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Research Article

Optimizing Joint Decisions of Dynamic Pricing and Ticket Allocation for High-Speed Railway with Operators' Risk Preference

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High-speed trains powered by electricity have low-carbon emissions and are the important intercity travel conveyances conducive to sustainable social development. The sustainable development of high-speed railway needs to improve the operating revenue and the resource utilization. Joint optimization of dynamic pricing and ticket allocation are often used to improve high-speed railway revenue and seat utilization in the literature. However, the uncertainty of demand makes joint decision-making difficult to be accurate. The risk preference of decision-makers also deeply affects the effect of joint decision-making. To cope with the uncertain demand and test the effect of operator risk preference on joint decision-making, this paper used chance constrained programming theory to optimize dynamic pricing and ticket allocation simultaneously. The presale period is divided into several stages. In each stage, ticket demand follows the normal distribution and changes with the price elasticity. Passengers choose tickets according to the multinomial logit model. The chance constrains reflect operators' risk preference. The chance constrained stochastic nonlinear programming is proposed for joint decision of dynamic pricing and ticket allocation. Then, the combination algorithm of particle swarm optimization algorithm and mixed-integer linear programming was designed to solve the model. Finally, the numerical experiments according to actual operating scale were design to validate the model and algorithm. The results indicate that under different confidence levels, the proposed model and algorithm increase the total revenue by 11.84%-13.40% compared with the ticket allocation under the single fixed fare. The model can help the high-speed railway operators understand the impact of risk preference on joint decision-making, and provide decision support for them.

1. Introduction

With the growth of social economy and breakthrough of technological innovation, the high-speed railway (HSR) has developed rapidly in China over the past more than 20 years. By the end of 2020, the operating length of HSR has reached 38,000 kilometers. With the advantages of high safety, comfort, convenience, and environmental-friendly, the HSR has become the important passengers' travel mode. A statistical communiqué from China State Railway Group Co., Ltd (China Railway), shows that the national railway passenger volume reached 2.167 billion in 2020. And the passenger volume of HSR accounts for about 70% of the passenger volume of railway transportation. However, behind the rapid development of HSR, China Railway has suffered losses year after year. Because the revenue is far from covering the high construction and operating costs, so, improving revenue is crucial for the sustainable development of HSR and is concerned by operators and researchers. Since 2016, China Railway has obtained pricing rights for HSR from National Development and Reform Commissions and met the conditions to implement revenue management strategy.

Revenue management technologies, including seat inventory control [1–4], dynamic pricing [5–8], and their joint optimization [9–13], are used to improve the revenue of railway operators. Seat inventory control is aimed at allocating optimal seat volumes for each fare class. Reasonable seat inventory control plays an important role in adjusting supply. Dynamic pricing decides optimal fare for each time to maximize the total revenue. Dynamic pricing strategy

1: Input: The maximum number of iterations $t_{\max}$, learning factor c_1 and c_2, inertia weight ω.
2: Generate randomly N particles, position matrix $\mathbf{s}(\mathbf{0})$, velocity matrix $\mathbf{v}(\mathbf{0})$, $p_i(0) \longleftarrow s_i(0)$,$p_g(0) = \max\{p_i(0)\}$
3: **for** $t = 0$ **to** $t = t_{\max}$ **do**
4: **for** $i = 1$ to $i = N$ **do**
5: Calculate $\lambda_{h,w,k}$ according to equation (1), (2), (3), (4);
6: Input $p_i(t)$, $\lambda_{h,w,k}$ and $\sigma_{h,w,k}$ into the joint model(8), (10)–(14), obtain the ticket allocation quantity $x_i(t)$ using CPLEX;
7: Compute $fitness_i(t)$;
If $fitness_i(t) > fitness_i(t-1)$
Update $p_i(t)$ and $p_g(t)$, record the price $p_i(t)$ and the corresponding ticket allocation scheme $x_i(t)$;
else
The optimal value still adopts the results obtained in generation $t-1$.
8: Update $\mathbf{v}(\mathbf{t})$ and $\mathbf{p}(\mathbf{t})$ according to equation (15) and (16), meanwhile ensure $\mathbf{p}(\mathbf{t})$ within the allowable range of ticket price.
9: Output: The ticket price scheme $\mathbf{p}$, the ticket allocation scheme $\mathbf{x}$, the optimal fitness function value $fitness$.

Algorithm 1: The algorithm steps.

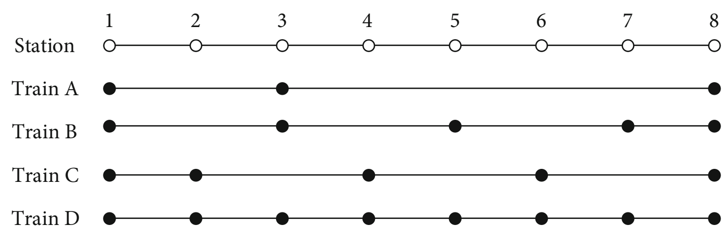

Figure 1: The train service network.

Table 1: Price elasticity coefficient η_k.

Time period	η_k
1	3.5
2	3
3	2.5
4	2
5	1

can adjust and stimulate passenger demand. Therefore, dynamic pricing and ticket allocation are two significant aspects to achieve the balance between supply and demand, which is beneficial to reduce the waste of transportation capacity and improve the income. Therefore, the joint optimization of the two has been explored a little. However, the uncertainty of demand makes joint decision-making difficult to be accurate. The existing literature often uses average demand or expected value according to a certain distribution to deal with demand uncertainty. And the operator's risk preference was not considered in the joint decision optimization.

To the best of our knowledge, the impact of operator's risk preference on joint decision of dynamic pricing and ticket allocation is not considered in the existing literature. Faced with unknown demands, the operators with different risk appetites will make different decisions. Risk-taking decision-makers tend to overestimate demand, while conservative decision-makers tend to underestimate demand. Therefore, this research tries to fill this gap by introducing chance constrained programming (CCP) into a joint decision model. We consider the situation of multiple trains operating on a line. The presale period is divided into several stages according to the change law of price elasticity of demand. Passenger demand obeys the normal distribution and passengers select the ticket according to the passenger's choice behavior model. We use the confidence level to reflect the risk preference of operators and propose a nonlinear chance constrained programming model to optimize dynamic pricing and ticket allocation simultaneously. The particle swarm optimization (PSO) algorithm and integer linear programming are combined to solve the model efficiently in speed and quality. The model and algorithm can provide decision support for railway operators in daily operation management.

2. Literature Review

Revenue management (RM) originated in the American aviation industry in the late 1970s. With abolishing air traffic control and opening of aviation market, airline company could determine price and schedule flights. Single and expensive ticket price cannot adapt to fierce market competition and rapid growth in passenger demand. To increase revenue, RM theory was introduced into the field of aviation. In 1972, Littlewood [14] proposed a discount cost criterion for air ticket reservation and the second-order classification model. This research prepared the ground for more mathematical models applied to RM. Belobaba [15] described application of the Expected Marginal Seat Revenue (EMSR) decision model to make reservation limits regularly before the date of flight departure. Then, a simple instance was given to verify the effectiveness of the probability model.

Table 2: Utility parameter $d_{h,w}$, $t_{h,w}$, p_w, average demand λ_w, and variance of demand σ^2.

OD	$d_{h,w}$ (yuan)/$t_{h,w}$ (minute) Train A	Train B	Train C	Train D	λ_w	σ_w^2	p_w (yuan)
1	0/0	0/0	31.6/73	32.8/75	455	42	144.5
2	37/82	40.4/106	0/0	40.9/101	249.7	44.1	184.5
3	0/0	0/0	62.9/159	62.9/153	241.9	46.3	284
4	0/0	67.7/171	0/0	68.5/173	387.2	48.6	309
5	0/0	0/0	98.3/256	98.3/250	175.5	51.1	443.5
6	0/0	114.7/312	0/0	116/312	368	53.6	523.5
7	110.6/268	121.2/346	122.5/342	122.5/342	411.5	56.3	553
8	0/0	0/0	0/0	8.9/26	168.6	42	40
9	0/0	0/0	30.9/86	30.9/78	282	44.1	139.5
10	0/0	0/0	0/0	36.4/98	162.7	46.3	164.5
11	0/0	0/0	66.2/183	66.2/175	199.8	48.6	299
12	0/0	0/0	0/0	84/237	144	51.1	379
13	0/0	0/0	90.5/269	90.5/267	297.5	53.6	408.5
14	0/0	0/0	0/0	22/52	181.4	42	99.5
15	0/0	27.3/65	0/0	27.6/72	208.4	44.1	124.5
16	0/0	0/0	0/0	57.4/149	159.7	46.3	259
17	0/0	74.3/206	0/0	75.1/211	202.6	48.6	339
18	73.7/186	80.8/240	0/0	81.6/189	172.2	51.1	368.5
19	0/0	0/0	0/0	5.5/20	254	42	25
20	0/0	0/0	35.3/97	35/97	181.6	44.1	159.5
21	0/0	0/0	0/0	52.5/159	299.5	46.3	239.5
22	0/0	0/0	59.6/183	59.6/189	406.3	48.6	269
23	0/0	0/0	0/0	29.5/77	455	42	134.5
24	0/0	42.9/141	0/0	47/139	249.7	44.1	214.5
25	0/0	48.8/175	0/0	53.5/169	241.9	46.3	244
26	0/0	0/0	0/0	17.5/62	387.2	42	80
27	0/0	0/0	24/86	24/92	175.5	44.1	109.5
28	0/0	5.9/34	0/0	6.5/30	368	42	29.5

On the basis of previous studies, Kimes [16] put forward a comprehensive and accurate definition of RM-4R theory, that is, to provide the right products or services at the right time and place with the right price to maximize the income of enterprises under resource constraints. Mcgill and Ryzin [17] did a survey to summarize the development of RM theory. They subdivided revenue management into seat inventory control, pricing, overbooking, and forecasting. Following the successful application in aviation, the RM has gradually formed a theoretical system. Then, RM theory was quickly applied and promoted in service industry such as hotels, banking, and transportation.

Ticket allocation is a specialization of inventory control which belongs to an essential part of revenue management. Scholars have done a lot of research and exploration. Ciancimino et al. [1] introduced revenue management into railway passenger transport for the first time, and a linear programming model and a probabilistic nonlinear programming model of seat allocation are established. Chang and Yeh [18] took an intercity rail line as research object; a two objective seat allocation model is constructed to maximize enterprise revenue and minimize passenger discomfort. The fuzzy mathematical programming method is used to solve the problem. A HSR to be built is used to verify the effectiveness of the multiobjective planning model. Ongprasert [19] adopted passenger choice behavior model to forecast the demand and determine the number of discount tickets. The discount tickets and seat allocation model are combined on the single-line-multistop line, which improves the income. You [2] considered the two-level fare system of full price ticket and discount ticket and constructs a nonlinear programming model. The PSO algorithm was designed to solve booking limit problem, which generated initial particles based on a solution under the determined demand. A numerical instance indicates that the heuristic algorithm is efficient and accurate. Wang et al. [3] used the multinomial logit (MNL) model to describe the probability of passengers choosing different products and established a single-stage and a multistage model to solve the problem of railway seat allocation. Li [20] used CCP to deal with stochastic demands and studies the optimization problem of ticket allocation under the certain decision preference. Yan et al. [4]

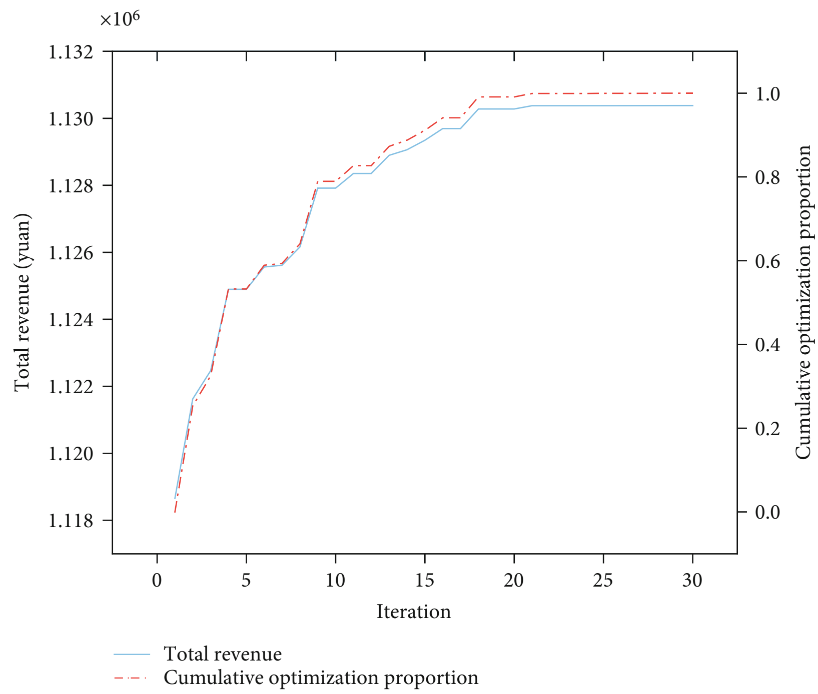

FIGURE 2: The convergence of total revenue.

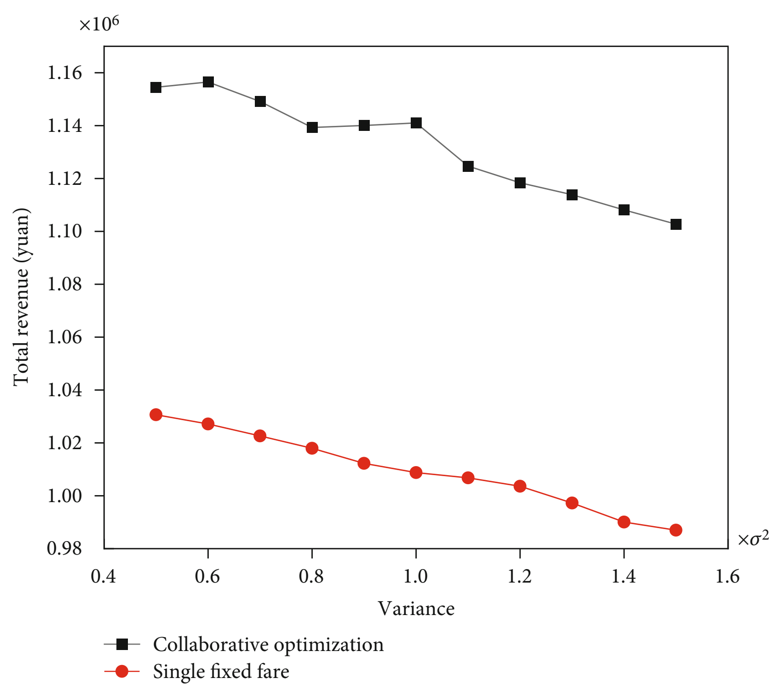

FIGURE 3: Total revenue under different variances.

constructed a probabilistic nonlinear programming model for HSR combing flexible train composition with seat inventory control. Compared with fixed train composition, this scheme improves the income of railway enterprises. It provides theoretical support for ticket allocation under flexible train composition.

Dynamic price is an essential tool of adjusting and regulating passenger flow. The study of dynamic pricing in railway is relatively late. Vuuren [5] studied the differential pricing of trains in peak hours and off-peak hours. This paper points out that maximizing social welfare is mainly considered in peak pricing. In the off-peak period, we should

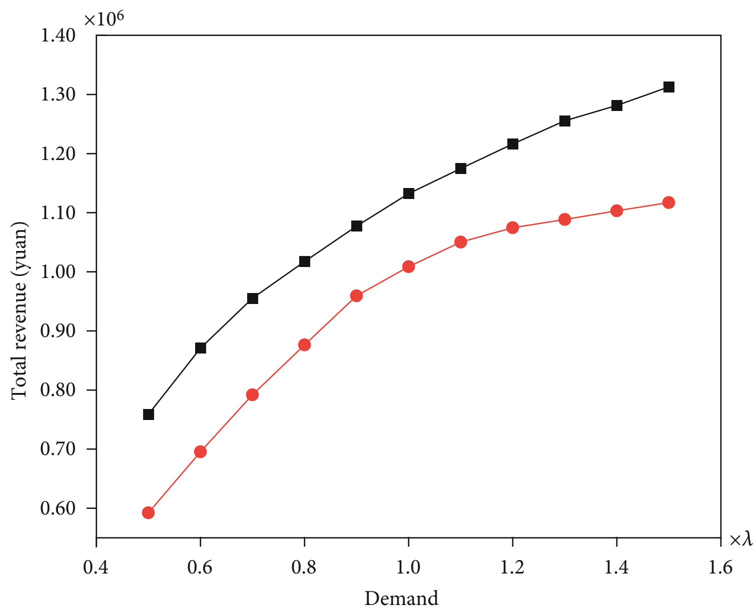

Figure 4: Total revenue under different demand.

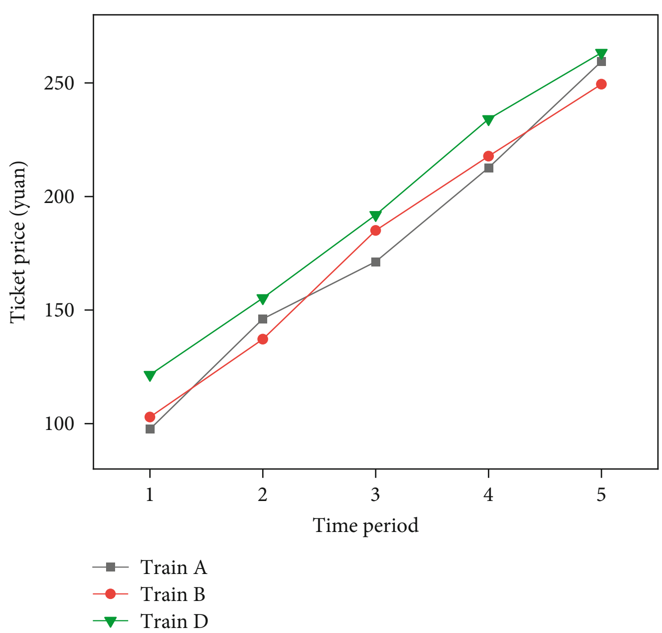

Figure 5: Ticket price for OD pair 2.

consider increasing the occupancy rate to improve the income. Gallego and Wang [21] used nest logit model to express passenger choice behavior and considered price sensitivity and arbitrary nest coefficient to make differential pricing for multiple trains. Yang et al. [6] introduced a cumulative distribution function of the maximum allowable price and discretized the whole presale time. A dynamic pricing model for HSR is constructed to determine the ticket price and selling time to satisfy the demand of different passengers. Given the seat inventory and presale range, Zhang et al. [7] established a two-stage dynamic pricing model of HSR considering individuals and groups, respectively. Qin et al. [8] classified passengers according to their sensitivity to time, price, and other relevant factors. Based on the expected travel cost, this paper constructed the differential pricing model of HSR by using the prospect theory. The simulated annealing algorithm is used to solve the model under the conditions of peak passenger flow and off-peak passenger flow. Zhan et al. [22] studied railway timetable from the perspective of promoting social equity. They established

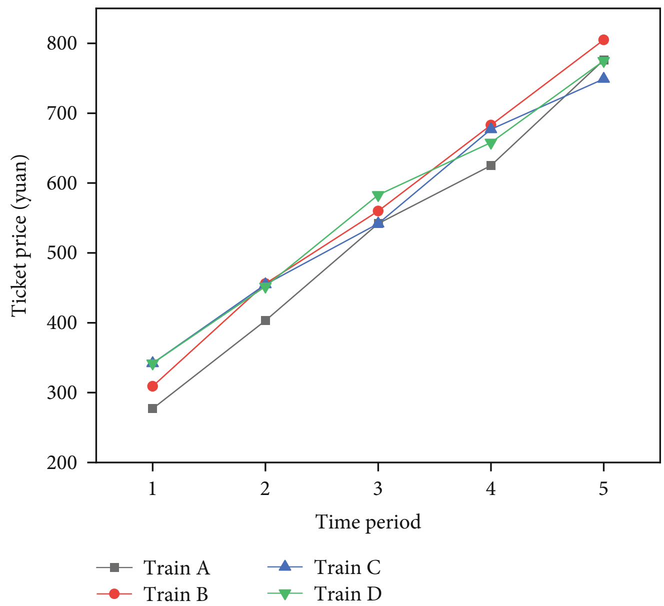

Figure 6: Ticket price for OD pair 7.

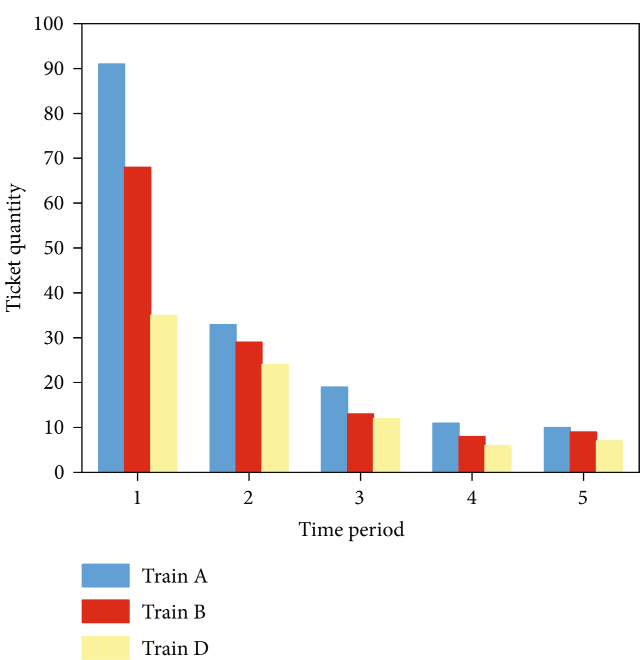

Figure 7: Ticket allocation quantity for OD pair 2.

a mixed-integer linear programming model considering enterprise profit, general travel cost of passenger, and government decision-making. It involved using dynamic pricing strategy to reduce the travel cost of low-income groups so that they can enjoy the same services as high-income groups. Chou et al. [23] constructed a bilevel model to explore optimum ticket fares for HSR. The upper-level model is to minimize the generalized cost of users considering passenger flow assignment. The lower-level model is to maximize operator's revenue containing passenger flow and the range of ticket fare transmitted by the upper-level model.

Pricing strategy and ticket allocation affect each other. The research of optimizing income considering ticket price and ticket assignment simultaneously has gradually rise in recent years. Hetrakul and Cirillo [24] put forward MNL model and latent class model to describe passenger choice behavior and established a joint optimization model of pricing and ticket allocation in railway for the first time. They considered the demand as deterministic. Lin [9] raised a bilevel programming HSR model for the joint optimization ticket price and seat allocation. The upper model is a nonlinear mathematical programming model aiming at maximizing the income of enterprises. The lower model is a two-stage stochastic programming model. The goal of the first part is to maximize the enterprise income, and the second part is to minimize the enterprise cost. Zhao et al. [10] constructed a joint optimization model to maximize revenue considering the choice behavior of passengers among different transportation modes under competition among road, air, and railway transport. They designed a hybrid heuristic algorithm to solve it based on artificial bee colony algorithm. Xu et al. and Qin et al. [11, 12] considered the price elasticity of demand, divided the entire presale period into several ticket selling stages, and built a joint optimization model based on the passenger choice behavior. Xu et al. [13] considered that the demand is affected by the service price and then established the joint optimization model of price and ticket allocation in the railway. Because the model is nonconcave and nonlinear, they developed the linearization of objective function and some constraints by taking logarithm. Furthermore, the other nonlinear constraints are linearized by relaxation. The final model is transformed into a mixed-integer linear programming which is easy to solve. In the joint decision optimization study of dynamic pricing and ticket allocation, the influence of decision-maker's preference has not been considered.

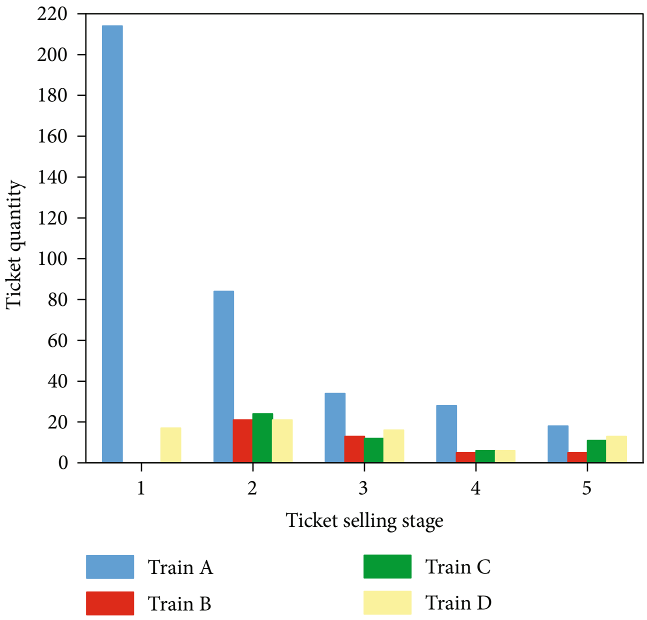

Figure 8: Ticket allocation quantity for OD pair 7.

This paper introduces CCP to build the joint decision model of dynamic pricing and ticket allocation. The theoretical contribution and practical value of this paper can be summarized as follows:

(1) Considering stochastic demand and the risk appetite of operators, this paper proposed the joint decision model of dynamic pricing and ticket allocation with CCP theory for HSR. By adjusting the confidence level, the model can optimize the ticket price and seat allocation scheme simultaneously under different risk preferences. The model can help the HSR operators understand the impact of risk preference on joint decision-making and provide decision support for them

(2) This paper provides a new solution idea for the joint decision-making stochastic nonlinear programming model considering stochastic demand, elastic demand, and passenger choice behavior. We use CCP to transform the stochastic programming model into a deterministic programming model. Then, the nonlinear model is decomposed into two layers using particle swarms. The upper layer uses particle swarms to iteratively optimize ticket prices, while the lower layer uses CPLEX to accurately solve the ticket allocation optimization model. The hybrid algorithm has satisfactory solution quality and solution speed

(3) Numerical experiments show that under different levels of demand, demand fluctuations, and risk preferences, joint decision-making can achieve higher returns than ticket allocation under fixed fares. As the confidence level increases, the total revenue decreases. When the confidence level equals to 0.5, the operator is in a neutral state and uses the average demand to make decision

The rest of the paper is organized as follows. The next part reports the problem description and propose the joint decision model. The principle of PSO algorithm and specific procedures of hybrid algorithm to tackle the problem are given in Section 4. Section 5 performs numerical experiments to verify the feasibility of model and algorithm and analyzed the experimental results in detail. The conclusions are obtained in the last part.

3. Model

Suppose a HSR line with several stations, serving the travel demand between origins and destinations. W denotes the set of the OD pairs and H denotes the set of the trains. The whole presale period is divided into K stages according to the demand elasticity. Set the period when the presale starts as the first stage, and the train departs at the end of the Kth stage. The HSR operators need to decide the fare and ticket allocation for each train and each OD in each stage to maximize the total revenue. Passengers choose ticket products according to the choice behavior. Demand is dynamic influenced by price and changes randomly. The operator has certain risk appetite. Under these conditions, the collaborative model is proposed to optimize fares and ticket allocations simultaneously. We introduce the symbol $<h, w, k>$ to uniquely identify ticket products, which means train h serves OD pair w, and the corresponding ticket is sold at stage k. In the following, a collaborative optimization model is established based on describing the passenger choice behavior, dynamic elastic demand, and decision-maker preferences.

3.1. Passenger Choice Behavior. There is a competitive relationship between trains on the same OD in the same time period. The mean passenger flow $q_{w,k}$ is allocated to the available trains according to passenger choice behavior. With reference for the results of previous studies [11, 25, 26], passengers mainly consider the factors of price, departure time, and travel time to make a choice. The utility of the product $<h, w, k>$ can be described by

$$V_{h,w,k} = -\tau t_{h,w} - p_{h,w,k} - d_{h,w}, \forall h \in H, w \in W, k = 1, \cdots, K, \quad (1)$$

where $t_{h,w}$ is the travel time of train h for the OD pair w. $p_{h,w,k}$ represents the fare of train h serving in OD w at the kth time period. $d_{h,w}$ denotes passengers' willingness to pay for the preferred departure time and can be quantified using SP surveys [23]. τ indicates passenger's time value. The MNL model can help us get the choice probability of each

Table 3: Ticket price scheme for confidence level $\alpha = 0.9$.

	1-A	1-B	1-C	1-D	2-A	2-B	2-C	2-D	3-A	3-B	3-C	3-D	4-A	4-B	4-C	4-D	5-A	5-B	5-C	5-D
1	0	0	85	83	0	0	111	128	0	0	144	136	0	0	174	176	0	0	204	206
2	98	103	0	122	146	137	0	155	171	185	0	192	213	218	0	234	259	249	0	263
3	0	0	160	142	0	0	243	225	0	0	274	262	0	0	335	350	0	0	387	400
4	0	205	0	177	0	237	0	247	0	305	0	337	0	354	0	361	0	462	0	432
5	0	0	297	267	0	0	333	321	0	0	421	433	0	0	511	531	0	0	627	636
6	0	324	0	345	0	428	0	469	0	535	0	511	0	633	0	603	0	689	0	750
7	277	309	342	342	403	456	455	452	542	560	542	583	625	683	677	658	776	805	749	775
8	0	0	0	27	0	0	0	32	0	0	0	41	0	0	0	49	0	0	0	53
9	0	0	90	83	0	0	117	119	0	0	135	135	0	0	170	162	0	0	199	204
10	0	0	0	89	0	0	0	140	0	0	0	168	0	0	0	210	0	0	0	225
11	0	0	195	197	0	0	242	241	0	0	293	277	0	0	366	338	0	0	427	401
12	0	0	0	212	0	0	0	281	0	0	0	371	0	0	0	436	0	0	0	551
13	0	0	239	256	0	0	310	335	0	0	418	408	0	0	490	508	0	0	597	599
14	0	0	0	57	0	0	0	87	0	0	0	104	0	0	0	115	0	0	0	133
15	0	73	0	75	0	99	0	97	0	121	0	123	0	158	0	143	0	181	0	178
16	0	0	0	159	0	0	0	209	0	0	0	244	0	0	0	304	0	0	0	372
17	0	216	0	178	0	265	0	275	0	331	0	329	0	426	0	417	0	480	0	467
18	213	235	0	219	284	318	0	292	388	348	0	393	451	445	0	433	518	500	0	523
19	0	0	0	14	0	0	0	19	0	0	0	26	0	0	0	31	0	0	0	34
20	0	0	92	96	0	0	135	122	0	0	148	169	0	0	187	187	0	0	225	225
21	0	0	0	138	0	0	0	173	0	0	0	249	0	0	0	274	0	0	0	326
22	0	0	153	166	0	0	197	209	0	0	255	263	0	0	321	333	0	0	359	396
23	0	0	0	73	0	0	0	95	0	0	0	129	0	0	0	163	0	0	0	198
24	0	136	0	108	0	174	0	186	0	218	0	221	0	253	0	254	0	293	0	285
25	0	168	0	130	0	194	0	171	0	231	0	240	0	292	0	288	0	353	0	355
26	0	0	0	47	0	0	0	67	0	0	0	75	0	0	0	101	0	0	0	115
27	0	0	64	69	0	0	92	90	0	0	113	113	0	0	129	136	0	0	147	156
28	0	17	0	20	0	22	0	21	0	28	0	32	0	36	0	35	0	43	0	43

train, as shown in the following equation:

$$\varphi_{h,w,k} = \frac{\exp\left(\theta V_{h,w,k}\right)}{\sum_{h' \in H_w} \exp\left(\theta V_{h',w,k}\right)}, \forall h \in H, w \in W, k = 1, \cdots, K, \tag{2}$$

where H_w denotes the set of trains serving OD pair w. $\varphi_{h,w,k}$ is the probability when $<h, w, k>$ is chosen by passengers. θ indicates the familiarity of the passengers with each product, and the parameters τ and θ can be estimated using the maximum likelihood method based on historical data or questionnaires.

The mean demand of $<h, w, k>$ is shown in the following equation:

$$\lambda_{h,w,k} = q_{w,k}\varphi_{h,w,k}, \forall h \in H, w \in W, k = 1, \cdots, K, \tag{3}$$

where $\lambda_{h,w,k}$ denotes mean demand of $<h, w, k>$.

3.2. Dynamic Elastic Demand. The buying demand for tickets is influenced by ticket prices. The price elasticity is usually used to describe the relationship between the buying demand for tickets and their prices. In the presale period, the closer to the departure time, the less passengers are sensitive to the price. Price sensitivity is expressed by elasticity parameter η_k and $\eta_1 > \eta_2 > \cdots > \eta_K$. Different OD markets will have different price elasticities. Different ticketing stages can be divided for different OD pairs according to the law of price elasticity. Here, for the convenience of model expression, the ticketing stages of all OD pairs are divided uniformly. The exponential function is used to describe the demand-price relationship, as in

$$\lambda_{h,w,k} = \lambda^0_{h,w,k} \exp\left[-\eta_k\left(\frac{p_{h,w,k}}{p^0_{h,w,k}} - 1\right)\right], \forall h \in H, w \in W, k = 1, \cdots, K, \tag{4}$$

where $p^0_{h,w,k}$ is the initial price for $<h, w, k>$ and the corresponding mean demand is $\lambda^0_{h,w,k}$. $p_{h,w,k}$ is the changed price for $<h, w, k>$, and $\lambda_{h,w,k}$ is the corresponding mean

Table 4: Ticket allocation scheme for confidence level $\alpha = 0.9$.

	1-A	1-B	1-C	1-D	2-A	2-B	2-C	2-D	3-A	3-B	3-C	3-D	4-A	4-B	4-C	4-D	5-A	5-B	5-C	5-D
1	0	0	156	50	0	0	83	66	0	0	44	48	0	0	27	26	0	0	27	26
2	91	68	0	35	33	29	0	24	19	13	0	12	11	8	0	6	10	9	0	7
3	0	0	0	0	0	0	33	0	0	0	22	27	0	0	14	12	0	0	14	12
4	0	84	0	0	0	76	0	0	0	39	0	25	0	27	0	23	0	17	0	26
5	0	0	0	0	0	0	32	0	0	0	17	15	0	0	10	8	0	0	8	8
6	0	0	0	0	0	68	0	0	0	29	0	38	0	18	0	27	0	31	0	13
7	214	0	17	0	84	21	21	0	34	13	16	9	28	5	6	8	18	5	13	9
8	0	0	0	0	0	0	0	0	0	0	0	28	0	0	0	18	0	0	0	20
9	0	0	58	0	0	0	41	0	0	0	27	0	0	0	15	17	0	0	15	15
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	14	0	0	0	18
11	0	0	0	0	0	0	31	0	0	0	16	0	0	0	8	13	0	0	8	13
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	0	0	0	14
13	0	0	0	0	0	0	61	0	0	0	24	0	0	0	18	14	0	0	15	15
14	0	0	0	0	0	0	0	0	0	0	0	16	0	0	0	22	0	0	0	22
15	0	0	0	0	0	0	0	0	0	20	0	0	0	10	0	12	0	10	0	10
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	18	0	0	0	16
17	0	0	0	0	0	20	0	0	0	19	0	0	0	9	0	10	0	9	0	11
18	66	0	0	0	29	11	0	0	9	10	0	5	6	4	0	5	7	5	0	3
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	19
20	0	0	0	0	0	0	13	0	0	0	18	0	0	0	10	0	0	0	9	9
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	40	0	0	0	37
22	0	0	0	0	0	0	89	0	0	0	44	0	0	0	26	21	0	0	30	17
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	26	0	0	0	25
24	0	0	0	0	0	30	0	0	0	15	0	14	0	10	0	10	0	10	0	11
25	0	50	0	0	0	74	0	0	0	45	0	12	0	25	0	26	0	24	0	23
26	0	0	0	0	0	0	0	15	0	0	0	30	0	0	0	13	0	0	0	15
27	0	0	96	0	0	0	37	36	0	0	19	18	0	0	14	12	0	0	14	12
28	0	66	0	109	0	104	0	108	0	43	0	42	0	28	0	29	0	27	0	27

Table 5: Total revenue at different confidence levels.

α	Collaborative optimization (yuan)	Single fixed fare (yuan)	Optimized proportion
0.1	1222884.83	1078352.5	13.40%
0.2	1196603.317	1069932.5	11.84%
0.3	1188489.005	1062035	11.91%
0.4	1183350.622	1055057.5	12.16%
0.5	1172751.496	1048107.5	11.89%
0.6	1170719.372	1041132.5	12.45%
0.7	1160459.997	1031720	12.48%
0.8	1152186.933	1022755	12.66%
0.9	1130381.824	1008787.5	12.05%

demand. The initial price and demand are often obtained according to the historical passenger flow data.

After the price adjustment, first, adjust the demand according to Formula (4). Then, considering the influence of passengers' choice behavior, the adjusted demand of each product is summed up and redistributed according to Formulas (2) and (3).

3.3. Description of Risk Preference. Charnes and Cooper [27] put forward the concept of CCP theory to solve venture capital decision-making in 1959. To a certain extent, the theory allows decision not to meet constraints. However, the decision should make the probability that the constraint conditions are satisfied not less than a certain confidence level α. The smaller the value of α, the greater the risk the decision-maker is willing to take in the face of random events. Equation (5) presents it here in its general form [28].

$$\begin{aligned} &\text{optimize}\, f(c, x), \\ &\Pr\{Ax \leq b\} \geq \alpha, \end{aligned} \tag{5}$$

where Pr means probability. A, b, and c do not have to be continuous, but at least one of them is a random variable that obeys a certain distribution. Decision variables represent the deterministic factors in the decision-making process. Random variables represent uncertain factors in decision-making.

It is necessary for transportation enterprises to consider the random characteristics of demand before pricing and

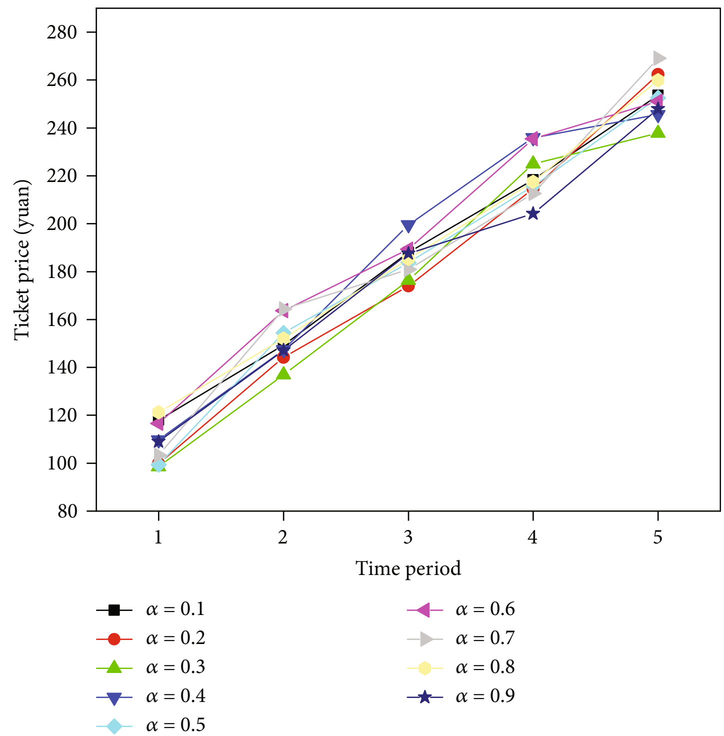

Figure 9: Ticket price of train A in OD 2.

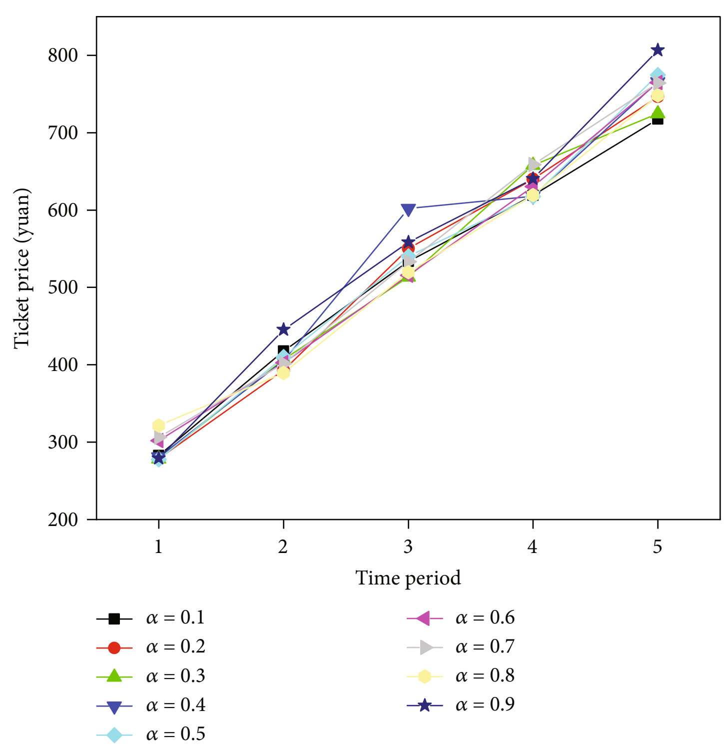

Figure 10: Ticket price of train A in OD 7.

ticket allocation. When faced with the influence of uncertain factors, different operation decision-makers have different risk appetites due to individual personality and cognition. In our study, we assume that passenger demand $q_{h,w,k}$ obeys normal distribution. $\lambda_{h,w,k}$ and $\sigma^2_{h,w,k}$ denote mean and variance of normal distribution, respectively, namely, $q_{h,w,k} \sim N$

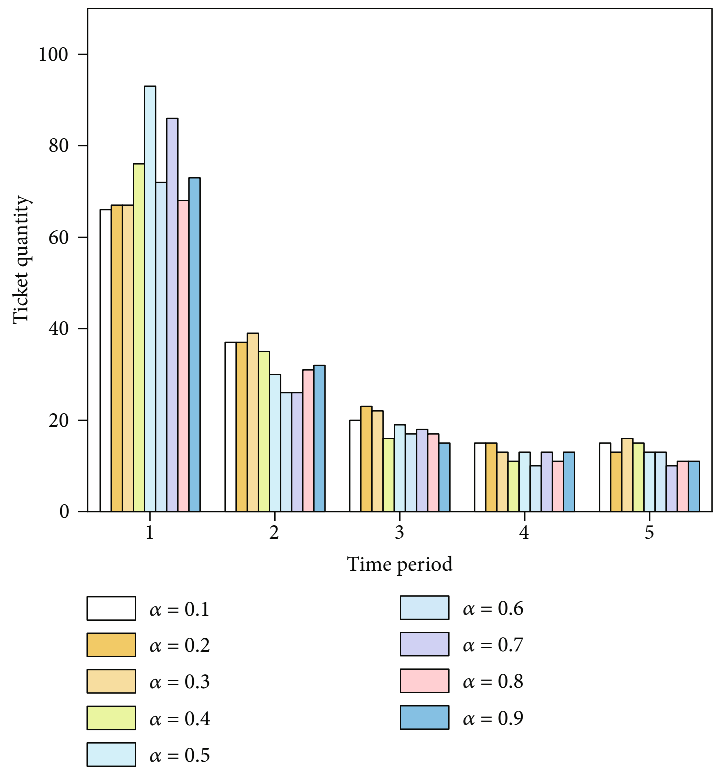

FIGURE 11: Ticket quantity of train A in OD 2.

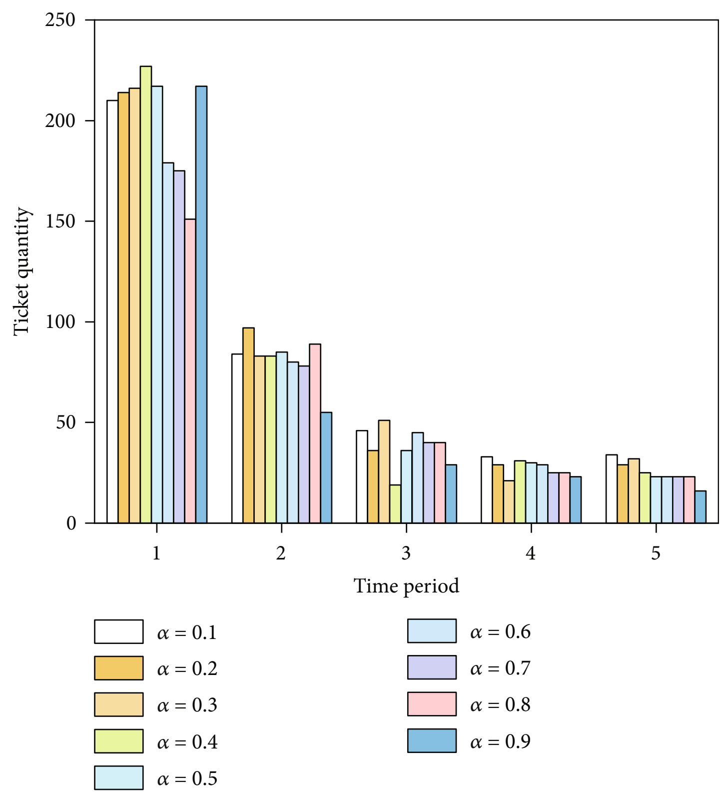

FIGURE 12: Ticket quantity of train A in OD 7.

Table 6: Comparison of the solutions under different algorithms.

Demand times	The total revenue (yuan) Lingo	Hybrid	PSO	Gap HL	PL
1	37160.62	35675.78	35069.47	-4.00%	-5.63%
2	76475.97	72353.88	71895.46	-5.39%	-5.99%
4	118954.8	115893.96	113181.82	-2.57%	-4.85%
6	148854.9	140604.81	133887.88	-5.54%	-10.05%
8	161845.2	157694.10	154744.22	-2.56%	-4.39%
10	167719.8	167043.52	161826.28	-0.40%	-3.51%

HL: hybrid-Lingo/Lingo; PL: PSO-Lingo/Lingo.

$(\lambda_{h,w,k}, \sigma^2_{h,w,k})$. Comprehensively considering the uncertainty of demand and the risk preference of operators, we construct the relationship between ticket allocation and demand as shown in the following:

$$\Pr\{x_{h,w,k} \le q_{h,w,k}\} \ge \alpha, \forall h \in H, w \in W, k = 1, \cdots, K, \quad (6)$$

where $x_{h,w,k}$ represents the amount of ticket allocated to $<h, w, k>$. According to the CCP theory, Equation (6) means the constraint $x_{h,w,k} \le q_{h,w,k}$ is at least established at the probability α when passenger demand changes randomly.

To facilitate solution and calculation, CCP needs to be transformed into deterministic programming. Many studies prefer to analyze random variables from the perspective of expectation and variance. Besides α, optimistic and pessimistic values can also be used to analyze uncertain factors. In this paper, we use α optimistic value of random variable, as shown in the following equation:

$$\xi_{\sup}(\alpha) = \sup \{r | \Pr\{\xi \ge r\} \ge \alpha\}, \quad (7)$$

where ξ is a random variable. α is within $(0, 1]$. $\xi_{\sup}(\alpha)$ is called α optimistic value of random variable ξ. α optimistic value is equal to the maximum value of r satisfying $\Pr\{\xi \ge r\} \ge \alpha$. $\xi_{\sup}(\alpha)$ is a decreasing function of confidence level α.

The following theorem exists for CCP.

Theorem 1. *Suppose ξ is a continuous and random variable, $\phi(x)$ is distribution function of ξ. When $0 < F(x) < 1$, $h(x, \xi) = g(x) - \xi$, then $Pr\{h(x, \xi) \le 0\} \ge \alpha$ if and only if $g(x) \le \xi_{\sup}(\alpha)$, where $\xi_{\sup}(\alpha) = F^{-1}(1-\alpha)$ [29].*

Based on Theorem 1, we can convert constraint (6) into its deterministic equivalent as shown in Equation (8), where $F^{-1}(x)$ is the inverse distribution function of the random variable $q_{h,w,k}$.

$$x_{h,w,k} \le F^{-1}(1-\alpha), \forall h \in H, w \in W, k = 1, \cdots, K. \quad (8)$$

Specially, when confidence level α equals 0.5, the constraint (8) can be converted into Formula (9) [20]. This means that neutral decision-makers make decisions based on the average demand.

$$x_{h,w,k} \le \lambda_{h,w,k}, \forall h \in H, w \in W, k = 1, \cdots, K. \quad (9)$$

3.4. Joint Decision Model. The operator decides the price $p_{h,w,k}$ and the ticket allocation $x_{h,w,k}$ for every $<h, w, k>$ to maximize the total revenue. The joint decision model is proposed as Formula (8) and (10)–(14).

$$R = \sum_h \sum_w \sum_k p_{h,w,k} x_{h,w,k}, \quad (10)$$

s.t. (8)

$$\sum_k \sum_w \sigma^s_{h,w} x_{h,w,k} \le C^s_h, h \in H, s \in S, \quad (11)$$

$$(1 - \mu_{h,w}) x_{h,w,k} = 0, \forall h \in H, w \in W, k = 1, \cdots, K, \quad (12)$$

$$p^-_{h,w,k} \le p_{h,w,k} \le p^+_{h,w,k}, \forall h \in H, w \in W, k = 1, \cdots, K, \quad (13)$$

$$x_{h,w,k} \in \{0\} \cup Z^+, \forall h \in H, w \in W, k = 1, \cdots, K, \quad (14)$$

where R represents the total revenue. The objective function (10) is to maximize the revenue of HSR network. Define S as the set of all sections. Constraint (11) means that the ticket amount of all OD pairs using section s cannot exceed the seat capacity of the train h in the section. $\sigma^s_{h,w}$ represents the relationship among train, OD, and section and is a 0-1 variable. Train h serves OD w and occupies section s, $\sigma^s_{h,w} = 1$, otherwise $\sigma^s_{h,w} = 0$. Constraint (12) shows that the relationship between train stop scheme and ticket allocation. $\mu_{h,w}$ also is a 0-1 variable. If train h provides service for OD w, $\mu_{h,w} = 1$. In this case, $x_{h,w,k} \ge 0$. Otherwise, $\mu_{h,w} = 0$ and $x_{h,w,k} = 0$. Considering the interests of enterprises and passengers, we set the price celling $p^+_{h,w,k}$ and price floor $p^-_{h,w,k}$. Constraint (13) is the upper and lower limits of the fare floating, which can be set according to pricing policies and operational needs. Constraint (14) makes sure the number of tickets is the positive integer.

4. Solution Algorithm

The proposed model in Section 3 is a nonlinear programming model. This model is difficult to solve directly and precisely. Comprehensively considering the model characteristics and solution efficiency, a hybrid algorithm integrating PSO and an exact algorithm for integer linear programming are designed in this paper. PSO [30] is a heuristic algorithm derived from the foraging behavior of birds. This algorithm possesses the advantages of fewer parameters, fast convergence speed, and easy implementation. Firstly, we use the PSO algorithm to generate particles randomly as ticket prices and iteratively optimize the fare by particle swarm. When the price $p_{h,w,k}$ is determined, the model can be transformed into a linear programming model which contains only one ticket allocation variable $x_{h,w,k}$. Then, we use CPLEX to solve the linear programming exactly. The algorithm settings and steps are described below.

4.1. Algorithm Settings

4.1.1. The Fitness Functions. The value of fitness function is an important index to evaluate the position of particles. In this article, the objective function is selected as the fitness function. If the fitness value is large, it represents that the revenue is high and the particle position is better.

4.1.2. Initial Particle Generation. The random numbers are uniformly generated in each interval $[a_{h,w,k}, b_{h,w,k}]$ as the initial particle position, that is, the price of the product $<h, w, k>$. We set the values of $a_{h,w,k}$ and $b_{h,w,k}$ increase with the time period k. The corresponding initial prices also increase as the time period increases. Keeping the initial price consistent with the law of dynamic elasticity helps the algorithm to obtain higher quality solutions.

4.1.3. Learning Factor and Inertia Weight. There are two learning factors in the PSO algorithm. c_1 is the flight step of the particle towards the individual best value. c_2 is the flight step of the particle towards the global best value. Let $c_1 = c_2 = 0.5$; ω is inertia weight and set as $\omega = 0.5$.

4.1.4. The Velocity Update Equation. The velocity is a key vector that determines the direction and distance of each particle. Updating the velocity is to determine which direction and how far the particle will fly in the next iteration. When a particle evolves from the tth iteration to the $t+1$th iteration, its velocity $v_i(t+1)$ can be updated according to Equation (15). It consists of three parts, including maintaining the motion habit of particles themselves, learning from individual experience, and exchanging with other particles. Therefore, the PSO algorithm makes full use of the information sharing of individuals in the group to make the movement of particles from disorder to order gradually.

$$v_i(t+1) = \omega v_i(t) + c_1 r_1 (p_i(t) - s_i(t)) + c_2 r_2 \left(p_g(t) - s_i(t)\right), \tag{15}$$

where $v_i(t)$ denotes the velocity of particle i in the tth iteration. $p_i(t)$ is the optimal value of particle i after the tth iteration. $p_g(t)$ is global optimal value after the tth iteration. $s_i(t)$ is the position of particle i in the tth iteration. r_1 and r_2 are random numbers in interval $(0, 1)$.

4.1.5. The Position Update Equation. According to the current position of the particle and the updated velocity, we can use Equation (16) to obtain the new position.

$$s_i(t+1) = s_i(t) + v_i(t+1). \tag{16}$$

4.1.6. The Termination Conditions. The termination conditions commonly used in PSO include the prespecified generation number as their terminations or stop the iteration when the fitness function value does not change. We adopt the former and set the maximum number of iterations as $t_{\max}$.

4.2. Algorithm Steps. The specific steps of the algorithm are shown in Algorithm 1.

5. Numerical Experiments

5.1. Data. A HSR line with 8 stations serves 28 OD pairs. The train service network is shown in Figure 1. We take the second-class seat as an example to conduct the experiments. The capacity of each train is 560, and the trains are not allowed to be overloaded.

The whole presale period is divided into 5 stages. The specific price elasticity coefficient of each ticket selling stage is shown in Table 1. The ticket demand of each OD pair follows normal distribution. The utility parameters $d_{h,w}$, $t_{h,w}$ and the original ticket price p_w [4], average demand λ_w, and variance of demand σ^2 are shown in Table 2. Assume that demand is evenly distributed over 5 time periods. The lower limits and upper limits of ticket prices in this paper are 0.5 and 1.5 times of the original ticket prices, respectively. The values of parameters θ and τ are 0.012 and 36, respectively.

5.2. Result and Discussion. We performed the numerical experiments on a laptop with Windows operating system, Intel Core i5 processor, and 16 G RAM. Set maximum number of iterations $t_{\max} = 30$. The confidence level of α is set as 0.9. The whole process took about 3.205 minutes to get the results. Figure 2 shows the convergence of total revenue and cumulative optimization proportion with the increase of iteration. The revenue growth rate is fast at the beginning and then gradually slows down. After 22 iterations, it has basically reached the optimal value. The total revenue under the collaborative optimization scheme is 1130381.824 yuan. Compared with the total revenue 1008787.5 yuan under the single fixed fare scheme, it has increased by 12.05%.

From the properties of normal distribution, we know that $\lambda_{h,w,k}$ determines the position of distribution function and the variance σ^2 determines the concentration of the distribution function. The smaller the variance is, the more concentrated the data distributes and the less the demand fluctuates. Figure 3 shows the total revenue under different variances when the confidence level is 0.9. A small variance means that the demand is more stable. We can see that as the variance increases, the total revenue decreases under fixed fare scheme. With the small fluctuation of passenger flow demand, operators tend to allocate more seats. In joint optimization, with the increase of demand fluctuation, the total revenue also shows a decreasing trend overall. But it is not strictly descending due to the dynamic characteristics of ticket prices. As the variance increases, the total revenue of the joint optimization scheme is always higher than that of the fixed fare scheme and the improved proportion is about 11.83%-12.64%. Figure 4 shows the revenue under different demand levels. With the increase of demand, the total revenue increase under both schemes. Compared with the fixed price, the increase in revenue mainly comes from two aspects. On the one hand, it is to meet the high-priced demand as much as possible, and on the other hand, it is to reduce the price to attract more demand to fill the empty seat. When the travel demand is high, the price increase is the main aspect of increasing revenue, and when the demand is small, the price reduction to attract demand is the main aspect of increasing revenue. It can be seen from Figure 4 that under different demand scales, joint optimization can better match supply and demand to achieve increased revenue. We also narrowed the price fluctuation range to 0.8-1.2 times of the original price for testing. The results show that the proportion of revenue increase decreases in all demand states. Under the original demand, the proportion of increase in revenue was reduced to 7.22%. A wider range of ticket price fluctuations can help boost revenue.

To observe the price changes, we select OD pair 2 and OD pair 7 to investigate the price of each train in each time period. It can be clearly seen from Figures 5 and 6 that trains on the same OD at the same time period have different prices. We find that ticket price shows an upward tendency as the departure time of train approaches. In the early stage of presale period, passengers are highly sensitive to ticket price. If the price is given an appropriate reduction, more passengers will be encouraged to choose HSR train. At the end of the presale period, passengers are less sensitive to ticket price and the operators can raise the price properly to increase income.

Figures 7 and 8 show the ticket allocation schemes of OD pair 2 and OD pair 7, respectively. As the period increases, the tickets allocated to each OD decreases. This is because the experiment assumes that the initial demand is evenly distributed in each period for the convenience of calculation. Such a setting can show the effect of price elasticity on ticket allocation. As the time period increases, the price elasticity decreases, and the joint decision model adopts the method of increasing the price to increase the revenue, so the ticket volume decreases. Train A has a higher ticket allocation at OD pair 7 than at OD pair 2. Most of the demand of OD pair 7 is satisfied by train A, while the demand of OD pair 2 is allocated to other trains in a higher proportion. The joint decision-making model can ensure that the big station express trains meet the long-distance demand as soon as possible, which conforms to the general rule of ticket allocation. Table 3 and Table 4, respectively, show the full version of the ticket price and ticket allocation plan when the confidence level is 0.9.

To explore the influence of decision-makers' risk preference, we keep other parameters unchanged and only change the value of α to conduct experiments. On the one hand, the confidence level can measure the degree to which constraint $x_{h,w,k} \leq q_{h,w,k}$ is met. On the other hand, it indirectly reflects the risk preference of the operator. We set different confidence levels, $\alpha = 0.1, 0.2, \cdots, 0.9$, successively under single pricing and joint optimization scheme, respectively. The results are shown in Table 5.

According to the data in Table 5, we made the following analysis.

When confidence level is less than 0.5, the HSR operator will organize more tickets to satisfy predictive passenger needs. That means decision-makers are heavily weighted in predictive demand, but actual passenger demand is uncertain. We can infer that the decision-maker is radical in this situation.

When confidence level is equal to 0.5, the demand fluctuation no longer exists according to the analysis in Section 3. The operator will use all the allocated tickets to meet the determined passenger demand. Currently, the HSR operator belongs to decision neutral type.

When confidence level is more than 0.5, the decision-maker will allocate a small amount of ticket to meet anticipated demand. That means decision-makers are not totally convinced anticipated passenger needs. Therefore, the HSR operator is conservative in this situation.

As confidence level decreases, the overall income shows an upward trend in Table 5. The total income is determined by ticket price and ticket amount. Taking train A as an example, Figures 9 and 10 show that the ticket prices increase with time periods under different confidence levels. Figures 11 and 12 show that the allocated tickets decrease with time period under different confidence levels. No matter under different time periods or different risk appetite levels, the number of tickets is lower when the price is high, and the number of tickets is higher when the price is low. The model optimizes the total expected revenue under the combined action of pricing and ticket allocations. The lower the confidence level, the more relaxed the capacity constraint and the greater the total expected revenue.

We can see that under the same confidence level α, the total income under the joint optimization scheme is always higher than that under a single fixed ticket price scheme. The specific value of the optimized proportion under each confidence level can be seen in the last column of Table 5. The percentage of revenue improvement is about 11.84%-13.40%. Therefore, the collaborative optimization is a better choice when formulating the ticket price and allocation scheme.

To test the algorithm performance, we intercepted the small network composed of stations 2-4 and trains C and D and their data to conduct experiments. The fare range is set to 0.8-1.2 times of the original fare. The confidence level is taken as 0.9. We multiply the average demand by different multiples and use Lingo, the particle swarm optimization algorithm, and the algorithm in this paper to solve the problem, and the results are shown in Table 6. Lingo adopted branch and bound algorithm to solve the model. A new particle swarm optimization algorithm in which two types of variables were simultaneously optimized as particles was designed to compare with the hybrid particle swarm optimization algorithm proposed in this paper. The results show that the gap between the solution of the algorithm in this paper and that of Lingo is about 0.4%~5.4% and the gap between the solution of the ordinary particle swarm and that of Lingo is about 3.5%~10.1%, indicating that the solution quality of the proposed algorithm is good. We also imputed the data in Section 5.1 into Lingo, and the calculation had not been completed for 24 hours. We tried the new algorithm with the data in Section 5.1 and found that the total revenue was about 2% less than the hybrid algorithm. In summary, for large-scale problems in practical applications, the hybrid algorithm proposed in this paper can provide a satisfactory scheme to improve revenue within a feasible time.

6. Conclusions

Considering the randomness of demand and the operator's risk preference, this paper introduces the CCP theory to study the joint decision of pricing and ticket allocation. Based on the changing law of price elasticity over time, this paper divides the whole presale period into several stages. After describing passenger choice behavior, elasticity of demand, and operator's decision preference, the joint optimization model of dynamic pricing and ticket allocation is established. Then, we design a hybrid algorithm, which combines PSO algorithm and the exact solution of integer linear programming algorithm. This algorithm has lower complexity and acceptable speed. The numerical experiment results show that introducing CCP theory, the total revenue under the joint optimization scheme is always higher than that under the single fixed fare scheme at the same confidence level. The percentage of revenue improvement is about 11.84%-13.40%. Under different levels of demand, demand fluctuations, and risk preferences, the joint decision-making always achieves higher returns than ticket allocation under fixed fares. In addition, a larger price range is more conducive to increasing revenue. It is recommended that the operator set a large fluctuation range as required. This study can provide a reference for the operators to reasonably deal with demand uncertainty and decision preferences to make the joint decision of dynamic pricing and ticket allocation, which is of great significance to improve enterprise revenue and alleviate losses. Future research can be expanded by considering overbooking, no-show, refunds, etc.

Data Availability

The data used to support the model and analysis of this study are presented within the article.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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