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Research Article

Design of Two-Regional Flexible-Route Bus Systems considering Interregional Demands

Siqing Wang, Jian Wang, Lang Song, Zhipeng Niu, and Xiaowei Hu

School of Transportation Science and Engineering, Harbin Institute of Technology, Harbin, Heilongjiang, China

Correspondence should be addressed to Jian Wang; wang_jian@hit.edu.cn

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The flexible-route bus system is a type of dynamic public transit service. Routes and timetables are not fixed during the operation process, and driving routes are planned according to passengers' reservation needs. This study develops a model that considers inter-regional travel demands. The optimal network layout is determined by minimizing an objective function that comprises operator and user costs. Then, two cases with and without loop-line buses are analyzed. In the case of the joint optimal solution, the parameter values of region side width, region angle, and cost components are compared. Results indicate that regional flexible transit is suitable for operation in areas with low demand density. Within certain ranges, increases in vehicle capacity and in the number of circle layers result in additional average total costs. Furthermore, adopting a mode with a loop is better when numerous inter-regional demands exist. The findings derived from numerical and sensitivity analyses can be used as planning guides for designing flexible-route bus systems.

1. Introduction

As cities expand, suburbs generate plenty of demands traveling to and from city centers. Meanwhile, cross-regional travel has increased. Facing growing demands, the traditional bus mode can no longer meet passengers' diverse needs [1]. Hence, the emerging regional flexible-route transit has become an attractive option [2]. As a supplement to land-side public transport, it provides flexible sharing services for passengers with similar destination and travel time, guided by the passenger reservation travel demands collected before departure. However, owing to the conflict between service efficiency and coverage, an effective regional design is required when planning a flexible bus network. This key factor makes the relationship between users and operators balanced [3]. It can help operators determine the service areas to make them efficient and attractive enough to the public. The analysis of the characteristics of flexible bus systems and the service scope of the operation area is beneficial to operator management [4]. It also reduces system costs, provides better services to passengers, and enhances travel experiences. Therefore, the design and operation scheme of regional flexible bus systems should be optimized to achieve an efficient collection and distribution of downtown passenger flows.

To design a flexible bus system for a large service area, a zoning strategy [5] is adopted to improve performance by dividing the entire service area into small regions. This study investigates how flexible-route bus services operating in multiple regions can be efficiently combined to meet intra- and inter-regional travel demands. It is also optimized to form a comprehensive flexible bus service system that provides complete many-to-many bus services for large cities and suburbs [6, 7]. For inter-regional travel, this study considers the optimal design of two travel cases, namely, with and without loop lines. In the loop-line case, inter-regional trips are made through the loops. In the no-loop-line case, trips between regions must pass through the city center. The proposed system is independent and closed and provides such flexible transit only in the study area. The system total cost formula expresses the comprehensive total cost of all regions considered, including bus operation and user costs. By minimizing it, the best operation strategy for the entire system can be found.

The rest of this paper is organized as follows. Section 2 provides literature review on regional flexible-route bus services. Section 3 presents system formulas, assumptions, and related variables. Section 4 performs numerical and sensitivity analyses. It also discusses the results. Section 5 gives the conclusion and future development direction.

2. Literature Review

Many previous studies have been devoted to operation strategies for flexible-route transit [8–11], bus scheduling, and route optimization [12–15]. The rise of transportation companies, such as Uber and Didi, provide a feasible demand response service that can be integrated into existing transit services. Stein [16], Calabrò et al. [17], Crainic et al. [18], and Cortés and Jayakrishnan [19] investigated the combination of fixed and flexible transit. Before configuring a hybrid transportation system, how to spatially couple the two services should be determined. Aldaihani et al. [20] combined fixed-route and flexible-demand-responsive services. Zhao et al. [21] formulated and solved the joint optimization of regular and flexible transit networks. Saeed and Kurauchi [22] designed a multimodal service that combines fixed-route services with flexible transit systems under stochastic demand. Li and Quadrifoglio [23] branched a line service that generally operates in a residential service area in accordance with the demand response mode, sending passengers to a connection point, which is connected to a main fixed route. Kim and Schonfeld [24] integrated conventional and flexible services with timed transfers. In terms of regional design, centered on a bus stop served by fixed routes, Stein [16] allocated demand response services to relatively small regions. In terms of route design, Chen and Nie [25] discussed the design of a new hybrid transit system to combine the flexibility of demand response services with the efficiency of traditional fixed-route services. In our study, a responsive service is operated at a stable speed to cover all stations on the paired fixed line.

For the service of flexible-route transit, studies have been conducted on the optimal structure and the design of service areas. Nourbakhsh and Ouyang [26] proposed a flexible transit system, in which each bus can serve passengers in a predetermined area, forming a hybrid large structure similar to spokes and grid networks. Li and Quadrifoglio [23] studied the division of feeder regions into multiple single lines for independent operation and determined the number of service regions to ensure the balance between service quality and operating cost. Using two types of vehicles for joint operation in each large residential area, Li and Quadrifoglio [5] recommended the most appropriate zoning service for the area under FRT and DRC policies and made optimal operational decisions to maximize the capacity of the entire transit system. Pan et al. [4] optimized irregular service areas and optimal routes with the objectives of maximizing the number of passengers served and minimizing operating costs. Wang et al. [3] designed a high-degree-of-freedom responsive transit system and proposed an optimization method for vehicle routing, scheduling and service area. They also constructed the model by considering factors such as vehicle capacity constraints.

However, the formulation of a zoning strategy should cater to inter-regional travel. A multiregion system with a transfer feature may require passengers to change vehicles across zones. Shen and Quadrifoglio [27] studied the nontransfer zoning design adopted by paratransit services in Texas. They determined that the decrease in the number of passenger trips could be due to the fact that more empty mileage is often generated in zoning systems without a transfer design. Therefore, the introduction of transfer between regions is a better way to reduce the empty mileage. The zoning system with transfer can coordinate vehicle scheduling at different transfer locations, which can improve not only its efficiency but also its mobility.

By simplifying reality and a simple network scheme, the analysis model adopts a continuous and compact formula, which makes studying the behaviors and designs of transportation systems easy. Several models differ in their objective functions, especially in the proposed network structure, such as radial [25], polar network or radial/ring [28], grid [29], and hub-and-spoke [30] types. Ordinary cities mainly have two structural modes: grid and radial types. For the grid type, Daganzo [31] proposed a grid/hub-and-spoke hybrid network on the basis of a square city with a uniform distribution of demands. Estrada et al. [32] applied the research results of Daganzo to Barcelona, abstracting Barcelona's urban structure as a rectangle with an aspect ratio of 2 : 1. For radial networks, Wirasinghe and Ho [33] analyzed the overall demand density of a radial bus system for transporting central business district (CBD) commuters in view of the demand of self-drive travel during peak hours between CBD and residential areas. They found that the demand density varies with time and space. Badia [28] and Chen et al. [34] considered a radial/ring bus system by finding the optimal spacing between radial and ring lines. They also made important contributions in finding the optimal spacing of loop lines. Badia et al. [35] proposed an accessible bus network oriented to the radial/ring urban structure. They revealed that even if the number of passengers' transfer times can increase, a bus network with simple lines, high station coverage, high frequency, and good accessibility still has a strong attraction. Research proves that demand-responsive services perform better in radial/ring networks than in grid networks [36]. Shi and Gao [37] redesigned the flexible traffic model developed by Nourbakhsh and Ouyang [26]. The model was also verified through numerical analysis.

From the literature review, many studies on flexible transit have focused on vehicle route planning and network design, and relatively few research has concentrated on service area selection and optimization. Most studies have modeled the service area as a rectangular area along a main road. The integration of different types of public transit services and the joint optimization of decision variables have been largely ignored in literature. In addition, many studies on the many-to-many mode of shared transit exist, but few have been performed on a flexible-route bus system that serves internal and inter-regional demands. Our work considers multiple regions, whereas Kim et al. [38] considered only one local region within a city. If all travels between regions must transfer through terminals, then certain resources may be wasted. We consider the introduction of loop-line buses, which can relieve terminal congestion by transferring some passengers to other transit points. In previous studies, multiple regions were not considered when passing through terminals. Meanwhile, when multiple regions were considered, no loop line was introduced. This research discusses the two cases and determines which is better.

3. Flexible Bus System Formulation

The flexible-route transit model developed by Kim et al. [38] is reformulated to accommodate many low-demand areas in the world, especially those characterized by radial/concentric street patterns. This section addresses assumptions for analyzing a general system with multiple local regions. The objective function is to minimize the total system cost, which includes vehicle operating and user costs. User cost comprises in-vehicle and waiting costs. The headway, region side widths and region angles are joint optimized as functions of demand density, bus speeds, bus fleet sizes and other relevant exogenous parameters. Detailed descriptions, assumptions and models for flexible-route bus systems are provided below.

3.1. System Description. The following is a description of a regional flexible-route transit service. Each regional flexible bus serves only one subregion. The driving route of the vehicle in a service region is shown in Figure 1. Each region has its own independent headway and line haul distance. To simplify the model, we divide the service area into several subregions with the same angle α as that of a terminal. All regions are annular sector. These regions may have different line haul distances. Taking subregions i, j and k as an example, as illustrated in Figure 2, subregion i is adjacent to the terminal, the other subregions j and k extend to the periphery of the city. Each region shares the boundary with other regions. The region side widths are W_i, W_j, and W_k, respectively. Line haul $J_i = 0$, J_j passes through region i, and J_k passes through regions i and j.

The line haul distance is assumed to be the shortest from the terminal to each region, and the connection points with each region are transfer stations. Given that the distance from the terminal to each point of the inner boundary of the region is equal to the radius of the arc, the location of the transfer point has minimal impact on the operation of the regional flexible bus. To facilitate bus operation, the transfer point is set as the midpoint of the boundary arc within each region. In this study, flexible transit services are provided between urban terminals and multiple regions, and each region is distributed around the terminal. The travel demands from one subregion to another are considered simultaneously in the system by combining each subregion.

This study considers the designs of two new flexible bus service cases.

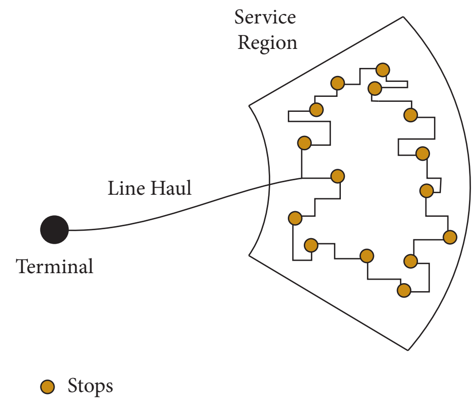

Figure 1: Route of the vehicle in a service region.

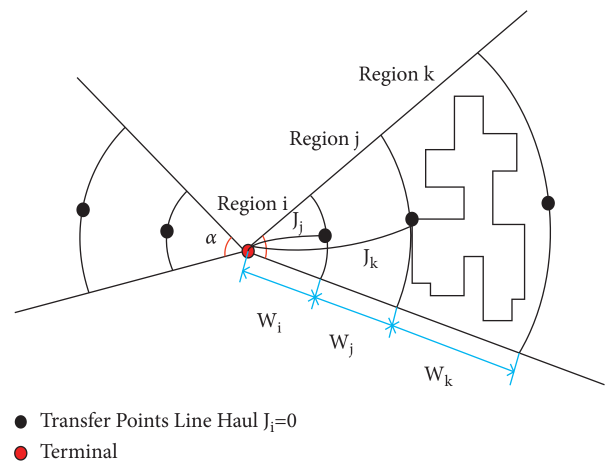

Figure 2: Multiregion system with transfer.

Case 1: Introduction of loop-line bus services. That is, passengers can travel from one region to another via the loop lines rather than transfer at the terminal, especially in the same direction of the terminal.

Case 2: Without loop-line bus services. That is, passengers must transfer at the terminal when traveling from one region to another.

In Case 1, a regional flexible bus system and a fixed loop-line bus system are included. Two kinds of fixed corridors, namely circular and inter-regional line haul corridors, are also designed. Vehicles stop and transfer at fixed transfer points in the corridors, whereas regional flexible bus services are adopted in the regions. For each cross-regional trip, passengers can only switch at a specific transfer point. The transfer place where vehicles may stop is usually at the zone boundary. Each transfer point is assumed to be the connection point between the flexible route and the region. The bus running only in the circular corridor is a fixed line bus with fixed stations and timetable, and the operation direction is two-way. In Case 2, only a regional flexible transit system is available. Vehicles stop and transfer only at the terminal and do not park in the line haul corridor. A regional flexible bus service is also adopted in the region. Figure 3

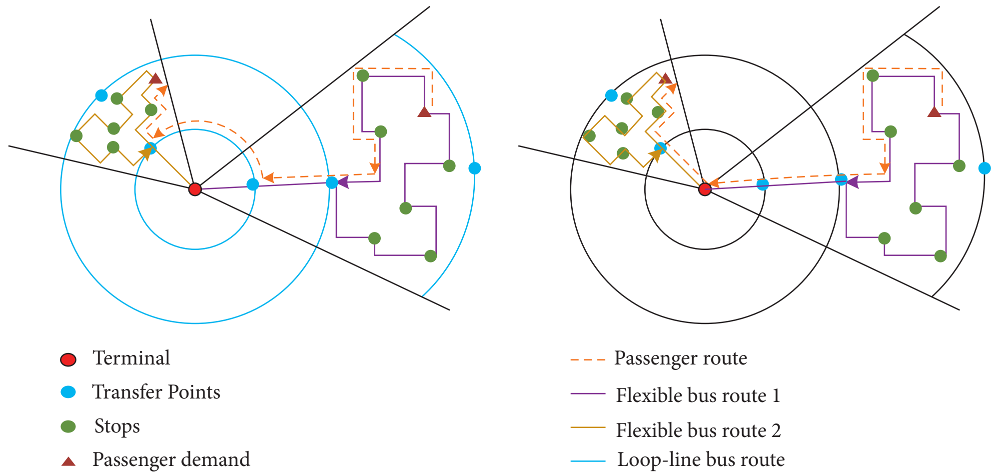

Figure 3: Overall structure of two service cases.

shows the overall structure of the two service cases, including the operation mode of each line, the relationships among service regions, and the locations of transfer points.

3.2. Regional Demand Settings. Each region has various travel demands. In accordance with passengers' starting points and destination locations, travel demands can be divided into three categories, namely intraregional, inter-regional, and terminal demands.

Intraregional demand: Travel starts and ends within a region, and the average travel distance of each passenger is half of the average travel distance of vehicles within the region

Inter-regional demand: Passengers transfer from one service area to another, that is, from one area to a transfer point.

Terminal demand: Users only travel between the region and the terminal, and the average travel distance of each passenger is half of the total travel distance of the vehicle in a round trip.

Passengers can be divided into transferring and non-transferring in accordance with whether they transfer. A trip without a transfer means that the trip starts and ends on the same line. For a transfer trip, passengers take a flexible bus to a fixed loop-line bus and then transfer to a flexible bus to reach their destination or take a flexible bus to another flexible bus.

In this study, combined with the starting and ending positions of passengers and the mode with or without transfer, the transfer trip is divided into multistage trips with the transfer point as the separation point, and the non-transfer trip is one-stage. Then, all travel segments can be divided into terminal-to-region segment, region-to-terminal segment, region-to-transfer point segment, transfer point-to-region segment, and transfer point-to-transfer point segment.

Henceforth, superscript i corresponds to regions, and superscripts rt and l represent round-trip lines. The relevant notation is defined in Table 1.

3.3. Hypotheses. Given that many complex conditions should be considered when calculating the total cost of multiple regions served by independent lines, the following assumptions are established in this study.

(1) The input values of variables are appropriately borrowed from existing similar studies [24].

(2) Bus routes and schedules shall be arranged in advance to optimize the path length.

(3) The average waiting time of a passenger is half of the headway of the route the passenger has selected.

(4) The arrival of vehicles at the transfer point is probabilistic and independent.

(5) The demand is given and evenly distributed in each region, and which is the same in each region.

(6) The transfer point of flexible-route transit and loop-line transit is one point, such that the transfer time is ignored.

(8) Each bus must stop at all stops in route; that is, the skipping strategy is not considered.

(9) The transfer point only serves transfer passengers; that is, the starting and ending points of the passengers are not regarded as the transfer points.

(10) The headway of all routes in a region is consistent.

3.4. Model. The cost calculation formulas for the two cases are as follows:

3.4.1. Case 1: Introduction of a Loop-Line Bus Service. The total cost for the entire system C^t_{sys} is the sum of flexible service cost C^t_{region} and round-trip service cost C^t_{rt}, as discussed below.

$$C^t_{\text{sys}} = C^t_{\text{region}} + C^t_{rt}. \tag{1}$$

(1) Flexible Service Cost C^t_{region}.

For a regional flexible-route bus service system, taking line i as an example, the demand includes terminal regional

Table 1: Notation.

Variable	Parameter	Definition	Base value	Units
	$C^t_{\text{sys}} C^{t'}_{\text{sys}}$	Total system cost in cases 1 and 2		\$/hour
	$C^t_{\text{ave}} C^{t'}_{\text{ave}}$	Average system cost in cases 1 and 2		\$/hour
	C^t_{region}	Total region cost in case 1		\$/hour
	C^t_{rt}	Total loop-line cost in case 1		\$/hour
	C^t_{trans}	Total transfer cost in case 1		\$/hour
	$C^o_i C^{o'}_i$	Operating cost for region i in cases 1 and 2		\$/hour
	$C^o_{rt,l,}$	Operating cost for loop line l		\$/hour
	$C^u_i C^{u'}_i$	User cost for region i in cases 1 and 2		\$/hour
	$C^u_{rt,l,}$	User cost for loop line l		\$/hour
	$C^v_i C^{v'}_i$	In-vehicle cost for region i in cases 1 and 2		\$/hour
	$C^v_{rt,l,}$	In-vehicle cost for loop line l		\$/hour
	$C^w_i C^{w'}_i$	Waiting cost for region i in cases 1 and 2		\$/hour
	$C^w_{rt,l,}$	Waiting cost for loop line l		\$/hour
	$C^{\text{trans}'}_i$	Transfer cost for region i in case 2		\$/hour
	c_u	Operating cost of each flexible bus		\$/bus hour
	c'_u	Operating cost of each loop-line bus		\$/bus hour
	a	Fixed cost coefficient for each bus	30	\$/bus hour
	b	Variable cost coefficient for bus operation	0.3	\$/seat hour
	c_s	Size of each flexible bus	20	Seats/bus
	c'_s	Size of each loop-line bus	30	Seats/bus
N_i		Fleet size of flexible buses for region i in case 1		Buses
$N_{rt,l}$		Fleet size of loop-line buses online l		Buses
N'_i		Fleet size of flexible buses for region i in case 2		Buses
h_i		Headway of flexible buses for region i		Hours
$h_{rt,l}$		Headway of loop-line buses online l		Hours
	T^{total}_i	Round trip time of flexible buses for region i in case 1		Hours
	T^{in}_i	Travel time of flexible buses within region i in case 1		Hours
	$T^{h,m}_i$	Round trip time of flexible buses between transfer point m and region i		Hours
	$T^{total'}_i$	Round trip time of flexible buses for region i in case 2		Hours
	H_i	Tour length within region i		Miles
A_i		Size of region i		Square miles
W_i		Side-width of region i		Miles
α		Angle of region i		Degrees
	Q_i	Total demand density for region i	10	Trips/square mile/hour
	Q^b_i	Demand density between terminal and region i		Trips/square mile/hour
	Q^h_i	Demand density between transfer point and region i		Trips/square mile/hour
	$Q^{h,m}_i$	Demand density between transfer point m and region i		Trips/square mile/hour
	Q^{in}_i	Demand density only in region i		Trips/square mile/hour
	$q^h_{rt,l}$	Transfer demand on loop-line l		Trips/hour
	u_i	Number of passengers per stop in region i	1	Number of passengers
	n_i	Number of stops in region i		Stops
	n^i_{trans}	Number of stops on the line haul connecting to region i		Stops
J_i		Line haul distance of region i		Miles
	δ	Stein's constant (1978)	1.15	Dimensionless
	v_1	Local regional speed		Miles/hour
	v_2	Line haul speed	30	Miles/hour
	v_3	Loop-line speed	30	Miles/hour
	γ	Value of in-vehicle time	12	\$/passenger hour
	τ	Value of waiting time	15	\$/passenger hour
	y	Ratio of local speed to express speed	0.9	Dimensionless
	w_i	Average waiting time		Hours
	t_s	Stopping times per stop	60/3600	Hours

demand Q^b_i, transfer point regional demand Q^h_i, and intraregional demand Q^{in}_i.

The flexible service cost consists of two components as shown in the following equation:

$$C^t_{\text{region}} = \frac{360}{\alpha} \cdot \left(\sum_i C^o_i + \sum_i C^u_i \right). \tag{2}$$

where C^o_i is the bus operating cost and C^u_i is the user cost of line i.

Operating cost C^o_i is expressed as follows:

$$C^o_i = N_i \times c_u, \tag{3}$$

where c_u denotes the unit operating cost. It can be assumed to be a linear function of bus size [39, 40]. Thus,

$$c_u = a + bc_s, \tag{4}$$

where a is the fixed cost (\$/bus/h) and b is the variable cost (\$/seat/h) with bus size c_s.

In equation (5), N_i represents the fleet size of flexible-route buses. For cases in which the round-trip time is longer than the headway, more than one service route is needed. Fleet size N_i is formulated by

$$N_i = \frac{T_i^{\text{total}}}{h_i}. \tag{5}$$

where T_i^{total} is the round-trip time and h_i is the headway.

Round-trip time T_i^{total} is the sum of distance traveled by vehicles and the total time spent by vehicles in making stops in one cycle, as presented as follows:

$$T_i^{\text{total}} = \frac{H_i}{v_1} + \frac{2J_i}{v_2} + \left(n_i + 2n_{\text{trans}}^i\right) \cdot t_s. \tag{6}$$

Similarly, T_i^{in} and $T_i^{h,m}$ are computed as follows:

$$\begin{aligned} T_i^{in} &= \frac{H_i}{v_1} + n_i \cdot t_s, \\ T_i^{h,m} &= \frac{H_i}{v_1} + \frac{2\left(J_i - J_m\right)}{v_2} + \left[n_i + 2\left(n_{\text{trans}}^i - m\right)\right] \cdot t_s, \\ m &= 1, 2, \ldots i. \end{aligned} \tag{7}$$

where H_i denotes the tour length within a zone. J_i is the line haul distance. n_i and n_{trans}^i represent the number of stops in the zone and on the line haul, respectively. t_s denotes the stopping time in every stop. The local and express speeds are v_1 and v_2, respectively. T_i^{in} represents the travel time of a flexible bus within region i in Case 1. $T_i^{h,m}$ represents the round-trip time of a flexible bus between transfer point m and region i.

Based on previous research data [24], the number of passengers boarding or alighting at each station tends to be the same. The number of stops during a tour n_i is formulated as follows:

$$n_i = \frac{Q_i A_i h_i}{u_i}, \tag{8}$$

where Q_i represents the total demand density, A_i denotes the zone size, h_i denotes the headway, and u_i is the number of passengers per stop.

With reference to the approximate expression [16] of the tour length within a zone, H_i is expressed as

$$\begin{aligned} H_i &= \delta\sqrt{n_i A_i}, \\ &= \delta\sqrt{\frac{Q_i A_i^2 h_i}{u_i}}. \end{aligned} \tag{9}$$

In accordance with the formula for calculating sector area, zone size A_i is formulated as follows:

$$\begin{aligned} A_i &= \frac{\alpha\pi \cdot \left(J_i + W_i\right)^2}{360} - \frac{\alpha\pi \cdot J_i^2}{360} \\ &= \frac{\alpha\pi}{360}\left(2J_i W_i + W_i^2\right), \end{aligned} \tag{10}$$

where α is the region angle and W_i is the region side width. In equation (10), it is assumed that the nearest distance from the terminal to the region (the radius is centered at the terminal) is approximately J_i.

To put simply, we assume that the local speed v_1 is a fraction of express speed v_2. We denote the ratio as y. Then,

$$v_1 = y v_2. \tag{11}$$

In-vehicle cost C_i^v is the product of the value of a passenger's in-vehicle time, the demand, and a passenger's trip time, which is assumed to be half of the vehicle round-trip time:

$$C_i^v = \gamma Q_i^b \cdot A_i \cdot \frac{T_i^t}{2} + \gamma Q_i^{in} \cdot A_i \cdot \frac{T_i^{in}}{2} + \gamma \sum_{m=1}^{i} Q_i^{h,m} \cdot A_i \cdot \frac{T_i^{h,m}}{2}. \tag{12}$$

Similarly, we can calculate in-vehicle cost C_i^w by the following equation:

$$C_i^w = \tau Q_i A_i w_i, \tag{13}$$

where γ is the value of a passenger's in-vehicle time and τ is the value of a passenger's waiting time. $Q_i^{h,m}$ represents the demand density between transfer point m and region i.

In accordance with Welding [41] and Osuna and Newell's [42] modes, a passenger's average waiting time w_i is expressed as the function of the mean and the variance of the headway:

$$w_i = \frac{E\left(h_i\right)}{2}\left(1 + \frac{\delta_i^2}{\left[E\left(h_i\right)\right]^2}\right). \tag{14}$$

where $E(h_i)$ is the expected value of route i headway and δ_i^2 is the variance of the headway on route i.

The headways in one zone are the same. If a passenger's arrival is random, waiting cost C_i^w can be simplified as follows:

$$C_i^w = \tau Q_i A_i \cdot \frac{h_i}{2}. \tag{15}$$

Thus, the user cost for flexible service C_i^u can be denoted as

$$C_i^u = C_i^v + C_i^w. \tag{16}$$

(2) Round-Trip Service Cost C_{rt}^t.

For a fixed loop-line bus system, taking line l as an example, demand $q_{rt,l}^h$ is the demand among transfer points.

The total cost for a round-trip service can be expressed as follows:

$$C_{rt}^t = \sum_l C_{rt,l}^o + \sum_l C_{rt,l}^u. \tag{17}$$

The operating cost $C^o_{rt,l}$ and user cost $C^u_{rt,l}$, which include in-vehicle cost $C^v_{rt,l}$ and waiting cost $C^w_{rt,l}$, of a round-trip service system are as follows:

$$C^o_{rt,l} = N_{rt,l} \cdot c'_u. \tag{18}$$

Similarly, c'_u is a linear function of round-trip bus size c'_s.

$$c'_u = a + bc'_s. \tag{19}$$

Other cost components of a round-trip service can also be converted to formulations as follows:

$$C^v_{rt,l} = \frac{1}{2}\gamma q^h_{rt,l} \cdot \frac{T_{rt,l}}{2}, \tag{20}$$

$$C^w_{rt,l} = \tau q^h_{rt,l} \cdot \frac{h_{rt,l}}{2}, \tag{21}$$

$$C^u_{rt,l} = C^v_{rt,l} + C^w_{rt,l}, \tag{22}$$

where $N_{rt,l}$ represents the fleet size, $T_{rt,l}$ is the round-trip time, and $h_{rt,l}$ is the headway of a round-trip bus. Fleet size and round-trip time are formulated by equations (23) and (24), respectively,

$$N_{rt,l} = \frac{T_{rt,l}}{h_{rt,l}}, \tag{23}$$

$$T_{rt,l} = \frac{2\pi J_l}{v_3} + \frac{360}{\alpha} \cdot t_s, \tag{24}$$

where J_l denotes the line haul distance of round-trip line l, and v_3 is the vehicle speed on the round line.

(3) Optimization Model.

The purpose of the regional flexible bus system design problem is to jointly optimize system variables while minimizing the average total system cost. Altogether, the transit system optimization problem can be expressed as

$$\min C^t_{ave} = \frac{C^t_{sys}}{360/\alpha \cdot \sum_i Q^h_i A_i}, \tag{25}$$

$$s.t.\ W_i = J_{i+1} - J_i > 0, \tag{26}$$

$$\frac{360}{\alpha} = \text{integer}, \tag{27}$$

$$N_i = \frac{T^{\text{total}}_i}{h_i} = \text{integer}, \tag{28}$$

$$N_{rt,l} = \frac{T_{rt,l}}{h_{rt,l}} = \text{integer}, \tag{29}$$

$$h_i \leq \frac{c_s}{Q_i \cdot A_i}, \tag{30}$$

$$h_{rt,l} \leq \frac{c'_s}{q^h_{rt,l}}, \tag{31}$$

$$\sum_l q^h_{rt,l} = \frac{1}{2} \times \frac{360}{\alpha} \cdot \sum_i Q^h_i A_i. \tag{32}$$

Equation (25) is the objective function. Equation (26) indicates that the more outward the region, the longer the haul line distance, i.e., each region side width is positive. Equation (27) ensures that the number of regions divided is an integer. The constraints in equations (28) and (29) should obtain integer values to calculate the fleet size for each route. Equations (30) and (31) are the maximum headway constraints. The transfer demands on all loop lines are half of the total demands between transfer point and every region. Thus, demands are restricted as equation (32). To calculate the total cost of integer solutions, the decision variables are first optimized, and then their neighboring integer solutions are compared to satisfy these constraints.

3.4.2. Case 2: Without Loop-Line Bus Service. The model for Case 2 is based on Kim et al.'s [38] single-region operating model. Only a regional flexible-route bus system exists, and passengers can only transfer at the terminal. The demand includes terminal regional demand $Q^b_i + Q^h_i$ and intraregional demand Q^{in}_i, and the transfer demand at the terminal is Q^h_i.

The total cost for flexible service $C^{t'}_{\text{sys}}$ is the sum of operating and user costs:

$$C^{t'}_{\text{sys}} = \frac{360}{\alpha} \cdot \left(\sum_i C^{o'}_i + \sum_i C^{u'}_i \right). \tag{33}$$

Detailed cost component derivations for operator and user costs are provided in Appendix.

3.5. Solution Approach. In this study, the solver optimization module in the MATLAB toolbox is used to determine the optimal layout of regional flexible-route transit systems. Solver optimization module is a free plug-in designed to solve mathematical planning problems. It allows solving problems with up to 200 variables and 600 constraints [43]. As the cost function is nonlinear and contains multiple continuous variables, the Fmincon solver is chosen to solve this numerical problem. It enables to solve the minimum value problem for constrained nonlinear multivariable functions [44].

Fmincon is a gradient-based function that is calculated using a sequential quadratic programming method. It can be used to search and find all possible local minimum values that satisfy a given objective. The iterative process starts with the initial guesses of the algorithm and stops once all the set conditions are satisfied. If the final iteration completes the first-order optimization, then the result is considered a local minimum value that satisfies the system requirements [45].

4. Numerical Analysis

A numerical example is used to calculate and explore the decision variables and cost components obtained from the

Table 2: Joint optimal solution in two cases.

	Region	1	2	3	4	5	6
Case 1	Line haul distance (miles)	0	8.44	13.36	16.74	19.85	22.60
	Side width of region (miles)	8.44	4.92	3.38	3.11	2.75	2.40
	Zone size (sq. miles)	37.28	56.12	53.41	59.47	61.07	59.91
	Angle of region (degree)	60					
	Average operating cost ($/h)	12.01					
	Average user cost ($/h)	8.61					
	Average total cost ($/h)	20.62					
Case 2	Line haul distance (miles)	0	7.29	13.32	17.51	21.16	23.72
	Side width of region (miles)	7.29	6.03	4.19	3.65	2.56	1.28
	Zone size (sq. miles)	33.44	78.05	81.13	88.76	72.25	39.08
	Angle of region (degree)	60					
	Average operating cost ($/h)	10.58					
	Average user cost ($/h)	8.91					
	Average total cost ($/h)	19.49					

joint optimization in both models. These variables are the region side width and angle. According to the actual situation, the study area for this investigation is set as a concentric circle with a radius of 25 miles. The number of circle layers is 6. The input values of variables for formulas and numerical cases are appropriately borrowed from existing similar studies [24, 38]. On these bases, the parameters are set as $N_i = N_i' = 20$ buses, $N_{rt,l} = 30$ buses, $Q_i = 10$ trips/mile^2/h, Q_i^b: Q_i^h: $Q_i^{\text{in}} = 5: 2: 3$, $a = 30$ $/bus/h, $b = 0.3$ $/seat/h, $v_2 = v_3 = 30$ miles/h, $\gamma = 12$ $/trip/h, $\tau = 15$ $/trip/h, $y = 0.9$, $\delta = 1.15$, $u_i = 1$, and $t_s = 60$ s.

In the case of the joint optimization of the optimal solution, the line haul distance, the side width, and the zone size of each region in the two modes are shown in Table 2. The more outward it is, the smaller the region side width will be. In Case 1, the optimal angle between an area and a terminal is 60 degrees; the average operating and user costs are 12.01 and 8.61 $/h, respectively; and the average total cost is 20.62 $/h. In Case 2, the optimal angle between an area and a terminal is also 60 degrees; the average operating and user costs are 10.58 and 8.91 $/h, respectively; and the average total cost is 19.49 $/h.

From the comparative analysis, the average operating cost and average total cost of Case 2 are lower than those of Case 1 under joint optimization. Nevertheless, the average user cost of Case 2 is higher. Due to the introduction of loop lines, the construction requires additional vehicles for new routes. As a result, the operating costs increase for corporations. Passengers traveling between regions do not necessarily have to transfer through terminals. Thus, terminal congestion is not only relieved but also passengers' travel time is shortened, leading to a reduction in the average user cost. Then, at a certain demand, the angle between the region and the terminal should be moderate. If the angle is too large or small, then an increase in average total system cost occurs. If the angle is too large, then an increase in passengers' in-vehicle time and a decrease in their satisfaction occur. If the angle is too small, then the number of zones divided increases. As a result, the number of vehicles required and the operating cost increase.

5. Sensitivity Analysis

In this section, a sensitivity analysis is conducted to discuss the impacts of several parameters on the operational planning of regional flexible transit systems. Table 3 shows the sensitivity of cost to demand density. From Table 3, with the increase in demand density, the optimal angle of regions and the average total cost decrease. In Case 1, the optimal angle decreases from 60 degrees to 15 degrees. In Case 2, the optimal angle decreases from 60 degrees to 20 degrees. The increased passenger intensity leads to a reduction in vehicle travel distance. Therefore, the regional service area should be properly narrowed. When the demand density increases from 10 persons/square miles to 35 persons/square miles, the average total cost decreases by 23% and 24%, that is, from 20.62 $/person to 15.88 $/person and from 19.49 $/person to 14.82 $/person, respectively. For both cases, the average operating costs are higher for Case 1 than for Case 2, whereas the average user costs are reversed. Exactly the introduction of loop lines has inevitably led to increased construction investment by bus companies. The loop lines improve the accessibility of inter-regional travel, resulting in an increased user experience of the journey. When the regional angle is used as a fixed input parameter, the average operating cost is reduced by 72.3% and 72.6%, and the average user cost is increased by 69.2% and 68.1%. When the demand density increases, the vehicle load ratio increases, whereas the average operating cost decreases. The increase in service passengers leads to the increase in passengers' in-vehicle time and waiting time and average user time cost. The average total cost first decreases and then increases, reaching the minimum at 20 and 25 persons/square miles, as presented in Table 4 and Figure 4.

We also explore the impact of the number of circle layers on cost, as shown in Table 5. The number of circle layers increases from 4 to 12, and the optimal region division angle obtained through joint optimization increases from 36 degrees to 90 degrees and from 30 degrees to 90 degrees. The average total cost increases by 20.1% and 26.9%. As the number of circle layers increases, the number of service areas also increases. Meanwhile, the increase in the number of routes inevitably leads to an increase in operating costs.

Table 3: Joint optimal solution for demand variations.

Demand density (persons/sq. mile)	Case 1				Case 2			
	Optimal angle	Average operating cost ($/h)	Average user cost ($/h)	Average total cost ($/h) joint optimal solution % change	Optimal angle	Average operating cost ($/h)	Average user cost ($/h)	Average total cost ($/h) joint optimal solution % change
5	45	17.04	8.03	25.07 21.6	60	16.08	8.37	24.45 25.4
10	60	12.01	8.61	20.62 0.0	60	10.58	8.91	19.49 0.0
15	45	10.65	8.18	18.83 −8.7	60	8.05	10.19	18.24 −6.4
20	30	10.38	7.47	17.85 −13.4	45	7.60	9.33	16.93 −13.1
25	36	7.30	9.43	16.73 −18.9	45	6.31	10.06	16.37 −16.0
30	20	10.34	6.68	17.02 −17.5	30	6.78	9.20	15.98 −18.0
35	15	8.60	7.28	15.88 −23.0	20	4.05	10.77	14.82 −24.0

Table 4: Effects of demand density on costs.

Demand density (persons/sq. mile)	Case 1			Case 2		
	Average operating cost ($/h)	Average user cost ($/h)	Average total cost ($/h)	Average operating cost ($/h)	Average user cost ($/h)	Average total cost ($/h)
5	16.85	8.87	25.72	16.08	8.37	24.45
10	12.01	8.61	20.62	10.58	8.91	19.49
15	9.17	9.91	19.08	8.05	10.19	18.24
20	7.43	10.93	18.36	6.53	11.11	17.64
25	6.46	12.12	18.58	5.61	11.57	17.18
30	5.75	13.19	18.94	4.96	12.70	17.66
35	4.67	15.01	19.68	4.40	14.07	18.47

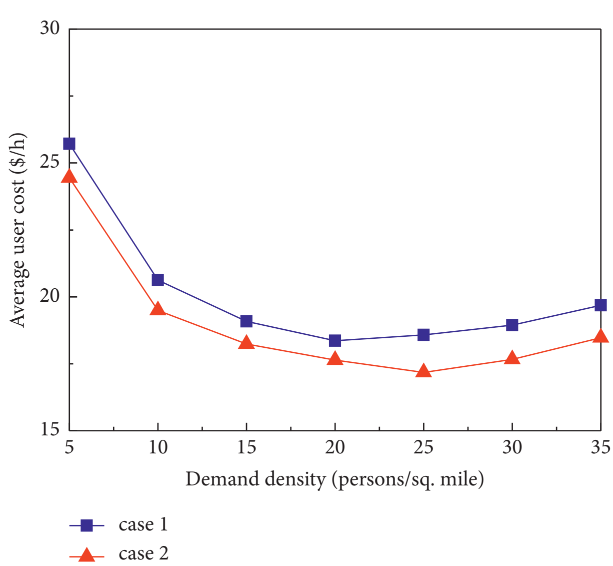

Figure 4: Average total cost variations over demand densities.

However, with a small divided service region, passengers take few detours, resulting in low user costs.

Vehicle capacity is the key design parameter in public transit operation planning and dispatching. In the two cases, when the vehicle capacity increases from 10 seats/bus to 45 seats/bus, the average total cost increases by 16.2% and 19.0%, respectively. Table 6 presents that the average operating cost increases as vehicle capacity increases. On the contrary, the average user time cost is insensitive to vehicle capacity and always changes minimally.

Assuming that the proportion of intraregional demand remains unchanged, the ratio of terminal regional demand to transfer point regional demand is adjusted, as shown in Table 7. For Case 2, the ratio of the sum of terminal regional demand and transfer point regional demand is constant. Hence, no matter how the ratio between them is changed, it has no effect on cost components. When the demand density between transfer points and regions is small, the average total cost of the flexible bus system with loop lines is higher than that of the system without loop lines. With the increase in demand proportion between transfer points and regions, the average total cost in Case 1 is lower than that in Case 2.

Table 5: Joint optimal solution for circle layers variations.

Number of circle layers	Case 1 Optimal angle	Average operating cost ($/h)	Average user cost ($/h)	Average total cost ($/h)	Case 2 Optimal angle	Average operating cost ($/h)	Average user cost ($/h)	Average total cost ($/h)
4	36	10.24	9.64	19.88	30	8.50	9.48	17.98
5	45	12.19	8.00	20.19	45	11.16	8.22	19.38
6	60	12.01	8.61	20.62	60	10.58	8.91	19.49
7	72	12.27	8.81	21.08	72	10.77	9.31	20.08
8	72	13.54	8.26	21.80	72	12.00	8.54	20.54
9	90	11.66	10.58	22.24	60	14.48	7.50	21.98
10	90	13.77	9.74	22.51	60	15.71	7.15	22.86
11	90	13.49	9.47	22.96	72	14.78	7.49	22.27
12	90	15.75	8.13	23.88	90	14.94	7.88	22.82

Table 6: Effects of vehicle capacity on costs.

Flexible bus capacity (seats)	Case 1 Average operating cost ($/h)	Average user cost ($/h)	Average total cost ($/h)	Case 2 Average operating cost ($/h)	Average user cost ($/h)	Average total cost ($/h)
10	10.73	9.25	19.98	10.15	8.81	18.96
15	11.72	8.52	20.24	10.56	8.78	19.34
20	12.01	8.61	20.62	10.58	8.91	19.49
25	12.57	8.65	21.22	11.45	8.78	20.23
30	13.24	8.51	21.75	11.94	8.67	20.61
35	13.41	8.82	22.23	12.16	8.86	21.02
40	14.04	8.58	22.62	12.75	8.89	21.64
45	14.72	8.50	23.22	13.58	8.98	22.56

Table 7: Effects of ratio of demands on costs.

$Q_i^b : Q_i^h : Q_i^{\text{in}}$	Case 1 Average operating cost ($/h)	Average user cost ($/h)	Average total cost ($/h)	Case 2 Average operating cost ($/h)	Average user cost ($/h)	Average total cost ($/h)	% change of average total cost (1) – (2)/(2)
6 : 1 : 3	12.05	9.17	21.24	10.58	8.91	19.49	9.0
5 : 2 : 3	12.01	8.61	20.62	10.58	8.91	19.49	5.8
4 : 3 : 3	11.17	9.03	20.20	10.58	8.91	19.49	3.6
3 : 4 : 3	11.03	7.64	18.67	10.58	8.91	19.49	−4.2
2 : 5 : 3	11.01	7.11	18.12	10.58	8.91	19.49	−7.0
1 : 6 : 3	10.99	6.70	17.69	10.58	8.91	19.49	−9.2

The difference ratio with or without loop line changes from 9.0% to 9.2%. When more travel demand exists between regions, Case 1 is better, and the cost is lower. As the terminal regional demand decreases, passenger trips do not have to go through terminals. The operating costs in Case 1 are always higher than in Case 2, and this result makes realistic sense. Due to the introduction of the loop lines, the investment of loop-line buses is required. It certainly leads to an increase in operating costs.

6. Conclusion

In this study, two regional flexible-route transit systems are designed with or without loop lines. Considering inter-regional demands, a model is developed to jointly optimize region angles, side widths, and area sizes. This model aims at minimizing the average total cost of each passenger. To determine the optimal network layout, a zoning strategy is used with Fmincon solver. Through numerical and sensitivity analyses, the model applicability is verified. Similarly, we compare the designed parameters of the two models when they are optimized. This study extends the research of Kim et al. [38] by integrating flexible routes optimized for a single region. It considers inter-regional demands to serve larger multiple regions in a many-to-many demand model. It also helps explore the possibility of designing a flexible-route transit network in large areas with low demands.

Moreover, this research provides flexibility options for passengers to travel across regions. The main findings are as follows.

First, the numerical example shows that choosing a moderate angle between an area and a terminal is desirable, whether loop lines exist. If the angle is too large or small, then the average total cost increases.

Second, as demand density increases, the optimal region angle gradually reduces and the average total cost decreases. The results confirm that the flexible transit system is more suitable for low-demand areas. This finding is similarly confirmed by Nourbakhsh and Ouyang [26].

Finally, the number of circle layers, vehicle capacity, and ratio of demands affect operational decisions and cost components to some extent. An increase in the number of circle layers results in a large optimum region angle and a low average total cost. The smaller the vehicle capacity within a certain range, the more efficient it is for the operation of flexible transit systems. Moreover, modes with loop lines are better than those without, in cases of great travel demands among regions.

The results obtained from the numerical analysis can be used as planning guides for designing flexible bus systems. However, some limitations remain in this research. To simplify the model, only a concentric circle structure is proposed to divide regions. A comparison with circular and grid network structures [46] can be made in future works. New variables can also be introduced to evaluate system cost components, such as land-use costs. As our insights are derived from hypothetical numerical examples, additional empirical studies should be conducted to verify our conclusions

Appendix

All the following formulas are the same as in the flexible transit service in Case 1. In Case 2, the user cost $C_i^{u'}$ is the sum of in-vehicle cost $C_i^{v'}$ and waiting cost $C_i^{w'}$:

$$
\begin{aligned}
C_i^{u'} &= C_i^{v'} + C_i^{w'}, \\
C_i^{o'} &= N_i' \cdot c_u, \\
N_i' &= \frac{T_i^{\text{total}'}}{h_i}, \\
T_i^{\text{total}'} &= \frac{H_i}{v_1} + \frac{2J_i}{v_2}, \\
C_i^{v'} &= \gamma\left(Q_i^b + Q_i^h\right)A_i\frac{T_i^{\text{total}'}}{2} + \gamma Q_i^{\text{in}} A_i \frac{T_i^{\text{in}}}{2}, \\
C_i^{w'} &= \tau Q_i A_i \frac{h_i}{2}.
\end{aligned}
\tag{A.1}
$$

The flexible-route transit system optimization problem in Case 2 can be expressed as follows:

$$
\begin{aligned}
&\min C_{\text{ave}}^{t'} = \frac{C_{\text{sys}}^{t'}}{360/\alpha \cdot \sum_i Q_i^h A_i}, \\
&s.t.\, W_i = J_{i+1} - J_{i+1} \geq 0, \\
&\frac{360}{\alpha} = \text{integer}, \\
&N_i' = \frac{T_i^{\text{total}'}}{h_i}, \\
&h_i \leq \frac{c_s}{Q_i \cdot A_i}.
\end{aligned}
\tag{A.2}
$$

Data Availability

The data used to support this study are available from the corresponding author upon request.

Disclosure

The authors declare that there are no potential conflicts of interest.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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