Combined Phase Design Model for Multileg Roundabout Intersections

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1. Introduction

A roundabout is one of the most common intersection forms of a modern road network and has the advantages of simple organization and fewer conflicts. Compared to non-signalized crossing intersections, roundabouts can substantially reduce traffic crashes and delays in the free flow state [1]. However, when a roundabout tends to saturate, traffic flow from each approach would obtain the right of way simultaneously, which results in the traffic flows’ conflict and blocks the roundabout from obtaining maximum efficiency/capacity. Therefore, signal control can be introduced to address this issue and further improve roundabout efficiency [2].

Although fixed-time signal control or multiperiod fixed-time control is a common signal control approach, it is only suitable for intersections with stable traffic. For multileg roundabouts with unstable traffic, actuated signal control has been adopted to adapt to the fluctuation of traffic flow.

However, the application of actuated signal control in roundabouts has failed to achieve satisfactory performance for a long time since roundabout traffic flows are too complex. According to the observation, the actuated signal control is not effective. Therefore, extensive research efforts from various traffic scholars have been put into this field, which can be mainly classified into the following two categories: roundabout capacity and roundabout signal control.

In terms of roundabout capacity, there are three capacity calculation methods for roundabouts: gap acceptance theory, empirical regression model, and interleaving theory. The gap acceptance theory [3, 4] mainly analyzes the capacity of each approach to the roundabout. The empirical regression model [5–7] calculates the roundabout capacity by establishing a regression equation between the entry capacity and the circulating flow. The interleaving theory [8] takes the maximum traffic in the weaving section as the roundabout capacity according to the short board effect.

As for roundabout signal control, it is mainly divided into control parameter optimization and phase optimization. Some existing studies considered optimizing parameters such as green time [9, 10] and signal cycle [11–13] to improve the traffic efficiency of roundabouts. Shiri and
Maleki [14] optimized the maximum green light time parameter based on fuzzy control to better adapt to traffic demand changes. Ma et al. [15] proposed a method that can optimize lane markings and signal timing simultaneously to obtain maximum capacity, minimum cycle time, and minimum delay at roundabouts. However, the above signal control research does not consider the optimization of the phase. In the study of phase optimization, Jiang et al. optimized the design of phases, such as considering the phase of the coordinated signal in the circulating lanes and the approach carriageways [16], as well as quantitatively analyzing a certain type of phase—the left-turn phase [17]. Wu et al. [18] introduced a new phase into the actuated signal control, which could control the traffic flow of reversed left-turns so as to maximize the utilization rate of both the left-turn lanes and the reversed lanes. Xia and Xu [19] took each phase as the research object and proposed a phase coordination control method based on the negotiation game optimization model. Considering the different functions of lanes, Nei and Ma [20] established a lane-based intersection lane function and signal phase optimization model. Shen et al. [21] proposed a method of traffic phase combination and signal timing optimization based on the improved K-Medoids algorithm. Liang et al. [22] proposed an overlapping phase-based signal control logic and a bus priority control algorithm under two-way signal coordination on arterial roads.

It is found from the above literature that two typical zones of a roundabout—either the approach zone or the weaving zone—were selected for capacity calculation. The roundabout capacity is calculated based on the vehicle characteristics of these zones. However, the above-mentioned existing studies did not consider the roundabout as a whole and ignored the negative impacts of other bottleneck areas on the traffic efficiency of the roundabout. Moreover, each structural part of the roundabout would also affect the overall capacity. As for the studies on roundabout signal control, most of the existing studies still use fixed phase schemes, which lack consideration of dynamic changes in traffic demand (unstable traffic fluctuations and complex turning) at roundabouts. The current research on phase optimization is mostly based on the relatively stable phase, which cannot be adjusted actively with the real-time traffic flow, so the control effect is compromised. The research on the signal phase of roundabouts is relatively rare, and most of them focus on only a specific traffic flow state, which make them difficult to be widely used.

To solve these problems, this study attempts to consider and analyze each structural component of the roundabout and calculate the capacity according to the characteristics of different zones. Then, a combined phase design model is proposed to maximize the roundabout capacity utilization by enabling dynamic phasing scheme changes based on traffic demand.

2. Roundabout Traffic Flow Characteristics and Capacity Analysis

A signal phase specifies a combination of one or more traffic movements receiving the right of way simultaneously during a signal interval [23]. Phase design is constrained by the structure, traffic flow characteristics, and traffic capacity of the intersection.

2.1. Roundabout Structure. A roundabout consists of the following structures: a central island, a circulating lane, an approach carriageway, a departure carriageway, and a splitter island. The different geometric parameters of these structures and the different interactions among vehicles would lead to very different vehicle operation characteristics, which would affect the traffic efficiency of the roundabout. Therefore, the division of multileg roundabouts into different areas is crucial [24].

In a typical roundabout, there are five areas with different traffic flow features: the approach area, the merge area, the lane-changing area, the diverge area, and the departure area. As Figure 1 shows, the five areas of the carriageway/leg i of a roundabout are respectively represented by $A_i$, $B_i$, $C_i$, $D_i$, and $E_i$.

Compared to straight roads, traffic flow operations in weaving zones would face shorter lane-changing distances and more complex conflicts because the traffic flow must merge, lane-change, and divert within such short distances and must slow down or even stop to wait for the right of way, which forms a bottleneck area. Therefore, in order to improve the roundabout’s operational efficiency, this paper first analyzes the capacity of each bottleneck area.

2.2. Capacity. Usually, the capacity of the approach area or departure area is similar to that of the straight road, and it is much larger than the capacity of the merge area, the lane-changing area, or the diverge area. Therefore, only the capacities of the latter three areas will be discussed in the following sections.

Assuming $T_{i,k}$ is the traffic volume generated from carriageway i and goes to the carriageway k (pcu/h). $T_i = \sum_{k=1}^{N} T_{i,k}$ is the total traffic volume generated from the approach of carriageway i. Then, the capacity of the merge area, the lane-changing area, and the diverge area can be acquired as follows.

2.2.1. Merge Area. The actual capacity of the merge area can be calculated as follows [25]:

$$Q_{Bi} = \sum_{x=1}^{y} \frac{3600}{h_x},$$  \hspace{1cm} (1)

where $Q_{Bi}$ is the actual capacity of merge area $B_i$ (pcu/h); $y$ is the number of circulating lanes; $h_x$ is the average time headway between vehicles for saturated traffic conditions in circulating lane $x(s)$.

When calculating the traffic volume in a particular merge area, the traffic flows originating from each approach should be considered, but only part of them will pass through that merge area. Take the merge area $B_1$ (See Figure 1) as an example. Traffic flows generated from carriageway 1 will all pass through the merge area $B_1$, and traffic flows generated from carriageway 2 will also pass through $B_1$ except for those
leaving the roundabout via departure carriageway 1. Similarly, it is easy to derive that traffic flows generated from the carriageway \( N \) will pass through the merge area \( B_1 \) except for those leaving the roundabout via departure carriageway 1, 2, 3, …, and \( N - 1 \). Therefore, the traffic volume of the merge area \( B_1 \) is
\[
\sum_{i=1}^{N} \sum_{k=1}^{N} T_{i,k}.
\]
Assume that \( m \) is an integer, one can also obtain the traffic volume of the merge area \( B_i \) as
\[
T_{B_i} = \sum_{m=i}^{N+i-1} \sum_{k=m}^{N+i-1} T_{m,k}.
\]

2.2.2. Lane-Changing Area. In this area, the actual road capacity is closely related to the drivers’ lane-changing behaviour, and it can be calculated as follows [26]:
\[
Q_{C_i} = \begin{cases} 
  v_{f_i} \rho_i, & 0 \leq \rho_i \leq \rho_{ai} \\
  \frac{\rho_{ai} - \rho_{bi}}{v_{f_i}^2} \rho_{bi} - \rho_i (1 + \varepsilon_i), & 1 + \varepsilon_i \leq \rho_i \leq \rho_{bi} / (1 + \varepsilon_i) \\
  \frac{\rho_{bi} - \rho_{ai}}{v_{f_i}^2}, & \rho_i \geq \rho_{bi} / (1 + \varepsilon_i) 
\end{cases}
\]
where \( Q_{C_i} \) is the actual capacity of the lane-changing area \( C_i \) (pcu/(h·ln)); \( v_{f_i} \) is the free flow speed in the area \( C_i \) (km/h); \( \rho_i \) is the total density of the lane-changing traffic density and nonlane-changing traffic density (pcu/(km·ln)); \( \rho_{ai} \) is the critical density of the area \( C_i \) (pcu/(km·ln)); \( \rho_{bi} \) is the jam density (pcu/(km·ln)); \( \varepsilon_i \) is the lane-changing intensity of area \( C_i \), which can be calculated as follows:
\[
\varepsilon_i = \begin{cases} 
  0, & \rho_i \leq \rho_{ai} \\
  2 - 2\rho_i / \rho_{bi}, & 15 + 2\rho_i / \rho_{bi} \leq \rho_i \leq \rho_{ai} \\
  \rho_i / \rho_{bi}, & \rho_i \leq \rho_{bi} / \rho_{ai} 
\end{cases}
\]

Since area \( C_i \) is adjacent to area \( B_i \), it is easy to derive that the traffic volume \( T_{C_i} \) equals the traffic volume \( T_{B_i} \), which is expressed as follows:
\[
T_{C_i} = T_{B_i}.
\]

2.2.3. Diverge Area. To simplify the combined phase design model, we assume that the drivers have completed lane changes before entering the diverge area. In addition, by assuming that the roundabout has a dedicated right-turn lane, vehicles in the diverge area will not conflict with each other. Therefore, the maximum actual capacity \( Q_{D_i} \) that passes through the diverge area \( D_i \) is the actual capacity

![Figure 1: Structure of a multileg roundabout.](image-url)
leaving the lane-changing area \( C_{i+1} \) which can be calculated using
\[
Q_{Di} = Q_{Ci+1}.
\]  
(6)

After analyzing the traffic flow in each direction, it was found that the vehicles passing through area \( C_{i+1} \) were flowing to the area \( D_i \). As such, the traffic volume \( T_{Di} \) can be acquired as follows:
\[
T_{Di} = T_{Gi+1}.
\]  
(7)

3. Combined Phase Design Model

3.1. Design Principles. For the nonoversaturated state, this paper proposed the following five regulations for the combined phase design model:

(1) Each roundabout approach must set a green light at least once in a cycle.

(2) The number of phases in a cycle should be kept as small as possible.

(3) The traffic volume in each roundabout area must not exceed its capacity.

(4) The red lights should be as few as possible in a single phase. Set red lights preferentially for approaches with smaller traffic volumes.

(5) The queuing vehicles in the corresponding approach shall be released within one green light.

3.2. Mathematical Expression

3.2.1. Traffic Volume. According to the definition of \( T_i \) in section “Capacity,” the traffic volume at the multileg roundabout is as follows:
\[
T = [T_1, T_2, \ldots, T_i, T_{i+1}, \ldots, T_N]^T = [T_{ik}]_{N \times N},
\]  
(8)
where \( i \) is the order of approach carriageways; \( k \) is the order of departure carriageways; \( N \) is the total number of approaches.

3.2.2. Phase. In this paper, we use 1 to indicate that the approach carriageway is set green light and 0 to indicate that the approach carriageway is set red/amber light, which means
\[
U_{ij} = \begin{cases} 
1, & \text{when Approach } i \text{ set green light in Phase } j, \\
0, & \text{when Approach } i \text{ set red/amber light in Phase } j,
\end{cases}
\]  
(9)
where \( j \) is the order of phase and \( U_{ij} \) refers to the light state of approach \( i \) in phase \( j \).

If we further define \( U_j \) as the light states of all the approach carriageways in phase \( j \), the phase matrix \( U \) can be expressed as
\[
U = [U_1, U_2, \ldots, U_j, U_{j+1}, \ldots, U_M]^T = [U_{ij}]_{M \times N},
\]  
(10)

where \( M \) is the total number of phases expressed as \( M = C_0 + C_1 + C_2 + \ldots + C_N^{-1} = 2^N - 1 \).

3.2.3. Phase Scheme. Since the signal phase scheme is different combination of phases in one cycle, it can be described as follows:
\[
P = \{p_1, p_2, \ldots, p_l, p_{l+1}, \ldots, p_L\},
\]  
(11)
where \( l \) is the order of phase scheme. Taking \( v \) for a full array, it is an integer and takes values in the range of \([1, 2^N - 1]\). We take the value of \( 2^N - 1 \) as an example, and the number of arrays is \( A^{2^N-1}_{2^N-1} \). However, it should be noted that \([1, 2, \ldots, 2^N - 1], [2, 3, \ldots, 2^N - 1, 1], [3, 4, \ldots, 2^N - 1, 1, 2], \ldots, [2^N - 1, 2^N - 2, \ldots, 2, 1]\) all refer to the same phase scheme. Hence, the actual number of phase schemes is \( A^{2^N-1}_{2^N} \). Thus, the total number of possible phase schemes \( L \) is described as follows:
\[
L = \frac{A^{2^N-1}_{2^N-1}}{1} + \frac{A^{2^N-1}_{2^N-1}}{2} + \ldots + \frac{A^{2^N-1}_{2^N-1}}{2^N - 1},
\]  
(12)

3.2.4. Passable Traffic Volume. The traffic that can pass in one cycle is
\[
T_V = \sum_{j=1}^{P} \sum_{i=1}^{N} \sum_{k=1}^{N} (T_{ik} \cdot U_{ij}),
\]  
(13)
where \( P \) is the order of phases in a cycle; \( T_V \) is the traffic that can pass in one cycle.

3.3. Model Formulation. According to the design principles introduced in section “Design Principles,” the proposed combined phase design model is described as follows:
\[
\text{max } T_V = \sum_{j=1}^{P} \sum_{i=1}^{N} \sum_{k=1}^{N} (T_{ik} \cdot U_{ij}),
\]
\[
\begin{align*}
T_{Bi} & \leq Q_{Bi}, \\
T_{Ci} & \leq Q_{Ci}, \\
T_{Di} & \leq Q_{Di}, \\
\sum_{j=1}^{N} U_{ij} & \geq N, \\
N & \geq Z(U_{j+1} - U_j) \geq 1
\end{align*}
\]  
(14)

3.4. Explanation of Model Conditions

3.4.1. Capacity of Each Roundabout Area. The traffic volume in each roundabout area must not exceed its capacity, which corresponds to
3.5. Model Objectives. The optimization objective of the combined phase design model is to make more vehicles pass through the roundabout in a single signal cycle, which is equivalent to

\[
T_{Bi} \leq Q_{Bi},
\]

\[
T_{Ci} \leq Q_{Ci},
\]

\[
T_{Di} \leq Q_{Di}.
\]

(15)

3.4.2. Restriction of Setting Green Light. Each roundabout approach should set a green light at least once in a cycle, which is equivalent to

\[
\sum_{j=1}^{P} \sum_{i=1}^{N} U_{i,j} \geq N.
\]

(16)

3.4.3. The Front Phase and Following Phase. To keep the signal cycle as short as possible, when setting up the next phase, at least one of the approach carriageways that was set as a red light in the previous phase should be set green, which means

\[
N \geq Z(U_{j+1} - U_{j}) \geq 1,
\]

where \(Z(U_{j+1} - U_{j})\) is the number of times accumulated when \(U_{i,j} = 1\).

3.4.4. Number of Phases. As the number of phases increases, the loss time will also increase. Therefore, the number of phases \(P\) in a cycle should be kept as small as possible.

\[
\min P.
\]

(18)

3.5. Model Objectives. The optimization objective of the combined phase design model is to make more vehicles \(T_{V}\) pass through the roundabout in a single signal cycle, which is

\[
\max T_{V} = \sum_{j=1}^{P} \sum_{i=1}^{N} \sum_{k=1}^{N} (T_{1k} \cdot U_{i,j}).
\]

(19)

3.6. Solution Algorithms. Since there are \(2^N - 1\) phase combination schemes at roundabouts with \(N\) legs, the number of phase schemes will increase exponentially with the number of legs. In addition, given that we also need to consider the variability of traffic flow directions and the real-time demand of phase schemes, it is impractical to solve the phase schemes manually. Instead, we use computer tools, such as Python, VB, and so on, to solve the schemes.

Due to the finiteness of the phase schemes, we use the Python programming language to build and solve the combined phase design model. Specifically, the programming flowchart is shown in Figure 2, and the detailed steps are also listed as follows:

Step 1 Input the traffic volume of each direction at the roundabout and the parameters related to the actual capacity of each area

Step 2 Combine all approach carriageways of the roundabout to generate \(M\) phases

Step 3 Calculate the actual capacity and traffic volume of each merge area, lane-changing area, and diverge area at phase \(j\)

Step 4 If the results from Step 3 for phase \(j\) meet the conditions formulated by equations (15), save phase \(j\) or else go to Step 5

Step 5 Let \(j = j + 1\), if \(j > 2^N - 1\), go to Step 6 or else return to Step 3

Step 6 Use all saved phases as Phase Scheme Set \(U\). Arrange the alternative phases and remove any identical phases to obtain a Phase Scheme Set \(p\)

Step 7 If Phase Scheme \(p_{l}\) meets the conditions formulated by equations (16)–(18), save \(p_{l}\) to \(p\) and calculate \(T_{C}\) of \(p_{l}\) or else go to Step 8

Step 8 Let \(l = l + 1\), if \(l > L\), go to Step 9 or else return to Step 7

Step 9 Comparing \(T_{C}\) of all phase schemes in \(p\), the phase scheme satisfying equation (19) is the model output

4. Case Study

4.1. Current Situation. The Hou-Cheng-Li Rd./Bei-Shan Rd. Roundabout (hereinafter referred to as the target roundabout) is a five-leg roundabout located on the 1st Ring Road of Jinhua. The west of the roundabout leads to the Jinhua Railway Station and the West Jinhua Highway Station, the north leads to Guang-fu Hospital, and the south leads to the high-density residential community and a thriving business district. In a word, the roundabout is a key node of the local road network.

At present, the roads in Jinhua city mostly use green wave control. However, the target roundabout not only has morning and evening peaks but also has significant traffic flow fluctuations throughout the day. Therefore, even the green wave control scheme did not work well at the roundabout.

Table 1 and Figure 3 summarize the current situation of the target roundabout.

4.2. Traffic Survey. The traffic flow data for the target roundabout was provided by the Traffic Police Brigade of Jinhua and selected for the morning peak period (7:00 am to 9:00 am). Table 2 summarizes the traffic volume for 2 cycles at the roundabout. As shown in Figure 4, the current timing scheme has two phases, and the signal cycle is 75 seconds.

4.3. Capacity Calculation. According to section “Capacity,” by substituting the mean value of the saturation headway into equation (1), the actual capacity \(Q_{Bi}\) of each merge area is obtained, and by substituting the free flow velocity \(v_{fj}\), the total density \(\rho_{i}\), the critical density \(\rho_{ai}\), the jam density \(\rho_{bi}\), and the lane-changing intensity \(\epsilon_{i}\) into equations (3) and (6), the actual capacity \(Q_{Ci}\) of each lane-changing area and the actual capacity \(Q_{Di}\) of each diverge area are obtained, which are shown in Table 3.
Meanwhile, the following 3 points should be noted.

### 4.3.1. Merge Area

The video survey method was used to derive the saturation headway for each merging area, as shown in Table 4.

### 4.3.2. Lane-Changing Area

According to the statistical analysis of the on-site traffic data at the target roundabout, the parameter values of free flow velocity $v_f$, total density $\rho_i$, critical density $\rho_{ai}$, jam density $\rho_{bi}$, and lane-changing intensity $\epsilon_i$ are shown in Table 5.

Other parameters can be expressed as

**Table 1: The number of lanes on each leg.**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of approach lanes</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Number of departure lanes</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Note: the Huan-Cheng West Rd. is marked as approach carriageway 1, and Bei-Shan Rd., Huan-Cheng North Rd., Feng-Ting West Rd., and Hou-Cheng-Li Rd. are labeled in clockwise direction as approach carriageway 2, 3, 4, and 5, respectively. The circulating lane closest to the central roundabout is marked as circulating lane 1, and the outer circulating lanes are marked as circulating lanes 2, 3, and 4 in sequence.
Figure 3: Current traffic organization of the roundabout.

Table 2: Traffic volume (pcu/h).

<table>
<thead>
<tr>
<th>Approach no.</th>
<th>Cycle no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Approach traffic volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>44</td>
<td>214</td>
<td>623</td>
<td>26</td>
<td>82</td>
<td>989</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>52</td>
<td>243</td>
<td>678</td>
<td>25</td>
<td>95</td>
<td>1093</td>
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<td>2</td>
<td>1</td>
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<td>81</td>
<td>22</td>
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<td>726</td>
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<tr>
<td></td>
<td>2</td>
<td>271</td>
<td>29</td>
<td>71</td>
<td>26</td>
<td>320</td>
<td>717</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>425</td>
<td>315</td>
<td>57</td>
<td>21</td>
<td>184</td>
<td>1002</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>364</td>
<td>293</td>
<td>35</td>
<td>13</td>
<td>179</td>
<td>884</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>6</td>
<td>16</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
<td>12</td>
<td>4</td>
<td>0</td>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>84</td>
<td>168</td>
<td>190</td>
<td>18</td>
<td>19</td>
<td>479</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>70</td>
<td>161</td>
<td>195</td>
<td>12</td>
<td>15</td>
<td>453</td>
</tr>
</tbody>
</table>

Figure 4: Current signal timing scheme.
4.3.3 Diverge Area. The source of the various parameters of the diverge area is the same as the lane-changing area. According to equation (6), the various parameter values of the diverge area $D_i$ are shown in Table 6.

4.4. Phase Design. In cycle 1, based on Step 1 of the flowchart in Figure 2, we input the traffic volume of each flow at the target roundabout and the relevant parameter values of each area into equation (14). In addition, since Huan-Cheng West Rd. has dedicated U-turn and right-turn lanes, the corresponding traffic flows ($T_{1,1}$ and $T_{1,5}$) will not pass through each merge area, lane-changing area, and diverge area. As such, $T_{1,1}$ and $T_{1,5}$ are set as 0.

According to Steps 3–6 of the flowchart, the Phase Scheme Set $\mathcal{U}$ that meets the capacity constraints of each area can be obtained as

\[
\mathcal{U} = \begin{bmatrix}
0 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\] (20)

Then, according to Steps 7-8 of the flowchart, the Phase Scheme Set $\mathcal{P}$ is formulated as follows:

\[
\mathcal{P} = \begin{bmatrix}
0 & 0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\] (21)

Note: the length of the lane-change area $L_{lc}$, passing time $T_{lc}$, traffic volume $Q_{lc}$, saturation traffic volume $Q_{slc}$, and distance headway $D_{lc}$ are obtained through the camera survey. $D_{lc}$ is 9.95 m. $\varepsilon_i$ is obtained from equation (4).
By substituting the traffic volume of each approach carriageway into equation (13), the traffic volume $T_V$ of each phase scheme in $p$ is obtained as

$$ T_V = \begin{pmatrix} 4300, 4252, 3821, 3773, 4252, 3773, 4204, \\ 3773, 3725, 3725, 3342, 3294, 3294, 3246 \end{pmatrix}. $$

At last, according to Step 9 of the flowchart, adopt the phase scheme with the maximum traffic volume as the final output, which is

$$ \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}. $$

Specifically, the first phase is 3, 4, and 5; the second phase is 1, 4, and 5; and the third phase is 2, 4, and 5. The actual signal timing for the above-mentioned phase scheme was obtained by using the Webster method [11]. The results are shown in Figure 5.

In cycle 2, following the above process, the phase scheme with the maximum traffic volume is obtained as

$$ \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}. $$

The actual signal timing for the above phase scheme was obtained by using the Webster method. The results are shown in Figure 6.

### 4.5. Traffic Organization Optimization

Before implementing the optimal phase scheme generated from the proposed model, the traffic organizations at the target roundabout need to be adjusted accordingly.

1. The circulating lane signal is removed
2. The circulating lane markers are also changed accordingly

The improved traffic organization of the target roundabout is shown in Figure 7.

### 4.6. Simulation and Performance Comparison

This section simulates and compares the performances of the current signal configuration and the optimized signal scheme for the target roundabout using VISSIM, as shown in Figure 8.

The input traffic volumes, the simulated current, and the simulated optimized traffic volumes of each approach carriageway of the target roundabout are summarized in Table 7.

### 5. Results and Discussion

Three common intersection evaluation indicators including average queue length, average number of stops, and average delay are adopted for performance comparison. After the
Figure 5: Optimized signal timing in cycle 1 for the model output scheme.

Figure 6: Optimized signal timing in cycle 2 for the model output scheme.

Figure 7: Optimized traffic organization of the target roundabout.
Figure 8: Simulation and comparison of the target roundabout performances using VISSIM.

Table 7: The input and simulated traffic volumes for each approach carriageway (veh).

<table>
<thead>
<tr>
<th>Approach no.</th>
<th>Cycle no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>1</td>
<td>991</td>
<td>726</td>
<td>1002</td>
<td>28</td>
<td>479</td>
<td>3246</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1093</td>
<td>717</td>
<td>884</td>
<td>24</td>
<td>453</td>
<td>3171</td>
</tr>
<tr>
<td>Current</td>
<td>1</td>
<td>897</td>
<td>704</td>
<td>945</td>
<td>18</td>
<td>412</td>
<td>2976</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>974</td>
<td>698</td>
<td>801</td>
<td>17</td>
<td>389</td>
<td>2879</td>
</tr>
<tr>
<td>Optimized</td>
<td>1</td>
<td>869</td>
<td>710</td>
<td>957</td>
<td>19</td>
<td>446</td>
<td>3001</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>983</td>
<td>711</td>
<td>846</td>
<td>20</td>
<td>405</td>
<td>2965</td>
</tr>
</tbody>
</table>

Table 8: Performance comparison results in cycle 1.

<table>
<thead>
<tr>
<th>Approach no.</th>
<th>Average queue length (m)</th>
<th>Average number of stops (times)</th>
<th>Delay (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current</td>
<td>Optimized</td>
<td>Current</td>
</tr>
<tr>
<td>1</td>
<td>39.0</td>
<td>42.2</td>
<td>0.38</td>
</tr>
<tr>
<td>2</td>
<td>26.1</td>
<td>23.4</td>
<td>1.33</td>
</tr>
<tr>
<td>3</td>
<td>67.3</td>
<td>45.3</td>
<td>1.29</td>
</tr>
<tr>
<td>4</td>
<td>0.6</td>
<td>0.1</td>
<td>0.23</td>
</tr>
<tr>
<td>5</td>
<td>19.2</td>
<td>9.7</td>
<td>0.43</td>
</tr>
<tr>
<td>Mean</td>
<td>30.4</td>
<td>24.1</td>
<td>0.73</td>
</tr>
</tbody>
</table>

Table 9: Performance comparison results in cycle 2.

<table>
<thead>
<tr>
<th>Approach no.</th>
<th>Average queue length (m)</th>
<th>Average number of stops (times)</th>
<th>Delay (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current</td>
<td>Optimized</td>
<td>Current</td>
</tr>
<tr>
<td>1</td>
<td>44.3</td>
<td>46.5</td>
<td>0.41</td>
</tr>
<tr>
<td>2</td>
<td>25.5</td>
<td>23.2</td>
<td>1.25</td>
</tr>
<tr>
<td>3</td>
<td>60.2</td>
<td>39.6</td>
<td>1.22</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>0.1</td>
<td>0.20</td>
</tr>
<tr>
<td>5</td>
<td>18.3</td>
<td>9.6</td>
<td>0.38</td>
</tr>
<tr>
<td>Mean</td>
<td>29.8</td>
<td>23.8</td>
<td>0.69</td>
</tr>
</tbody>
</table>
VISSIM simulation, the corresponding performance comparison results are shown in Tables 8 and 9. In addition, the average queue length and the average number of stops are used as arithmetic means to calculate the mean values, while the mean value of the average delay is obtained based on the following equation:

$$\Omega = \frac{\sum_{i=1}^{N} d_i e_i}{\sum_{i=1}^{N} e_i},$$

(24)

where $\Omega$ is the mean value of the average delay (s); $d_i$ refers to the average delay of the approach carriageway $i$ (s); $e_i$ is the $s_i\Omega$ simulated traffic volume of the approach carriageway $i$ (veh).

By comparing the simulation result of the current configuration with that of the optimized configuration, the following conclusions can be drawn:

1. In cycle 1 and cycle 2, the average queue length of the target roundabout was reduced by 20.72% and 20.13%, respectively. In cycle 1, the Huan-Cheng West Rd. is the only exception where the queue length has slightly increased by 3.2m due to the conflict between vehicles entering from Huan-Cheng West Rd. and vehicles exiting to Hou-Cheng-Li Rd., which is why the average queue length at the Huan-Cheng West Rd. also increased in cycle 2. Beyond that, the average queueing length of all other approach carriageways has been substantially reduced.

2. In cycle 1 and cycle 2, the average number of stops at the target roundabout has been substantially reduced by 30.14% and 27.54%, respectively. The only exception is still the average number of stops at Huan-Cheng West Rd., which has increased by only 0.06 in cycle 1 and 0.04 in cycle 2 due to the aforementioned vehicle movement conflicts.

3. In cycle 1 and cycle 2, the average delay at the target roundabout has been reduced by 14.73% and 11.66%, respectively. Similarly, the only exception is the delay at Huan-Cheng West Rd. which has increased by 7.5 s in cycle 1 and 5.8 s in cycle 2. Apart from the optimized signal scheme generated by the proposed model, the cancellation of traffic lights in the circulating lanes may also contribute to the average delay reduction since the traffic flows in the circulating lanes no longer need to slow down or stop to wait for the green time.

Based on the actual traffic demand during the morning peak period (7:00 to 9:00), the combined phase design model can be used to calculate the capacity of each bottleneck area (especially the $B_1$ area) and the geometric characteristics of the target roundabout and transform the current dual signal control into a single signal control to make the phase scheme more suitable for the actual traffic conditions, which is the reason the performances of the target roundabout in terms of all three indicators have been greatly improved. In other words, the optimized phase scheme was proved feasible, and the traffic efficiency of the roundabout was substantially increased. Therefore, the combined phase design model can indeed improve the current unreasonable phase setting at many multileg roundabouts by adapting to real-time traffic flow fluctuations.

Pedestrian crossing situation: the sidewalks are equipped with stop signs, which affect the traffic flow inside the roundabout to varying degrees. When a small number of pedestrians conflict with the traffic flow, the proposed approach can still have satisfactory performances. If the number of pedestrians further increases, the conflict between pedestrians and traffic flow will cause additional delays, resulting in the change of road capacity, and the model performances will be inevitably compromised.

6. Conclusions

This paper proposed a combined phase design model to optimize the signal control at multileg roundabouts with the aim of achieving the maximum utilization of roundabout capacity, resulting in the following conclusions:

1. The proposed combined phase design model for the multileg roundabout is developed based on the traffic operation characteristics and the capacity of each area of the roundabout.

2. The proposed model is validated by the case of the Hou-Cheng-Li Rd./Bei-Shan Rd. Roundabout to Jinhua. Experimental results indicate that the combined phase design model can reduce the average queue length, the average number of stops, and the average delay by 20.72%, 30.14%, and 14.73% in cycle 1, respectively, and in cycle 2, the three metrics are reduced by 20.13%, 27.54%, and 11.66%, respectively. Thus, the proposed model can effectively improve the traffic efficiency of the roundabout.

3. Furthermore, this model can be used in actuated signal control to better cope with the real-time traffic demands at roundabouts. Thus, it can contribute to a better control effect in green wave control and area control for roundabouts with unstable traffic.

At last, it is also worth noting that this paper only studies motor vehicle traffic flow and does not consider the impact of mixed traffic flows. In other words, the proposed model cannot be used to solve the optimal phase schemes when roundabout pedestrian traffic flow is involved. In addition, the proposed model is only applicable to cases where excessive traffic delays at roundabouts are caused by large traffic volumes (but still below roundabout capacity). If the traffic volume is small, the roundabout does not need any signal control. On the other hand, the roundabout should be reconstructed if the traffic volume is even larger than its capacity. As such, future research directions include improving the combined phase design model to further solve the pedestrian crossing safety and efficiency issues at roundabouts and to study the phase optimization and signal timing problems of multileg intersections other than roundabouts.
Data Availability
Data cannot be shared publicly because of the confidentiality requirements of the Traffic Police Brigade of Jinhua, Zhejiang, China, who imposed data sharing restrictions on the data underlying our study. Data are only available for researchers who meet the criteria for access to confidential data.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

Authors’ Contributions
Cheng-yuan MAO designed the study; Qin WANG conducted data analysis and wrote the report; Wen-jiao XU and Sheng-de YANG collected the data; Xin CHENG and Pei-ran LI contributed to the rationale and discussion of the results. All authors have read and agreed to the published version of the manuscript.

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