Research Article

An Optimization Method of High-Speed Railway Rescheduling to Meet Unexpected Large Passenger Flow

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In the rapidly changing multimodal transportation market, not only the obligation in providing train services but also the responsibility in helping other transportation modes to meet emergencies are instrumental for maximizing the generalized benefit of railway companies, leading to an increased utilization. This paper proposes an optimization framework, which includes multiple dispatching measures of arranging residual seats, modifying stopping plans, and inserting additional trains, to design a rescheduling operation plan on a high-speed rail corridor to meet the passenger demand of unexpected large passenger flow (ULPF), which is generated from the disruption of other transportation modes. Considering the revenues of shifting passengers and the costs of the three dispatching measures as two objectives, we formulate a linear integer programming (LIP) model by employing a time-space network to obtain an optimization rescheduling operation plan. Several experiments based on the Beijing-Shanghai high-speed railway corridor are conducted to evaluate the effectiveness and efficiency of the proposed model. The experimental results demonstrate that the proposed model can be used to obtain a reasonable rescheduling operation plan for serving the passengers from ULPF within an acceptable calculation timeframe.

1. Introduction

High-speed railway, with its stability and capacity of transporting a large volume of passenger flows, plays an important role in the rapidly changing multimodal transportation market, not only in the field of providing train services but also in undertaking social service responsibilities in helping other transportation modes to meet emergencies. How to optimize the operation plan to meet the passenger demand that is implemented during the emergency of other transportation modes and further enlarge the social benefit is of great importance for railway companies and their operators. To this end, effectively rescheduling to meet unexpected large passenger flow (ULPF) has become an increasingly demanded-research issue in recent years.

1.1. Motivation. The primary motivation of this study is the low response of high-speed railway operation to the ULPF that is generated during the emergency of other transportation modes. Consider the situation in China as an example: the average delay rate of Shanghai Hongqiao airport in the city of Shanghai is 14.19%, and the average of passenger throughput per day is 850 millions of person-time in the year of 2020. The potential shifting demand from air transportation to high-speed railway is significant. Although the transferring from airport to high-speed railway station is difficult in most cities, there are many cities, such as Shanghai Hongqiao station, that can provide a convenient transfer for passengers. Thus, the rescheduling operation for serving passengers from the disruptions of other transportation modes has become important for railway companies to enlarge the...
transportation market. Therefore, the design of a satisfactory rescheduling operation plan for serving the ULPF is a significant issue that railway operators faced in practical operations.

Traditionally, a sequential process consisting of line planning, train timetabling, rolling stock, and crew scheduling is used for planning train operations. The outcome of each stage is used as an input for the following stage. While a planned timetable is put into operation, unavoidable stochastic perturbations (e.g., bad weather, large passenger flow, and capacity breakdowns) may influence the train running and dwelling times, causing delays. If the perturbation is slight, only the train arrival and departure times need to be rescheduled. Nevertheless, if the perturbation is serious, the rescheduling operation plan with more dispatching measures, such as reordering trains, inserting additional trains, modifying stopping plans and trains cancellation, should be generated.

In this paper, we focus on how to design a rescheduling operation plan to serve passengers from ULPF effectively. We address the train timetables problem (TTP) through an optimization approach to consider multiple dispatching measures simultaneously, which include arranging residual seats, modifying stopping plans, and inserting additional trains. A linear integer programming (LIP) model is formulated to solve the TTP problem in this study, and several experiments based on the Beijing-Shanghai high-speed railway corridors are conducted using a standard CPLEX solver.

The remainder of this paper proceeds as follows: a literature review and the statement of the contribution of this paper are provided later in this section. In Section 2, the problem statement and illustrative example are presented. In Section 3, a binary integer programming model is proposed to optimize the rescheduling operation plan for serving ULPF, with the statement of assumptions and notations. Section 4 conducts several experiments to test the effectiveness and efficiency of the proposed model, followed by a conclusion and future research in Section 5.

1.2. Literature Review. The problem of this paper is a train timetabling problem (TTP), which has been studied in the past few decades.

Szpiegel [2] was concerned with a single-track railroad train timetabling problem as a general job-shop scheduling problem and formulated an integer programming model to solve the problem. Carey and Lockwood [3] presented a mixed integer programming model and solution algorithms for the train timetabling problem on a double-track rail line. Carey [4] further developed an extended model to consider more general and more complex rail networks with possible choices of lines and station platforms. A companion paper by Carey [5] proposed an extension from one-way to two-way rail lines. Mascis and Pacciarelli [6] concentrated on the job-shop scheduling problem with blocking and no-wait constraints and formulated an optimization model using a generalization of the disjunctive graph called alternative graph to solve the problem. Caprara et al. [7] proposed a graph-theoretic formulation for the periodic-timetabling problem using a directed multigraph by incompatible arcs and forbid the simultaneous selection of such arcs through a novel concept of clique constraints. This formulation is used to derive an integer linear programming model that is relaxed in a Lagrangian way, which is embedded within a heuristic algorithm that makes extensive use of the dual information associated with the Lagrangian multipliers. Depending on the basic problem of TTP, Caprara et al. [8] proposed a mathematical model incorporating several additional constraints (e.g., manual block signaling for managing station capacities and prescribed timetable for a subset of trains and maintenance operations). Meng and Zhou [9] developed an integer programming model for the problem of train dispatching on an N-track network by means of simultaneously rerouting and rescheduling trains. A vector of cumulative flow variables was introduced by them to reformulate the track occupancy so that they can decompose the original complex rerouting and rescheduling problem efficiently into a sequence of single train optimization subproblems. The decompose mechanism provides us a method to deal with large-scale optimal problems of train dispatching.

On the other hand, modifying stopping plans and inserting additional trains into existing timetables are two critical measures in rescheduling the TTP problem. Burdett and Kozan [10] considered the problem of scheduling additional trains as a hybrid job-shop scheduling problem with time window constraints. The original timetable was finetuned according to the operations and the demand of various customers or operators. A constructive algorithm and a simulated annealing approach were used to solve this problem. Cacchiani et al. [11] described a problem for inserting new freight trains, which send requests for infrastructure usage, to existing passenger trains timetables. An integer linear programming (ILP) model with the objective of total deviation between the actual timetable and the ideal one of all the freight trains is proposed, and it is solved by the Lagrangian heuristic solution. It is a large-scale dispatching problem since timetables should be rescheduled associating with new trains added. Gao et al. [12] considered the problem of scheduling additional trains on a high-speed rail corridor where only passenger trains were run. To add more trains, the timetable of the original trains may need to be modified in that study. A biobjective mixed integer linear programming model was formulated, of which the objectives were to minimize the total travel times of the additional trains and to adjust the timetable of the original trains. Pouryousef and Lautala [13] presented the hybrid simulation framework to improve the capacity utilization of the railway and timetable. The hybrid simulation experiments in this paper were implemented based on two types of developed simulation software, i.e., the timetable simulation system and nontimetable simulation. The hybrid simulation approach is to make use of the complementary features of nontimetable and timetable and use the output from a simulation system as input for the other simulation system. Yang et al. [14] first proposed the collaborative optimization framework for both stop planning and timetable problems. In the previous research, the timetable needs to be regenerated to meet the pre-specified stop plan constraints.
Although all of these studies have considered the TTP problem with multiple dispatching strategies, they did not consider the passenger demands. In addition, these studies focused on the timetable of additional trains without considering the suitable rescheduling measures according to passenger demand.

Furthermore, studies on demand-oriented train timetabling problems have become an active area among the researchers. Chierici et al. [15] focused on the regular timetable problem and constructed a mix-integer non-linear model that takes into account the logit model of passenger choice behaviors. The model is solved by a branch-and-bound algorithm based on outer approximation and a heuristic algorithm. Niu and Zhou [16] focused on a passenger train timetable optimization problem in a heavily congested urban rail corridor. Based on time-dependent, origin-to-destination trip records from an automatic fare collection system, a nonlinear optimization model is developed to formulate the problem, and the model is solved through a genetic algorithm. Kaspi and Raviv [17] focused on the service-oriented line planning problem and timetabling problem (SOLPTP), and an integer programming model with two objectives on the microscopic level was constructed to formulate the problem. The model is solved using a cross-entropy metaheuristic. Sun et al. [18] formulated three optimization models to design demand-sensitive timetables by demonstrating the train operation using an equivalent time. Niu et al. [19] focused on how to minimize the total passenger waiting time at stations by computing and adjusting train timetables for a rail corridor with a given time-varying, origin-to-destination passenger demand matrices. A unified quadratic integer programming model with linear constraints is developed to jointly synchronize effective passenger loading time windows and train arrival and departure times at each station, and the model is solved through an integer programming optimization solver. Meng and Zhou [20] developed an integrated demand/service/resource optimization model to manage the dynamic choice behaviors of passengers and the service patterns and timetables of trains with a special focus on passengers' responses to time-dependent service interval times or frequencies. The proposed model is decomposed into a novel team-based train service search subproblem through a Lagrangian relaxation solution framework. Liu et al. [21] proposed a new optimization framework for designing the operation plan, which includes the number of additional trains, train type, stop plan, and timetable, for additional trains in a peak period to satisfy the increasing passenger demand. A biobjective integer programming model is formulated for generating a cost and passenger responsible primary operation plan. The model is solved through a standard CPLEX solver.

In summary, in spite of the rich body of existing train rescheduling studies, there is still a significant gap in the literature, when referring to the rescheduling of online train with the consideration of 4 levels of dispatching strategies of retiming, reordering, rerouting, and reserving simultaneously.

1.3. Contribution. This paper makes three main contributions.

Firstly, this paper focuses on a dispatching problem aiming to serve unexpected additional passengers in helping other transportation modes to meet emergencies. It is different from the previous studies, which considered the existing passengers. Thus, an optimization framework is proposed to solve this new problem. The study of this paper makes a step forward in perfecting rescheduling in multimodal transportation market.

Secondly, we construct an optimization model that considers the dispatching strategies of retiming, reordering, rerouting, and reserving simultaneously. Multiple dispatching measures, including arranging residual seats, modifying stopping plans, and inserting additional trains, are introduced jointly to meet the passenger demand of ULPF. Two parts of objectives that maximize the generalized benefit of shifting passengers from ULPF and minimize the total cost of dispatching measures are formulated in the model as the primary and secondary objectives, respectively, to generate the rescheduling operation plan that is economical for railway companies.

Thirdly, a practical detail of extra running time constraints is considered in this model. We design a team of constraints to impose train acceleration and deceleration time restriction on the segments before and after the station, which modified the stopping plans. Thereupon, the proposed model can describe the problem more precisely to the dispatching process of real-time management.

2. Problem Description

2.1. The Problem in Time-Space Network. In this paper, a typical time-space network is introduced to describe a double-track railway corridor of the problem. The physical railway line is represented by nodes and links in the time-space network. The nodes that are indexed by \( i, j, k \in N \{1, 2, \ldots, N\} \) represent the stations in the physical railway line, in which the train can stop for loading and unloading passengers or pass through. The link that is generated by two nodes \( e = (i, j) \) represents the segment between the two adjacent stations, \( e \in E \), and \( E \) is the set of all links.

The set of trains being considered in the model is \( F = F^c \cup F^r \), where \( F^c \) denotes the set of existing trains, and \( F^c \) denotes the set of candidate additional trains. In this paper, we study the problem of designing a rescheduling operation plan that includes one or more dispatching measures to improve the transport capacity and serve as many passengers from ULPF as possible.

In this study, all the passengers from ULPF have the same origin station, which is not far from the initial terminal of other transportation modes inside the city, and they have several different destination stations that are located along the high-speed railway corridor. The passengers from ULPF are decomposed into several groups, which are called the passenger OD pairs. Each passenger OD pair \( p \) has the same origin and destination stations and the preferred departure
time windows. $P$ is the set of all passenger OD pairs, $p \in P$. Passengers from ULPF may choose any of the trains that departure from the origin station between the time windows and stop at their destination station. Obviously, passengers would prefer to take the earlier trains.

2.2. Multiple Dispatching Measures. The dispatching measures should satisfy the passenger demand of ULPF as much as possible but not be so large as to cause a waste of transport capacity. Therefore, we construct three dispatching measures more complex in sequence to obtain a benefit-cost friendly rescheduling operation plan. The candidate-dispatching measures include the following: (1) arranging the residual seats on existing trains, (2) modifying the stopping plans of existing trains, and (3) inserting additional trains.

(1) Arranging residual seats. While the existing trains that correspond well to the passenger demand have sufficient residual seats, we only need to assign the passengers to existing trains with the consideration of train capacity, so that the cost of additional passengers is the least.

(2) Modifying stopping plans. Nevertheless, not all the existing trains stopped at the OD stations of ULPF. Hence, we should add the measure of modifying the stopping plans of existing trains to increase the number of residual seats for transporting passengers. The dispatching measure of modifying stopping plans indicates whether an existing train adds stop at the destination station of ULPF as required, whereas the modification of stopping plans may affect the service quality and operation efficiency. Thus, we discuss this dispatching measure in two aspects, which include train choice and train delay.

Since not all the existing trains stopped at the destination stations of passenger OD pair $p$, we construct a candidate train set $F_p$ to contain the trains that do not stop at the destination station of passenger OD pair $p$. Only the trains in set $F_p$ can modify the stopping plans from passing through to stopping at the destination station of passenger OD pair $p$.

Because of the additional stop at the destination station of $p$, the extra dwelling time and train acceleration and deceleration time would prolong the train travel time. The extension of train travel time may not only influence the punctuality of its existing passengers but also disturb the adjacent trains causing consecutive delays. Therefore, the delays of existing passengers can be treated as dispatching cost, which is calculated in the objective functions.

(3) Inserting additional trains. Furthermore, in most situations, the trains do not have enough residual seats on busy high-speed railway corridors. Then, we should insert additional trains to meet the passenger demand of ULPF. The dispatching measure of inserting additional trains is a complex problem with the consideration of the time-space number of additional trains, train timetables, and stopping plans.

Obviously, the capacity for serving ULPF enlarges with the increasing number of additional trains. However, the number of trains to be additionally inserted is subject to the candidate multiple unit trains. Meanwhile, the use of candidate multiple unit trains is also limited by various factors. Thus, not only the total number but also the earliest departure time of candidate trains is used to limit the time-space number of inserting additional trains in this paper.

The timetable determines the arrival, departure, and dwelling times of each additional train at each station. In the peak period, inserting additional trains in the existing timetables may influence the departure and arrival times of existing trains according to the scarce capacity. Thus, we should consider the cost of existing trains that are interfered by the additional trains.

Identically, the additional trains being inserted also need to modify the stopping plans at the destination station of $p$, causing additional dwelling time and train acceleration and deceleration time.

These three strategies are frequently used on high-speed railways, where a seat reservation system is applied. While some airlines are disrupted, passengers may obtain the information of disruption and suggestions of new itinerary from the airport. In addition, they may purchase the tickets through the seat reservation of high-speed railway on apps to reroute their trips. As the preparation of the candidate train units need a certain time span, passengers have sufficient time to purchase the tickets and transfer between two transportation terminals.

2.3. An Illustrative Example. A small-scale time-space network is illustrated in Figure 1 to explain the problem in this paper. We consider a physical line that consists of three stations and two segments from station A (node 1) to station C (node 3). There are 4 high-speed trains prescribed on the line from station A to station C. Two of them, signed $f_1$ and $f_3$, stop at station B, and the others two, $f_2$ and $f_4$, do not. Two passenger OD pairs, $p$ and $p'$, of ULPF are generated at station A, with the different destination stations B and C. The departure time windows of $p$ and $p'$ at their origin stations are $(t_1, t_2)$ and $(t_3, t_4)$, respectively. Assume that all the trains have a certain number of residual seats to serve ULPF.

Based on the problem presentation above, a feasible solution for transporting passengers is generated (see Figure 2). As shown in Figure 2, there are a total of five trains that are put into operation, including four original trains (signed $f_1, f_2, f_3$, and $f_4$) and one additional train (signed $f$). In an attempt to satisfy the passenger demand of $p$, train $f$ is added, and the stopping plan of train $f_2$ at station B is modified from passing through to stopping. Since the stopping plan was modified, the travel time of train $f_2$ is increased. As train $f$ adds, train $f_3$ is delayed to keep the specified interval times for the transportation safety. The dotted lines are the predetermined paths of train $f_2$ and $f_3$. Therefore, the passengers of the OD pair $p$ can take the train $f_2, f_3$, and $f$ to reach their destination. For the same reason, passengers of $p'$ can travel through train $f_3$ and $f_4$, whose
Figure 1: An illustration of the primary problem in the time-space network.

Figure 2: An illustration of rescheduling operation plan in time-space network.
departure times are falling in the time windows \((t_3, t_4)\) of the passenger OD pair \(p'\).

According to the example, the problem stated above can be transformed into a multiple-train path planning problem in a time-space network. Furthermore, if two groups of binary decision variables are introduced to indicate whether the infrastructure resources are used by trains and whether the trains are selected by passengers from ULPF, this problem can be formulated as a binary integer programming model.

### 3. Problem Formulation and Method

In this section, a linear integer programming (LIP) model is constructed to obtain the optimal rescheduling operation plan for serving passengers from ULPF.

#### 3.1. Assumptions

Three assumptions are considered to simplify the problem in the following formulations:

**Assumption 1.** A station is assumed as a node in the formulation, without considering the routes in the station.

**Assumption 2.** The length of a train is assumed to be zero.

**Assumption 3.** The strategy of passenger transfer is not considered in this paper, which means that passengers can only take direct trains to their destinations.

**Assumption 4.** The rolling stock circulation is not considered, which means that all the train units are available. In the real condition, several candidate train units that distributed in several stations along the high-speed railway corridor, are available.

#### 3.2. Notations

For reading convenience, all the parameters and variables used in the study are defined in Tables 1 and 2.

#### 3.3. Objective Function

To help the other transportation modes to meet the ULPF for emergencies, the objective of serving the revenues of passengers from ULPF is preferred. Thus, multiple dispatching measures that include arranging residual seats, modifying stopping plans, and inserting additional trains are considered in the model simultaneously to enlarge the capacity for serving ULPF. In addition, the revenues of serving passengers should be maximized as shown in Eq. 1.

\[
C_{\text{passenger}} = \sum_{p \in P} \sum_{f \in F} \mu_{f,p}.
\]  

However, the rescheduling operation plan for serving ULPF is not a cost regardless of dispatching, and it is not a delay regardless of existing trains. Thus, we consider the cost of negative effects of dispatching measures as shown in equations (2)–(4).

\[
C_{\text{insert}} = \sum_{f \in F} \delta_{f}.
\]  
\[
C_{\text{modify}} = \sum_{p \in P} \sum_{f \in F_p} y_{f,p}.
\]  
\[
C_{\text{delay}} = \sum_{f \in F} \mu_{f}.
\]

Equation (2) is the number of inserting additional trains. Equation (3) is the total number of adding stops of existing trains considering its adding organizing workload in the corresponding stations. (4) is the number of existing trains, which are delayed because of the strategies of inserting additional trains and modifying stopping plans.

Since it is a multiobjective optimization problem with the four contradictory parts of objectives, we introduced the \(\varepsilon\)-constraint method to solve the problem. In the \(\varepsilon\)-constraint method, we optimize one of the objective functions using the other objective functions as constraints.

The main objective in the \(\varepsilon\)-constraint method is the revenues of serving passengers, for the following reasons: (1) the main purpose of the model is to reschedule the trains to serve the passengers from ULPF, and (2) the number of additional trains inserted, stops added to the existing trains, and the number of delayed existing trains can be easily constrained according to the real dispatching experiences. Thus, the objective function of the model is formulated as follows:

\[
\text{Max} C = \sum_{p \in P} \sum_{f \in F} \mu_{f,p}.
\]  

The objective functions in equations (2)–(4) are treated as constraints incorporating in the constraint part of the model as below.

\[
\sum_{f \in F} \delta_{f} \leq \alpha_{i},
\]

\[
\sum_{p \in P} \sum_{f \in F_p} y_{f,p} \leq \alpha_{m},
\]

\[
\sum_{f \in F} \mu_{f} \leq \alpha_{d}.
\]

Note that \(\alpha_i\) represents the boundary value of the number of additional trains inserted, which can be valued according to the number of reserved multiple unit trains in the real dispatching situations. \(\alpha_m\) represents the boundary value of the total number of adding extra stops, which can be valued considering the organizing capacity in the corresponding stations. At last, as the delay of existing trains in this problem is caused by the strategies of inserting additional trains and modifying stopping plans of existing trains, the boundary value of the total number of delayed trains \(\alpha_d\) can be evaluated according to the number of inserting additional trains and adding stops.
3.4. Constraints. Five sets of constraints are considered in the process of rescheduling the operation plan, which are as follows: (I) flow balance constraints, (II) running and dwelling time constraints, (III) time-space network constraints, (IV) train order and capacity constraints, and (V) passenger demand constraints.

\[ \sum_{j \in \{i,j\} \in E_f} x_{f,o,j,i} = 1, \forall f \in F^e, \tag{7} \]

\[ \sum_{i \in \{i,j\} \in E_f} x_{f,i,j} = \sum_{k \in \{j,k\} \in E_f} x_{f,j,k}, \forall f \in F, j \in N_{f,j} \tag{8} \]

\[ \sum_{i \in \{i,j\} \in E_f} x_{f,i,j} = 1, \forall f \in F^e, \tag{9} \]

\[ \sum_{j \in \{i,j\} \in E_f} x_{f,o,j,i} = \delta_f, \forall f \in F^e m, \tag{10} \]

\[ \sum_{i \in \{i,j\} \in E_f} x_{f,i,j} = \delta_f, \forall f \in F^e. \tag{11} \]

3.4.1. Flow Balance Constraints. To guarantee that at most one path is generated from the origin to the destination for each existing train \( f \in F^e \) in the time-space network, constraints (7)–(9) are presented. Furthermore, to ensure the consistency of path generating and inserting for each candidate additional train \( f \in F^e \), we construct constraints (10)–(11), in which the train’s path is not equal to one.

### Table 1: Input parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_f )</td>
<td>Set of station nodes train ( f ) needs to stop for loading/unloading, ( N_f \in N )</td>
</tr>
<tr>
<td>( E_f )</td>
<td>Set of links train ( f ) may use, ( E_f \in E )</td>
</tr>
<tr>
<td>( w_{i,f} )</td>
<td>Minimum dwell time for train ( f ) at station node ( i )</td>
</tr>
<tr>
<td>( w_{i,f} )</td>
<td>Additional dwell time for train ( f ) at station node ( i ) caused by adding a stop</td>
</tr>
<tr>
<td>( a_{f,i,j} )</td>
<td>Acceleration time of train ( f ) on link ( (i,j) ) caused by adding a stop at station node ( i )</td>
</tr>
<tr>
<td>( \tau_{f,i,j} )</td>
<td>Deceleration time of train ( f ) on link ( (i,j) ) caused by adding a stop at station node ( j )</td>
</tr>
<tr>
<td>( r_{f,i,j} )</td>
<td>Free flow running time for train ( f ) to drive through the link ( (i,j) )</td>
</tr>
<tr>
<td>( h )</td>
<td>Minimum interval time between two adjacent trains running on the same link</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>Maximum number of inserting additional trains restricted by the plans of multiple unit trains</td>
</tr>
</tbody>
</table>

### Table 2: Decision variables.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{f,i,j} )</td>
<td>0-1 binary routing variables, ( x_{f,i,j} = 1 ), if train ( f ) used the link ( (i,j) ) at some time. Otherwise, ( x_{f,i,j} = 0 )</td>
</tr>
<tr>
<td>( a_{f,i,j} )</td>
<td>Arrival time of train ( f ) on link ( (i,j) )</td>
</tr>
<tr>
<td>( d_{f,i,j} )</td>
<td>Departure time of train ( f ) on link ( (i,j) )</td>
</tr>
<tr>
<td>( \theta_{f,f,i,j} )</td>
<td>0-1 binary train ordering variables, ( \theta_{f,f,i,j} = 1 ), if train ( f' ) arrive at link ( (i,j) ) after train ( f ). Otherwise, ( \theta_{f,f,i,j} = 0 )</td>
</tr>
<tr>
<td>( y_{f,p} )</td>
<td>0-1 binary stopping plan modifying variables, ( y_{f,p} = 1 ), if the stopping plan of train ( f ) at the destination station of passenger OD pair ( p ) modified from passing through to stopping. Otherwise, ( y_{f,p} = 0, f \in F_p )</td>
</tr>
<tr>
<td>( \delta_f )</td>
<td>0-1 binary inserting variables, ( \delta_f = 1 ), if train ( f ) is inserted. Otherwise, ( \delta_f = 0, f \in F^e )</td>
</tr>
<tr>
<td>( \mu_f )</td>
<td>0-1 binary train delay variables, ( \mu_f = 1 ), if train ( f ) arrive at destination later than the predetermined arrival time. Otherwise, ( \mu_f = 0, f \in F^e )</td>
</tr>
<tr>
<td>( \gamma_{f,p} )</td>
<td>0-1 binary passenger to train assignment variables, ( \gamma_{f,p} = 1 ), if passenger OD pair ( p ) takes train ( f ) to finish its trip. Otherwise, ( \gamma_{f,p} = 0 )</td>
</tr>
<tr>
<td>( \varphi_{f,p} )</td>
<td>Volume of passenger OD pair ( p ) transporting by train ( f ) to finish its trip</td>
</tr>
</tbody>
</table>
3.4.2. Running and Dwelling Time Constraints. To correspond with the real condition of train operation, several running and dwelling time restrictions must be respected as constraints (12)–(16) formulated.

\[ d_{f,i,j} - a_{f,i,j} \geq x_{f,i,j} \times r_{f,i,j}, \forall f \in F, (i, j) \in E_f, \]  
\[ d_{f,i,j} + w_{f,j} \leq a_{f,i,j} \times N_f \times (i, j), (j, k) \in E_f, \]  
\[ d_{f,i,j} + w_{f,j} \times y_{f,j,p} \leq a_{f,j,s_p} \times k, \forall p \in P, f \in F_p, (i, s_p), (s_p, k) \in E_f, \]  
\[ d_{f,i,j} + w_{f,j} \times y_{f,j,p} \geq a_{f,j,s_p} \times \tau_{f,i,i}, \forall p \in P, f \in F_p, (i, s_p) \in E_f, \]  
\[ d_{f,i,j} + w_{f,j} \times y_{f,j,p} \geq a_{f,j,s_p} \times \sigma_{f,j,p}, \forall p \in P, f \in F_p, (s_p, j) \in E_f. \]  

Constraints (12) and (13) enforce the minimum running time on each link and minimum dwelling time at each station, respectively. Furthermore, while the train modified the stopping plan at a station from passing through to stop, the running time of the segments before and after the station would prolong than before, for the reason of decelerating and accelerating. Meanwhile, the train should stop at the station no shorter than the specified time span for loading/unloading passengers. Constraints (14)–(16) restrict the necessary stopping time and train acceleration and deceleration time.

3.4.3. Time-Space Network Constraints. In the process of rescheduling, the timetables of existing trains should not disrupt the normal schedules of passengers that have already purchased fares and should not violate the in-use plan of rolling stock and crew.

\[ a_{f,o_f,j} + \left(1 - x_{f,o_f,j}\right) \times M \geq \varepsilon_f, \forall f \in F, (o_f, j) \in E_f, \]  
\[ d_{f,i,s_f} + \left(1 - x_{f,i,s_f}\right) \times M \leq \varepsilon_f, \forall f \in F, (i, s_f) \in E_f, \]  
\[ a_{f,j,s_f} \geq x_{f,j,i} \times \alpha_{f,j,i}, \forall f \in F, \]  
\[ x_{f,i,j} - 1 \leq a_{f,i,j} \leq x_{f,i,j} \times M, \forall f \in F, (i, j) \in E_f, \]  
\[ x_{f,i,j} - 1 \leq d_{f,i,j} \leq x_{f,i,j} \times M, \forall f \in F, (i, j) \in E_f. \]

Constraints (20) and (21) are imposed to map the arrival and departure time in time-space network to the link usage variables in physical network to describe the relationship between the links selection of a train and its departure and arrival time.

\[ \frac{|d_{f,i,j} - a_{f,i,j}|}{M} \leq \mu_f \leq \frac{|d_{f,i,j} - a_{f,i,j}|}{M}, \forall f \in F, (i, s_f) \in E_f. \]

Constraint (22) links the train delay variables and the train departure time variables at the final segments. If and only if train \( f \) departs from its final segment at the predetermined time, the delay variable \( \mu_f = 1 \). Otherwise, \( \mu_f = 0 \).
3.4.4. Train Order and Capacity Constraints. The train order variables $\theta_{f,f',i,j}$ are introduced in this paper to represent the sequence of two different trains using one link.

$$x_{f,i,j} + x_{f',i,j} - 1 \leq \theta_{f,f',i,j} + \theta_{f',f,i,j} \leq 3 - x_{f,i,j} - x_{f',i,j}, \forall f \in F, f' \in F, f \neq f', (i,j) \in E_f \cap E_{f'},$$

(23)

$$\theta_{f,f',i,j} \leq x_{f,i,j}, \forall f \in F, f' \in F, f \neq f', (i,j) \in E_f \cap E_{f'},$$

(24)

$$\theta_{f,f',i,j} \leq x_{f',i,j}, \forall f \in F, f' \in F, f \neq f', (i,j) \in E_f \cap E_{f'},$$

(25)

Constraints (23) link train order variables and usage variables. Additionally, if and only if train $f$ and train $f'$ use link $(i,j)$, the two trains have the sequential order $\theta_{f,f',i,j} = 1$ or $\theta_{f',f,i,j} = 1$, i.e., either train $f$ arrives at link after train $f'$ or the opposite condition. If $x_{f,i,j} = 0, x_{f',i,j} = 1$,

$$a_{f',i,j} + (3 - x_{f,i,j} - x_{f',i,j} - \theta_{f,f',i,j}) \times M \geq a_{f,i,j} + h, \forall f \in F, f' \in F, f \neq f', (i,j) \in E_f \cap E_{f'},$$

(26)

$$d_{f',i,j} + (3 - x_{f,i,j} - x_{f',i,j} - \theta_{f',f,i,j}) \times M \geq d_{f,i,j} + h, \forall f \in F, f' \in F, f \neq f', (i,j) \in E_f \cap E_{f'},$$

(27)

$$\sum_{f \in F^c} \delta_f \leq \Omega.$$  

(28)

Constraints (26)–(28) explicitly make sure that any train occupies the link within a safety interval time. Note that for trains $f$ and $f'$ traversing on link $(i,j)$, i.e., $x_{f,i,j} = x_{f',i,j} = 1$, constraints (26) and (27) can be reduced to common if-then conditions as follow: if train $f$ arrives at link after train $f'$, i.e., $\theta_{f,f',i,j} = 1$, then the arrival and departure time of train $f'$ should be no earlier than the corresponding time of train $f$ on link $(i,j)$, respectively, else the constraints reduce to nonactive inequalities. Constraint (28) restricts the total number of additional trains.

3.4.5. Passenger Demand Constraints

(1) Loading relationship for each train. The implicit variables $\theta_{f,p}$ are introduced to describe the assignment between passengers and trains. Thus, the passenger assignment variables are restricted to the stopping plans of existing trains, as formulated below.

$$\theta_{f,p} \leq \phi_{f,p} + y_{f,p}, \forall p \in P, f \in F^c.$$  

(29)

However, the passenger assignment variables of each additional train are relevant to whether the trains are inserted and whether the trains stopped at their destination station, as $\phi_{f,p} \leq y_{f,p} \times \delta_f$. To solve the nonlinear problem of this constraint, we extend the characteristic of stopping plans to the candidate additional train set $F^c$, namely, splitting a candidate train in set $F^c$ into several trains by the characteristic of different stopping plans and using 0-1 binary parameters $\phi_{f,p}$ to represent whether train $f$ stops at the destination station of $p$. Thus, the passenger assignment restriction of additional trains is formulated as follows:

$$\theta_{f,p} \leq \phi_{f,p} \times \delta_f, \forall p \in P, f \in F^c.$$  

(30)

However, passengers from ULPF may choose other transportation modes or abandon shifting to wait for the recovering of initial transportation mode on the condition that the demands are not responded. Thus, we introduce a passenger departure time window $[\psi_p, \psi_p]$ to represent passengers’ earliest and latest departure time expectations. Passengers cannot be assigned to the trains with departure from the origin station out of the time windows, as formulated in constraints (31) and (32).

$$(\theta_{f,p} - 1) \times M \leq a_{f,o,p} - \psi_p, \forall p \in P, f \in F, (o_p, j) \in E_f,$$

(31)

$$(\theta_{f,p} - 1) \times M \leq \psi_p - a_{f,o,p}, \forall p \in P, f \in F, (o_p, j) \in E_f.$$  

(32)
(2) Loading number for each train. The loading number of passenger OD pair \( p \) depends on the following: (a) loading relationship between passengers and trains, (b) trains’ loading capacities, and (c) total passenger number of \( p \).

\[
\begin{align*}
\rho_{f,p} &\leq \theta_{f,p} \times M, \forall p \in P, \ f \in F, \quad (33) \\
\phi_{f,p} &\leq \xi_{f,p}, \forall p \in P, \ f \in F, \quad (34) \\
\sum_{f \in F} \rho_{f,p} &\leq \Lambda_p, \forall p \in P. \quad (35)
\end{align*}
\]

Constraint (33) makes sure that the loading number is dependent on the loading relationship. Constraint (34) enforces that the loading number of passengers in each train does not exceed the loading capacity of the train. Constraint (35) further makes sure that the total number of serving passengers is not more than the number of each passenger OD pair \( p \).

(3) Passenger choice behavior. Normally, various factors would be taken into account in the passenger choice behavior of trains, such as the arrival and departure times, the travel time, and so on. However, because of the passenger demand in this study, which is generated from the disruption of other transportation modes, they tend to choose the train as early as possible to return to the normal schedule as much as possible. Thus, only the departure time factor is considered in this study to depict passengers’ choice behavior.

\[
\begin{align*}
\frac{\partial_{f,p}}{\xi_{f,p}} \left( \sum_{(o,p,j) \in E_f \cup E_{f'}} \left( 3 - \theta_{f,f',o,p,j} - \theta_{f,p} - \theta_{f',p} \right) \right) \times M &\geq \frac{\rho_{f,p}}{\xi_{f,p}} \\
\forall p \in P, \ e \in F, \ f' \in F, \ f \neq f', (o,p,j) &\in E_f \cup E_{f'}.
\end{align*}
\] (36)

We introduce the concept of attendance of residual seats to represent the passengers’ choice of different trains. Constraint (36) guarantees that the attendance of residual seats in the earlier train is not less than that in the later train, while the train departures in the time windows of passenger OD pair \( p \). Additionally, if and only if passenger OD pair \( p \) chooses both train \( f \) and \( f' \) to finish the trip and the two trains have the sequential order \( \theta_{f,f',o,p,j} = 1 \), then train \( f \) has the high attendance of residual seats for \( p \) than train \( f' \). If \( \theta_{f,p} = 0, \theta_{f',p} = 1, \theta_{f,p} = 1, \theta_{f',p} = 0, \) or \( \theta_{f,p} = 0, \theta_{f',p} = 0, \) constraint (36) is reduced to a nonactive inequality.

4. Numerical Experiments

In this section, several numerical experiments are conducted to evaluate the effectiveness and efficiency of our proposed model, which is implemented through the commercial integer programming solver IBM ILOG CPLEX 12.5. All of the following experiments are performed on a Lenovo PC with 2.3 GHz Intel i5-6200U CPU and 8 GB memory.

IBM ILOG CPLEX offers C, C++, Java, .NET, and Python libraries that can solve the linear programming problem (LP), linear integer programming problem (LIP), and related problems that may have millions of variables and constraints. To solve the linear integer programming problem (LIP), CPLEX implements optimizers based on the primary simplex algorithm, dual simplex algorithm, primary-dual logarithmic barrier algorithm, and sifting algorithm.

4.1. Basic Experiment

4.1.1. Experimental Setup. In this study, a basic experiment on the operational environment of the Beijing-Shanghai high-speed railway corridor in China is conducted, in which a rescheduling operation plan, including three dispatching measures, is designed for serving passengers from ULPF.

The Beijing-Shanghai high-speed railway corridor consists of 24 stations and 23 segments, and in total, it is 1318 km in length, as shown in Figure 3. Because of the passenger demand of ULPF that is generated from the disruption of flights, only the city that has an airport located in it is considered in this study for simplicity. Thus, five main stations, which are Shanghai Hongqiao, Nanjingnan, Xuzhoudong, Jinanxi, and Beijingnan, are considered in this experiment, as shown by big red dots in Figure 3. The time horizon of this experiment is 500 minutes, and only one type of train, namely G-train, is considered in this experiment. The length of each segment and the minimum travel time of G-train at each segment are listed in Table 3. The minimum dwelling time of the trains at each station is set to 2 minutes. The minimum departure and arrival headways are set to 3 minutes.

In this experiment, we discuss the passenger demand generated from the disruption of three flights at Shanghai Hongqiao airport, which is adjacent to Shanghai Hongqiao high-speed railway station. All of them are heading to Jinanxi. Each of the three flights has 500 passengers. Obviously, the passenger demand for high-speed railway is divided into six passenger OD pairs according to the different OD time. The departure time windows of the six passenger OD pairs at their origin station are (30, 60), (30, 90), (60, 90), (60, 120), (90, 120), and (90, 150), respectively, and the passenger volume of each pair is 300, 300, 300, 400, 400, 400, and 100.

We design 40 existing trains in this experiment, labeled as G2 to G80, between Shanghai Hongqiao and Beijingnan, on the up orientation. The stopping plan and prescribed timetables of these trains are illustrated in Figures 4 and 5, respectively. However, we cannot obtain the data of residual seats of each existing train because of the protection of commercial data. Thus, we assume that each of the existing trains has 50 residual seats along its whole journey. To fully utilize the transportation capacity, we assume that all candidate additional trains are CR400 and composed of 8 vehicles, with the maximum loading capacity of 500 passengers in this experiment, and at most, 1 train is permitted to be added.
To better rationalize the experiment, we refer to the relevant literature to set the boundary values. Since the extra multiple unit trains that are reserved for one railway line are not more and are distributed in several stations along the line, the extra multiple unit trains used for one disturbance is not more than one in most situations. Thus, the boundary value of the number of inserting additional trains in this experiment is set to $\alpha_i = 1$. The boundary values of the number of additional stops and the number of delayed trains are set to $\alpha_m = 6$ and $\alpha_d = 7$, respectively, according to the parameter tests in Section 4.2.

4.1.2. Experimental Result. The experiment is conducted on the CPLEX solver to obtain the optimal rescheduling operation plan for serving ULPF based on the Beijing-Shanghai high-speed railway corridor. The number of variables and constraints is 4523 and 30,064, respectively. The computational time is 6.35 seconds on the platform stated above, with the optimal objective value of 1100. The resulting optimal train timetable and passenger assignment are illustrated in Figure 6, in which the trains that start from Shanghai Hongqiao station are numbered according to their departure times, i.e., the black lines (including black dotted lines) from left to right are numbered from G2 to G70, and the trains that do not start from Shanghai Hongqiao station are numbered from G72 to G80 from left to right. The numbers below the timetables are the volume of the passengers from ULPF transported by each train, and each line represents the passenger OD pair $p_1$ to $p_6$ from top to bottom.

According to the resulting optimal solution, the stopping plans of the train numbered G16, G24, G28, G36, G42, and G48 are modified at Jinanxi station, which is the destination station of passenger OD pairs. Thus, all these trains are late at their destination stations for the reason of adding the dwelling time and train acceleration and deceleration time, as illustrated in Figure 6. Moreover, to maintain the safety departure and arrival headways with train G48, the timetables of train G50 are also postponed, causing delay. The red line in Figure 6 represents the partial trips of the train that are different from the predetermined time-space path, and the black dotted lines are the predetermined time-space path. The stopping plans of all the trains in the rescheduling operation plan are illustrated in Figure 7, including the additional train.

According to the resulting optimal solution, one additional train is inserted into the existing timetables, which is numbered G82, and it is labeled with a blue line in Figure 6. To disturb the existing trains as little as possible, train G82 stopped at all intermediate stations of Nanjingnan and

<table>
<thead>
<tr>
<th>Segment</th>
<th>Length (km)</th>
<th>Minimum travel times (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shanghai Hongqiao – Nanjingnan</td>
<td>284</td>
<td>60</td>
</tr>
<tr>
<td>Nanjingnan – Xuzhoudong</td>
<td>330</td>
<td>80</td>
</tr>
<tr>
<td>Xuzhoudong – Jinanxi</td>
<td>269</td>
<td>70</td>
</tr>
<tr>
<td>Jinanxi - Beijingnan</td>
<td>419</td>
<td>90</td>
</tr>
</tbody>
</table>

Figure 3: Map of Beijing-Shanghai high-speed railway corridor.
Figure 4: The stopping plan of existing trains in the experiment.

Figure 5: The prescribed timetables of existing trains in the experiment.
Xuzhoudong to avoid the existing trains and stopped at Jinanxi to unload passengers.

According to the resulting optimal solution, the total passengers served by the rescheduling measures are 1100, including 300 passengers served by residual seats, 300 passengers by the measures of modifying stopping plans, and 500 passengers by inserting additional trains. The rescheduling operation plan, which satisfied approximately 73.3% of the passengers from ULPF, increased the average attendance of all trains from 87.8% to 93.2%.

Although the operation plan can satisfy most of the passenger demand from ULPF, there are, in total, 400 passengers of OD pairs $p_1$, $p_2$, and $p_5$ that have not been served, as sufficient capacity is not provided. The passengers of $p_1$ and $p_2$ are directed from Shanghai Hongqiao to Jinanxi with the time windows (30, 60) and (30, 90). There are 2 and 6 trains between the time windows of them, respectively. Thus, there are only 100 and 300 residual seats for serving the passenger OD pair $p_1$ and $p_2$. The residual seats of train G24 is used to serve passenger OD pair $p_3$, and G16, G24, G28 are used by modifying the stopping plan at the Jinanxi station. Although the inserted additional train G82 departs from Shanghai Hongqiao in the time window of $p_2$, most of the seats on it are used to serve $p_3$ and $p_4$ for the optimal purpose. Meanwhile, the passengers of $p_5$ are directed from Shanghai Hongqiao to Jinanxi with the time windows [90, 120]. There are totally 5 trains between the time windows, however, only 2 of them, numbered G38 and G40, pass through the OD stations of $p_5$. The residual seats of trains G38 and G40 are used to serve passenger OD pair $p_5$, and G36, G42, and G48 are used by modifying the stopping plan at Jinanxi station.

4.2. Additional Experiments with respect to the Boundary Values of ε-Constraint Method. In the $\varepsilon$-constraint method, three boundary values, namely $\alpha_i$, $\alpha_m$, and $\alpha_d$, are introduced to restrict the secondary objectives to obtain the pareto optimal solution set. As we stated above, the boundary value of the number of additional trains inserted, $\alpha_i$, can be valued according to the number of reserved multiple unit trains in the real dispatching situations. In this section, we conduct several experiments to examine the influence of the boundary values of the number of adding stops $\alpha_m$ and the number of delayed trains $\alpha_d$ to obtain the best empirical value to solve the problem. We set $\alpha_i$ = 1 fixedly in these experiments and set $\alpha_m$ and $\alpha_d$ from 1 to 10, respectively, to test the results of the formulations. The test results of these experiments are shown in Table 4 and Figure 8.

Three conclusions can be drawn from the analysis of the results of experiments, which are summarized as follows:

(1) At most, 1100 passengers from ULPF can be served by all the trains in this rescheduling operation plan, including the insertion of additional trains, while the boundary values of the number of adding stops $\alpha_m$ and the number of delayed trains $\alpha_d$ are sufficient. As shown in Figure 8, while $\alpha_m \geq 6$, the observed values of experiments are no longer increased at any value point of $\alpha_d$. Thus, we can infer that the pareto optimal solution is under 6 on the objective of the number of adding stops, and $\alpha_m = 6$ is reasonable in the experiment.

(2) As illustrated in Figure 8, while the boundary value of $\alpha_m$ is fixed, the observed values are increased stepwise along with the boundary value of the
number of delayed trains $\alpha_d$. While adding a stop at an intermediate station of a train, the travel time of this train is increased as the extra time for acceleration and deceleration and dwelling time are added, and the train would delay at its destination station. Therefore, the number of delayed trains permitted that can influence the strategy of modifying the stopping plans come into play. Thus, the boundary value of the number of delayed trains $\alpha_d$ cannot be set less than the boundary value of $\alpha_m$. From Figure 8, we can find that while $\alpha_d \geq 7$, the observed values of the experiments are no longer increased at
any value point of \( \alpha_m \). Consequently, the setup of \( \alpha_d = 7 \) is reasonable in the experiment.

(3) All the results of these experiments that test above are influenced by the structure of train timetables and the demands of passengers from ULPF. Therefore, we should set the boundary values of the number of additional stops and the number of delayed trains in the \( \varepsilon \)-constraint method, according to the parameters through the method above.

5. Conclusion and Future Research

This paper solved the problem of designing a rescheduling operation plan on a high-speed rail corridor to meet the passenger demand of unexpected large passenger flow (ULPF) that is generated from the disruption of other transportation modes. An optimization framework is proposed for solving the problem. In the framework, multiple dispatching measures that include arranging residual seats, modifying stopping plans, and inserting additional trains, are optimized simultaneously to provide sufficient transport capacity, which was less considered in previous studies on meeting the passenger demand. To be consistent with the unexpected passenger demand in real-time operation, the passenger demand from ULPF is divided into several passenger OD pairs, and each passenger OD pair has the same origin and destination stations and departure time windows. To obtain the optimal rescheduling operation plan for a railway company, four parts of objectives that maximize the number of shifting passengers from ULPF and minimize the total number of the three dispatching measures are constructed and addressed through an \( \varepsilon \)-constraint method to obtain the Pareto solution set. Meanwhile, to optimize the train timetables, train insertion, and train stopping plan modifications simultaneously, three-dimensional variables, including infrastructure resource using binary variables, passenger loading binary variables, and passenger volume integer variables, which increase the complexity of the model, were introduced. Thus, we formulated the problem as a linear integer programming (LIP) model by employing a time-space network and using the variables stated above. Furthermore, a practical detail of extra time constraints is considered in this model to impose the additional dwelling time and train acceleration and deceleration time restrictions while modifying the stopping plans. Several experiments based on the Beijing-Shanghai high-speed railway with 5 stations and 40 trains were conducted to evaluate the effectiveness and efficiency of the proposed model. The experimental results demonstrate that the proposed model can be used to obtain a reasonable rescheduling operation plan for serving the passengers from ULPF within an acceptable calculation timeframe.

The rescheduling model can be used to obtain an optimization rescheduling operation plan for helping the dispatchers of the railway operation companies in the real dispatching situations, while the condition of unexpected large passenger flow (ULPF) would have an impact. Dispatchers can also set the parameters of the boundary values of the \( \varepsilon \)-constraint method according to the real restrictions to obtain a better rescheduling operation plan.

In future research, we will extend the problem to the high-speed railway network, in which passengers will choose the routes and transfer methods. We will also consider the passenger demand on the microscopic level, with passenger choices of seat grade and other preferences. Thus, the design of an efficient and intelligent algorithm for solving a more complicated model is necessary.
Data Availability
Some or all data, models, or code generated or used during the study are available from the corresponding author upon request (list items).

Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

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