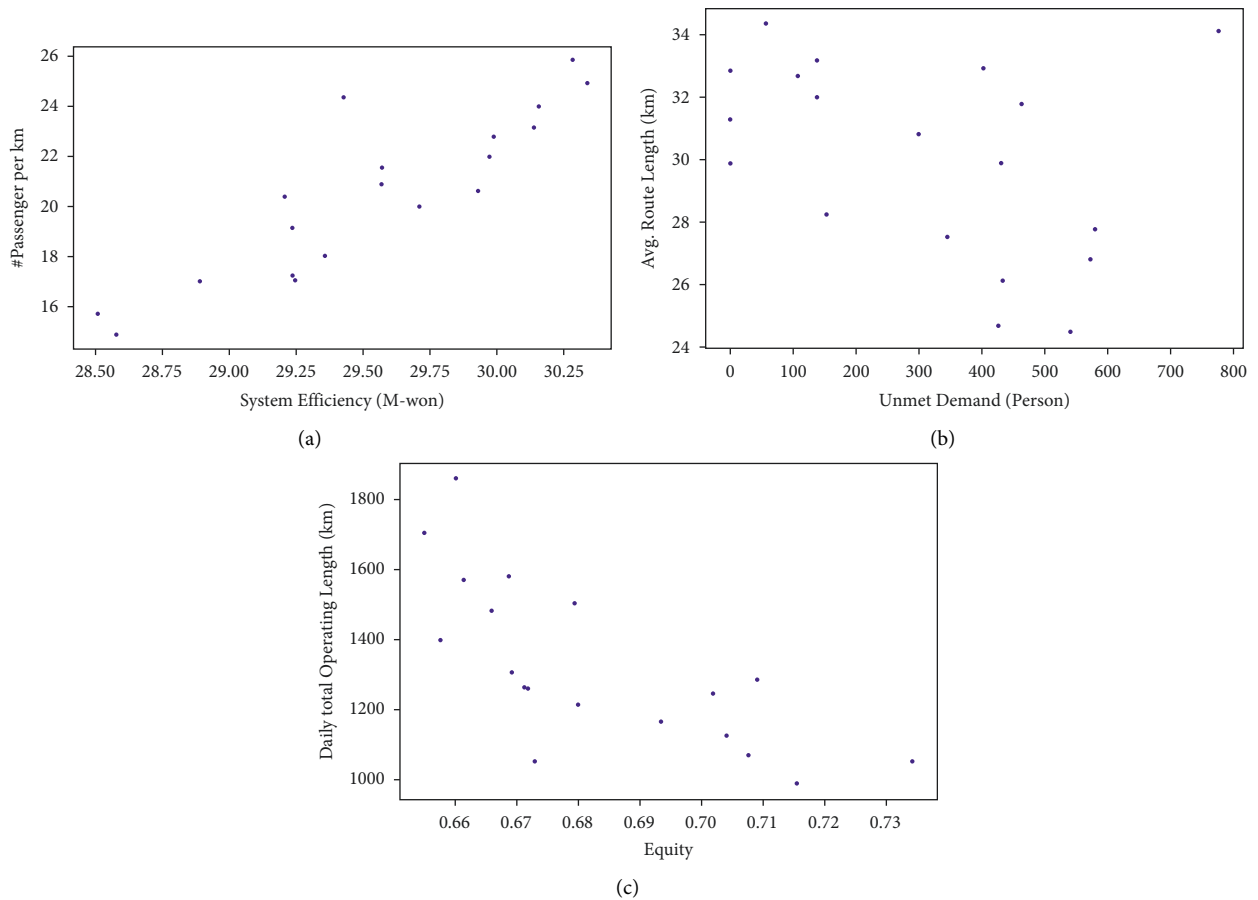


FIGURE 9: Changes in the objectives. (a) System efficiency. (b) Unmet demand. (c) Equity.

TABLE 5: Pareto optimal solutions with and without equity.

		System efficiency (M-won)	Unmet demand (person, % *)	Equity	Car (%)	Transit (%)	Total length (km)	No. Lines
With equity	S*	30.334	539 (2.0)	0.673	67.1	32.9	416	17
	—	29.571	137 (0.5)	0.702	67.2	32.8	544	17
	U*	29.427	—	0.734	67.9	32.1	460	14
	—	30.282	427 (1.6)	0.715	67.3	32.7	420	17
	E*	28.510	775 (2.9)	0.655	66.5	33.5	580	17
Without equity	S*	30.729	308 (1.2)	0.753	67.4	32.6	364	10
	—	30.641	182 (0.7)	0.754	67.7	32.3	372	10
	U*	28.959	—	0.769	68.4	31.6	388	11
	—	29.143	41 (0.2)	0.971	69.5	30.5	368	10
	E*	30.529	51 (0.2)	0.751	67.5	32.5	376	10

S\*: Best system efficiency solution, U\*: Best unmet demand solution, E\*: Best equity solution, “—” Solutions other than S\*, U\*, or E\*, \*: Ratio of unmet demand to transit demand.

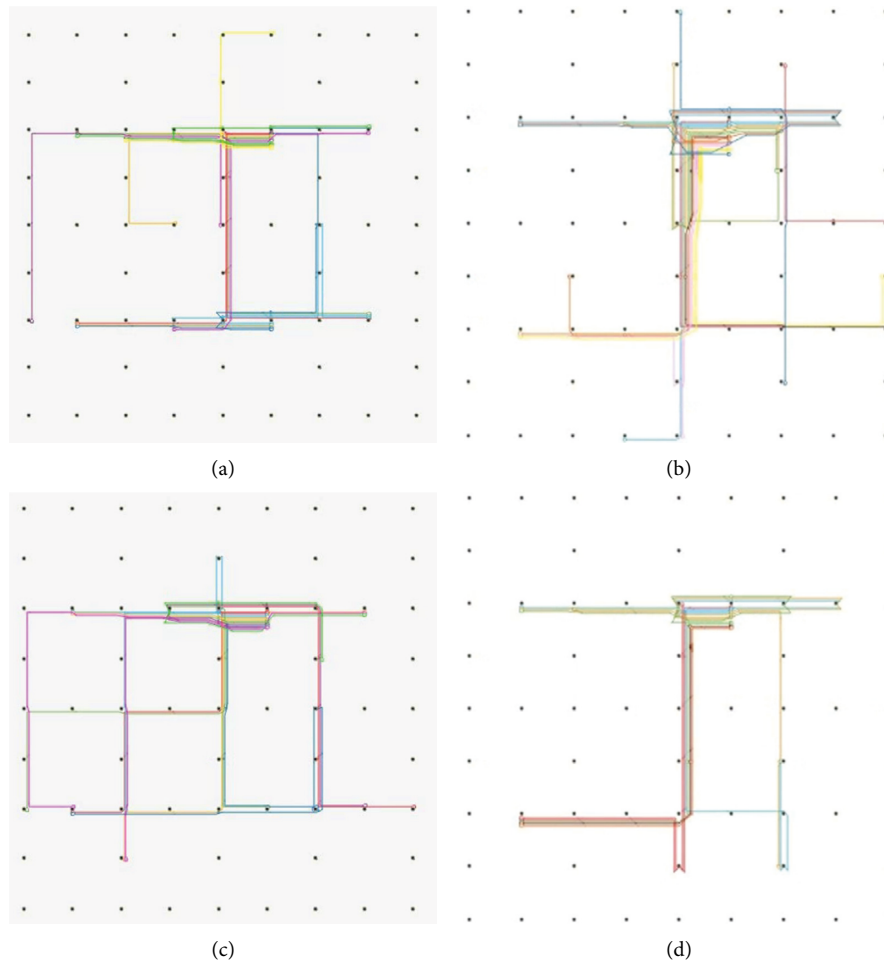


FIGURE 10: Optimal transit network: (a) Best system efficiency (with equity). (b) Best system efficiency (without equity). (c) Best unmet demand (with equity). (d) Best unmet demand (without equity).

For a large network, the average computation time takes 34 h, but this study is geared toward long-term transit systems in large cities that do not require near real-time solutions. Given that the typical interval between the redesign of transit routes is several years, the computational time is not a significant factor [53].

Figure 14 presents the transit equity of the existing network, the network based on maximizing system efficiency only, and three groups of Pareto solution sets against the two other objective functions. Table 6 summarizes the associated numerical values. The existing network performs poorly in terms of the objective functions and is thus shown to be

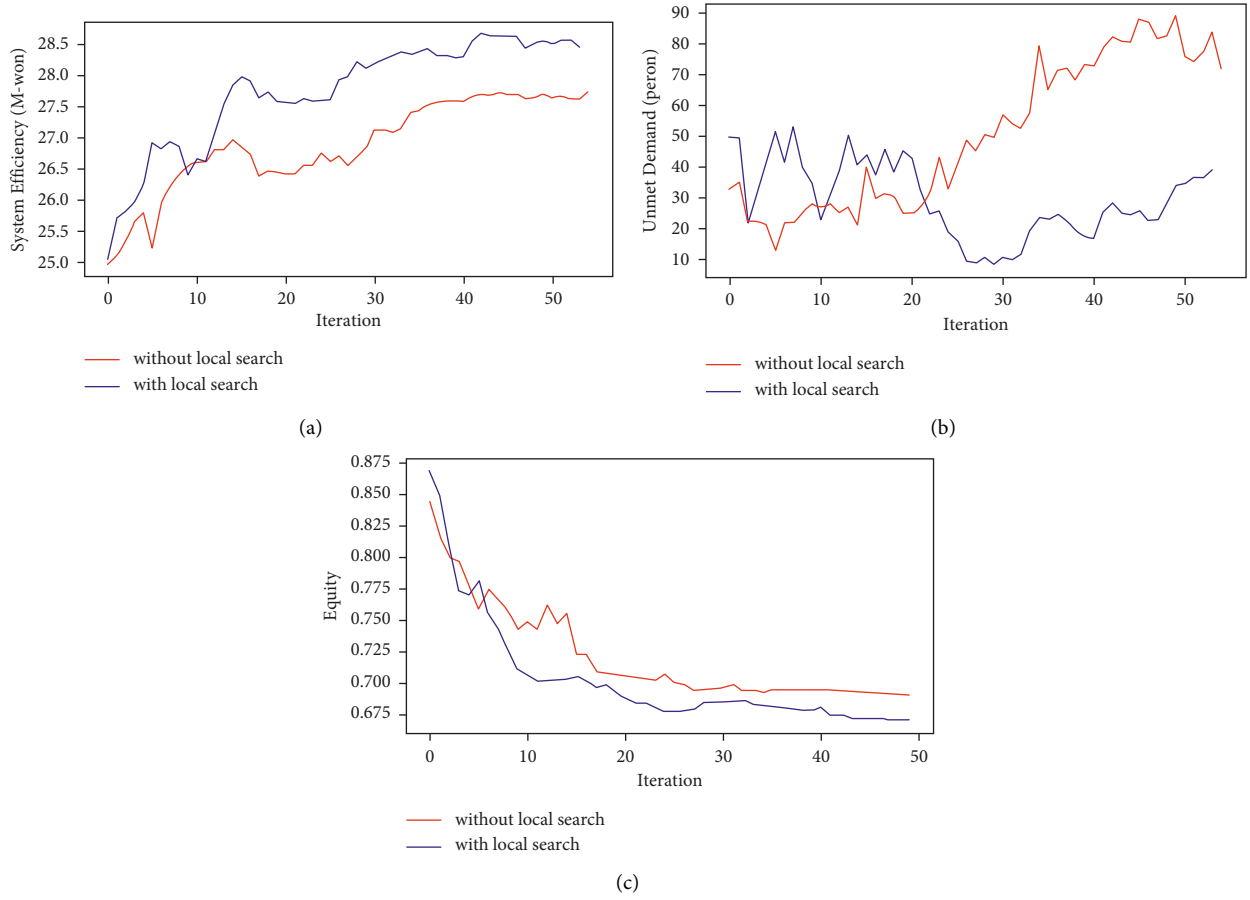


FIGURE 11: Objective Function values by iteration with and without neighborhood local search, (a) System efficiency, (b) Unmet demand, (c) Equity.

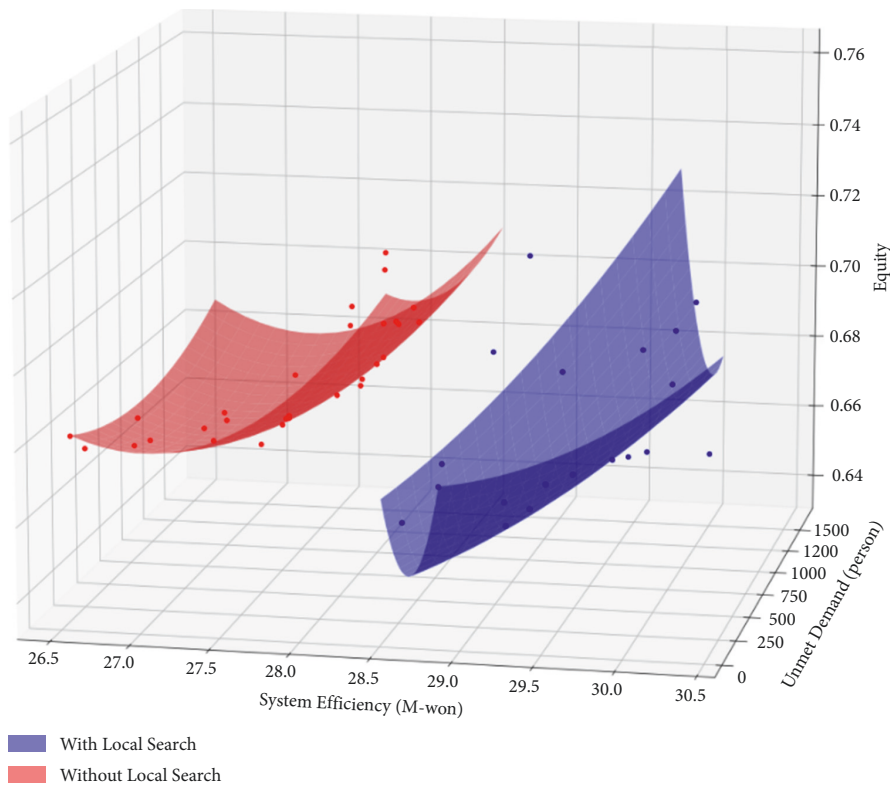


FIGURE 12: Pareto optimal comparison with and without local search.

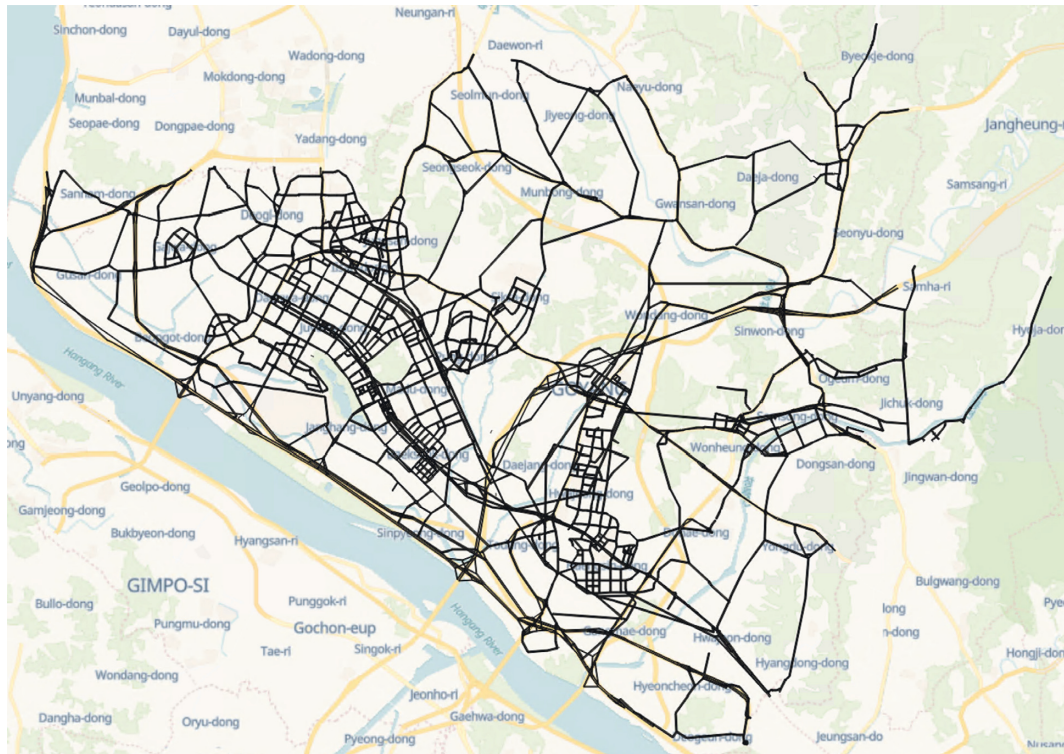


FIGURE 13: Goyang transit network (Gyeonggi province, South Korea).

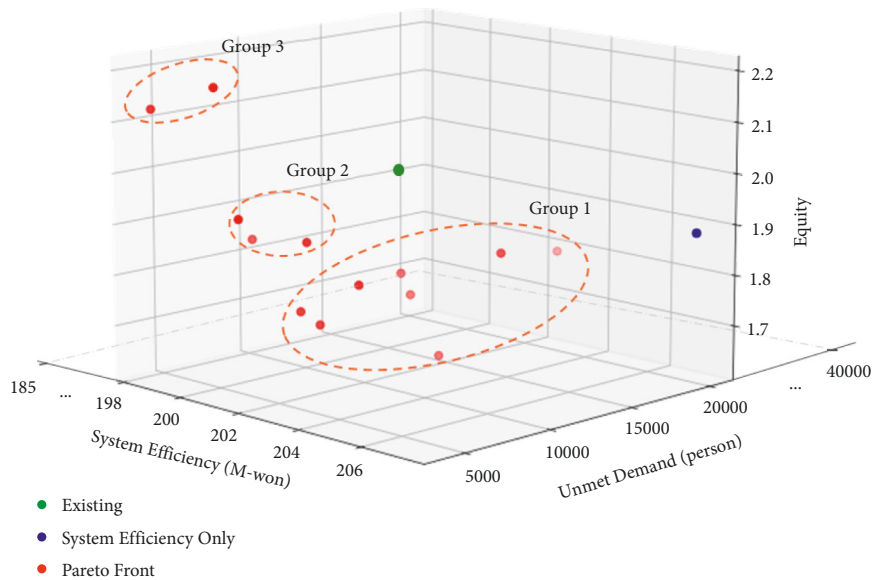


FIGURE 14: Comparison of the existing network and Pareto optimal solution sets.

ineffective. In field operations, it is common to only consider the system efficiency. Maximizing the system efficiency increases it by 11.5% compared to the existing network. However, because this does not consider either unmet demand or equity, the transit services tend to be offered in regions with high demand, resulting in an uneven geographic distribution of transit routes. When all three objective functions are considered together, though the system

efficiency is slightly lower, both unmet demand and equity improve.

As shown in Figure 14, the solutions found via our proposed method can be categorized into three groups. Group 1 results in high equity and high system efficiency while the unmet demand is relatively high. In Group 2, the equity varies only slightly, while the unmet demand improves and the system efficiency worsens. Thus, Group 2

TABLE 6: Summary of the solutions for the transit networks.

	System efficiency ( $M$ -won)	Unmet demand (person, % *)	Equity	Car (%)	Transit (%)	Total length (km)	No. lines	
Existing	185,846	42,014	(31.2) 1.721	54.9	45.1	1,164	67	
System efficiency only	207,301	20,335	(12.1) 1.885	43.7	56.3	1,772	51	
Pareto solution sets	$S^*$	204,035	14,544 (8.7)	1.843	43.8	56.2	1,421	61
	—	201,341	7,409 (4.3)	1.743	42.8	57.2	1,549	65
	$U^*$	198,352	3,887 (2.3)	2.126	44.0	56.0	1,530	64
	—	199,736	7,340 (4.4)	1.865	43.6	56.4	1,573	65
	$E^*$	202,788	13,070 (7.6)	1.633	42.6	57.4	1,503	62

$S^*$ : Best System Efficiency Solution,  $U^*$ : Best Unmet Demand Solution,  $E^*$ : Best Equity Solution, “—” Solutions other than  $S^*$ ,  $U^*$ , or  $E^*$ , \*: Ratio of Unmet Demand to Transit Demand.

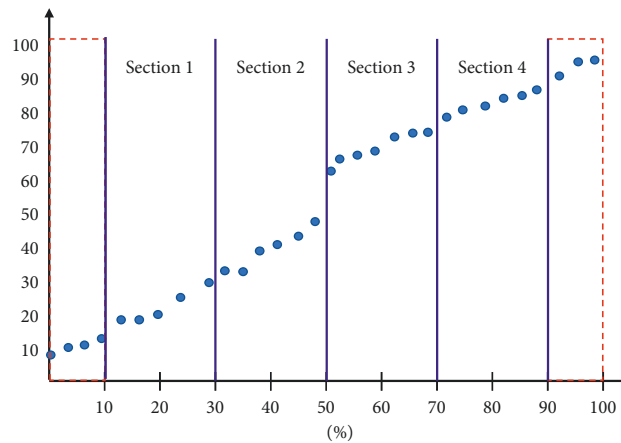


FIGURE 15: Interval-based average calculation.

overlaps with the solution region in which all three objectives are well-balanced. Group 3 has the lowest unmet demand and the highest equity but with significantly lower system efficiency. These observed trends were expected because it is difficult to optimize multiple nonprioritized objective functions, especially when they have trade-off relationships with each other. It is recommended that planners choose a feasible solution group based on their target operational policies and social outcomes.

## 6. Sensitivity Analysis

The sensitivity of the objective functions to the constraint on the number of lines, the total demand, and the operating cost is also analyzed. For this, it is not possible to use the average of the objective values for the Pareto solutions because the solutions may not be distributed evenly. To minimize the bias from uneven distributions, the top and bottom 10% are excluded, and the range between 10% and 90% is divided evenly into four sections. The averages of these sections are then compared (Figure 15).

The previously identified Pareto solutions were subject to a constraint on the maximum number of lines and consequently, they all had a number of lines that were below the constraint. Therefore, the number of allowed lines is reduced by 0% to 50%. As the number of lines allowed decreases, the average route length increases (Figure 16(b)), which indicates that longer routes can cover a larger service area and

passengers do not need to transfer as often, reducing the unmet demand and the daily total operating length. However, as shown in Figure 16(a), the system efficiency is unaffected by the reduction in the number of lines allowed because route frequencies can be increased on some routes to compensate for loss.

In the long term, total regional demand can increase as a city grows and decrease if the city shrinks. Therefore, sensitivity analysis for changes in total demand is conducted. As shown in Figure 17(a), unmet demand and system efficiency both have a positive correlation with total demand. In addition, transit equity improves as the total demand increases (Figure 17(b)), possibly due to the expansion in transit services to match the growing demand.

The effect of operating costs, which typically include fuel and labor costs, on the objective functions is also analyzed (Figure 18). As the operating costs increase and the total revenue remains constant, the level of service and modal split for transit decreases, resulting in a reduction in transit equity and system efficiency. However, unmet demand does not appear to have a correlation with operating costs. Figure 18(b) shows that the relationship between operating costs and the daily total operating length is nonlinear.

Based on the sensitivity analyses, the optimal transit network solutions vary in terms of their route length, fleet size, and the number of lines, thus affecting the objective functions. The system efficiency is influenced by transit demand, unmet demand is influenced by the average route

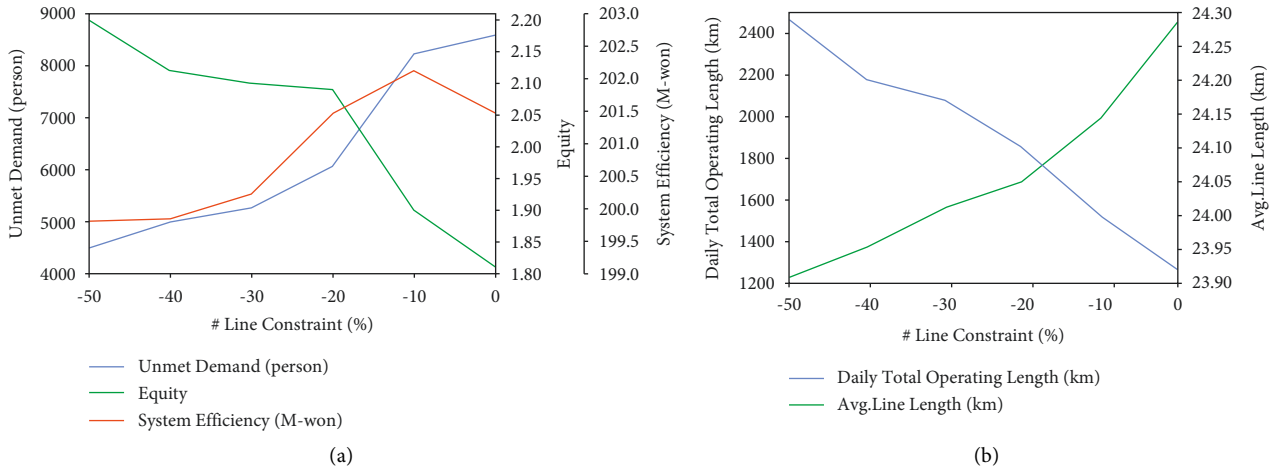


FIGURE 16: Sensitivity analysis for the constraint on the number of lines. (a) The three objective functions. (b) Daily total operating length and average line length.

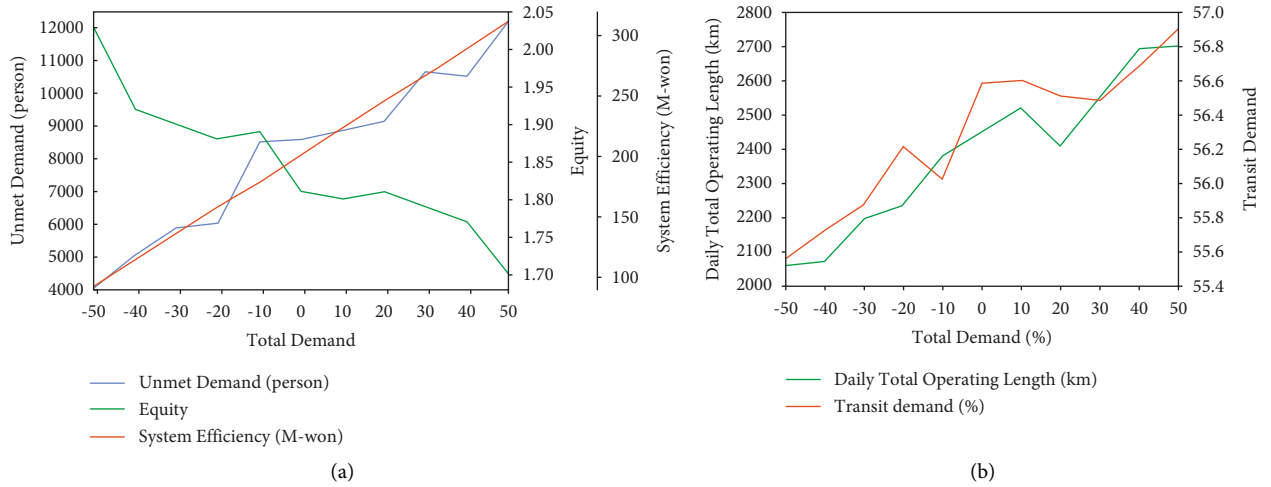


FIGURE 17: Sensitivity analysis for total demand. (a) The three objective functions. (b) Daily total operating length and transit demand.

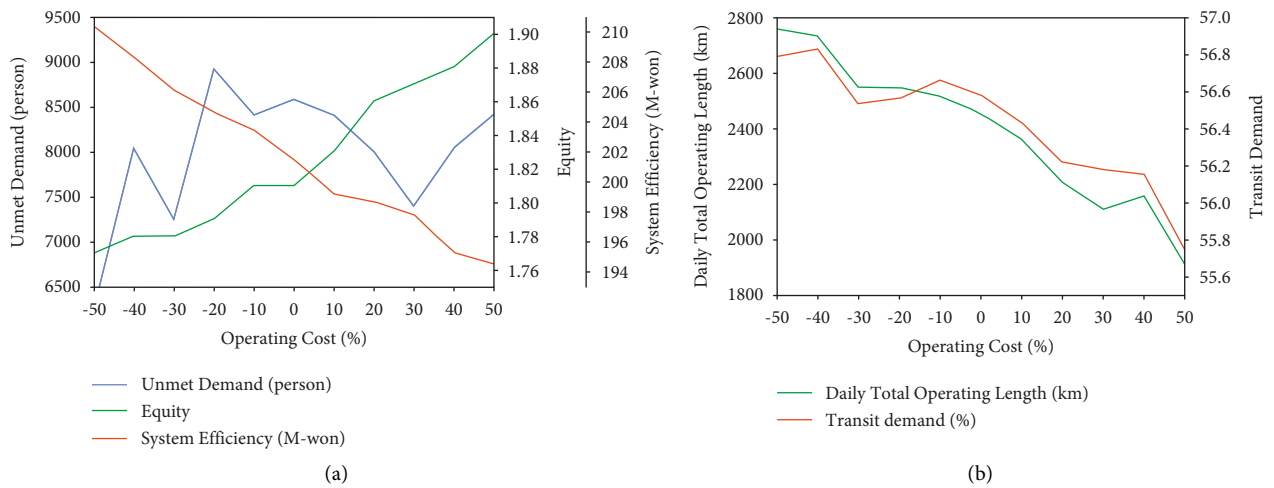


FIGURE 18: Sensitivity analysis for operating costs. (a) The three objective functions. (b) Daily total operating length and transit demand.

length of the transit network, and transit equity is influenced by the daily total operating length, which represents the extent of transit services. These findings can serve as a guideline for determining the optimal transit network for individual cities or regions based on their geographic, economic, and policy-related characteristics.

## 7. Conclusions

The design and operation of transit networks require various elements, including public policies and demand, to be considered. Therefore, the TNDFSP is considered a multi-objective problem [1, 50, 64]. Existing literature has focused mainly on the perspective of operators and transit passengers. In this paper, an innovative transit network design methodology was proposed that also considered transit equity in addition to the operator and users without applying subjective and/or qualitative weights to the multiple objective functions.

Our proposed method seeks a range of solutions while simultaneously trying to maximize system efficiency, minimize unmet demand, and improve transit equity to minimize the difference in transit service levels between regions. Using a toy network, the relationships between the three objective functions were delineated, and we identified potential factors that may influence these relationships. By applying the proposed approach to an existing real-life network in Goyang, Korea, using key information such as the location of bus stops, the service coverage, and the number of lines, the applicability of the method was verified. It was found that it was not possible to optimize all three objective functions simultaneously; rather, our proposed approach produced a set of solutions from which a particular transit network can be designed based on the goals of the decision-makers.

In addition to analyses based on the current state of the existing network, sensitivity analyses were conducted for potential future changes in the number of possible lines, total demand, and operating costs and their effect on the objective functions. By considering the results of these sensitivity analyses, it is expected that decision-makers will be able to assess regional characteristics in more detail in order to make well-informed decisions about their target transit network.

In this paper, a novel methodology for a multiobjective transit route network design algorithm that considers transit equity and variable demand was developed. This line of research suggests some useful future research directions. For example, optimal solutions for specific seasons or months could be explored according to changing demand over time. In addition, the proposed approach can be refined by comparing results from other cities and be generalized by introducing differentiating parameters to develop a more universal methodology.

## Data Availability

The experiment dataset used to support the findings of this study is available at [https://github.com/jennapark/TRNDFSP\\_SJ.git](https://github.com/jennapark/TRNDFSP_SJ.git).

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Acknowledgments

This paper is a revision of the first author's doctoral dissertation for Seoul National University. This research was supported by Jungseok Logistics Foundation grant and the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (2020R1A6A1A03045059) and Ministry of Science and ICT (2021R1A2C2004125).

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