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Research Article

Analysis of Perception Variance in Regret Choice Modeling Based on GPS Data Considering Building Environment Effects

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The purpose of this paper is to deal with the perception variance problem in regret model by scaling perception variance and relaxing the distribution assumption of the random error term, which are corresponding to the attribute and alternative differences, respectively. Specifically, the power coefficient in regret function can capture individual perception variance of specific attributes and latent class structure is used to analyze individual heterogeneity of alternative differences by defining weight function. Accordingly, perceiving behaviors of individuals are analyzed in depth and generating absolute and relative behavior interpretation and asymmetric property. The proposed models are estimated and analyzed using GPS data of taxis in Guangzhou, and bicycle household surveys are collected in Tel Aviv metropolitan area. The results show both significant effects of attributes on drivers' and cyclists' route choice behavior and identification of different types (absolute or relative) of perceiving attribute and alternative regret differences of the two travelers by incorporating the ratio of length between chosen route and the shortest route.

1. Introduction

Perception variance is a traditional problem in transportation planning and route choice modeling firstly introduced with respect to trips of different lengths [1], which is generated by the logit model that assumes the unobserved terms have independent and identical *Gumbel* distribution. Therefore, it also exists in the regret model using logit structure [2]. However, it is more complicated in regret function, because not only the alternative part can have perception variance stemming from logit model but also within the regret function, the max operator or logarithmic transformation does, which is identified by attribute perception variance.

For now, there are some researches using Weber's law and psycho-physical mapping function to generate the non-identical perception variance to solve the problem of linear mapping variance in regret problem [3]. Based on this theory, we explore the two types of perceptions that are alternative- and attribute-based. Note that the idea of regret is incorporated into the same discrete choice framework of random utility maximization which is driven by the contributions in large measures [4, 5]. Therefore, this paper proposes the methods to deal with the perception variance problem in regret function by exploring the previous methods used in utility analysis.

1.1. Literature Review

1.1.1. Behavior Consideration. There are mainly three ways for analysts to capture heterogeneous perception variance of individuals. The first one is to use multinomial probit route choice model, by assuming the covariance between unobserved error terms of alternative routes [6]. However, this model does not have a closed-form mathematical formulation, and it is computationally burdensome, especially for regret function using the max operator with nonsmooth likelihood function or logarithmic transformation with complex calculation compared with linear mapping for utility. What is more, when the choice sets are comprised of extensive routes, it cannot work efficiently.

The second one is to explicitly scale the perception variance by individual OD pair, which allows the perception variance to increase and decline according to the specific attributes, in general to the travel distance, such as multinomial logit model with scaling, the C-logit model with scaling, path-size logit model with scaling, the paired combinatorial logit model with scaling, and so on [7–10]. This method can be incorporated in the regret function to generate nonidentical perception variance, which will be discussed later as attribute perception differences.

Finally, the third one is to relax the homogeneous perception variance assumption, such as *Weibull* and *Fréchet* distribution [11], which constitute generalized extreme value distribution based on parameters to capture the shape and underlying properties as a complex and flexible distribution [12, 13], which is called multinomial multiplicative model (MNM). Under the independence assumption, the models have the alternative-specific nonidentical perception regard to the trips with different attributes. This way will be used as alternative perception differences and discussed later. However, they have the limitation of insensitive to any arbitrary scale changing on the regret of the alternatives [8, 14].

It should be noted that the actual magnitude of the perception variance should be determined on a case-specific basis. Accordingly, the application situation using regret based model should be previewed.

1.1.2. Regret Model Development. Traditionally, regret can be generated when one or more nonchosen alternatives outperform the chosen one in terms of the corresponding attributes. For the deterministic part, there are some different versions. The original formulation makes use of two max operators across attributes and alternatives in Equation (1), which means regret only depends on the worst nonchosen alternative in the choice set [5], while the classical model is given by Equation (2) as a smoothed approximation of the original model [4]. Rsum was proposed to avoid the emergence of regret in the domain where regret does not actually exist caused by the logarithmic part of the Rlog function [15], which is listed as Equation (3).

$$R\max_{ni} = \max_{j \neq i} \left[\sum_{m} \max\left\{0, \beta_m \left(x_{njm} - x_{nim}\right)\right\} \right], \tag{1}$$

$$R\log_{ni} = \sum_{j \neq i} \sum_{m} \ln\left(1 + \exp\left[\beta_m \left(x_{njm} - x_{nim}\right)\right]\right), \tag{2}$$

$$R\mathrm{sum}_{ni} = \sum_{j \neq i} \left[\sum_{m} \max\left\{0, \beta_m \left(x_{njm} - x_{nim}\right)\right\} \right]. \tag{3}$$

In Equations ((1)–(3)), $x_{njm} - x_{nim}$ is the difference in attribute value if alternative j performs better than i in terms of attribute m. Parameters β_m in equations above are to be estimated from observed choices for attribute m.

What is more, in traffic network design, regret theory can also be incorporated into the user equilibrium problem, generating new VI formulation and behavior interpretations [16, 17]. In these years, regret based models have been developed considering psycho-physical mapping, asymmetric impact of perception, attribute difference tolerance, error generation and so on [3, 18–21]. However, of the most regret formulation in the previous researches, the perception variance cannot be considered both on attribute and alternative level, which is one of the motivations of this research to propose the hybrid model using latent class in Section 2.3.

1.1.3. Application Consideration. Route choice modeling is mainly based on discrete choice theories [22], which is to identify the behavior of travelers perceiving the corresponding attributes and their preferences. This can help that, when faced with serious traffic problems, such as congestions, pollutions and infrastructure constructions, urban planners, and administrations can make management on the aggregate level.

Traditionally, there is a handful of literature on problems of car route choice modeling using regret based models [23–26]. However, as the popularity of bike sharing nowadays, several bicycle route choice models have been proposed using either survey (i.e., *Onderzoek Verplaatsingen in Nederland* (OVIN)) [27] or GPS data [28–30]. As we all know, there are significant differences between drivers and cyclists route choice behavior on perceiving attributes and alternatives.

Therefore, it is necessary to propose new regret models considering nonidentical perception variance to compare the two types of traveler's behavior. According to our best knowledge, there are no previous studies focusing on this topic.

1.2. Contributions. Motivated by the above issues, this paper aims to deal with the perception variance problem in regret function by proposing an analytical framework. The main contributions are twofold.

(1) From the behavioral and methodology aspects, both attribute and alternative differences are incorporated into the regret model using latent class structure to capture nonidentical perception variance, which can generate absolute and relative behavior interpretation and asymmetric property helping us to analyze the behavior of decision makers

(2) From the application aspects, the proposed regret route choice models are used to automobile and bicycle behavior modeling, the former one using GPS data of taxis in Guangzhou and the latter one using bicycle household surveys collected in Tel Aviv metropolitan area. Accordingly, some separable and comparable findings are obtained

Figure 1 shows the basic structure of this paper as a summary of above. The remainder of the paper is organized as

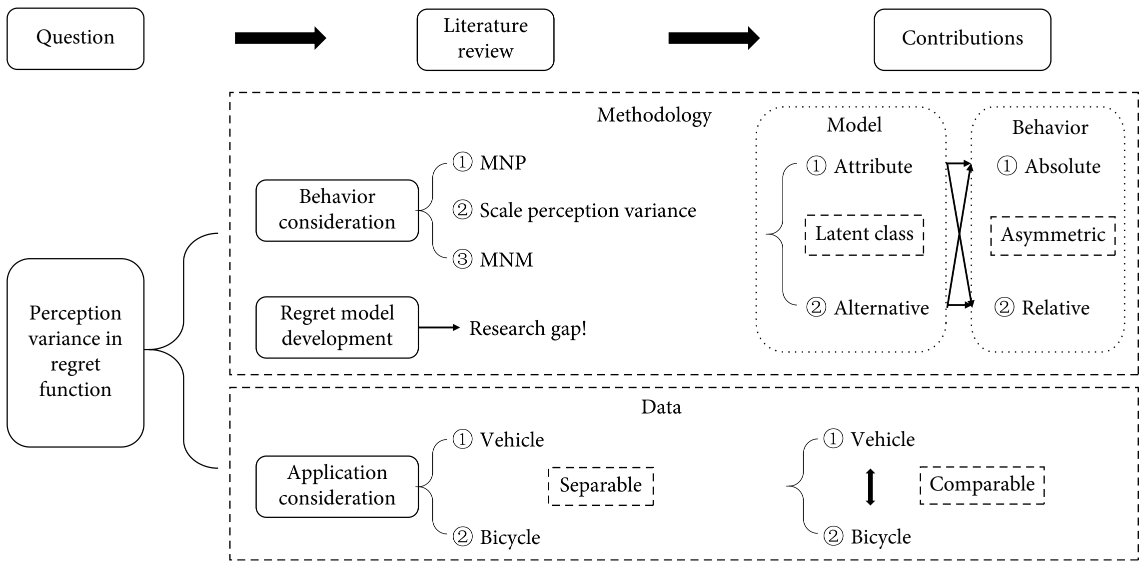

Figure 1: Ideas and basic structure of this research.

follows. Section 2 presents the regret choice model considering attribute and alternative differences and the hybrid latent class structure. Case studies of automobile and bicycle are provided in Section 3 and Section 4, respectively. Section 5 provides the comparison results of behavior between drivers and cyclists and conclusions are presented in Section 6.

2. Model Specification

In this study, we develop choice models within the structure of random regret minimization, which are derived as follows.

The route choice problem can be described in terms of a set of alternative routes in the choice set C_n of individual n, characterized by a nonempty set of m attributes. The set of attributes may be identical for the different routes or maybe specific. The decision makers will obtain a certain level of regret for each alternative route $j \in C_n$, which are denoted by R_{nj}. Regret-based model assumes that they prefer the route that has the minimum regret, that is:

$$P^R(i|C_n) = P\left(R_{ni} < R_{nj}, \forall j \neq i\right). \quad (4)$$

R_{ni} is the function of observed and unobserved alternative attributes and individual characteristics, which is comprised of deterministic part structured with observed variables of alternatives and random error term indicating the perception variance.

$$R_{nj} = f^R\left(r_{nj}, \varepsilon_{nj}\right). \quad (5)$$

In the real choice setting, for instance, a decision maker experiences one route costing him 100 minutes, while the nonchosen route travel time is 95 minutes. He would feel a certain small amount of regret. However, when he travels for 10 minutes and the nonchosen route is 5 minutes, he would have a sporting chance feeling a large regret. According to the traditional regret function, he will get the same regret. It is the model which can only capture absolute differences of attributes that is insensitive to a shift that leads to the problematic results.

To overcome the inability to account for the perception variance, here we introduce two types of methods according to the introduction.

2.1. Attribute Specific Difference. According to the introduction, the first method we can use here is to scale perception variance. Intuitively, $x_{njm} - x_{nim}$ should be replaced by x_{njm}/x_{nim}. However, this can generate two more questions. On the one hand, the relative difference is insensitive to an arbitrary scale, which can also have unreal results in some situations. On the other hand, because of the same sign within attributes in general, relative differences are always positive. Therefore, these values always fall into the same area of regret model curve, either all are regret domain or rejoice domain, which lost the important properties of regret model, such as compromise effect and asymmetric marginal effect [31]. Considering the ratios of relative differences of two attributes are reciprocal, we can subtract them by one to generate the two values opposite to each other, which has the same effect of the absolute difference. Here we propose the new formulation of regret model in Equation (6).

$$R_{ni} = \sum_{j \neq i} \sum_{m} \ln\left(1 + \exp\left[\beta_m \left(\frac{x_{njm}}{x_{nim}} - 1\right)\right]\right). \quad (6)$$

From this viewpoint, it has two specifications, $x_{njm} - x_{nim}$ and $x_{njm}/x_{nim} - 1$, corresponding to absolute and relative differences, respectively. However, in real world, no decision maker perceives the attribute difference according to the only one rule, either absolute or relative perception.

Then a natural way of better representing travelers' behavior and improving model fits is to develop some hybrid models, which consider absolute and relative attribute differences, simultaneously.

Here the power coefficient α that varies from 0 to 1 is used to capture the perception variance of decision makers. See the Equation (7) as follows.

$$R_{ni} = \sum_{j \neq i} \sum_{m} \ln \left(1 + \exp \left[\beta_m \left(\frac{x_{njm} - x_{nim}}{(x_{nim})^{\alpha}} \right) \right] \right). \tag{7}$$

It should be noted that when the value of attribute is nonpositive, the denominator may not be a real number. To avoid this, the data can be first normalized to a specific range.

To be concrete, when α approaches one, it means the decision maker is more likely to perceive the relative difference of this attribute. When α approaches zero on the contrary, decision maker is more likely to perceive the absolute difference of this attribute. However, if we assume a fixed value of α, different decision makers will have the same perception variance on the attribute. To make the model more applicable, it should be noted that perception values can vary as a function of the size of attributes among different decision makers. Traditionally, when the size or intensity of attribute is small, individuals tend to perceive more absolute differences than relative ones of attributes between alternatives, which can be expressed in terms of the low-level of α. As the intensity of attribute increases to the middle part of the range, the bigger part of the relative difference can be perceived by decision makers and represented by a value of the power coefficient closing to one. Finally, when the size of the attribute becomes too large, individuals' perception of attribute differences becomes absolute, implying that the value of α becomes smaller again. This can help us elaborate on the power coefficient by using a mapping function incorporated into the regret function, which depends on the size of the attribute of the specific alternative.

It should be noted that for one thing, the mapping function should be truncated by the range of power coefficient, that is, zero to one. For another, in real settings, individual perception variance around the middle point may be asymmetric, which means that the curve of the power coefficient should be skewed on the left or right side of the middle point. For now, we propose the final regret function which is asymmetric and can capture relative and absolute differences of attributes.

$$\begin{aligned} \alpha_{nim} &= \exp\left(X - e^{X-1}\right), \\ X &= \gamma_m (x_{nim} - \delta_m), \end{aligned} \tag{8}$$

$$R_{ni} = \sum_{j \neq i} \sum_{m} \ln \left(1 + \exp \left(\beta_m \left(\frac{x_{njm} - x_{nim}}{(x_{nim})^{\exp\left[\gamma_m (x_{nim} - \delta_m) - e^{\gamma_m (x_{nim} - \delta_m) - 1}\right]}} \right) \right) \right). \tag{9}$$

From this viewpoint, the differences between two simple exponential and linear functions are used here to capture the asymmetry of perception variance around the middle point. What is more, two adjusting parameters are added on the independent variable in order to affect the shape of asymmetry. δ_m is named shift parameter to reflect the level of attribute and γ_m is named scale parameter to capture the arbitrary scale of attributes. Figure 2 shows the relationship between adjusted independent variables and the asymmetric differences. The two curves intersect at the point (1,1), and they generate negative differences anywhere else according to Equation (8). However, as the adjusted independent variable increases, the difference grows exponentially, while in the other part of the curve, the difference grows linearly.

What is more, in Figure 3, the relationship between power coefficient and attribute intensity can be described. It should be noted that we assume the shift parameter is fixed at 5. Therefore, the middle point for each curve can be calculated easily, which corresponds to the biggest value of the power coefficient as one. Generally, the middle point is equal to $1/\gamma_m + \delta m$, which is affected by the two parameters. And in the light of the relationship, firstly, scale parameter γ_m decides the degree of asymmetry. When it is estimated as positive, individuals' perception is more sensitive (reflected by the larger scope on the curve) to the attribute larger than the middle point with the curve skewed on the right side of the point. Otherwise, when it is estimated as negative, the graph is skewed to the left as the more sensitive part. What is more, if it is close to zero, the shape of the curve is going to be symmetric and flat, however, the power coefficient is approaching the reciprocal of the natural logarithm, which means that decision makers perceive larger relative part of the attribute with little absolute effect.

Here we should note that we only use the Rlog function because it is the classical regret model and it has a continuous function for easy estimation. What is more, the analogous form has been put forward in the previous research [3, 32]. The pointcuts of the two researches are different. This research considers the relative difference in contrast to the traditional absolute difference, while another one incorporates psycho-physical effects and nonlinear representation of the perception of attributes, which considers the magnitude of attributes.

2.2. Alternative Specific Difference. The second method used in this research to overcome the drawback of inability to account for perception variance is to assume that regret is structured with multiplication of deterministic part with random error term [33]. Traditional multinomial logit model cannot handle the perception variance issue because, assumingly, each alternative has the same perception variance of $\pi^2/6\theta$, where θ is the scale parameter. However, if we assume the random error term has *Weibull* or *Fréchet* distribution, which is called multinomial multiplicative model (MNM), it can handle the issue because the perception variance is the function of the attribute and its shape and location parameters, which is shown as follows (only for *Weibull* distribution):

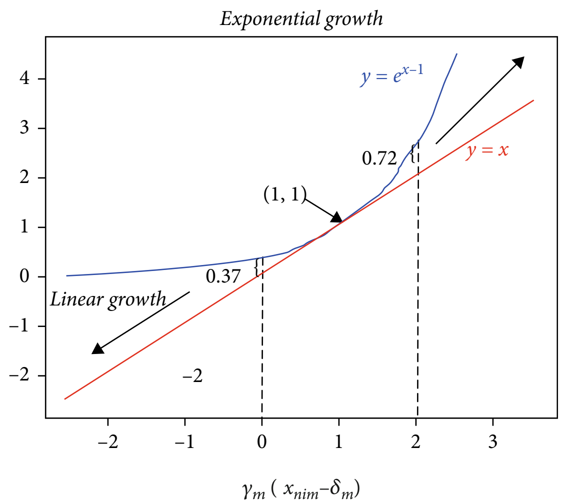

FIGURE 2: Relationship between adjusted independent variables and the asymmetric differences.

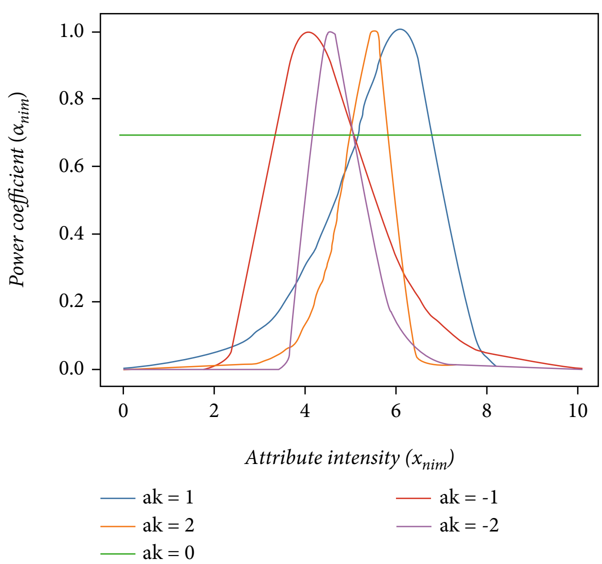

FIGURE 3: Relationship between power coefficient and attribute intensity with the variation of scale parameter.

$$(\sigma_i^s)^2 = \left[\frac{x_i^s}{\Gamma(1+(1/\beta_s))}\right]^2 \left[\Gamma\left(1+\frac{2}{\beta_s}\right) - \Gamma^2\left(1+\frac{1}{\beta_s}\right)\right], \tag{10}$$

where $\Gamma()$ is the gamma function, s denotes the specific choice set, i is the alternative, x_i^s denotes the corresponding attribute and β_s is the shape parameter. For the derivation details, refer to ([8, 12].

As regret is always positive and decision maker chooses route that has the minimum regret, we use only one form of MNM model, that is, assuming the random error terms have *Weibull* distributions (see the appendix). The probability function is listed as follows:

$$P(i|C_n) = \frac{R_{ni}^{-\lambda}}{\sum_j R_{nj}^{-\lambda}} = \frac{1}{1+\sum_{j\neq i}\left(\dfrac{R_{nj}}{R_{ni}}\right)^{-\lambda}}. \tag{11}$$

Here we should note that the shape parameter of *Weibull* distribution (scale parameter of regret) can be estimated which is different from the situation under the utility function. What is more, in MNM model, we do not need to transform the attribute because the logarithmic form of regret can guarantee the positive property of regret. It is another reason for us to use Rlog in Equation (9) rather than other functions. Figure 4 shows the relationship among the probability, shape parameter and relative difference of regret. If the ratio of two regret greater than one, which means alternative i is better than alternative j, the probability of choosing alternative i becomes large as the increase of the shape parameter. If the ratio of two regret smaller than one, which means alternative i is worse than alternative j, the probability of choosing alternative i becomes large as the decrease of the shape parameter. What is more, the probability of choosing the alternative i becomes large as the increase of the ratio of two regret with the same shape parameter. However, the growth rate at the scope of the ratio near one, which means that the alternative i becomes better from worse than alternative j gradually, is going to be large as the increase of shape parameter. When the regret of alternative j is many times larger than that of alternative i, the probability varies from 0.5 to 1 with the increase of shape parameter. Under this condition, the larger the shape parameter, the more decisive the individual is.

However, from the view of behavior according to the Equation (11), it can only capture relative differences of alternatives. Although it can handle the perception issue, it is insensitive to a shift (level of regret). Therefore we need to recall the general regret model that captures absolute differences of alternatives. For the identifiable scale parameter, here we use μRRM [34]. The specification is as follows:

$$R_{ni} = \sum_{j\neq i}\sum_{m} \ln\left(1+\exp\left[\frac{\beta_m}{\mu}\left(\Delta x_{njm\leftrightarrow nim}\right)\right]\right),$$
$$P_{ni} = \frac{e^{-\mu R_{ni}}}{\sum_j e^{-\mu R_{nj}}} = \frac{1}{1+\sum_{j\neq i} e^{-\mu\left(R_{nj}-R_{ni}\right)}}. \tag{12}$$

Here $\Delta x_{njm\leftrightarrow nim}$ means aforementioned asymmetric absolute and relative differences in Equation (9). The scale parameter varies from zero to infinite. When it reaches zero, the log-likelihood of the μRRM approaches P-RRM (using Rsum) and approaches RUM when it becomes large.

From the view of asymmetry, according to Figure 5, a simple binary route choice scenario is used here. The regret of the second route is fixed at 5, while regret of the first one varies from zero to ten. On the basis of the shape of curves, it can be found that logit model capturing absolute differences of alternatives is symmetric, which means that the increase and decrease of independent variable from the

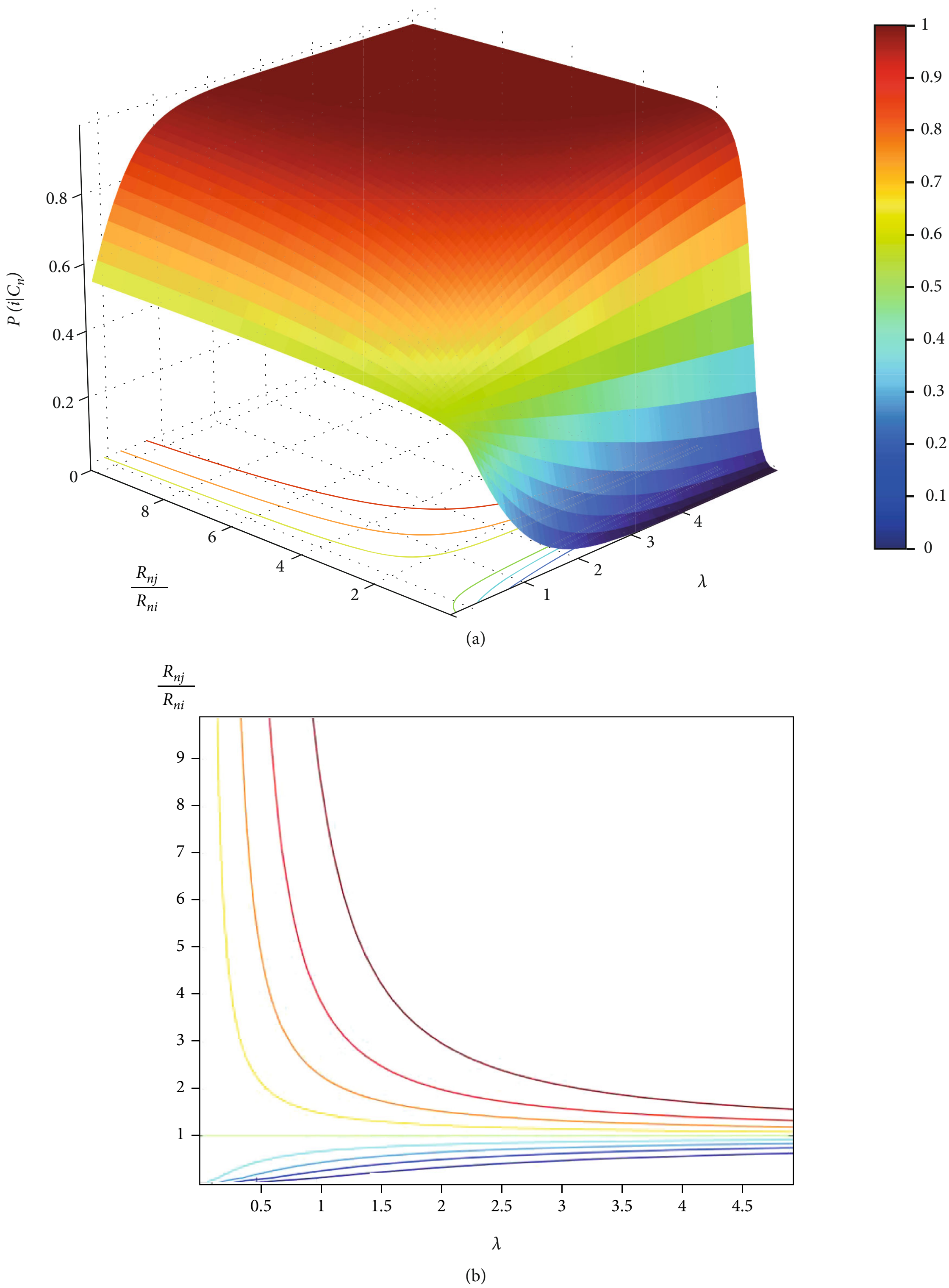

FIGURE 4: Relationship among the probability, shape parameter, and relative difference (a) surface plot (b) contour plot.

middle point can generate the same effect on the chosen probability, while the multiplicative model reflects the different changing rates at the both side of the middle part, which represents relative differences of alternatives. What is more, as the shape parameter increases, the property of asymmetry is going to vanish.

Therefore, in the next subsection, a hybrid method is used here to combine the two models according to the

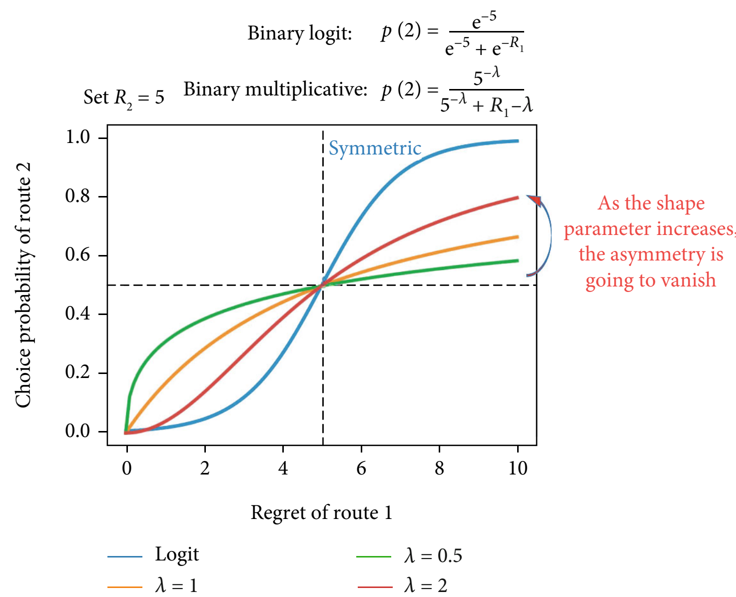

FIGURE 5: An example to show the symmetric and asymmetric properties of BNL and BNM models.

behavior interpretation, that is, one is symmetric and reflects absolute effects and the other is asymmetric and captures relative differences.

2.3. Latent Class Model for Absolute and Relative Alternative Differences. To analyze individual heterogeneity of alternative differences, here we use the latent class model for the sake of convenience instead of proposing a new formulation combining both multinomial logit and multinomial multiplicative models. According to the several literatures [35–38], the latent class model used in this research is shown as follow:

$$P_{ni} = \sum_{s=1}^{S} w_{ns}(P_{ni}|s). \tag{13}$$

The probability of individual n choosing alternative i is comprised of the sum of probability this decision maker belonging to class s multiply the probability of choosing alternative i conditional on the individual belonging to this class. The conditional probability takes the familiar MNL and MNM forms in Equation (12) and Equation (11). And the class weight can be written as the function of a vector x_n of sociodemographic and trip-related variables associated with the traveler n. Traditionally, the multinomial logit formulation is used here.

$$w_{ns} = \frac{\exp(\beta_s x_n)}{\sum_k \exp(\beta_k x_n)}. \tag{14}$$

In this research, we have two different classes, MNL and MNM models. So the combined probability is:

$$\begin{aligned}
P_n(i|C_n) &= w_{MNL} \times P_{MNL}(i|C_n) + w_{MNM} \times P_{MNM}(i|C_n) \\
&= w_{MNL} \int_{R_i} \partial F_{MNL}(R_1, \cdots, R_k)/\partial R_i|_{R_i=R_k,\forall k} dR_i \\
&\quad + w_{MNM} \int_{R_i} \partial F_{MNM}(R_1, \cdots, R_k)/\partial R_i|_{R_i=R_k,\forall k} dR_i \\
&= \int_{R_i} [w_{MNL} \partial F_{MNL}(R_1, \cdots, R_k) \\
&\quad + w_{MNM} \partial F_{MNM}(R_1, \cdots, R_k)]/\partial R_i|_{R_i=R_k,\forall k} dR_i \\
&= \int_{R_i} \partial F(R_1, \cdots, R_k)/\partial R_i|_{R_i=R_k,\forall k} dR_i,
\end{aligned} \tag{15}$$

where F denotes the cumulative distribution functions of the different distributions.

What is more, this latent class model seems like the CoRUM models proposed in the literature [37, 38]. It is a sum of absolutely continuous functions characterized by the same expected value. However, Equation (13) is mixed by CDFs of an additive and a multiplicative random regret with error term which have different expected values. Therefore, this model can capture different behavior interpretations of perceiving alternative differences by decision makers.

3. Case Study of Automobile

Here we use two case studies to test the proposed regret models and illustrate the behavior interpretation of automobile users and bicycle users in this section and the next section, respectively.

Figure 6: The overall perspective of study area and an example route choice set between an OD pair.

Table 1: Statistics on the attributes.

Eigenvalues	Length	Traffic light	Artery road	Path size	Shortest path
	(L)	(TL)	(AR)	(PS)	(SP)
Max.	48.04	36.00	1.00	-0.02	18.64
Min.	0.50	0	0.01	-3.45	0.47
Mean	5.26	3.57	0.75	-1.20	4.30
SD.	4.64	4.04	0.30	0.68	2.88

3.1. Data Description. GPS data of taxis in Guangzhou is used to analyze drivers' route choice behavior. Guangzhou is the comprehensive transportation hub and is one of four first-tier cities in China. It has eleven districts. This study uses only main urban area of Guangzhou, covering nearly 500 square kilometers, as shown in Figure 6. The data was collected from four weeks in 2014 with GPS devices being used for monitoring and management but not for navigation. Here a total of 2489 observations from 292 OD pairs are used. The statistics on the attributes of the chosen alternative are shown in Table 1.

It should be noted that the unit of Length is km. Artery Road means that the ratio of the length of the arterial road compared to the total length of route. To overcome the overlapping problem, path size factor is calculated by comparison of the degree of independence of alternatives. Accordingly, the path size correction can be added to the regret function in order to consider travelers' perception correlation when paths overlap. The path size term is given by:

$$PS_{ni} = \ln \sum_{a\in\Gamma_i} \frac{L_a}{L_i} \frac{1}{\sum_{j\in C_n}\delta_{aj}}, \tag{16}$$

where L_a is the length of link a; L_i is the length of route i; δ_{aj} is the link-path incidence dummy, being equal to one if route j uses link a and zero otherwise. For the behavior interpretation of the sign of β_{ps}, it should be noted that regret function enters into the probability with the opposite value compared with the utility, that is, regret minimization theory. Actually, the expected sign of it should be positive to indicate that when this route is more independent than others, the path size correction is approaching one and the regret diminishes and vice versa.

What is more, Figure 7 presents the distribution of the relative differences of length between chosen route and the shortest route, which shows that nearly 61% of decision makers chose the shortest path, while 57 observations chose the route more than 5 times longer than the shortest path. However, there is a small group of people who chose a much

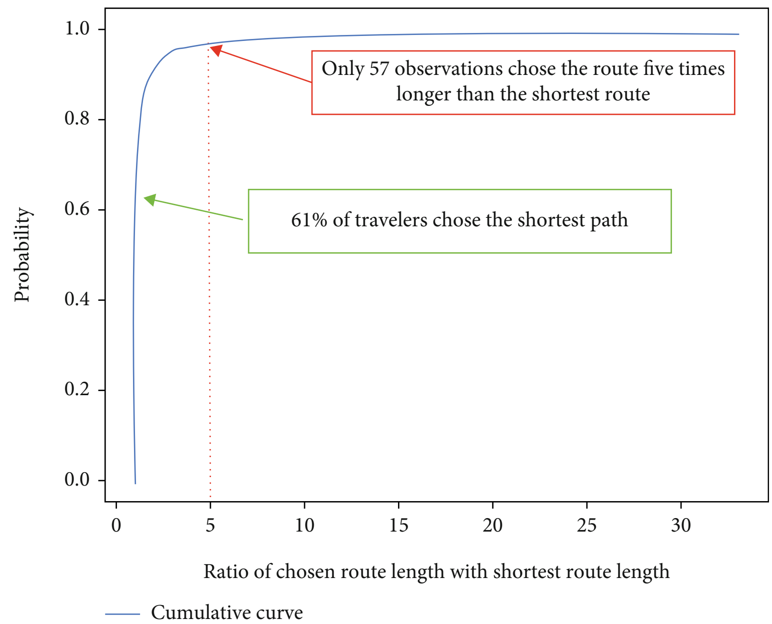

FIGURE 7: Distribution of the relative differences of length between chosen route and the shortest route.

longer route than the shortest one, even some are 10 or 20 times. For more information about the data, please check our previous research [21].

3.2. Route Choice Set Generation. Here we use the heuristic enumeration method to generate alternative routes between OD pairs [39]. Then we set the maximum size of choice set to 40 considering the automobile travelers' ability to process route choice information. Then, the maximum, minimum, and average numbers of trips observed between an OD are 35, 5, and 9, respectively, and the maximum, minimum, and average numbers of different routes observed between an OD are 14, 1, and 4, respectively [40].

3.3. Model Specifications. According to the proposed structures of model shown in Section 2, and the available data described above, various specifications of different models have been estimated. Considering the coefficient significances, finally, the regret function specifications and latent class structure are shown as follows:

$$R_i = \sum_{j\neq i} \ln\left(1 + \exp\left(\beta_L \times \Delta L_{nj\leftrightarrow ni}\right)\right) + \sum_{j\neq i} \ln\left(1 + \exp\left(\beta_{TL} \times \Delta TL_{nj\leftrightarrow ni}\right)\right) + \sum_{j\neq i} \ln\left(1 + \exp\left(\beta_{AR} \times \Delta AR_{nj\leftrightarrow ni}\right)\right) + \sum_{j\neq i} \ln\left(1 + \exp\left(\beta_{ps} \times \Delta PS_{nj\leftrightarrow ni}\right)\right),$$

$$P_{latent,i} = \frac{1}{1 + \exp\left(\theta \times (L_i/SP) - \eta\right)} \times P_{MNL}(i|C_n) + \frac{\exp\left(\theta \times (L_i/SP) - \eta\right)}{1 + \exp\left(\theta \times (L_i/SP) - \eta\right)} \times P_{MNM}(i|C_n), \quad (17)$$

where L_i is the length of route i; SP denotes the shortest path length among all alternatives under certain conditions. TL_i is the number of traffic lights on the route i; and AR_i is the ratio of the length of the artery road compared to the total length of route i. PS_i is the path size correction to account for the overlapping problem. $\Delta x_{njm\leftrightarrow nim}$ means the corresponding hybrid relative and absolute differences of attribute m. (For the function about absolute alternative differences, that is the μRRM model, the regret scale parameter μ is included in this expression instead of the coefficient for simplicity.) θ and η are parameters that need to be estimated. What is more, the probability of different models that capture alternative specific differences (MNL and MNM) are shown in Equations (11) and (12). It should be noted that in the Equation (9), the denominator part x_{nim} should be positive. Therefore, we need to normalize the path size correction into positive.

Here we should explain our consideration of variables included in the function. Firstly, route length is selected rather than travel time because it is easy for travelers to estimate length than time. Secondly, we expect that travelers will choose routes that have fewer traffic lights to avoid stops. What is more, according to the artery roads, we expect that people would prefer to maximize the proportion of them for higher travel speed and less congestion.

3.4. Parameter Estimation and Behavioral Analysis. This research uses maximum likelihood equation to estimate the parameters related to the attributes:

$$\vec{\beta} = \arg\max\left(LL(\beta)\right) = \arg\max\left(\sum_n \sum_i y_{ni} \times \ln\left(P\left(ni\middle|\vec{\beta}\right)\right)\right), \quad (18)$$

where y_{ni} equals 1 if the alternative i is chosen by decision maker n, and 0 otherwise. All of these estimations in this study are done in MATLAB. The estimation results of these

Table 2: Estimation results.

Latent class	Estimates (t – value)								
MNL part	β_L	-0.151 (-6.95)	γ_L	0.123 (3.95)	MNM part	β'_L	-1.029 (-3.78)	γ'_L	-0.131 (-4.32)
			δ_L	23.4 (12.2)				δ'_L	38.5 (9.98)
	β_{TL}	-0.031 (-3.23)	γ_{TL}	0.049 (2.98)		β'_{TL}	-0.034 (-3.23)	γ'_{TL}	-0.043 (-1.78)
			δ_{TL}	15.2 (3.12)				δ'_{TL}	25.23 (3.29)
	β_{AR}	0.392 (4.30)	γ_{AR}	1.20 (5.44)		β'_{AR}	4.312 (1.78)	γ'_{AR}	1.049 (2.12)
			δ_{AR}	0.523 (1.99)				δ'_{AR}	0.502 (2.45)
	β_{ps}	0.015 (2.29)	γ_{ps}	1.01 (2.98)		β'_{ps}	0.023 (2.11)	γ'_{ps}	-0.921 (-2.93)
			δ_{ps}	0.579 (2.10)				δ'_{ps}	-3.25 (-4.38)
	μ	5.30 (10.22)				λ	1.20 (12.50)		
Weight function	η	-1.96 (-3.39)				θ	1.01 (2.89)		
Model fits	Null LL	-2973.35							
	Final LL	-2323.98							
	$\bar{\rho}^2$	0.209							
	BIC	4867							

models are shown in Table 2, including estimates, t-values, final log-likelihoods and Bayesian information criterion [41]. The following are some analyses of the results.

3.4.1. Route Choice Behavior. Based on the signs of the estimated parameters, it can be found that the results are corresponding to our real-world settings. Drivers prefer to travel on shorter paths with fewer traffic lights indicated by both coefficients of length (L) and traffic light (TL) having negative signs. However, they are willing to drive on the artery road, indicated by the parameters associated with artery road (AR). As the level of road becomes higher, the limited velocity is going to increase and their driving experience is more comfortable compared to the low-grade road with more stops and delays. The positive sign of path size (PS) indicates that when one route is more independent than others, the path size correction approaches one and the regret diminishes and vice versa.

However, by comparing the magnitude of coefficients between MNL part and MNM part, it can be found that the absolute values of the latter part are larger than that of the former part. This indicates that, when individuals consider and perceive the relative differences of alternatives, they are more deterministic to choose the best route that has the minimum regret according to the specific attribute.

3.4.2. Attribute-Specific Behavior Interpretation. In Figure 8, there are four subplots corresponding to the four variables showing the power coefficients of all individuals (2489 observations). In each subplot, one curve is MNL part and the other one is MNM part. According to the asymmetry and absolute or relative perception behavior, there are some distinctions shown as follows.

From the perspective of absolute and relative behavior, firstly, Figure 9 shows the average power coefficient of each independent variable with both MNL part and MNM part. For instance, the average travel length of the individuals from Table 1 is 5.26 kilometers and the corresponding shape parameter and shift parameter from Table 2 are 0.123 and 23.4, respectively. According to the Equation (8), the power value 0.103 of length (L) in MNL part can be generated. The same calculation can be used to obtain other values. It can be easily found that for length (L) and path size (PS), decision makers are more likely to perceive absolute differences of attributes within regret function, regardless of either MNL part or MNM part. While for the traffic light (TL), in the MNL part, it is not obvious that people consider which behavior rule to choose, or both absolute and relative differences. For MNM part, it is a very clear process of perceiving relative differences. What is more, when travelers think about artery road (AR), they always consider the relative effects of attributes. This implies that people will not be attracted by only one or two more main roads compared with other routes. In other words, artery roads always appear simultaneous in their chosen routes if possible.

Secondly, Figure 8 shows all 2489 individuals' behavior according to the relative and absolute differences of attributes. Only the length (L) curve can be generated completely. That is to say, it comprises both the left and right part from the middle point. For one thing, the travel length of route varies from 500 meters to 48 kilometers while the middle point is near 30 kilometers. For another, it is difficult for analysts to actually depict how the travelers perceive path length, which is the dominant variable in most route choice situations. However, aggregate results can help us to generate general interpretations. There is a trend that individuals perceive absolute differences of length under low intensity. As the length increases, people focus more attention on the relative difference. For automobile is a mode with high driving and traveling capability, the absolute

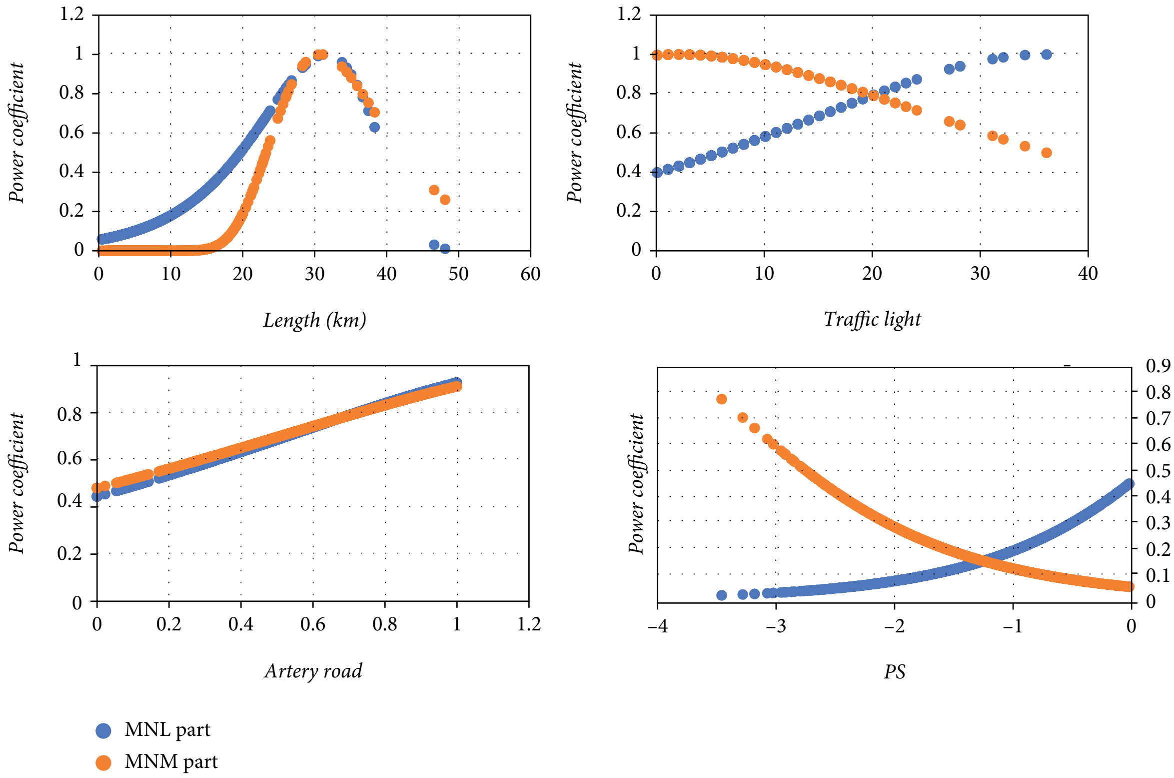

FIGURE 8: Scatter plots of individual power coefficients of different variables.

difference acts as the leading role in the small network while the relative difference plays the dominant part in the large network. This finding is in line with the results reported by previous researches [8, 42].

Thirdly, the other three variables, traffic light (TL), artery road (AR), and path size (PS) capture only one side of the whole graph, either left or right side of the middle point. It should be noted that as the traffic light is a discrete variable, the curve seems sparser than other curves. Therefore, it is not obvious that which behavior rule for people to perceive according to traffic light. For instance, in the MNL part, the average power coefficient is 0.459, however, most points locate at the above of this value as the result of traffic light in most cases is near five. For MNM part of the traffic light and two parts of the artery road, individual always perceive relative differences as the same results with the above discussion. According to the path size (*PS*), absolute part dominates because overlapping problem can be observed absolutely, such as this route has fewer common links than another route. The relative difference is just generated by calculating path size corrections; however, decision maker cannot take it in this way.

From the perspective of asymmetric, it can be found from both the sign of shape parameters of variables and the shape of the curves in Figure 8. For perceiving path length, the MNL part and MNM part generate the opposite trend. The increasing speed of power coefficient with length is faster in the high-level attributes than low-level in MNL part, while slower of the high-level in MNM part. Although individuals consider relative differences in the large network, they are more sensitive in the domain part, which is the right side of the curve when considering absolute differences of alternatives and left side when perceiving relative differences of alternatives. The alternative specific difference is discussed later in the next part because length is incorporated in weight function as the variable.

The other six curves of three variables only capture one side of the whole graph. For traffic light (TL) and path size (PS), both parts locate at the insensitive area of the graph meaning that individuals are more deterministic and less changeable when they are faced with the best route according to these two variables. In this dataset, decision makers seem not to change their behavior rules, either absolutely or relatively. For artery road (AR), both shape parameters are positive and both parts locate at the insensitive area. However, the asymmetry is not significant compared with the above two variables, that is to say, the shape of the two curves is analogous. People have the same perceiving behavior considering the main road rate of the alternative routes.

3.4.3. Alternative-Specific Behavior Interpretation. Although individuals can only perceive attributes of alternatives and regret (or utility) is used by analysts of systematically modeling, it is useful in dealing with the questions of alternative behavior rules (absolute and relative alternative differences) and asymmetric property.

From the perspective of absolute and relative behavior, the relationship between the weight of latent class and the

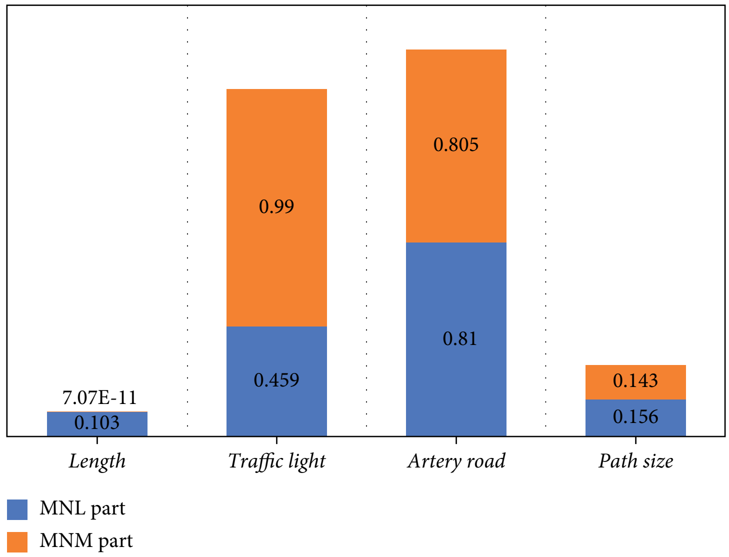

Figure 9: Average power coefficient of each independent variable with both MNL part and MNM part.

ratio of chosen route length with the shortest route length using the estimated weight parameters η and θ is depicted in Figure 10. Generally, we consider two different behavior interpretations by using heterogeneous structures: MNL and MNM. It is found that the ratio of chosen route length with the shortest route length between OD pairs has a significant effect on the segmentations of the two parts.

There are mainly three parts divided by the changing trends of the two curves. The horizon axis from left to right means the variation of the ratio between chosen route length with the shortest route length from small to large. The first part is composed of 61% of individuals who chose the shortest path, according to Figure 10. It is found that the absolute part (MNL structure) is dominant in this area. It is obvious that taking a little longer driving can also generate a great deal of regret for these cases. Relative part of regret can only hold a small fraction. As the ratio increases, the percentage of absolute part declines and the relative part rises gradually, which makes up the second part of the graph. However, the absolute part is also larger than the relative part until the ratio reaches 1.96, at which time the two parts have the same weight. Under this division, there are nearly 30.6% of decision makers in the second part. As a whole, there are more than 90% people in this dataset perceiving absolute regret differences more than relative differences. This is in line with the above discussion about attribute differences on length, which shows the preference for absolute length differences under low intensity and for relative length differences under high intensity. When the ratio is further than 1.96 (8.4% of the observations), drivers almost only consider relative differences, since the weight of MNM part grows rapidly.

Therefore, under the small network or large network, the people who choose the shortest path mainly consider the absolute differences of alternatives. If the length of the chosen path is two times larger than the shortest route length, they are more likely to perceive relative alternative differences.

From the perspective of asymmetric, according to Section 2, in order to overcome the drawback of inability to account for perception variance, MNM is used. As MNL part is symmetric, therefore, the property of asymmetry is captured only by the shape parameter λ, which is estimated as 1.2. It should be noted that for one thing, only the people who chose the path much longer than the shortest path can have significant asymmetry. For another, according to Figures 4 and 5, the slope of the probability curve is going to be smaller and smaller when the ratio of two regrets of alternatives increases from one. Therefore, the change of regret (increase or decrease) generating the asymmetry is more significant within the ratio range around one than the large ratio range, which means that changes through ratio one can obtain remarkable asymmetry.

4. Case Study of Bicycle

4.1. Data Description. The data was collected by household surveys conducted on weekdays in 14 major cities in Tel Aviv metropolitan area from December 2013 to June 2014. The survey contained two main steps. Firstly, the surveyor visited households and provided GPS data loggers for the members older than 14 and they were instructed to carry the devices for 24 consecutive hours. In order to ensure the normal use of devices, they have enough battery for 24 hours. Secondly, the travelers were asked to retrieve the GPS readings and complete the questionnaire about their daily activities on the recording day. At the same time, the GPS data was uploaded on the laptop computers to identify the true travel information.

The overall sample included 8515 persons living in 2896 households. A total of 39952 trips were recorded. After

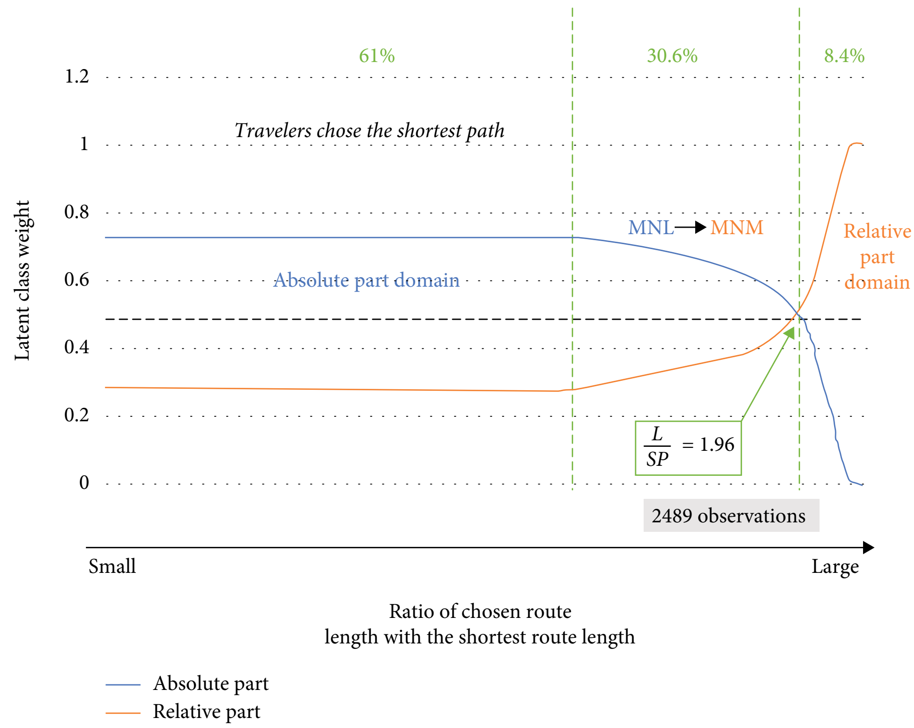

FIGURE 10: Relationship between the weight of latent class and ratio of chosen route length with the shortest route length.

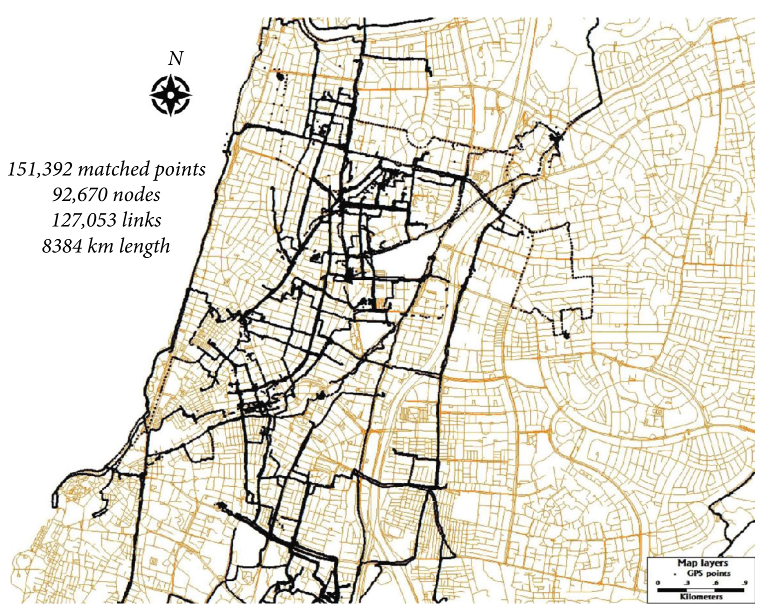

FIGURE 11: GPS points of bicycle trips.

modifying gross errors in GPS, there were 516 bicycle trips performed by 221 persons were correctly recorded. Figure 11 shows the GPS points of this study. Figure 12 provides the distribution of route length that has been chosen by cyclists, which shows that more than 80% of trip distances are less than 2 km. Descriptions of the explanatory variables are shown in Table 3. Table 4 presents the descriptive statistics of the route chosen by each individual for later analysis. More information can be found on the previous research [28].

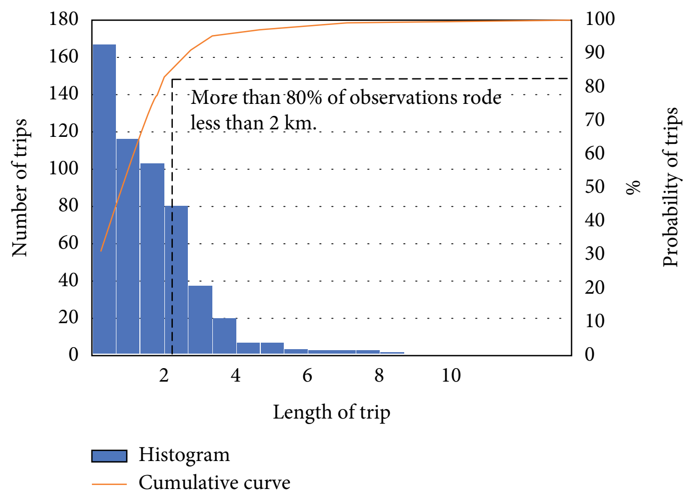

FIGURE 12: Distribution of route length that has been chosen.

TABLE 3: Descriptions of the explanatory variables.

Variables	Descriptions
Length	Total length of route (km)
Length_*A*	Total length of category *A* segments of route, i.e., bike paths (km)
Length_*C*	Total length of category *C* segments of route, i.e., urban arterials and highways (km)
Link	Average of street length (m)
Dwell	Number of dwelling units per meter along the route
Near_Sea	Total length of route segments passing along or up to 100 m of the seashore (km)
Near_Park	Total length of route segments passing along or up to 100 m near park (km)
lnPS	Natural logarithm path size

TABLE 4: Statistics on the attributes of the route chosen by individuals.

Eigenvalues	Length	Length_A	Length_C	Link	Dwell	Near_Sea	Near_Park	lnPS
Max.	12.5	3.21	2.91	546.56	4.15	11.33	6.73	-0.08
Min.	0.2	0	0	26.68	0.01	0	0	-1.94
Mean	1.61	0.22	0.12	60.79	0.82	0.51	0.24	-0.56
SD.	1.50	0.42	0.36	27.75	0.72	0.92	0.54	0.26

4.2. Choice Set Generation. According to our survey, there are nearly 3.8% bicycle facilities in the Tel Aviv city. And then we omitted 2% network roads which prohibited the travel of bicycle. We use three main methods: link elimination, link penalty, and simulation method in sequence. Finally, for each OD pair, we get a total of 20 routes for model estimation [28].

4.3. Model Specification. According to the proposed model structure in Section 2, the observed regret function specifications of bicycle route choice and latent class structure in this study are shown as follows.

$$R_{ni} = \sum_{m}\sum_{j\neq i}\ln\left(1+\exp\left(\beta_m \times \Delta x_{nim\leftrightarrow njm}\right)\right),$$
$$P_{latent,i} = \frac{1}{1+\exp\left(c+\gamma L_i\right)} \times P_{MNL}(i|C_n) + \frac{\exp\left(c+\gamma L_i\right)}{1+\exp\left(c+\gamma L_i\right)} \times P_{MNM}(i|C_n), \tag{19}$$

where m denotes the eight independent variables for the sake of simplicity and L_i is the length of route i (in the regret function the unit of length is km while in the weight function the unit is m).

4.4. Estimation Results and Analysis

4.4.1. Route Choice Behavior. All parameters are successfully and significantly estimated, and the results are presented in Table 5. According to the signs of coefficients, cyclists prefer to choose shorter routes generally. The positive sign of Length_A suggests that riders are more likely to extend their trips on bicycle paths. While they dislike travel on the urban arterials and highways for more safety and less delay indicated by the parameters associated with Length _C. What is more, for Link, people prefer to choose streets with more links, which is the characteristic of cyclists to use low-level roads for travel propose with more residential areas, corresponding to the estimates associated with Dwell.

Both Near_Sea and Near_Park have positive effects indicating that riders enjoy experiencing a pleasant environment along the seashore and park street. What is more, according to the lnPS, people prefer to ride along the routes with many independent links to avoid heavy traffics.

However, the absolute value of coefficients of MNM part is smaller than MNL part except for Near_Sea, which is different from the automobile case. This indicates that, when individuals consider and perceive the absolute differences of alternatives, they are more deterministic to choose the best route that has the minimum regret according to the specific attribute.

4.4.2. Attribute-Specific Behavior Interpretation. First of all, Figure 13 shows the average power coefficients of all variables like Figure 9 corresponding to the mean value in Table 4. It can be found that people perceive absolute regret differences of all attributes in this dataset. However, there are some behavior findings shown as follows. The analysis can also be divided into two parts.

From the perspective of absolute and relative behavior, Figure 14 shows eight subplots corresponding to the estimated variables. There are mainly three trends of different curves. Firstly, both Length and Link capture absolute attribute differences in most cases. What is more, as the value of route length and average street length increase, the power coefficients are going to decline, and the absolute effect is more obvious. When riding between further OD pairs or longer links, they are more likely to perceive absolute attribute differences, such as considering an additional 200 meters rather than the ratio of length of new route with the old route.

What is more, for the Length_A, Length_*C*, Dwell, Near _Sea, and Near_Park, which are related to travel environment, all the power coefficients are close to zero, which means that individuals perceive absolute attribute differences. For these variables can be directly observed, the absolute difference is more intuitive than relative difference. For instance, people just prefer to ride on bike paths and streets along the seashore or close to the park, whereby dislike the arterial roads regardless of either large or small network. However, all these variables capture fewer absolute differences as the value of them increases except Length_*C*. Because the coefficients of these variables are all positive, that is, people prefer these higher values no matter for MNL part or MNM part. As the relative parts take more weights, individuals generate less regret for specific attributes. For the negative variable Length_*C*, people perceive absolute differences to generate more regret.

Finally, although lnPS capture absolute effect in most cases, the MNL part locates at the left part of the middle point while MNM part locates at the right side, which means that as the increase of the path size, relative weight grows in the former part and declines in the latter part. However, both trends obtain less regret for individuals.

From the perspective of asymmetric, firstly, as the important properties of road, Length and Link have the opposite trend. The former one locates at the sensitive area with the positive shape parameter meaning that individuals perceive a rapid downward trend of power coefficient and upward of regret, while the latter one has a moderate trend of both values. Therefore, the total length of a route is more influential in perceiving absolute regret than the average length of streets.

What is more, the variables which are related to travel environment have the positive shape parameter of both MNL and MNM part and locate at the left side of the middle points except Length_*C*, which is the undesirable variable. Decision makers have a faster upward or downward trend of MNM part than MNL part except Near_Park. This indicates that people prefer changes (increase or decline) of bike paths length, arterials and highways length, range of residential area, and link segment along the seashore. For the percentage of streets near park and near sea, the dominant parts have the absolute perceiving meaning because people do not always have a pleasant environment to cycle.

Finally, lnPS has the same trend of curve compared with the automobile case. Specifically, it is more common for cyclists to perceive absolute regret differences of the degree in independence of routes.

4.4.3. Alternative-Specific Behavior Interpretation. In the weight function of the latent class, both models incorporated by the length and the ratio of chosen route length with the shortest route length are estimated. Fortunately, they generate analogous results about coefficients (except constant c) and relationship graph of perceiving absolute and relative behavior. In this part, the chosen route length is used to analyze the behavior of cyclists, according to the results from Table 5 and Figure 15. In the next section, the ratio is used to compare both automobile and bicycle cases. It should be noted that constant c is 2105 in the length weight function and 1.10 in the ratio weight function.

From the perspective of absolute and relative behavior, Figure 15 shows that the boundary of two domains locates at the 2105 meters of the chosen route. In the left part, cyclists are more likely to perceive relative alternative differences while in the right side, individuals consider absolute differences. From Figure 12, it can be found that the CDF curve of OD pair distances also becomes flatter around 2 km, and more than 80% of the bicycle trips are shorter than 2 km, which means that cyclists prefer to consider relative differences by bike. What is more, as the length

Table 5: Estimation results.

Latent class		Estimates (t – value)							
MNL part	Length	-9.20 (-5.23)	γ_1	1.12 (3.21)	MNM part	Length	-6.04 (-4.20)	γ'_1	1.01 (2.73)
			δ_1	-1.5 (-2.39)				δ'_1	-2.34 (-3.12)
	Length_A	7.43 (4.23)	γ_2	0.35 (3.98)		Length_A	2.74 (3.83)	γ'_2	0.57 (3.21)
			δ_2	5.5 (4.23)				δ'_2	4.35 (2.54)
	Length_C	-9.98 (-3.23)	γ_3	-1.04 (-4.40)		Length_C	-6.87 (-2.98)	γ'_3	2.43 (3.13)
			δ_3	-1.5 (-4.43)				δ'_3	-0.98 (-1.93)
	Link	-12.32 (-5.40)	γ_4	-2.02 (-2.23)		Link	-9.29 (-4.96)	γ'_4	-0.20 (-3.23)
			δ_4	29.2 (9.34)				δ'_4	19.2 (5.44)
	Dwell	3.21 (3.04)	γ_5	3.20 (3.05)		Dwell	2.19 (5.48)	γ'_5	1.23 (2.91)
			δ_5	4.21 (2.65)				δ'_5	5.12 (3.12)
	Near_Sea	1.96 (2.90)	γ_6	2.34 (1.70)		Near_Sea	2.75 (2.09)	γ'_6	3.48 (2.25)
			δ_6	7.85 (4.32)				δ'_6	3.32 (2.91)
	Near_Park	5.23 (2.13)	γ_7	0.98 (2.00)		Near_Park	3.31 (4.27)	γ'_7	3.34 (4.12)
			δ_7	4.01 (1.69)				δ'_7	10.35 (5.23)
	lnPS	5.34 (1.99)	γ_8	0.53 (2.21)		lnPS	4.20 (1.81)	γ'_8	1.53 (1.95)
			δ_8	1.85 (3.31)				δ'_8	-3.14 (-1.98)
	μ	0.98 (3.48)				λ	3.12 (9.15)		
Weight function	c	2105 (9.21)				γ	-0.99 (-3.12)		
Model fits	Null L-L				-1346.00				
	Final L-L				-812.12				
	$\bar{\rho}^2$				0.358				
	BIC				1949				

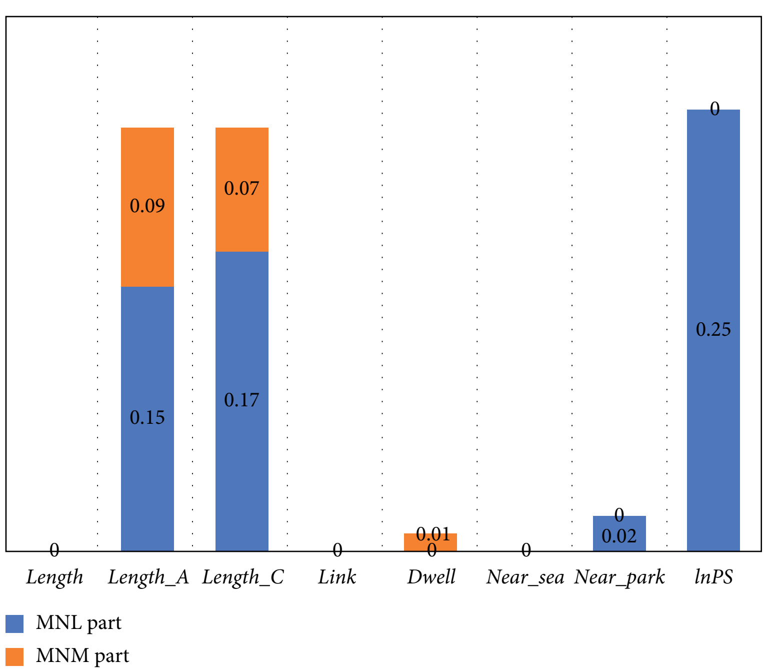

Figure 13: Average power coefficient of each independent variable with both MNL part and MNM part.

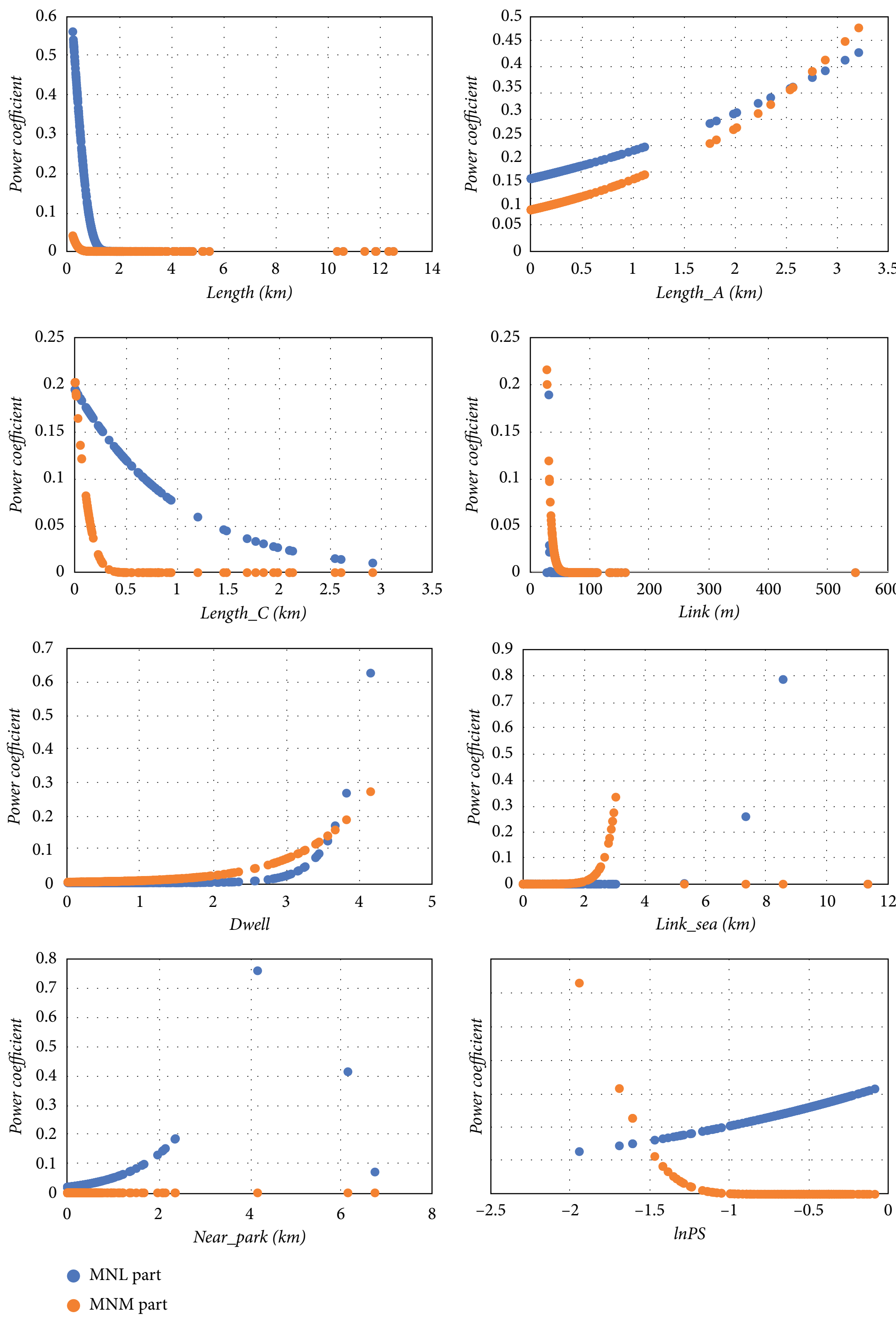

Figure 14: Scatter plots of individual power coefficients of different variables.

increases, this effect vanishes when the chosen route length is more than 2 km.

Because travel by bike is usually as the mode for the last mile not for the whole commute, individuals should consider the traditional problems of the small or large network to choose whether take a bike or not when the length is not long. However, the effect of an additional 200 meters (absolute difference) after having ridden a long distance

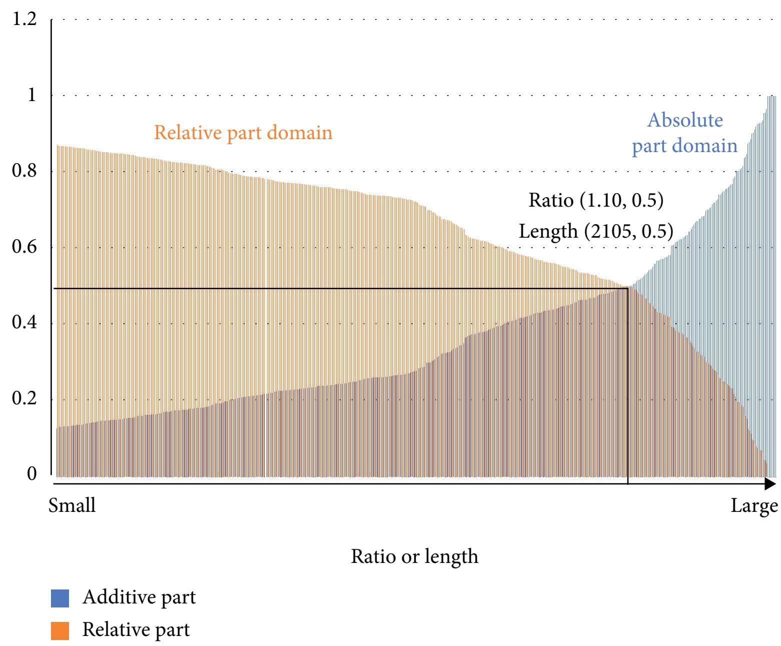

Figure 15: Relationship between the weight of latent class and ratio or length.

(over 2105 meters in this dataset) will be even higher, which is in line with our previous research using utility function.

From the perspective of asymmetric, as MNL part is symmetric, therefore, the property of asymmetry is captured by the shape parameter λ, which is estimated as 3.12. It should be noted that only the people who chose the path closing to the shortest path according to the length that can obtain asymmetry, which is opposite to automobile travelers. What is more, significant asymmetry can be generated within the regret ratio changing through one, which, however, less obvious than automobile case. This implies that individuals become more deterministic when they are faced with some analogous routes by bicycle than by automobile.

5. Comparison of Case Studies

In Sections 3 and 4, two cases are used to test the hybrid regret model. Goodness of fit of models can be guaranteed with the behavior interpretation about asymmetry and relative and absolute differences perception. Some behavior findings are shown as follows comparing drivers and cyclists.

Firstly, the most important and interesting finding is that as the route length increases, individuals prefer to consider relative regret differences when they travel by automobile while cyclists are more likely to perceive absolute regret differences. It should be noted that because the travel lengths of the two modes vary dramatically, for automobile case from 0.5 km to 48.4 km with the mean length of 5.26 km while for bicycle case from 0.2 km to 12.5 km with the average route length of 1.61 km. It is inapplicable to directly compare the length. Therefore, in the weight function, the ratio of the chosen route length with the shortest length is used to capture the degree of absolute and relative alternative regret differences.

As the common mode for travel, when the travel length is not very long, travelers by car should consider the additional length of the route rather than large or small network problem, which is captured by the absolute regret differences of alternatives. If the length of the chosen path is two times longer than the shortest route length, they are more likely to perceive relative alternative differences to deal with the large network perception problem. However, for the people travel by bike, the last mile problem should be considered in the short network because of the main purpose of cyclists. And then the additional regret effect will be higher with the increase of the length indicating the absolute behavior considering alternative regret.

Secondly, in the automobile case, it can be found that the absolute values of the MNM part are larger than that of the MNL part by comparing the magnitude of coefficients. This indicates that, when individuals consider and perceive the relative differences of alternatives, they are more deterministic to choose the best route that has the minimum regret according to the specific attributes. In the bicycle case, it has the opposite results except Near_Sea. *Ceteris paribus*, absolute differences of alternative part of regret dominates the behavior selection process.

Finally, according to the power coefficients of variables about either whole individuals or average values, cyclists are more likely to perceive absolute attribute differences than drivers except considering route length which has complex behavior interpretations. The bicycle seems like a simpler way for people to take, to consider route regret and to perceive differences than car, which are the characteristics of absolute differences.

Table 6: Different calculations of three extreme value distributions under maximum and minimum theory.

Distribution		Case1		Case2		Specification	Constrains
		1	2	-1	-2		
Gumbel	Max	e^1/e^1+e^2	e^2/e^1+e^2	$e^{-1}/e^{-1}+e^{-2}$	$e^{-2}/e^{-1}+e^{-2}$	$\exp(\beta v_i)/\sum\exp(\beta v_k)$	$\beta>0$
	Min	$e^{-1}/e^{-1}+e^{-2}$	$e^{-2}/e^{-1}+e^{-2}$	e^1/e^1+e^2	e^2/e^1+e^2	$\exp(-\beta v_i)/\sum\exp(-\beta v_k)$	
Weibull	Max	—	—	$1/1+0.5$	$0.5/1+0.5$	$(-v_i)^{-\beta}/\sum(-v_k)^{-\beta}$	$\beta>0$ $v_{i\cdots k}<0$
	Min	$1/1+0.5$	$0.5/1+0.5$	—	—	$v_i^{-\beta}/\sum v_k^{-\beta}$	$\beta, v_{i\cdots k}>0$
Fréchet	Max	$1/1+2$	$2/1+2$	—	—	$v_i^{\beta}/\sum v_k^{\beta}$	$\beta, v_{i\cdots k}>0$
	Min	—	—	$1/1+2$	$2/1+2$	$(-v_i)^{\beta}/\sum(-v_k)^{\beta}$	$\beta>0$ $v_{i\cdots k}<0$

6. Conclusions

In this paper, we presented an analytical framework to deal with the problem of identical perception variance in regret model. From the methodology aspect, as indicated in the introduction section, two methods (scaling perception variance and relaxing the error term distribution assumption) can be incorporated in the regret-based model, which are corresponding to the attribute and alternative specific differences. From the behavior aspect, perceiving actions of decision makers are analyzed in-depth, generating absolute and relative behavior interpretation and asymmetric property. Latent class structure is used to analyze individual heterogeneity of alternative differences by defining weight function, while the power coefficient in regret function can capture individual perception variance of specific attributes.

To test the proposed regret route choice models, we applied two data sources. One is the GPS data of taxis in Guangzhou to capture the behavior of drivers and the other is bicycle household surveys collected in Tel Aviv metropolitan area. All parameters are successfully and significantly estimated. Accordingly, some separable and comparable findings can be obtained. More specifically, we have the following main observations from the comparison of two datasets.

(1) As the route length increases, individuals prefer to consider relative regret differences when they travel by automobile while cyclists are more likely to perceive absolute regret differences, as a result of the real settings that compared with car trips, the bicycle trips will cost much more physical effort

(2) MNM part is mainly larger than that of the MNL part by comparing the magnitude of coefficients in the automobile case. The opposite happens in the bicycle case. This implies that for the importance of alternative differences, cyclists and drivers perceive striking differences

(3) According to the attribute perception variance affected by the power coefficients, cyclists are more likely to perceive absolute attribute differences than drivers except considering route length which has complex behavior interpretations

For further research, on the one hand, it is possible to extend the model in a multimodal application, which needs to collect data based on household survey or mobile phone information. On the other hand, regret model can be explored through the property of asymmetry, to overcome the drawback that each exponential term depends on the attributes of two or more alternatives [43].

Appendix

A. Different Functions of Three Extreme Value Distributions under Maximum and Minimum Theory

The three extreme value distributions (*Gumbel*, *Weibull* and *Fréchet* distributions) are commonly used in transportation research. We know that if a random variable X is *Gumbel* distributed, then e^X is *Fréchet* distributed and e^{-X} is *Weibull* distributed. Decision maker can have many choice strategies, such as utility maximum and regret minimum. When faced with both max and min policies and values with different signs, the specifications will have a little variation.

Previous research has shown the framework of comparative-statics exercises and effects on the choice probability and resulting distribution of the achieved utility and value [13]. Here, for our research objectives, we illustrate more details about different distributions of the two theories. See Table 6 for details.

The grid which does not have any content means that under this case, it cannot be calculated by this theory. Although it seems that we can generate them by reversing positions of two values, it cannot be true when alternatives are more than two.

Data Availability

The GPS data of taxis in Guangzhou used to support the findings of this study have not been made available because of privacy issues. The bicycle data are from previously reported studies and datasets, which have been cited.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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