Research Article

A Slack Departure Strategy for Demand Responsive Transit Based on Bounded Rationality

Hongfei Wang,1,2 Hongzhi Guan,1,2 Huanmei Qin,1,2 Wanying Li,1,2 and Junze Zhu1,2

1Faculty of Urban Construction, Beijing University of Technology, Beijing 100124, China
2Beijing Key Laboratory of Traffic Engineering, Beijing University of Technology, Beijing 100124, China

Correspondence should be addressed to Huanmei Qin; hmqin@bjut.edu.cn

Received 4 January 2022; Revised 10 March 2022; Accepted 15 March 2022; Published 24 May 2022

1. Introduction

Urban sprawl and the explosion of private car ownership have led to numerous challenges, such as traffic congestion, noise pollution, and carbon emissions [1]. In the context of global resource intensification and carbon neutrality, public transport (PT) is considered a promising solution for alleviating the excessive reliance on private cars [2]. However, the limitations of traditional public transport are becoming increasingly apparent due to the lack of flexibility [3]. To accommodate the diversified travel demands of passengers, transit operators provide more flexible, comfortable, and intelligent services [4]. With the support of information technology and smartphones, demand responsive transit (DRT) service is bringing the remarkable potential for the public transport system.

DRT is a flexible route system that provides service for any individual according to the Americans with Disabilities Act. Passengers submit their requirements, and then, the system provides personalized services by matching supply and demand. The natural DRT system needs to fulfill the flexible demands of all passengers; hence, the system operation cost is excessively expensive [5, 6]. Derived from a retrospective study [7], the results after the implementation of 120 projects from 19 countries indicate that the failure rate of DRT is exceptionally high.

To improve the operating profit and decrease the cost, most existing studies [2, 3, 5, 8, 9] applied the space-time clustering of origins and destinations to achieve integration regardless of passengers’ willingness. Nevertheless, the real world is complicated, and passengers are not “rational economic men” [10]. The above approaches enhance the operating profit but degrade the service quality of DRT,
which is detrimental to maintaining its passengers in the long run. Research considering passengers’ decision-making psychology in the optimization of DRT problem is valuable but relatively scarce.

This study proposes a slack departure strategy to promote the operating profit and the quality of service. The strategy adjusts passengers’ departure time to adjacent time windows. Changing departure time will decrease passenger satisfaction, and the discount-incentive mechanism can be designed to compensate passengers and encourage them to use DRT service. Passengers are not completely rational to abide by the utility maximization rule in realistic departure time choices. Therefore, the theory of bounded rationality [11] is applied to describe the decision-making psychology of passengers in this paper. The specific process is shown in Figure 1. This study aims to answer the following questions:

1. What benefits will the slack departure strategy bring for passengers and DRT operating companies?
2. How to develop a discount-incentive mechanism to promote the operators’ profit and attract more passengers to use DRT for a long time?
3. How will passengers’ aspiration level and time window influence the effect of the slack departure strategy?

To address three problems, a multiobjective programming model is established to analyze the interaction between passengers and the operation organization. We propose a two-phase heuristic algorithm to obtain a Pareto solution set for the model. Furthermore, the simplified Sioux Falls network is explored to identify the effectiveness of the proposed model and algorithm. Ultimately, a case study of Beijing is highlighted to verify the benefit of the slack departure strategy.

The remainder of this study is organized as follows: Section 2 presents a literature review on DRT service systems. The problem description and mathematical model are proposed in Section 3. Section 4 provides the heuristic solution framework in detail. Section 5 discusses the numerical experiment and the case study of Beijing to evaluate the effect of implementing the slack departure strategy for DRT. Eventually, the results and directions for future research are concluded in Section 6.

2. Literature Review

In essence, the problem we study is the optimal design of DRT system based on bounded rationality. We review the latest research progress on DRT systems and then study the application of discount-incentive mechanisms and bounded rationality in the traffic field.

2.1. Research on the DRT Model. DRT initially appeared as a form of Dial-a-Ride in the 1970s, which provided services for the disabled and the elderly [12]. Stein [13] proposed the first model for the planning and scheduling problem of DAR systems. Subsequently, the DRT service receives widespread attention from the public. DRT problems can be divided into four basic categories [12]: static deterministic [14], static stochastic [15], dynamic deterministic [16], and dynamic stochastic [17]. This study is in the reservation model, so we identify it as a static-deterministic DRT system.

Request-oriented DRT service can be viewed as the pickup and delivery problem (PDP) [18]. Dumas [19] presented pickup and delivery problem, which meets not only the capacity constraint but also the time window constraint. Exact algorithms are applied to resolve the pickup and delivery problem with time windows (PDDTW). On the basis of PDDTW, MT-PDDTW is formed by adding the concept of multiple times (MT). The vehicle is arranged multiple times to improve the efficiency of utilizing the vehicle. In addition, the model of multiple pickup and delivery problems is established in the previous study [20]. Each of the requests can be categorized into various pickup locations, and the service time window is also diversified [21]. The passenger’s time window in this paper can be adjusted between adjacent periods, which is more complicated than the existing PDDTW.

As shown in Table 1, the most commonly used objectives of the DRT model are the operating cost (e.g., transportation cost, total travel time, number of service vehicles) as well as an evaluation index of passengers (e.g., travel cost, travel time). Previous DRT problems mainly focus on single-objective functions, but multiobjective functions and hierarchical objective functions can convey the conflict between cost-effective operation and high-quality service, which is the research trend in the future [22, 23]. The solutions of multiobjective DRT are classified into three categories, including weighted average method, lexicographic functions, and Pareto optimality. In this paper, Pareto optimality is utilized to understand all potential optimal solutions of DRT.

2.2. Discount-Incentive Mechanism. The discount-incentive mechanism is designed to motivate the individual’s choice behavior toward desired outcomes. In recent years, many scholars have studied the discount to stimulate people’s willingness to participate and improve the service satisfaction in the transport field. For instance, Cheng [27] estimated the impact of bus price discounts on multimodal transport. The results show that higher discounts can induce more potential passengers. To improve ridership and profit, Li [28] applied a discount on trip fares to stimulate riders to participate in shared traffic. Expected from material reward [29, 30], Raghubir [31] took the coupon as another incentive mechanism to increase the profit. Additionally, the study [32] attracted people to adjust their route choices in the form of adaptive coupons, contrasting with the incentive effect of the traditional material reward. Therefore, the discount-incentive mechanism is implemented in this paper to diminish the dissatisfaction caused by changing departure time, while ensuring the service level of DRT.

2.3. Bounded Rationality Behavior in the Traffic Field. Simon puts forward the concept of “bounded rationality” in 1955. Henceforth, many articles have been published in
Bounded rationality was incorporated into the transportation field for the first time. Mahmassani [33] analyzed the departure time and route choice behavior of travelers. The boundedly rational behavioral framework can primarily be divided into four categories: indifference threshold, satisficing theory, regret theory, and prospect theory. Aavineri [34] applied prospect theory to describe the user equilibrium model of route choice. The results show that network equilibrium has a significant reference dependence effect.

From the perspective of the indifference threshold, Liu [35] investigated that complete rationality cannot reflect people’s route choice behavior. A new modeling framework is developed to study the dynamic choice in the congested network. Based on satisficing theory, Li [28] proposed a multistage integer program to solve the one-to-two matching problem of online car-hailing, and bounded rationality is investigated to simulate the selection process of passengers. Zhao [36] developed a stochastic user equilibrium model based on the improved random regret model (RRM), and the decision feature of traveler aversion is estimated.

Furthermore, bounded rationality is extensively applied to traffic planning [37], traffic assignment [38, 39], and traffic safety [40]. In the field of the DRT system, passengers’ choice behaviors are described by utility maximization. The related research of bounded rationality is rare.

### Table 1: The latest researches on the DRT system.

<table>
<thead>
<tr>
<th>Demand pattern</th>
<th>Objective</th>
<th>Decision variables</th>
<th>Constraints</th>
<th>Solution method</th>
<th>Paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static</td>
<td>Number of unserved passengers; routing costs.</td>
<td>Route; assignment</td>
<td>PA; VC; OP; TW; FB</td>
<td>Lagrangian decomposition</td>
<td>Tong et al. (2017) [1]</td>
</tr>
<tr>
<td>Static</td>
<td>Excess travel time; operating profit</td>
<td>Route; assignment</td>
<td>PA; TW; VC; TD; FB;</td>
<td>Two-phase approach heuristic solution</td>
<td>Chen et al. (2021) [2]</td>
</tr>
<tr>
<td>Dynamic; static</td>
<td>Operating profit; operating cost</td>
<td>Route; schedule; assignment</td>
<td>VC; TW; PA; PD; OP</td>
<td>Dynamic insertion method; branch-and-bound (B&amp;B) algorithm</td>
<td>Huang et al. (2020) [8]</td>
</tr>
<tr>
<td>Static</td>
<td>Transportation cost</td>
<td>Route; assignment</td>
<td>PD; TW; VC</td>
<td>Hybrid adaptive large neighborhood search (ALNS)</td>
<td>Naccache et al. (2018) [21]</td>
</tr>
<tr>
<td>Static</td>
<td>Transportation cost</td>
<td>Route; assignment</td>
<td>PD; VC; FB;</td>
<td>Hybrid genetic algorithm</td>
<td>Park et al. (2021) [24]</td>
</tr>
<tr>
<td>Static</td>
<td>Transportation cost</td>
<td>Route; assignment</td>
<td>PA; PD; VC; FB</td>
<td>ALNS</td>
<td>Hornstra et al. (2020) [25]</td>
</tr>
<tr>
<td>Static</td>
<td>Operating profit</td>
<td>Route; schedule; assignment</td>
<td>FB; PA; VC</td>
<td>Three-phase approach heuristic solution</td>
<td>Yan et al. (2019) [26]</td>
</tr>
<tr>
<td>Static</td>
<td>Operating profit; number of passengers departing at the expected time; general travel cost</td>
<td>Route; assignment; schedule; time adjusted</td>
<td>FB; LF; VC; TW; PA; PD</td>
<td>Two-phase approach heuristic solution</td>
<td>This paper</td>
</tr>
</tbody>
</table>


**Figure 1:** The framework of DRT with the slack departure strategy.
The three main contributions of this study are as follows: (1) We propose a new slack departure strategy with the discount-incentive mechanism, which can improve operating profit and attract passengers to use DRT service for a long time. (2) The application of bounded rationality in DRT. Bounded rationality can better describe the passenger’s choice decision. (3) Pareto optimality is used to resolve multiobjective DARP. In comparison with the frequent weighted average method, the operation manager can handle the full picture of feasible optimal schemes, which is beneficial to establishing the price standard and the service standard.

3. Problem Description and Modeling

In this section, bounded rationality is introduced to the DRT model with a slack departure strategy. We present the problem description in Section 3.1. Section 3.2 describes the objective function and constraints of the multiobjective programming model, which develops a framework that considers the decision-making processes of boundedly rational passengers and the DRT operator.

3.1. Problem Description. \( G = (N_0, A), N_0 = \{1, 2, 3, \ldots, n, n+1\} \) is the set of nodes, and \( A = \{(i, j), i, j \in N_0, i \neq j\} \) is the set of arcs connecting node \( i \) and node \( j \). As is illustrated in Figure 2, green node and red node represent the departure point and arrival point, respectively. The passenger who adjusts departure time to the adjacent time window is depicted as the yellow node. Each node in the set \( N \) can be not only the departure point but also the arrival point. Moreover, \( K \) is defined as a set of vehicles. Each vehicle in set \( K \) has a capacity \( Q_k \).

In contrast with the other VRP, the PDP model that considers adjusting the passenger’s departure time is substantially more complex. Table 2 shows the sets, parameters, and decision variables involved in the model.

Hypothesis 1

H1: this study is a static-deterministic problem based on the reservation mechanism, in which the origin/destination and departure/arrival times are not allowed to be modified after system matching

H2: after a passenger gets off, his/her seat will be allocated to the other passenger

H3: only when passengers’ travel cost does not exceed his/her maximum acceptable cost, his/her departure time can be adjusted

H4: the time for the passenger to get on and off is ignored

3.2. Mathematical Model. The proposed model involves two decision-making problems: (1) how to design the service system to maximize the operating profit while ensuring the service levels; (2) how do boundedly rational passengers make departure time choices. Three objective functions are established to describe the problem.

3.2.1. Operating Profit. The first optimization objective of operators is to maximize profits, which is calculated as the sum of fares minus the operating cost. Operating cost is divided into two components: dispatching cost and routing cost. \( c_v \) represents the fixed dispatching cost of a vehicle, which is composed of the vehicle purchase cost, driver salary, and vehicle insurance cost; \( c_d \) represents the distance-based fare rate of a vehicle, including fuel cost and depreciation. Note that node 1 is the depot.

\[
\max F_1 = \sum_{p \in P} c_v \sum_{k \in K} \sum_{j \in N} y_{ij}^k - c_d \sum_{p \in P} \sum_{j \in N} \sum_{k \in K} y_{ij}^k d_{ij}. \tag{1}
\]

3.2.2. Passengers Departing at the Expected Time. The second objective function can be formulated as the maximization of the number of passengers who travel on the expected departure time. In the process of supply and demand matching, the operator needs to ensure that passengers get on their expected travel schedule to the greatest extent. Only when passengers cannot be assigned or matched with the appropriate vehicle, their departure times will be modified. \( M \) represents the total number of passengers in the system.

\[
\max F_2 = m - \sum_{p \in P} \mu_p. \tag{2}
\]

3.2.3. General Travel Cost. The general travel cost of passengers can be categorized into four parts: travel cost, time cost, penalty cost for changing the departure time (DC\( P \)), and penalty cost for deviation from the expected arrival time...


**Table 2: Notation for the sets, parameters, and decision variables.**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sets</td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>Set of nodes, $N = {1, 2, \ldots, n}$</td>
</tr>
<tr>
<td>$N_0$</td>
<td>Set of nodes, $N_0 = N \cup {n+1}$ node $n+1$ is dummy destination</td>
</tr>
<tr>
<td>$A$</td>
<td>Set of arcs</td>
</tr>
<tr>
<td>$P$</td>
<td>Set of passengers, $P = {1, 2, \ldots, m}$</td>
</tr>
<tr>
<td>$K$</td>
<td>Set of service vehicles, $K = {1, 2, \ldots, s}$</td>
</tr>
<tr>
<td>Parameters</td>
<td></td>
</tr>
<tr>
<td>$c_R$</td>
<td>Regular fare of passenger $p$ calculated according to the shortest distance</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Fare of passenger $p$ considering the operator’s profit</td>
</tr>
<tr>
<td>$\Delta c$</td>
<td>Additional fare, when the number of passengers is less than the minimum load $Q_{\text{min}}$</td>
</tr>
<tr>
<td>$c_v$</td>
<td>Fixed dispatching cost of a vehicle</td>
</tr>
<tr>
<td>$c_d$</td>
<td>Distance-based cost of a vehicle</td>
</tr>
<tr>
<td>$d_{ij}$</td>
<td>Distance between node $i$ and node $j$</td>
</tr>
<tr>
<td>$m$</td>
<td>Total number of passengers</td>
</tr>
<tr>
<td>$A$</td>
<td>Value coefficient</td>
</tr>
<tr>
<td>$B$</td>
<td>Penalty coefficient for changing the departure time</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Penalty coefficient for deviation from the expected arrival time</td>
</tr>
<tr>
<td>$\delta_1, \delta_2$</td>
<td>Concave-convex extent of the value function in prospect theory</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Varying risk preference for late arrival and late departure</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Aspiration level of passengers</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Price discount obtained by changing departure time</td>
</tr>
<tr>
<td>$t_{ij}$</td>
<td>Travel time along arc $(i, j) \in A$</td>
</tr>
<tr>
<td>$(ED_p, LD_p)$</td>
<td>Expected departure time window of passengers $p$, he/she submits (earliest departure time, latest departure time) in DRT system</td>
</tr>
<tr>
<td>$(EA_p, LA_p)$</td>
<td>Expected arrival time window of passengers $p$, he/she submits (earliest arrival time, latest arrival time) in DRT system</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>Maximum acceptable travel cost of passenger $p$</td>
</tr>
<tr>
<td>$\tau^p_c$</td>
<td>Minimum time cost for passenger $p$</td>
</tr>
<tr>
<td>$mc^p$</td>
<td>Minimum money cost for passenger $p$</td>
</tr>
<tr>
<td>$Q_i^k$</td>
<td>Load of vehicle $k$ before arriving at node $i$</td>
</tr>
<tr>
<td>$Q_{\text{min}}$</td>
<td>Minimum load of vehicle $k$</td>
</tr>
<tr>
<td>$Q_{\max}$</td>
<td>Capacity of vehicle $k$</td>
</tr>
<tr>
<td>$w$</td>
<td>Length of the time window</td>
</tr>
<tr>
<td>$t_{\text{max}}$</td>
<td>Threshold of detour time</td>
</tr>
<tr>
<td>$D_p^i, A_p^j$</td>
<td>If passenger $p$ gets on at node $i$, $D_p^i = 1$; otherwise, $D_p^i = 0$</td>
</tr>
<tr>
<td>$D_p^i, A_p^j$</td>
<td>If passenger $p$ gets off at node $j$, $A_p^j = 1$; otherwise, $A_p^j = 0$</td>
</tr>
<tr>
<td>Decision variables</td>
<td></td>
</tr>
<tr>
<td>$x_p^k$</td>
<td>Passenger assignment variable (if passenger $p$ is assigned to vehicle $k$, $x_p^k = 1$; otherwise, $x_p^k = 0$)</td>
</tr>
<tr>
<td>$y_{ij}^k$</td>
<td>Vehicle routing variable (if the vehicle $k$ performs the task of arc $(i, j)$, $y_{ij}^k = 1$; otherwise, $y_{ij}^k = 0$)</td>
</tr>
<tr>
<td>$z_p$</td>
<td>Time adjusted variable (the departure time of passenger $p$ has been adjusted, $z_p = 1$; otherwise, $z_p = 0$)</td>
</tr>
<tr>
<td>$\bar{t}_p$</td>
<td>Final price of passenger $p$</td>
</tr>
<tr>
<td>$DT_p$, $AT_p$</td>
<td>Actual departure time and arrival time of passenger $p$</td>
</tr>
<tr>
<td>$DC_p$, $AC_p$</td>
<td>Penalty cost of passenger $p$ for changing the departure time</td>
</tr>
<tr>
<td>$DC_p$, $AC_p$</td>
<td>Penalty cost of passenger $p$ for deviation from the expected arrival time</td>
</tr>
<tr>
<td>$t_i^k$</td>
<td>Arrival time of vehicle $k$ at node $i$</td>
</tr>
<tr>
<td>$Q_i^k$</td>
<td>Load of vehicle $k$ arriving at node $i$</td>
</tr>
</tbody>
</table>

(AC$_p$). Only the passenger whose departure time is changed has DC$_p$ in this study. Similarly, AC$_p$ will arise if the vehicle’s arrival time is outside the expected time window.

\[
\min F_3 = \sum_{p \in P} \alpha + \sum_{p \in P} (AT_p - DT_p) + \sum_{p \in P} DC_p + \sum_{p \in P} AC_p.
\]  

(3)

To describe the impact of time deviation on passengers, prospect theory [41] is applied to estimate the penalty costs of passengers. The expected departure time/arrival time is defined as the reference point. An enormous difference exists in the penalty for early arrival and late arrival. The psychological effect of late arrival for the same deviation time is significantly greater than that of early arrival. Consequently, parameters $\delta_1$ and $\delta_2$ indicate the concave-convex extent of the value function in prospect theory, and $\epsilon$ indicates the varying risk preference for late arrival and late departure. The calculation process is expressed as follows:
3.2.4. Objective Function. A multiobjective mathematical programming model is established to analyze the interaction between the operation organization and passengers. The operation organization takes the operating profits as the main research objective, which ensures maximizing the profit and the number of passengers departing at the expected time. General travel cost is defined as the objective for passengers.

The operation organization’s objectives are as follows:

\[
\max F_1, \max F_2. 
\]  
(6)

The passengers’ objective is as follows:

\[
\min F_3.
\]  
(7)

3.2.5. Departure Time Choice Based on Bounded Rationality. The slack departure strategy changes the departure times of passengers. According to the satisficing theory of bounded rationality, passengers do not always pursue the optimal solution, rather than a relatively satisfactory route. Therefore, they are willing to adjust their departure time for the discount, only if the travel cost is less than their maximum acceptable travel cost \( \theta_p \).

The maximum acceptable travel cost is the product of the minimum travel cost of the passenger and the aspiration level \( \lambda \) [28]. Formula (8) describes the minimum travel cost of the passenger. If the passenger is an economic man (\( \lambda = 1 \)), he/she only accepts the option of the lowest travel cost. This paper focuses on the departure time choice of bounded rational passengers under the diversified aspiration levels. The linearized formula (9) represents the boundedly rational travel decision rule in the system.

\[
\theta_p = \lambda (mc_p + tc_p). 
\]  
(8)

\[
\overline{\theta}_p + (AT_p - DT_p) \leq \lambda (mc_p + tc_p) + M(1 - z_p), \forall p \in P. 
\]  
(9)

3.2.6. Profit Principle and Discount-Incentive Mechanism. Fare is designed to fulfill the need of the operator’s profit and the discount-incentive mechanism. Considering the operator’s profit, a low load factor of the vehicle may lead to the risk of deficit. We set the threshold for the number of passengers in order to maintain breakeven. When the number of passengers in a vehicle is less than the threshold \( Q_{\text{min}} \), passengers have to pay an additional fare \( \Delta c \). The initial fare \( c_p \) of passenger \( p \) is obtained by formula (10). Additionally, passengers change their departure time in exchange for the discount \( \mu \). Formula (11) demonstrates the final price \( \overline{c}_p \) of passenger \( p \) after discount.

\[
c_p' = \begin{cases} 
  c_p + \Delta c & Q_k^i \leq Q_{\text{min}} \\
  c_p & Q_k^i > Q_{\text{min}}.
\end{cases} 
\]  
(10)

\[
\overline{c}_p = \begin{cases} 
  \mu \times c_p', z_p = 1 \\
  c_p', z_p = 0.
\end{cases} 
\]  
(11)

3.2.7. Model Constraints

Assignment Constraint. Constraint (12) ensures the passenger \( p \) is arranged to exactly one vehicle \( k \).

\[
\sum_{k \in K} x_{pk} = 1, \forall p \in P. 
\]  
(12)

Vehicle routing Constraints. Constraint (13) represents that the vehicle starts from the depot (node 1) and finally returns to dummy destination (node \( n+1 \)). Constraint (14) indicates the flow balance. Constraint (15) represents the origin and destination of a passenger is served by the same vehicle.

\[
\sum_{j \in N \setminus \{1\}} y_{ij}^k = 1, \sum_{i \in N} y_{i,n+1}^k = 1, \forall k \in K. 
\]  
(13)

\[
\sum_{i \in N} y_{ih}^k - \sum_{j \in N_i} y_{bh}^k = 0, \forall h \in N \setminus \{1\}, k \in K. 
\]  
(14)

\[
\sum_{i \in N} \left( \sum_{j \in N} y_{ij}^k \times D_{ij}^p + \sum_{j \in N} y_{ij}^k \times A_{ij}^p \right) \geq x_{pk}, \forall p \in P, k \in K. 
\]  
(15)

Vehicle Capacity Constraints. To provide a safe and comfortable service for passengers, constraints (16 and 17) are defined as the “one-person-one-seat” [1, 2]. The maximum number of passengers at any node in vehicle \( k \) cannot surpass the capacity of the vehicle. Constraint (17) represents the number of vehicles \( k \) at node \( j \).
Time Window Constraints. Constraints (18 and 19) are the linearized forms of the time when vehicle $k$ arrives at node $j$. Constraints (20 and 21), and constraints (22 and 23), respectively, show the calculation of departure time and arrival time of passenger $p$. This paper expands the scope of the departure time window. To identify the slack departure strategy, constraints (24 and 25) indicate the departure time window is extended from $(ED_p, LD_p)$ to $(ED_p - w, LD_p + w)$.

$$t_j^k \geq t_i^k + t_{ij} - M(1 - y_{ij}^k), \; \forall i, j \in N, k \in K. \tag{18}$$

$$t_j^k \leq t_i^k + t_{ij} + M(1 - y_{ij}^k), \; \forall i, j \in N, k \in K. \tag{19}$$

$$DT_p \geq \sum_{j \in N} t_j^k D_p^j - M(1 - x_{ij}^k), \; \forall p \in P, k \in K. \tag{20}$$

$$DT_p \leq \sum_{j \in N} t_j^k D_p^j - M(1 - x_{ij}^k), \; \forall p \in P, k \in K. \tag{21}$$

$$AT_p \geq \sum_{j \in N} t_j^k A_p^j - M(1 - x_{ij}^k), \; \forall p \in P, k \in K. \tag{22}$$

$$AT_p \leq \sum_{j \in N} t_j^k A_p^j - M(1 - x_{ij}^k), \; \forall p \in P, k \in K. \tag{23}$$

$$ED_p - w - \sum_{i \in N} t_i^k D_p^i \leq M(1 - x_{ij}^k), \; \forall p \in P, k \in K. \tag{24}$$

$$\sum_{i \in N} t_i^k D_p^i - (LD_p + w) \leq M(1 - x_{ij}^k), \; \forall p \in P, k \in K. \tag{25}$$

If passengers alter their departure time, $z_p = 1$; otherwise, $z_p = 0$.

$$ED_p - M \cdot z_p \leq DT_p, \; \forall p \in P. \tag{26}$$

$$LD_p + M \cdot z_p \geq DT_p, \; \forall p \in P. \tag{27}$$

Driver and Passenger Constraints. From the perspective of the driver, they should avoid fatigue driving. Constraint (28) imposes the driver’s maximum service time.

$$\sum_{i \in N} \sum_{j \in N} t_{ij} y_{ij}^k \leq t_{max}, \; \forall k \in K. \tag{28}$$

From the perspective of passengers, constraint (29) indicates the passenger’s maximum detour time [28].

$$AT_p - DT_p - t_{c_p} \leq t_{max}, \; \forall p \in P. \tag{29}$$

4. Solution Algorithms

Based on the studies [8, 42], VRP, DARP, and PDPTW are all proved to be NP-hard problem. As mentioned in Section 2, the problem we study is a special form of PDPTW. Therefore, this study is also an NP-hard problem. The feasible solutions to the NP-hard problem increase exponentially with the increasing number of passengers. Due to the complexity of the NP-hard problem, it is difficult to obtain an accurate solution to the large-scale computation in polynomial time.

Since the mathematical model is an NP-hard problem, this section established a two-phase heuristic algorithm to get the Pareto solutions set. The first stage is travel demand classification. Based on k-means algorithm, the initial request set is aggregated into $k$ independent subsets. Moreover, the second stage is the pickup and delivery problem. The variable neighborhood search (VNS) algorithm is adopted to acquire the optimal solution.

4.1. Travel Demand Classification. Travel demand classification is the critical step in solving the model. The aim is not to cluster adjacent stations to achieve spatial integration but to divide the passengers into $k$ independent subsets according to their origin and destination. Space partitioning clustering is studied extensively in solving efficient heuristic algorithms [43, 44]. The classical K-means is appropriate for this study.
Basic principles of classification: If a group of passengers has similar route directions, they can be clustered into a class according to distance attributes. The distance between any two points in different classes should be greater than that in the same class. Euclidean distance is the large-scale deployment in various classification algorithms [45] and is used in this part.

4.2. VNS Algorithm. Relying on the above classification, the research is transformed into a smaller scale problem. In this section, variable neighborhood search (VNS) is designed to seek the Pareto optimal solution set. Compared with genetic algorithms [46], tabu search [47, 48], simulated annealing [49], VNS is suitable for various optimization problems because of its simple structure [50, 51]. The main idea is to change the neighborhood regularly in the local search procedure to improve the searchability of the algorithm. Specific steps are presented in Algorithm 2.

4.2.1. Generation of Initial Set. During the initialization phase, we construct a set of feasible initial routes with high service quality. The initial solution is generated by the insertion method. The procedure stops until all the passengers have been inserted in. Furthermore, the initial number of service vehicles is obtained. Generation of the initial set is as Algorithm 3.

4.2.2. Shaking Procedure. The shaking procedure is a crucial stage in designing an effective VNS algorithm. The aim is to perturb solutions in defined neighborhoods while avoiding the search being trapped in the local optimum [52]. 1–1 exchange and 1–0 shift are adopted to generate neighboring solutions.

1–1 exchange: Two passengers are randomly swapped from varying paths. Figure 3(a) illustrates the process of 1–1 exchange. The first route consists of 8 passengers, and the second route consists of 5 passengers. According to the principle of 1–1 exchange, passenger 5 of the first route and passenger 2 of the second route are randomly chosen, and the two passengers are exchanged and combined again to form a new route. If all routes are feasible, the exchange is stopped, and the local search is entered directly. Otherwise, the exchange process is repeated until two new routes are feasible.

1–0 shift: A passenger from the original path is deleted randomly and removed from the new path. The 1–0 shift process is shown in Figure 3(b). If two new routes are effective, move on to the next step.

4.2.3. Variable Neighborhood Descent. The neighborhood operators involved in the search process include the swap operator, insert operator, and inverse operator. In the search process, the algorithm considers both intraroute search and interroute search. The weight of each operator in the iteration is determined according to roulette.

Swap: Figures 4(a) and 5(a) show swap two passengers to generate a new path. As is shown in Figure 4(a), passenger 7 of the first route and passenger 2 of the second route are swapped.

Insert: A customer is selected randomly in the path. Figure 5(b) and Figure 4(b) reveal the passenger is deleted from the current location and then inserted into other locations.

Inverse: Figure 5(c) and Figure 4(c) describe swapping two passengers, and the passenger sequence is reversed.

4.2.4. Pareto Solutions. Pareto solutions can provide the picture with all potential optimal routes for the operation manager, which is beneficial to establish a reasonable fare and service standard. In the local search process, we set up the number of iterations in advance. A set of nondominated solutions is to be achieved by updating the elite pool, where Pareto set solutions are stored. First of all, the objective function of the initial solution is calculated. The second step is to update the elite pool as highlighted in section 4.2.2 and section 4.2.3. A new solution by selecting the neighborhood operators has been generated if its objective function values are better than the existing Pareto solution’s one. The above process is repeated to seek the nondominant solution and update the Pareto solutions.

5. Experimental Results

In this section, a numerical experiment on a small-scale network and the real-world circumstance of Beijing are discussed to evaluate the effectiveness of the proposed model and two-phase heuristic algorithm. The aim is to answer the aforementioned three questions in the introduction. Sensitivity analysis is to evaluate the impact of parameters on system performance including passenger’s bounded rationality, time window, and discount-incentive mechanism. The experiments are conducted in MATLAB on the computer (Intel Core i5 CPU @ 2.2 GHz).

Based on the traffic analysis report of major cities in China [52], the operating speed of the service vehicle is 26 Km/h. Other parameter values refer to previous studies [2, 20, 28]. Parameter values for the analysis are listed in Table 3.

5.1. Numerical Test

5.1.1. Sioux Falls Network. The Sioux Falls network is regarded as a small-scale network, which is widely used in numerous studies. As displayed in Figure 6, the network is composed of 24 nodes and 38 bidirectional arcs. The number on the arc indicates travel time. Passengers can choose any node as the origin and destination. We set the vehicle capacity as 8, the minimum load factor $q_{\text{min}}$ is 4.20, and hypothetic travel demands are randomly generated in this paper. Green nodes represent the demand station of passengers.

5.1.2. Comparison of Optimal Solutions and Pareto Solutions. Four scenarios are defined to distinguish the effect of the proposed model. Scenario 1 adopts a fixed departure time window; namely, the departure time of passengers is not
Input: A set of passengers’ locations  
Output: A set of feasible travel demand classifications.  
Step 1: Input the position information of \( n \) passengers.  
Step 2: Randomly generate \( k \) passengers from \( n \) passengers as the number of passenger groups.  
Step 3: Calculate the distance between each passenger and \( k \) cluster centers. Assign passengers to the cluster centers with the smallest distance.  
Step 4: For the formed passenger group, recalculate the sum of the distance from all the positions to their nearest cluster center. Renew the cluster center.  
Step 5: Repeat step 3. If the position of the cluster is no longer changed, output the clustering result.

**Algorithm 1: Travel demand classification.**

Input: A set of travel demands  
Output: A set of Pareto solutions \( S^* \)  
Initial set  
Set parameters (\( NP \), maximum number of iterations \( K_{\text{max}} \), \( L_{\text{max}} \))  
Get Nondominated (FX)  
for \( k = 1 \) to \( K_{\text{max}} \)  
for \( l = 1 \) to \( L_{\text{max}} \)  
\( S^* \leftarrow S^* \)  
Pick a random solution \( S_b \) in the neighborhood of \( S \) \( // \) Shaking  
Perform a neighborhood search \( S_a \) to get \( S_b \) \( // \) generate New \( S \)  
If \( f(S_b) < f(S^*) \)  
\( S^* \leftarrow S_b \)  
break;  
ext else if \( f(S_b) = f(S^*) \)  
choose one of them randomly;  
end if  
end for  
end for  
report \( S^* \);  
end

**Algorithm 2: VNS algorithm.**

Input: A set of travel demands.  
Output: A set of initial routes.  
Step 1: Randomly choose the passenger’s origin with the earliest departure time.  
Step 2: Take this point as the center to randomly select the next passenger. The constraints of time window and vehicle capacity must be satisfied. Repeat this step if the constraint cannot be met.  
Until an initial vehicle is generated.  
Step 3: Repeat step 1. If all customers in the system complete the insertion, the formed set of feasible routes are the initial solution.

**Algorithm 3: Generation of initial set.**

![Image](image-url)  
**Figure 3:** Neighborhood operators. (a) 1–1 exchange neighbourhood. (b) 1–0 shift neighbourhood.
Figure 4: Interroute search. (a) Swap operator. (b) Insert operator. (c) Inverse operator.

Figure 5: Intraroute search. (a) Swap operator. (b) Insert operator. (c) Inverse operator.

Table 3: The value of parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating speed</td>
<td>26 km/h</td>
</tr>
<tr>
<td>Vehicle capacity</td>
<td>8 persons/vehicle</td>
</tr>
<tr>
<td>Time window</td>
<td>5 min</td>
</tr>
<tr>
<td>Discount rate</td>
<td>0.8</td>
</tr>
<tr>
<td>Passengers’ aspiration level</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Figure 6: Simplified Sioux Falls network.
allowed to be changed. Scenarios 2–4 simulate the effect of the slack departure strategy. Scenario 2 is the highest operating profit in the Pareto solutions. Scenario 3 is the maximum number of passengers departing at the expected time in the Pareto solutions. Scenario 4 is the minimum general travel cost in the Pareto solutions.

We provide a comparison between optimal solutions and Pareto solutions. The DRT routes and departure times created by the two-phase heuristic algorithm are similar to the optimal solutions. The detailed information is displayed in (available here). The deviation mainly exists in scenario 4, in which the DRT system refuses to serve a passenger. Consequently, the optimality gap of operating profit is 4.72%. As the results show, the two-stage algorithm can effectively solve the proposed model, and Pareto solutions are suitable for the DRT optimization problems.

5.2. Case Study. Beijing case study is proposed to confirm the effect of the slack departure strategy. First of all, the travel demands of passengers are obtained through the recruitment survey in Yi Zhuang. Subsequently, by comparison with the various scenarios, the results demonstrate that the slack departure strategy benefits both the operator and passengers. Eventually, we conduct a sensitivity analysis of system parameters.

5.2.1. Survey of Beijing. On October 7, 2021, and October 24, 2021, we recruited 89 clients in Yi Zhuang to conduct a face-to-face survey. The survey involves travel characteristics of clients, including the origin/destination, departure time/arrival time, expected charging standard, expected time window, and accepted discount. Referring to the travel information achieved from the survey, we generate 300 passenger demands, respectively. DRT is especially expensive compared with the fixed bus and subway. Therefore, to improve the authenticity of the experiment, the passenger’s demands that can be directly reached by bus and subway were deleted. The final number of passenger’s demands was 256. Figure 7 illustrates the OD pairs of passengers and grouping results. The parameters of the survey are listed in Table 4.

Note: This survey was conducted during the period of COVID-19, so we cannot eliminate the impact of the epidemic on the psychology of travelers in this paper.

5.2.2. Benefit of the Slack Departure Strategy. To prove the benefits of the strategy, this section provides a comparison between the fixed departure time and the slack departure strategy. The same as Section 5.1.1, scenario 1 adopts a fixed time window. Scenario 2–4 simulates the slack departure strategy, using a dynamic variable time window. The results are depicted in Table 5.

These results indicate that the slack departure strategy with the discount-incentive mechanism can benefit both the operator and passengers. In comparison with scenario 1, scenarios 2–4 significantly increase the operational profit and load factor. Meanwhile, from the passenger’s perspective, fare as well as travel cost has decreased steadily. Take scenario 4 as an instance, the operating profit is promoted by 63% compared to scenario 1. Moreover, the load factor has increased by as high as 30%. The average fare declines from 13.07 RMB to 11.21 RMB, while the general travel cost of passengers reduces by roughly 12%. Overall, the slack arrival strategy with the discount-incentive mechanism is not only positive to improve the operator’s benefit and system performance but also to decrease passengers’ travel costs.

5.2.3. Bounded Rationality of Passengers. To better understand the association between passengers’ bounded rationality and implementation effect, we evaluate the impact of passengers’ aspiration level on the operating profit, load factor, general travel cost, and average fare. \( \lambda \) is defined as the aspiration level of passengers. When the passenger is an economic man, that is, \( \lambda = 1.0 \). The rationality degree of passengers gradually decreases with the increase of \( \lambda \). In compliance with the paper [28], the range of values for \( \lambda \) is 1.0 to 2.0.

Figure 8(a) demonstrates the impact of the aspiration level of passengers on the operating profit. There has been a significant surge from 467 RMB to 924 RMB in the operating profit. The load factor follows a similar tendency in Figure 8(b), indicating that the aspiration level of passengers is higher, and the load factor is higher. By contrast, Figure 8(c) describes the steady decline by 15% in general travel cost. Low levels of rationality may give rise to accepting the slack departure strategy. In spite of some fluctuation in fare, Figure 8(d) displays a downward trend with variations in the aspiration level. In other words, operating profit and load factor substantially decline with the increase of passenger’s rationality degree, while the general travel cost and fare are expected to rise moderately. The more passengers with a low rationality degree, the more remarkable the improvement effect is.

5.2.4. Time Window. To evaluate the influence of time windows on system performance, we further estimate four varying time windows \((w = 3, 5, 8, 10\text{ min})\). As detailed in Figure 9(a), expanding the scope of the time window will increase the operating profit. When the value of \( w \) ascends from 3 to 8, the operating profit increases to 872 RMB. Nevertheless, after this point, there is a tremendous upward in the operating profit with the variation of the time window. Therefore, the most economical time window may be 10 minutes. But the most expected time window for passengers is 5 minutes as shown in Table 3. In response to the passenger satisfaction, we set the time window as 5 min. In addition, Figure 9(b) displays load factor gradually increases by 39% with the expansion of the time window. Figure 9(c) and Figure 9(d) indicate that general travel cost and average fare are slightly declining as time windows rise from 3 min to 10 min.
5.2.5. Discount-Incentive Mechanism. This section further investigates the interactive impact between charging standard and discount rate in the discount-incentive mechanism. In order to reduce passengers’ dissatisfaction while maintaining the quality of service, the operator provided specific discounts for passengers who are willing to alter their departure time. Figure 10(a) reveals the impact of charging standard and discount rate on the operating profit. The operating profit remarkably ascends to 1828 RMB (the highest value) as the charging standard improves from 0.75 to 1.5. Moreover, when the discount rate changes from 0 to 0.2, there has been a significant rise in profit. After a certain point, the operating profit drops despite promoting the discount rate. The reason is that excessive discounts may result in losses for the operating company. Consequently, the turning point in Figure 10(a) is the optimal discount rate. Interestingly, the result is in accord with the data obtained from the survey in Beijing. Operators have the lowest profit when there is no discount, which demonstrates the effectiveness of the discount-incentive mechanism.

Furthermore, Figure 10(b) indicates the load factor stays relatively stable with the increasing charging standard. As opposed to the charging standard, the discount rate has a remarkable effect on the load factor. The load factor gradually rises by improving the discount rate. However, Figure 10(c) represents an upward trend in general travel cost with the increase in charging standards. Meanwhile, the higher the discount rate is, the lower the general travel cost is. In terms of discounts and price, the results in Figure 10(d) prove fare is the same variation trend as that of general travel cost.

The results confirm that the impacts of the passengers’ bounded rationality and discount rate on the system are interdependent. The discount-incentive mechanism may be an interesting option that is inclined to develop a more efficient DRT system.

Table 5: Scenario comparison for the case study of Beijing.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Operating profit</th>
<th>Passengers departing at the expected time</th>
<th>Load factor (%)</th>
<th>Average fare</th>
<th>Average general travel cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>467</td>
<td>189</td>
<td>33.75</td>
<td>13.07</td>
<td>27.34</td>
</tr>
<tr>
<td>2</td>
<td>867</td>
<td>142</td>
<td>60.14</td>
<td>12.26</td>
<td>26.11</td>
</tr>
<tr>
<td>3</td>
<td>831</td>
<td>148</td>
<td>54.03</td>
<td>12.12</td>
<td>25.81</td>
</tr>
<tr>
<td>4</td>
<td>763</td>
<td>133</td>
<td>63.75</td>
<td>11.21</td>
<td>24.06</td>
</tr>
</tbody>
</table>

Table 4: Survey parameters and results.

<table>
<thead>
<tr>
<th>Expected charging standard (RMB/km)</th>
<th>Expected time window (min)</th>
<th>Accepted discount (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>5</td>
<td>20</td>
</tr>
</tbody>
</table>
Figure 8: Sensitivity analysis of the aspiration level. (a) Operating profit. (b) Load factor. (c) General travel cost. (d) Fare.

Figure 9: Sensitivity analysis of the time window. (a) Operating profit. (b) Load factor. (c) General travel cost. (d) Fare.
6. Conclusion

In this paper, a slack departure strategy for DRT based on bounded rationality is formulated to improve the operating profit while ensuring service quality. Meanwhile, in response to the negative effects of modifying departure times, the discount-incentive mechanism is proposed to encourage passengers to accept the changes. Passengers are no longer merely passive recipients. Bounded rationality is investigated in the model to represent their decision-making process of departure time. Furthermore, to express the impact of time deviation on passengers, we incorporate the prospect theory to improve the penalty function of passengers. Ultimately, DRT system performance can be promoted by maximizing the operating profit, the number of passengers departing at the expected time, as well as minimizing the generalized cost of passengers.

A two-phase heuristic algorithm is applied to resolve the multiobjective programming model. We utilize the simplified Sioux Falls network to prove the effectiveness of the proposed model and algorithm. The case study of Beijing demonstrates that the strategy can significantly enhance the operating profit and decrease the passenger travel cost. Simultaneously, the load factor has increased by about 30%. The data show that the rationality degree of passengers is lower, and the effect of the strategy is better. Despite increasing the discount rate, the system performance cannot be promoted after a certain point. Therefore, the best discount rate of the incentive mechanism is 20%. Why most passengers choose the accepted discount is 20%, which is "80% of the price" in Chinese. Whether it is related to the lucky number 8 is an interesting topic. Overall, the results indicate that the slack departure strategy with a reasonable incentive mechanism can benefit both DRT operators and passengers. Considering passengers' decision-making psychology in the optimization of DRT problem is conducive to the research more consistent with the actual situation.

For future research, the potential enhancements could focus on the following directions: (1) taking into account the heterogeneous vehicle schedule, fleet size and vehicle type.
are needed to optimize. (2) Carbon emission is a topic worthy of study. A model is established to test whether the slack departure strategy can effectively reduce carbon emissions. (3) A scalable nonmyopic dynamic DRT problem may be discussed to promote the system’s performance [53]. (4) The solution algorithm should be improved to calculate a larger-scale DRT service network. [54].

Data Availability
The data used to support the findings of this study are included within the article and the supplementary information file.

Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments
This research was funded by the National Natural Science Foundation of China (No. 71971005), the Beijing Municipal Natural Science Foundation (No. 8202003, No. 8212002). The authors would like to thank Jie Xiong, Peifei Zhao, and Yuan Zhang for providing guidance.

Supplementary Materials
Appendix A: Hypothetic travel demands, Appendix B: Optimality gaps of the fixed departure time window, Appendix C: Optimality gaps of the slack departure strategy. (Supplementary Materials)

References

Journal of Advanced Transportation 15


