Research Article

Synergistic Optimization Method for URT Network Train Connection Scheme in Peak and Off-Peak Periods

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In this study, we developed a method for coordinating and optimizing the train connection plans of different lines under the conditions of urban rail transit (URT) network operation. The method allows trains of different lines to form good connections at transfer stations, which can shorten the waiting time of passengers for transfers and reduce passenger retention. A mathematical model was developed to simulate the interaction between passengers and trains. Two optimization models were developed for the train connection plan of network transfer stations based on different optimization objectives during peak and off-peak hours. Subsequently, a corresponding solution method based on a genetic algorithm and simulation was designed. Finally, the Suzhou URT network was used as a case study, and the passenger flow of the transfer station was simulated and calculated using relevant automatic fare collection (AFC) data. The results indicated that the average waiting time and the number of passengers stranded were reduced using the proposed method. The calculation example demonstrated the effectiveness of the model and algorithm, which can guide the coordinated preparation of a network train connection plan.

1. Introduction

With the development of the urban rail transit (URT) network, the number of passenger flow in the network has increased significantly. Thus, the problem of transfer matching in a network has become more complex. Many operation companies require the operation plans of different lines of the network to be coordinated and optimized according to the passenger travel demand and passenger flow distribution to fully exploit the overall benefits of the rail transit network and ensure its efficient and reliable operation. In practical applications, the operation plans of most operating companies are limited to a single line. For complex and large transfer demands, the operator should consider coordinating the train operation plans of different lines at the network level. The train plan should achieve a good connection between trains of different lines at transfer stations to shorten the waiting time of passengers for transfers and improve the interchange service.

When the connection scheme is formed during the actual operation, it is mainly designed through manual adjustment. However, many unknown problems remain. Two-directional trains on the same line should be prevented from arriving at the same station simultaneously. However, if there are multiple transfer stations on the same line, there will be conflicts between the multiple transfer stations, which cannot meet the train connection of all transfer stations simultaneously. The workload required for manual adjustment was significant. To solve the connection problem at transfer stations, the train operation plan must consider the network passenger flow. The train operation plan affects the flow and time of passengers arriving at a transfer station. In addition, the train operation plan should be optimized according to the space-time requirements of the transfer passenger flow.

Regarding train connections, many scholars have focused on optimizing the first and last train timetable connections, which are undoubtedly crucial. The optimization goal for the first train is usually to improve the satisfaction of the first train transfer time so that passengers who take the first train in the morning do not wait too long [1–3]. Kang et al. [3] aimed to minimize the train arrival time differences
and the number of missed trains. With the closure of daily services, the last train is unable to reach certain destinations because the connection service may be closed when passengers arrive at the transfer station. The last train connection optimization goal is to maximize the accessibility of the last train destinations in the network and ensure that more passengers can take the last train to reach more places on the network [4–9]. Optimization is usually accomplished through timetable synchronization of the last trains for urban rail networks. This approach involves optimizing the URT during particular hours. The last train optimization model aims to maximize the number of successful transfers. The goal of the first train optimization model is to reduce the transfer time, but there should be no passenger retention caused by insufficient transport capacity. Therefore, these models cannot be applied to the peak and off-peak periods because of their different objective functions.

Researchers have optimized the coordinated transportation of trains at a single transfer station. Li et al. [10] considered the capacity coordination and platform safety between two lines of a transfer station. They optimized the train operation plan for the two lines by adjusting the departure time and departure interval. Liu et al. [11] developed a multiobjective optimization model to minimize the total transfer waiting time, train operating cost, and fluctuation of the departure interval. However, the connection optimization of a single transfer station does not consider the impact on other transfer stations in the network. Cao et al. [12] considered the constraint of timetable symmetry and adjusted the departure times of upward and downward trains on each line of a transfer station to minimize the transfer waiting time of all transfer passengers. Tsang et al. [13] developed an agent negotiation model to coordinate the schedules of trains belonging to two different operating companies at a transfer station. They developed three different negotiation strategies to improve coordination with the goal of maximizing revenue. Wang et al. [14] proposed timetable synchronization optimization methods for optimizing passenger transfer waiting times according to the time-dependent demand and train capacity.

In the other studies, the coordination among multiple transfer stations was optimized. Most of these studies aimed to minimize the waiting time of transfer passengers. Wong et al. [15] constructed an optimization model by adjusting the joint travel time, dwell time, dispatching time, and headway design to reduce the transfer waiting time and discussed the effects of different parameters on the waiting time for the Hong Kong Metro. Kwan et al. [16] proposed two indicators to measure the total dissatisfaction of passengers and total deviation from the original schedule. A multiobjective model based on these indicators was developed to minimize the waiting time of passengers on the premise of minimizing schedule adjustment. Wu et al. [17] reduced the maximum waiting time for each transfer direction at transfer stations, passenger waiting time, and schedule robustness. Liu et al. [18] minimized the waiting time of the transfer passengers in a network by adjusting the departure time of the network train. On this basis, Li et al. [19] considered the waiting psychology of passengers, established a cost function for the transfer waiting time, proposed a calculation method for the waiting time, and optimized the train arrival and departure times to reduce the total waiting time of passengers. Zhou et al. [20] considered minimizing the total waiting time and passenger dissatisfaction as coordination objectives. They proposed a synchronized and coordinated control process and a method for passenger flow organization and train connection in the URT network. Guo et al. [21] considered a significant change in travel demand during the transitional period from peak to off-peak hours or vice versa. A mixed-integer nonlinear programming model was developed to adjust the train timetables according to the time-varying travel demand and to optimize the transfer synchronization in metro transit networks.

All the above connection scheme optimization methods aim to realize the smooth organization of passenger flow in URT networks. Table 1 presents the considerations and objective functions of previous studies. As shown, in most of the studies, the minimum transfer waiting time was selected as the objective function or one of the objective functions. In some studies, the company’s profitability, number of failed transfers, and passenger satisfaction were considered as objective functions. Owing to the complexity of the URT network, it is necessary to consider its impact on other transfer stations after adjusting the train diagram. Therefore, we believe that the optimization objective must consider the effects of transfer stations on the entire network. Additionally, in the previous studies, few researchers defined the applicable period for their model. Guo et al. [21] defined the research period as the transition period from peak to off-peak hours or off-peak-to-peak hours, and Liu et al. [18] found that the effect of off-peak optimization is better than that of peak optimization. However, according to a survey [22], the number of passenger flow fluctuates between peak and off-peak. In addition, our communication with URT operators reveals that passengers have different transfer goals during different periods. During off-peak hours, every passenger can transfer successfully, and the goal of the transfer is to minimize the waiting time. During peak hours, there is a large passenger flow, and passengers cannot transfer successfully at the transfer station, resulting in multiline train delays [23]. At this time, the goal of passengers is to transfer successfully without being stranded. Owing to the different operation strategies and passenger flow characteristics in the peak and off-peak periods, there should be different coordination and optimization objectives and connection strategies in the different periods. In this study, according to the different passenger demands and characteristics of different periods, along with the constraints of train operation and the collaborative optimization of multiple transfer stations, a time-divisional collaborative optimization method for a network train connection scheme was designed.

The remainder of this paper is organized as follows. Section 2 explains the optimization objectives in different periods and presents the basic assumptions. In Section 3, an optimization model for the connection scheme is established. In Section 4, the solution algorithm is designed using
shown in Figure 1, passengers can transfer from line \( l \) to line \( l' \) at the earliest. Arranging the arrival times of the trains at the transfer station is crucial for the coordination and optimization of the network train operation plan. Owing to the different characteristics of passenger flow in different periods, the optimization objectives of the train operation plan are also different.

During off-peak hours, the average transfer waiting time is longer because of the relatively small number of transfer passengers and the long intervals. Therefore, the goal of train operation plan coordination during off-peak hours is to achieve a reasonable connection between trains on different lines and with different directions at the transfer station. This approach can shorten the waiting time of passengers and prevent a situation where to transfer train leaves the station after passengers arrive at the platform of another line. However, the train operation interval is small during peak hours. Even if trains traveling in different directions at the transfer station do not form a good connection, the passenger waiting time for transfer will not be too long. Passengers are often stranded on platforms with large passenger flows and limited train capacity during peak hours. Suppose that multiple groups of passengers gather simultaneously in a short period on one platform from different directions and lines; the load on the platform increases, and the difficulty of organizing the passenger flow at the station increases, which affects the safety of passengers. Therefore, it is necessary to ensure that passengers can transfer to trains during peak hours to the maximum possible extent to avoid secondary retention due to the inability of passengers to board trains at the station platforms.

Considering the different needs of the aforementioned periods, the adjustment targets should also be different. Moreover, the adjustment of the operation plan should have a minimal impact on the original plan. Simply adjusting the times at which trains arrive at the transfer station during off-peak hours can reduce the waiting time for passengers; i.e., the departure time of the first train during this period is adjusted. The trains continue to maintain the original interval. The essence of passenger retention during peak hours is the insufficient capacity of the connecting trains. Therefore, during peak hours, in addition to adjusting the train departure time, it is also necessary to adjust the train departure interval. The decision variables and objective functions for the different models are presented in Table 2.

When the departure interval is adjusted, the load rate of each section in each period of the entire line changes. Once the stranded passengers transfer successfully, the saturated load rate may be reduced, which increases the overall transfer success rate of transfer passengers in the network. The effect of the adjustment can be evaluated through a train operation simulation to simulate the interaction between passengers and the trains.

In this study, two mathematical models for different periods were developed to solve the connection problem of transfer stations. The two models share a component, which

<table>
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<tr>
<th>Article</th>
<th>Target</th>
<th>Objective function</th>
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<tbody>
<tr>
<td>Li et al. [10]</td>
<td>Single transfer station</td>
<td>Optimize matching of the transfer demand and capacity during peak hours</td>
</tr>
<tr>
<td>Liu et al. [11]</td>
<td>Single transfer station</td>
<td>Minimize the total transfer waiting time, train operating cost, and fluctuating departure interval.</td>
</tr>
<tr>
<td>Cao et al. [12]</td>
<td>Single transfer station</td>
<td>Minimize the transfer waiting time of all transfer passengers.</td>
</tr>
<tr>
<td>Tsang et al. [13]</td>
<td>Single transfer station</td>
<td>Maximize the revenue.</td>
</tr>
<tr>
<td>Wang et al. [14]</td>
<td>Single transfer station</td>
<td>Minimize the passenger total waiting time and the number of passengers who fail to transfer.</td>
</tr>
<tr>
<td>Wong et al. [15]</td>
<td>Multiple transfer stations</td>
<td>Minimize the transfer waiting time of all transfer passengers.</td>
</tr>
<tr>
<td>Kwan et al. [16]</td>
<td>Multiple transfer stations</td>
<td>Minimize the total passenger dissatisfaction index and total deviation index.</td>
</tr>
<tr>
<td>Wu et al. [17]</td>
<td>Multiple transfer stations</td>
<td>Reduce the worst weighted transfer waiting time as well as the probability and propagation of delay in the urban subway network.</td>
</tr>
<tr>
<td>Liu et al. [18]</td>
<td>Multiple transfer stations</td>
<td>Minimize the transfer waiting time of all transfer passengers.</td>
</tr>
<tr>
<td>Li et al. [19]</td>
<td>Multiple transfer stations</td>
<td>Minimize the total waiting-time cost of passengers.</td>
</tr>
<tr>
<td>Zhou et al. [20]</td>
<td>Multiple transfer stations</td>
<td>Minimize the total waiting time and passenger dissatisfaction.</td>
</tr>
<tr>
<td>Guo et al. [21]</td>
<td>Multiple transfer stations</td>
<td>Maximize the transfer synchronization in the transitional period.</td>
</tr>
</tbody>
</table>
reduces the complexity of the overall model. Suppose that at a station \( k \), \( M \) lines intersect; \( M \in N^* \), and when \( M > 1 \), \( k \) is a transfer station. A certain direction \( d \) of each line \( i \) is denoted as \( i_d \), \( d \in \{0, 1\} \), where 0 represents upward and 1 represents downward. The upward and downward directions of each line are regarded as independent lines in the coordination process, and there is a corresponding platform in each direction.

The assumptions of this study are presented below, and the model-related notation is presented in Table 3.

1. During peak or off-peak hours, the departure intervals of trains running in the same direction on the same line are uniform.
2. Passengers board the train sequentially, in a “first come, first served” manner.
3. Passengers boarding a train wait for the other passengers on the train to alight.
4. In the process of passenger transfer, the average transfer walking time of passengers during peak and off-peak hours is taken regardless of the fluctuations in passenger walking speed.
5. The compositions and capacities of trains on the same line are fixed; that is, the trains have the same passenger carrying capacity. Considering the train capacity limit, when the number of passengers reaches the maximum capacity of the train, the remaining passengers are unable to board the train and must wait in line for the next train.
6. During off-peak hours, there is no passenger retention, owing to sufficient capacity. During peak hours, passengers become stranded on the platform no more than once in each queue.
7. Owing to the complexity of the problem, the rolling stock cycle is not considered.

### 3. Modeling

#### 3.1. Timestamp Constraints

In a certain research period, the departure time of the first train in the direction \( d \) of the line \( i \) is the time offset from the starting time of the period, i.e., \( X_{i1}^d \in [0, I^d) \). The departure time of the \( j \)th train line \( i \) from the first station in the \( d \) direction during the research period can be calculated using the following formula:

\[
X_{ij}^d = X_{i1}^d + (j - 1)I^d. \tag{1}
\]

The arrival timestamp \( A_{ijk}^d \) and departure timestamp \( L_{ijk}^d \) of the \( j \)th train line \( i \) at a station \( k \) in the direction \( d \) during the research period are calculated as follows:

\[
A_{ijk}^d = X_{ij}^d + \theta^d_k + \sum_{k=1}^{h} r^d_{ik}, \tag{2}
\]

\[
L_{ijk}^d = A_{ijk}^d + x^d_{ik}. \tag{3}
\]

In equations (1)–(3), \( \theta^d_k \) and \( x^d_{ik} \) are known parameters that can be obtained from the actual train diagram data. The decision variables are the departure timestamp of the first train \( X_{i1}^d \) and the departure interval (constant during off-peak periods) in the study period. This method is applicable even if the line involves complex operations. For example, in
3.2. Passenger-Flow Constraints. It is difficult to predict the number of passengers boarding and alighting from each train at each station. Because of data privacy, detailed personal AFC data cannot be obtained. According to the OD data, after sorting (including the inbound time, outbound time, inbound station, outbound station, and the number of passengers), we approximate the passenger flow entering the station. It is assumed that the inbound and outbound rates of passengers within 30 min are based on AFC data. Additionally, it is assumed that the average number of passengers outside the station arriving at the platform is \(V_{i,d}^k\). In the period \((T_1, T_2)\), the number of passengers outside the station that take the train on line \(i\) in the \(d\) direction is given as follows:

\[
V_{i,d}^{(T_1, T_2)} = V_{i,d}^k \times (T_2 - T_1).
\]  

(4)

It is assumed that if the train capacity is sufficient, all passengers who have been waiting since the departure of the \((j-1)\)th train can board the \(j\)th train. Then, after the \(j\)th train arrives at the station \(k\), the number of passengers from outside the station that can successfully board the train is

\[
V_{i,d}^{(T_1, T_2)} = V_{i,d}^k \times P_i^d.
\]  

(5)

Similarly, after the train arrives at the platform, it is assumed that passengers exit the train at the same time and leave the station at a uniform rate. It is assumed that the average outbound rate of passengers is \(O_{i,d}^k\). In the period \((T_1, T_2)\), the number of passengers leaving the station is given as follows:

\[
O_{i,d}^{(T_1, T_2)} = O_{i,d}^k \times (T_2 - T_1).
\]  

(6)

After the \(j\)th train arrives at the station \(k\), the number of passengers leaving the station is given as follows:

\[
O_{i,d}^{(T_1, T_2)} = O_{i,d}^k \times P_i^d.
\]  

(7)

Suppose that station \(k\) is a transfer station where \(M\) lines intersect. \(F_{i,d}^{m} \rightarrow m_b\) represents the number of passengers

<table>
<thead>
<tr>
<th>Notations</th>
<th>Type</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d \in [0, 1])</td>
<td>Index</td>
<td>Train running direction: 0 represents upward, and 1 represents downward.</td>
</tr>
<tr>
<td>(X_{i,j}^d)</td>
<td>Variable</td>
<td>Departure timestamp of the (j)th train in the direction (d) of line (i) (when (j = 0), this variable represents the departure timestamp of the last train before the research period)</td>
</tr>
<tr>
<td>(I_i)</td>
<td>Variable</td>
<td>Departure interval in the direction (d) of line (i)</td>
</tr>
<tr>
<td>(A_{i,k,j}^d)</td>
<td>Variable</td>
<td>Arrival timestamp of the (j)th train in the direction (d) of line (i) at station (k) (when (j = \text{last + 1}), this variable represents the arrival timestamp of the first train after the research period)</td>
</tr>
<tr>
<td>(F_{i,d}^{m} \rightarrow m_b)</td>
<td>Variable</td>
<td>Departure timestamp of the (j)th train in the direction (d) of line (i) at station (k)</td>
</tr>
<tr>
<td>(F_{i,d}^{m} \rightarrow m_b)</td>
<td>Variable</td>
<td>Number of passengers on the (j)th train in the direction (d) of line (i) who transfer to the direction (b) of line (m) at station (k)</td>
</tr>
<tr>
<td>(F_{i,d}^{m} \rightarrow m_b)</td>
<td>Variable</td>
<td>Arrival rate of transferred passengers to the direction (b) of line (m) from the direction (d) of line (i)</td>
</tr>
<tr>
<td>(m_{b} \rightarrow i_d)</td>
<td>Variable</td>
<td>Transfer walking time from the platform of the direction (b) of line (m) to the platform of the direction (d) of line (i) at station (k)</td>
</tr>
<tr>
<td>(O_{i,d}^{(T_1, T_2)})</td>
<td>Variable</td>
<td>Number of outbound passengers from the direction (d) of line (i) at station (k) in the period ((T_1, T_2))</td>
</tr>
<tr>
<td>(O_{i,d}^{(T_1, T_2)})</td>
<td>Variable</td>
<td>Average outbound rate of passengers from the direction (d) of line (i) at station (k)</td>
</tr>
<tr>
<td>(V_{i,d}^{(T_1, T_2)})</td>
<td>Variable</td>
<td>Number of passengers outside the station who take the train on line (i) in the direction (d) in the period ((T_1, T_2))</td>
</tr>
<tr>
<td>(V_{i,d}^{(T_1, T_2)})</td>
<td>Variable</td>
<td>Average ratio of passengers outside the station arriving at the platform of the direction (d) of line (i) at station (k)</td>
</tr>
<tr>
<td>(U_{i,d}^{(T_1, T_2)})</td>
<td>Variable</td>
<td>Number of passengers who transfer to (i_d) at station (k) with the period ((T_1, T_2))</td>
</tr>
<tr>
<td>(W_{i,d}^{(T_1, T_2)})</td>
<td>Variable</td>
<td>Number of passengers waiting for the upward or downward platform before the departure of the (j)th train on line (i) at station (k)</td>
</tr>
<tr>
<td>(R_{i,j}^{(T_1, T_2)})</td>
<td>Variable</td>
<td>Number of passengers stranded on the platform of direction (d) before the ((j-1))th train leaves the station on line (i) at station (k)</td>
</tr>
<tr>
<td>(O_{i,d}^{(T_1, T_2)})</td>
<td>Variable</td>
<td>Number of passengers deboarding from the (j)th train arriving at the direction (d) of line (i) at station (k)</td>
</tr>
<tr>
<td>(L_i)</td>
<td>Variable</td>
<td>Number of passengers boarding the (j)th train arriving at the direction (d) of line (i) at station (k)</td>
</tr>
<tr>
<td>(T_{i,d}^k)</td>
<td>Variable</td>
<td>Passenger waiting time on the platform before the departure of the (j)th train</td>
</tr>
<tr>
<td>(N_i^d)</td>
<td>Variable</td>
<td>Residual capacity of the (j)th train of line (i) in the direction (d)</td>
</tr>
<tr>
<td>(\lambda_{i,d}^k)</td>
<td>Variable</td>
<td>Average arrival rate of transfer-in passengers to (i_d) during the period ((T_1, T_2))</td>
</tr>
<tr>
<td>(C_{net}^k)</td>
<td>Variable</td>
<td>Network transportation capacity after optimization</td>
</tr>
<tr>
<td>(C_{net}^k)</td>
<td>Constant</td>
<td>Network transportation capacity before optimization</td>
</tr>
<tr>
<td>(P_i^d)</td>
<td>Constant</td>
<td>Passenger capacity limit of the direction (d) of line (i) at station (k)</td>
</tr>
<tr>
<td>(\alpha_i^d)</td>
<td>Constant</td>
<td>Maximum load factor of the train</td>
</tr>
<tr>
<td>(\alpha_{i,d}^d)</td>
<td>Constant</td>
<td>Minimum or maximum interval constraint value</td>
</tr>
<tr>
<td>(\alpha_{i,d}^d)</td>
<td>Constant</td>
<td>Rated capacity of each train in the direction (d) of line (i)</td>
</tr>
<tr>
<td>(\beta_i^d)</td>
<td>Constant</td>
<td>Running time for the direction (d) of line (i) from the departure station to station (k)</td>
</tr>
<tr>
<td>(\tau_i^d)</td>
<td>Constant</td>
<td>Dwell time for the direction (d) of line (i) at station (k)</td>
</tr>
<tr>
<td>(\psi_i^d)</td>
<td>Constant</td>
<td>Average inbound rate of passengers from outside the station entering the platform of direction (d) of line (i) at station (k)</td>
</tr>
<tr>
<td>(\psi_i^d)</td>
<td>Constant</td>
<td>Average outbound rate of passengers from the direction (d) of line (i) at station (k)</td>
</tr>
<tr>
<td>(\phi_i^d)</td>
<td>Constant</td>
<td>Importance value of transfer station (k)</td>
</tr>
</tbody>
</table>
transferring from the $j^{th}$ train of $i_d$ to $m_o$, which can be calculated as follows:

$$f_{k}^{j,i_d,m_o} = f_{k}^{i_d} - m_o \times f_{j}^{i_d}.$$  \tag{8}

3.3. Train Capacity Constraints. If the station $k$ is a transfer station, $G^{j,i}_j$ represents the number of passengers alighting, which can be expressed as the sum of the transfer out passenger flow $F$ and outbound passenger flow $O$:

$$G^{j,i}_j = \sum_{m=1}^{M} \sum_{b \in \{0,1\}} F_{k}^{i_d,m} - m_o + O_{i_k,k}^j \left( t_{i_k,k}^{j,i_d} + t_{i_k,k}^{j,i} \right). \tag{9}$$

Then, $E_{j,k}^{j,i}$ represents the number of passengers boarding, which can be expressed as the sum of the transfer in passenger flow $F$ and inbound passenger flow $V$:

$$E_{j,k}^{j,i} = \sum_{m=1}^{M} \sum_{b \in \{0,1\}} F_{k}^{j,i_d,m} + m_o + V_{i_k,k}^{j,i} \left( t_{i_k,k}^{j,i_d} + t_{i_k,k}^{j,i} \right). \tag{10}$$

If station $k$ is not a transfer station, we have

$$\begin{align*}
G^{j,i}_j &= O_{i_k,k}^j \left( t_{i_k,k}^{j,i_d} \right), \\
E_{j,k}^{j,i} &= V_{i_k,k}^j \left( t_{i_k,k}^{j,i_d} \right). \tag{11}
\end{align*}$$

The residual train capacity is the maximum number of passengers that can board the train, which depends on the remaining capacity after passengers exit the train when it arrives at a station $k$. The residual capacity of the train before it arrives at the station $k$ is determined by the number of boarding and alighting passengers at each station. There are $h$ stations in front of the station $k$, and the station passing by is denoted as $k_r$. $G^{j,i}_j$ represents the rated capacity of the train and $\alpha_{\text{max}}$ is the maximum load factor. The residual capacity of the $j^{th}$ train of Line $i$ in the $d$ direction can be calculated using the following formula:

$$N_{j,k}^{i_d} = C_i^d \times \alpha_{\text{max}} - \sum_{k_r=1}^{h} E_{j,k}^{i_d} + \sum_{k_r=1}^{h} G^{j,i}_j.$$  \tag{12}

3.4. Stranded-Passenger Constraints during the Peak Period. The transfer passenger flow is related to the arrival and departure times of trains on connected lines at the transfer station $k$. If the passengers on the $j^{th}$ train $m_o$ can successfully transfer to $i_d$ within the period $(T_1, T_2)$, the following constraint should be satisfied:

$$T_1 < A_{m_k}^{i_d} + t_{k_d}^{m_o} - i_d \leq T_2,$$  \tag{13}

where $t_{k_d}^{m_o} - i_d$ represents the transfer walking time. Then, the number of passengers transferred to $i_d$ within the period $(T_1, T_2)$ is given as follows:

$$U_{j,k}^{(T_1, T_2)} = \sum_{m=1}^{M} \sum_{b \in \{0,1\}} \sum_{i_d < A_{m_k}^{i_d} + t_{k_d}^{m_o} - i_d \leq T_2} F_{k}^{m_o,n} - i_d.$$  \tag{14}

The average arrival rate of transfer-in passengers to $i_d$ during the period $(T_1, T_2)$ is given as follows:

$$\lambda_{j,k}^{i_d} = \frac{U_{j,k}^{(T_1, T_2)}}{T_2 - T_1}. \tag{15}$$

Before the departure of each train, passengers waiting on the platform include those transferred from other lines, inbound passengers from outside the station, and stranded passengers on the platform. The number of passengers waiting at the top or bottom platform before the departure of the $j^{th}$ train on line $i$ is given as follows:

$$W_{j,k}^{i_d} = U_{j,k}^{i_d} + V_{j,k}^{i_d} + R_{j-1,k}^i,$$  \tag{16}

where $W_{j,k}^{i_d}$ represents the number of passengers waiting for the upward or downward platform before the departure of the $j^{th}$ train on Line $i$ at the station $k$, and $R_{j-1,k}^i$ represents the number of people stranded on the platform of direction $d$ before the $(j-1)^{th}$ train leaves the station on Line $i$ at station $k$.

Passenger retention depends on whether the transport capacity of the train satisfies the demand for passenger flow. When the passenger flow demand exceeds the residual capacity of the train, passenger retention occurs, resulting in an undesirable waiting environment and security risks. The number of passengers stranded on the $d$ direction platform of the line $i$ after the departure of the $j^{th}$ train is given as follows:

$$R_{j,k}^i = \max \left\{ W_{j,k}^{i_d} - N_{j,k}^{i_d}, 0 \right\}.$$  \tag{17}

3.5. Waiting-Time Constraints during the off-Peak Period. At the transfer station on the $d$-directional platform of the line $i$, from the departure time of the $(j-1)^{th}$ train to the departure time of the $j^{th}$ train, the passenger waiting time on the platform before the departure of the $j^{th}$ train is given as follows:

$$T_{j,k}^{i_d} = \sum_{T=0}^{T} \left( \int_{T}^{T} dt \cdot \left( \lambda_{k}^{i_d} + v_{k}^{i_d} \right) \right) + R_{j-1,k}^i,$$  \tag{18}

According to the basic requirements of URT network operation, the network train connection scheme must ensure that passengers on the network have an optimal experience. Additionally, owing to special operational requirements, the operator may require the network train connection scheme to consider specific connection constraints. Moreover, in the case of a complex network with many transfer stations, the transfer needs of each station may not be satisfied. We can only consider the connection priority based on the importance of the transfer station. In research on timetable synchronization [3, 15, 28], weight scoring is often used to determine the priority. Therefore, when developing the model, it is necessary to consider that different stations have different connection importance levels, and the importance of each transfer station is
determined via expert scoring \( \phi_k \). The importance of transfer stations is determined by their locations (downtown or suburban). A more critical transfer station has a more significant effect on the value of the objective function. In equations (17) and (18), the target value increases with an increase in the transfer passenger flow; thus, the number of transfer directions (a larger number corresponds to a larger passenger flow) and the transfer passenger flow are not considered in the weight setting, to avoid the Matthew effect.

In addition, the model must consider cost constraints; otherwise, the results may change toward the minimum interval. This may increase blindly the network’s transport capacity, ensuring smooth transfer and safe transport. While coordinating the train plans, it is necessary to ensure an economy [28]. Moreover, the impact of the adjustment coordinations on the section demand of the line should be minimized. Thus, in this study, we limit the network transportation capacity \( C_{\text{net}} \), which is the sum of the rated capacities of the trains running in all directions in the network within 1 h. We assume that the floating range after the optimization of \( C_{\text{net}} \) is limited to 80%–120%. Assuming that there are \( s \) lines in the network, the network transportation capacity is calculated as follows:

\[
C_{\text{net}} = \sum_{i=1}^{s} \sum_{d \in \{0,1\}} C_i^d \times \frac{3600}{I_i^d}
\]  

(19)

The following formulas define the model’s objective functions. The equations corresponding to the off-peak and peak periods, respectively are given as follows:

\[
\min Z = \sum_{i}^{M} \sum_{j}^{N} \sum_{k}^{K} \phi_k \cdot T_{jk}^i
\]  

(20)

\[
\min Y = \sum_{i}^{M} \sum_{j}^{N} \sum_{k}^{K} \phi_k \cdot R_{jk}^i
\]  

(21)

The following formulas give the model limitations:

\[
s.t \ W_{jk}^i \leq P_{jk}^i \forall k, \forall i, \forall j, \forall d,
\]  

(22)

\[
T_E - T_i^d \leq A_{\text{last},k}^i \leq T_E \forall k, \forall i, \forall d,
\]  

(23)

\[
I_{\min} \leq A_{\text{last},k}^i - A_{\text{last},k}^i \leq I_{\max} \forall k, \forall i, \forall d,
\]  

(24)

\[
A_{jk}^i \neq A_{jk}^i \forall k, \forall i, \forall j, \forall d,
\]  

(25)

\[
0.8C_{\text{net},l} \leq C_{\text{net}} \leq 1.2C_{\text{net},l},
\]  

(26)

\[
0 \leq X_i^j < I_i^j \forall i, \forall d,
\]  

(27)

\[
I_{\min} \leq I_i^d \leq I_{\max} \forall i, \forall d,
\]  

(28)

\[
I_{\min} \leq X_i^j - X_i^j \leq I_{\max} \forall i, \forall d,
\]  

(29)

\[
I_i^d, X_i^j \in N^+ \forall i, \forall d,
\]  

(30)

where \( P_{jk}^i \) represents the passenger capacity limit of the up or down platform of line \( i \) at station \( k \), \( A_{\text{last},k}^i \) represents the arrival time of the last train of \( i_d \) in the research period, \( A_{\text{last},k}^i \) represents the arrival time of the first train after the research period, \( \{T_S, T_E\} \) represents the periods in which the train operation plan must be adjusted, and \( C_{\text{net}} \) represents the network transportation capacity before optimization.

In the two periods, the two models had the same constraints but the objective functions differed. Equations (20) and (21) are objective functions, and equations (22)–(30) are constraints. Equation (20) gives the shortest total waiting time for passengers in the study period for off-peak hours. Equation (21) gives the minimum number of stranded passengers in the study period for peak hours. Equation (22) is the platform safety constraint; the number of people waiting on the platform cannot exceed the platform capacity limit. Generally, the platform capacities in the up and down directions at the same station are equal. Equation (23) is the constraint on the arrival time of the last train. Equation (24) specifies the interval between the arrival times of the last train in the optimization period and the subsequent train. Equation (25) is the arrival-time constraint for avoiding the simultaneous arrival of trains in two directions on the same line. Equations (26)–(28) are decision variable constraints, and the departure time of the first train is the offset from the starting timestamp of the optimization period, which is an integer. Equation (28) is the basic constraint of the train departure interval, where \( I_{\min} \) mainly depends on the technical conditions of the line. The departure interval must exceed the minimum tracking interval. To ensure the line service level, the departure interval must be smaller than the maximum departure interval. In general, \( I_{\max} \) is determined by the service level. Equation (29) specifies the departure time interval between the first train in the optimization period and the previous train. Equation (30) is considered only during the peak period; \( I_i^d \) is known during the off-peak period.

4. Solution Algorithm

The model presented above is a nonlinear integer programming model with complex constraints and a large set of feasible solutions. It is difficult to obtain an optimal solution quickly and accurately if the enumeration method is used to solve this model. Because the model involves the process of train–passenger interaction, it cannot be linearized. We believe that commercial solvers (such as CPLEX) are unsuitable for solving this model. Thus, a heuristic algorithm was used to solve the model. The GA is a mature and widely used heuristic algorithm. It is an efficient random search and optimization method based on the theory of biological evolution. Its main characteristics are group search strategies and information exchange among individuals in the population. The search does not depend on the gradient information.

In this study, a two-level algorithm was designed to solve the problem. The upper layer is based on the GA of the train operation adjustment. The lower layer is based on the interactive simulation of network passengers and trains to...
adjust the train operation parameters and obtain the calculation indicators and fitness values. The upper layer takes the minimum number of stranded passengers or waiting time as the goal and adjusts the train operation parameters (the departure time and departure interval of the first train). At the bottom level, the model simulates the passenger carrying rate of the train, the number of passengers waiting on the platform, and the number of passengers stranded at the transfer station, and then returns to the upper level to continue adjusting and optimizing. Then, the lower layer simulates the scheme calculated by the upper layer to obtain the fitness value and cycles until the optimal solution is obtained. The logic of this algorithm is presented in Figure 2.

4.1. GA Parameter Setting. The GA has good global convergence, high calculation efficiency, and high robustness [29]. The algorithm design is described as follows.

4.1.1. Coding Method. In GA, binary symbol strings are often used to represent individuals in a population. In this study, the departure timestamp of the first train in two periods and the departure interval only in the peak period are the decision variables. According to the constraint condition of equation (27), the departure timestamp of the first train cannot exceed the departure interval. The maximum departure interval is 480 s. Therefore, according to the ranges of the decision variables, the variables correspond to two 9-bit binary codes. An example of a chromosome is shown in Figure 3.

4.1.2. Fitness Function. Fitness is used to measure the quality of individuals in a population (degree of conformity) and is usually expressed in the form of numerical values. In general, a lower (or higher) fitness value indicates a higher quality solution and a higher probability of the individual being selected. Therefore, the selection of the fitness function is important as it affects the convergence speed of the GA and determines whether the optimal solution is obtained. In the foregoing model, constraints (22)–(30) limit the range of the solution set. The fitness-function value \( Y' \) is the sum of the objective-function values \( Y \) and \( Z \):

\[
Y' = \mu_Z \cdot Z + \mu_Y \cdot Y,
\]  

(31)
where $\mu_Z$ and $\mu_Y$ are binary variables (0 or 1); $\{\mu_Z = 0, \mu_Y = 1\}$ in the peak period and $\{\mu_Z = 1, \mu_Y = 0\}$ in the off-peak period.

4.1.3. Control Parameters. The selection of control parameters, including the population size and genetic operators, affects the speed and accuracy of the GA. The population size affects the convergence of the GA, and the number of control parameters is generally 20–100. Better chromosome adaptability yields a higher probability of survival and inheritance. Crossover is the operation of exchanging one or more genes on the parent chromosome to generate new individuals. The crossover probability is generally 0.4–0.99. A mutation is a random change introduced by an individual. Under certain conditions, one or more genes on a chromosome are randomly changed. The probability of variation is generally 0.0001–0.1. The generation gap indicates the proportion of each generation that is selected to change. The gap is generally set as 1.

4.1.4. Algorithm Termination Rule. The termination conditions are different for the different periods. During the off-peak period, the algorithm terminates the calculation if a better solution cannot be obtained for a continuous period. During the peak period, the calculation time is long; thus, a maximum genetic threshold $\text{MAXGEN}$ is provided. The iteration of the algorithm stops when it reaches $\text{MAXGEN}$.

4.2. Interactive Simulation Mechanism between Passengers and Trains. The migration of passengers from one station to another requires the train’s help; thus, the interaction between passengers and trains is crucial for passenger travel.

After passengers choose the boarding scheme according to the actual situation, they form a queue on the platform according to the principle of “first come, first served” to wait for the arrival of the target train. After the train arrives, they complete the boarding and alighting process according to the principle of “first alight, then board.”

Owing to the limitations of the train capacity, in the morning and evening peak hours, for stations with large discrepancies between the train capacity and passenger flow demand, the residual capacities of arriving trains may not satisfy the needs of waiting passengers, and some passengers will be stranded. Taking the travel process of a passenger $P$ in Figure 4 as an example, because of the full load of train1, the passenger $P$ who arrives at the platform cannot complete the boarding behavior at time $T_1$; therefore, he can only wait for the subsequent train (Train2) to arrive and complete the boarding process at time $T_2$. The waiting time of passenger $P$ on the platform is extended by $T_{\text{Delay}}$.

According to the foregoing analysis of the passenger boarding and alighting processes, the interaction between passengers and trains during line operation is shown in Figure 5.

The passenger–train interaction process is summarized below.

(1) Interaction process of passengers alighting the train

Step 1: The train releases arrival information (such as the station name and type, i.e., transfer station or general station) to all passengers onboard.

Step 2: The system randomly assigns routes to passengers according to a sorting proportion table.

Step 3: The passengers on the train decide whether the station is the target station (destination station or transfer station) according to their travel plans.
Step 4: Passengers form a line to complete the alighting process.

(2) Interaction process of passengers boarding the train

Step 1: Passengers form a line on the platform according to the “first come, first served” principle.
Step 2: After the train arrives at the station, passengers decide whether to board the train according to their travel plans and the running information of the train.
Step 3: Passengers who wish to board the train queue up to complete the boarding action under the constraints of the upper limit of the train capacity.
Step 4: If the load capacity rate of the train reaches the upper limit, the subsequent passengers will be retained and cannot complete the boarding action; they must wait for the arrival of the subsequent train.

5. Case Study

5.1. Scenario. The Suzhou URT network was used as an example to validate the model and solution method. As shown in Figure 6, there are four lines and nine transfer stations, of which the branch line of line 4 operates independently and S9 serves as the transfer station of the branch line. According to the AFC data and train timetable data for April 30, 2021, the peak period (07:00-08:00) and off-peak period (10:00-11:00) were taken as the research periods for analysis. The objective functions are different in different periods; thus, they were calculated separately. The model was solved using Visual Studio 2015 with C# and a PL/SQL editing program on a personal computer with an Intel Core i5-8700 central processing unit.

For the GA parameters, if the population size is too small, the precision is insufficient, and the solution is unstable. If the population size is too large, the performance deteriorates. The probability of variation is too low, and the diversity of the population decreases too quickly. If the mutation probability is too high and the probability of high-order mode destruction is high, it is easy to destroy the existing favorable pattern, increase the randomness and miss the best individual. The mutation probability is too low to effectively update the population. There is no fixed standard for GA parameters; they must be adjusted continuously.
Table 4: Setting model parameters.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Line i</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_i$</td>
<td>Rated capacity of each train of line $i$</td>
<td>1</td>
<td>942</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>1187</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>1436</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>1504</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>360 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>405 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>420 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>405 s</td>
</tr>
<tr>
<td></td>
<td>4-branch line</td>
<td></td>
<td>415 s</td>
</tr>
<tr>
<td>$I^{o}$</td>
<td>Departure interval in the direction $d$ of line $i$ (off-peak period)</td>
<td>1</td>
<td>360 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>405 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>420 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>405 s</td>
</tr>
<tr>
<td></td>
<td>4-branch line</td>
<td></td>
<td>415 s</td>
</tr>
<tr>
<td>$I^{p}$</td>
<td>Departure interval in the direction $d$ of line $i$ (peak period)</td>
<td>1</td>
<td>140 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>220 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>350 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>285 s</td>
</tr>
<tr>
<td></td>
<td>4-branch line</td>
<td></td>
<td>415 s</td>
</tr>
<tr>
<td>$\tau_k$</td>
<td>Dwell time for the direction $d$ of line $i$ at station $k$</td>
<td>1, 2, 3, 4</td>
<td>40 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>120 s</td>
</tr>
<tr>
<td>$\theta_k^i$</td>
<td>Running time for the direction $d$ of line $i$ from the departure station to station $k$</td>
<td>According to the actual performance timetable data</td>
<td></td>
</tr>
<tr>
<td>$\alpha_{max}$</td>
<td>Maximum load factor of the train</td>
<td>1, 2, 3, 4</td>
<td>100%</td>
</tr>
<tr>
<td>$P_d^i$</td>
<td>Passenger capacity limit of the platform</td>
<td>1, 2, 3, 4</td>
<td>1000</td>
</tr>
</tbody>
</table>

Table 5: Average transfer walking time.

<table>
<thead>
<tr>
<th>Station</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
<th>S9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average transfer walking time</td>
<td>40 s</td>
<td>40 s</td>
<td>40 s</td>
<td>65 s</td>
<td>220 s</td>
<td>35 s</td>
<td>45 s</td>
<td>190 s</td>
<td>60 s</td>
</tr>
</tbody>
</table>

Figure 7: Convergence process of the fitness in the off-peak period.
through practice and selected according to different scenarios. The parameter selection standard used in this study is that the model can have a more obvious optimization effect in a shorter time, that is, the fitness function decreases faster. According to many tests, the parameters with the best genetic iteration effect (fastest convergence) were selected as follows: population size = 20, crossover probability = 0.8, mutation probability = 0.1, and MAXGEN = 100 (peak period). In off-peak cases, the model converged naturally, approximately 3000–5000 times.

According to the actual operation, the detailed train capacity and interval parameters were set, as shown in Table 4. We considered the priority of the connection according to the importance of the transfer station. According to the transfer passenger flow of each transfer station in the network, the weight values obtained by the on-site operation staff were as follows: S9 (256), S4 (128), S3 (128), S8 (64), S1 (32), S7 (16), S2 (8), S5 (4), and S6 (2).

At the transfer station, the passenger transfer walking time approximately follows a normal distribution, and its mean and variance are related to the transfer walking distance. Moreover, an increase in congestion increases the average value [30]; therefore, the transfer walking time is affected by the number of passengers traveling in each direction. To simplify the calculation, the average transfer walking time in each direction during the off-peak period was determined, as shown in Table 5. Du et al. [30] found that the average transfer walking time during peak hours is approximately 11% longer than that in the off-peak period; thus, we set the transfer walking time in the peak period to 1.11 times that in off-peak hours.

5.2. Off-Peak Period. In the URT network, from 10:00 to 11:00, the optimization objects were lines 1, 2, 3, 4, and 4-branch line. The departure timestamp for each direction of the line before optimization was 0 (10:00:00). In this study, the departure timestamp of the first train in each direction of each line, i.e., $X_{i}^{d} = 0$, was used as the initial scheme. There were 20 solutions in each generation, and the solution with the smallest fitness value was the optimal solution for the generation. The calculation took approximately 2 min; the process converged after approximately the 3250th generation, and the optimal solution was obtained. The variations in optimal fitness for each generation are presented in Figure 7. After 3250 iterations, the optimal solution was obtained, as shown in Table 6. The fitness value was the result of the weighted transfer waiting time. The fitness value of the network was reduced by 7.17% compared with the initial value.

5.3. Peak Period. In the URT network, from 07:00 to 08:00, the optimization objects were lines 1, 2, 3, 4, and 4-branch lines. The departure timestamp for each direction of the line before optimization was 0 (07:00:00). For the morning peak period, the optimization objective of the model was to
reduce the retention of passengers. After 100 iterations, the optimal solution was obtained, and the optimal solution is presented in Table 7; the iterative process is shown in Figure 8. In solving this double-layer algorithm, the entire iterative process of 100 iterations took approximately 2 h because the calculation process involved many passengers. Therefore, the objective function had not converged when the iteration was completed. However, it was verified that the algorithm could optimize the scheme in a short time and obtain a feasible solution.

Figure 9 shows the number of people stranded on the platform at all transfer stations before and after optimization (07:00-08:00). The ordinate indicates the weighted cumulative number of people stranded, and the abscissa indicates time (in min). After the optimization of the connection plan, the number of stranded passengers was significantly
reduced. The cumulative number of stranded passengers in 1 h decreased by approximately 9.2%.

6. Conclusions

The operation management department must design the network train connection plan with careful consideration of the characteristics of the transfer passenger flow in the network to improve the passenger transport service level of the URT. In this study, a mathematical model was developed for simulating the interaction between passengers and trains. On this basis, an optimization model for the network train connection plan was proposed. Different objective functions were established for the peak and off-peak periods. The minimum waiting time is taken as the goal in the off-peak period; in the peak period, the number of stranded passengers is minimized. The train operation plan was adjusted under the constraints of the operational process. A corresponding solution algorithm was designed, and a case study based on the Suzhou URT network was examined to validate the model and algorithm.

The train connections between multiple lines are complex. In this study, we did not consider the negative feedback of network passenger flow path selection after train operation adjustment, which may lead to changes in the station entry and exit volumes and passenger travel paths. Rather, an approximate simulation analysis based on the historical passenger flow was performed. In addition, when describing the transfer process of passengers at a transfer station, the differences in the transfer processes of passengers were ignored. The transfer travel time of passengers was set according to the average value. The transfer walking time of passengers is closely related to the number of passengers and the passengers’ personal attributes. Therefore, describing the dynamic interaction between passengers and train operations according to the changing characteristics of passenger choice behavior is a critical topic for future research. And, if the transfer station is a railway hub or an aviation hub, it is also necessary to consider the impact of the fluctuating large passenger flow brought by the railway or aviation on the URT [31]. Finally, we will attempt to increase computational efficiency in future research.

Data Availability

The acquisition of the research data for the study was supported by the Suzhou Metro Operation Co., Ltd. The research data used to support the findings of this study have not been made available because it is company confidential data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

Study conception and design was performed by Ruihua Xu (RX). Data collection was performed by Feng Zhou (FZ) and analysis and interpretation of results was done by Xuyang Song (XS) and Fangsheng Wang (FW). Draft manuscript preparation was done by XS and FW. All the authors reviewed the results and approved the final version of the manuscript.

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