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Research Article

Performance Assessment of a Rehabilitation Transportation Reservation Matching Service with Market Design Mechanisms

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Government agencies provide huge amount of subsidies to support the rehabilitation transportation service over the past decade in eastern Taiwan; however, low demand request fulfillment rate, limited medical and transportation resources, long travel distances, and an extremely high percentage of dead mileages are still the main challenges faced by rehabilitation transportation service providers. This study applies the market design theory to match the rehabilitation buses with the requests of patients, so as to improve resource utilization efficiency in rural areas. The developed market design mechanisms aim to allocate resources to those who need them most in a matching manner, by using the deferred acceptance algorithm and the top trading cycle approach. The model is initialized with the requests of those who choose the rehabilitation bus based on their desired boarding time slots. On the other hand, the service providers of the rehabilitation bus would determine patients' schedule based on their disability level, willingness to share the ride, number of fulfilled appointments during this month, and the travel distance of this trip as the order of preference. Since the current vehicle dispatching rule is to reserve seats of a rehabilitation bus on the "first-come-first-served" basis, and it cannot fully satisfy patients need. In accordance with the historical data, 63 of 72 demand requests could successfully reserve the seats. In the "first-come, first-served" mode, 48 requests obtained the first-ranking shift (i.e., their desired time slots), and the sum of their disability level score is 155. In the market design matching mode, 57 requests obtained the first-ranking shift, and the sum of their disability level score is 170, which demonstrates that the proposed market design matching mechanism outperforms than the conventional rules.

1. Introduction

According to the definition of the World Health Organization, Taiwan has become an aged society (i.e., a population over 65 years old exceeding 14%) by 2013, with 14.5% of the population over 65, and is expected to become a super-aged society (i.e., a population over 65 years old exceeding 20%) by 2026, with an expected 20.7% of the population over 65 [1]. In accordance with the statistical results provided by the Ministry of Health and Welfare, Taiwan, R.O.C. [2], there were 1,195,448 people in Taiwan with certificates of disabilities in the third quarter of 2022, comprising around 5.15% of the total population. It should be noted that the credentials of disabilities are divided into four levels: extremely severe, severe, moderate, and mild, which are 11.7%, 16.9%, 32.2%, and 39.2%, respectively, of the current overall disabled population in Taiwan. In 2021, both the elderly population ratio and the disability ratio of Hualien County (26.09% and 8.21%) were significantly higher than the national average (23.81% and 5.15%), as shown in Figures 1 and 2, respectively. Figure 1 shows the elderly population ratio in Taiwan from 1996 to 2021, while Figure 2 displays the elderly population ratio in Hualien County from 1996 to 2021.

Hualien County is located in eastern Taiwan, with a low residential density of around 70 people per square kilometer [3] and a wide service area. It is the largest county-level administrative region in Taiwan. According to the statistics

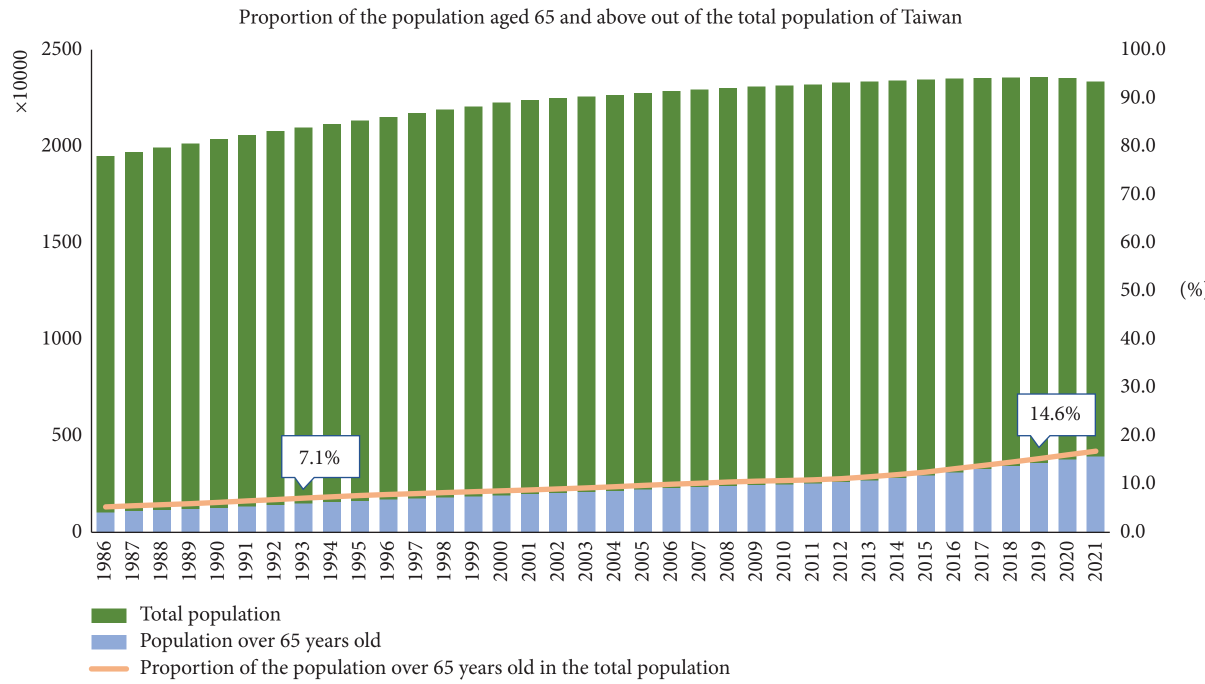

Figure 1: The proportion of the aged 65+ out of the total population of Taiwan.

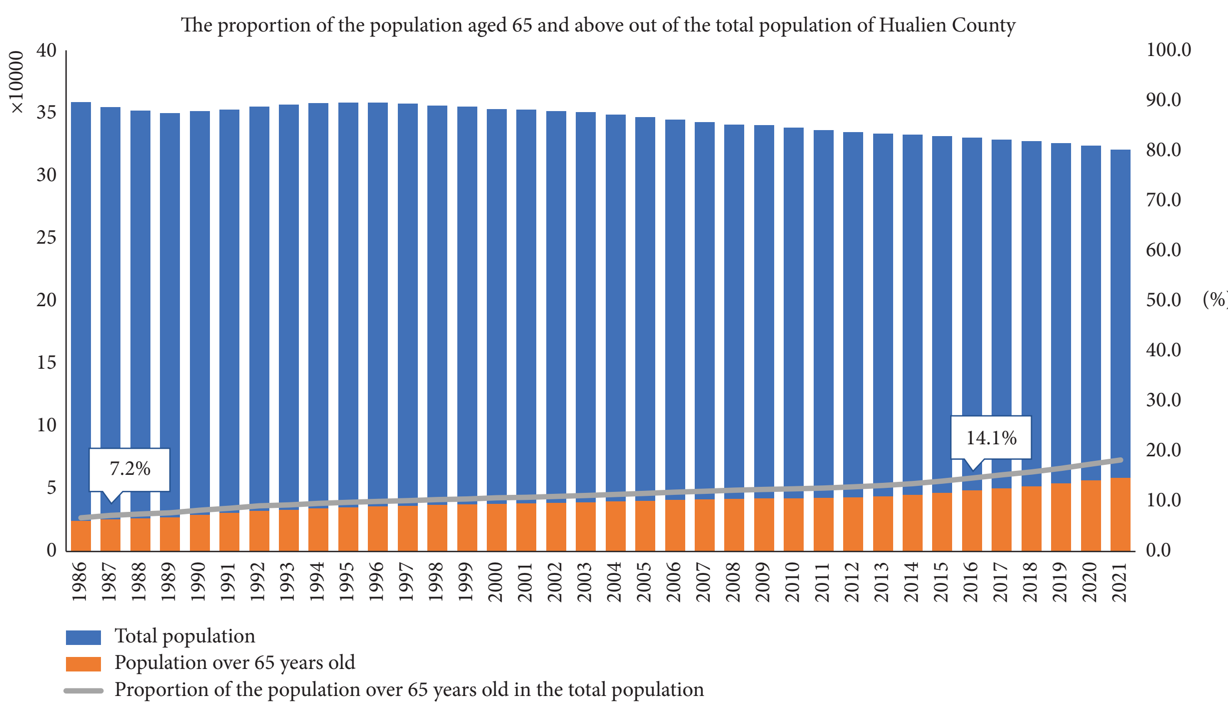

Figure 2: The proportion of the aged 65+ out of the total population of Hualien County.

from the Hualien County Government, in March 2022, the elderly population (over 65 years old) accounts for 18.5% of the total population. In addition, there are 26,489 people with disabilities, accounting for 8.2% of the total population.

There are three types of medical resources in Hualien County [4], namely: medical centers, regional hospitals, and district hospitals. Figure 3 illustrates the distribution of medical resources in Hualien County. Medical centers have the most professional staff and resources, while district hospitals provide the fundamental medical services. Despite the long travel distances from southern to northern Hualien, people prefer to visit large hospitals for treatment. It should be noted that most depots for dispatching service vehicles are also located in the northern region in Hualien. Demand requests from the southern and central areas going toward northern hospitals definitely suffer from the ineffective transportation service and increasing dead mileages.

The existing key performance indicators (KPIs) also significantly affect system operations. There are two main KPIs that monitor and control the social welfare transportation service: (1) the annual total number of service trips and (2) the effective service distance rate (i.e., the reduction of the dead mile rate). The effective service distance rate refers to the distance traveled by people with disabilities on the rehabilitation bus. It specifically calculates the round-trip distance between the disabled person's house and the medical facility, excluding the round trip distance between the bus depot and the disabled person's house when the people with disabilities is not on the rehabilitation bus. Both KPIs induce local service providers to favor accepting demand requests within short distances rather than those requiring promising requests with long travel distances. It should be noted that "the number of trips" is not differentiated between short and long travel distance trips, which may result in spatial inequity issues in rural areas.

The Hualien County Government entrusts the Mennonite Social Welfare Foundation to operate rehabilitation and long-term care buses [5]. The foundation owns 25 rehabilitation buses and 12 long-term care buses; in addition, six parent-care institutions have accepted the government entrusting them with the operation of long-term care buses. Taking the 2020 data provided by the Mennonite Foundation as an example, Hualien is divided into three districts: the northern district includes Hualien City, Ji'an Township, Xincheng Township, Xiulin Township, and Fengbin Township; the central district covers Shoufeng Township, Fenglin Township, Guangfu Township, and Wanrong Township; and the southern district includes Yuli Township, Ruisui Township, Zhuoxi Township, and Fuli Township. The numbers of patients carried by the Northern, Central, and Southern Districts accounted, respectively, for 73.6%, 10.6%, and 14.5% of the total number of patients, with an average of 2,498 trips per month based on five working days per week, averaging more than 120 trips a day. In April 2019, the news revealed that Hualien County's rehabilitation buses were obsolete and the county faced a shortage of vehicles [6]. Thus, enhancing the vehicle utilization becomes a significant issue.

Most of previous studies focused on platform integration (e.g., [7, 8], route planning (e.g., [9, 10], and/or service quality (e.g., [7, 11], but rarely focused on the resource allocation by reservation systems. This study analyzes the relation between rehabilitation buses and patients from the market design perspective. In the past, market design was primarily used in university admissions, student selection, pairing residents with hospitals, and matching kidney transplants among nonrelatives. The seat allocation of the rehabilitation bus is also the distribution of intangible assets from the perspective of the patients who use the rehabilitation buses. This study aims at making preference and process improvements, in order to increase the utilization rate of rehabilitation buses and reduce their operating costs.

2. Literature Review

This study plans the rehabilitation bus reservation system from the market design perspective. This part of the literature review discusses the current situation of Taiwan rehabilitation buses and customized buses with similar operating models in foreign countries. In addition, algorithms of market design and preference settings are developed based on the needs of the patients to achieve the effective utilization rate of rehabilitation vehicles.

Taiwan's rehabilitation bus fleet was first evaluated and promoted by the Eden Social Welfare Foundation, which was entrusted by the government to handle its business in 1990, to provide barrier-free transportation services for the physically and mentally disabled. The rehabilitation bus service in Hualien County started in 2004 with the Mennonite Social Welfare Charity Foundation and operates 25 rehabilitation buses now.

Huang and Lin [12] mentioned that there were three difficulties in developing Taiwan's rehabilitation buses: insufficient numbers of service vehicles, limited appointment times and myopic service targets of local governments, and a lack of flexibility due to the first-come, first-served reservation rules. The study mentioned that the reservation software should be improved via education and training for rehabilitation bus drivers to assist persons with disabilities in boarding smoothly and provide direct assistance when needed. Furthermore, it suggests collaborating with private enterprises to integrate rehabilitation bus services and offer diverse solutions. Wu and Chen [13] took Taiwan's Duofu Rehabilitation Bus as an example to explore the operation of rehabilitation buses with a multiincome model and provide more flexible and high-quality services for the disabled. Hanson et al. [14] used a community volunteer driver program and transportation planning tools to assist the elderly in solving their transportation needs. Wu et al. [7] established a customer-centered multireservation transportation platform with the Taiwan Eden Foundation. Through the disclosure of information, users can obtain the required resources in real time, and practitioners can manage them more easily. The above study points out the related difficulties of rehabilitation buses, such as an insufficient number of vehicles, a deficient reservation platform, and related problems in operation and management.

Some rehabilitation buses are customized, unlike traditional buses. Wang et al. [15] compared customized buses with conventional buses in terms of travel time, speed, number of stops, and differences in arrival times. The researchers believe that customized buses offer several advantages, including shorter travel times, faster speeds, fewer stops, and minimal differences between the actual arrival time and the scheduled arrival time. In addition, they can reach a balance between travel costs and service quality, as well as attract users to switch from private cars to using customized buses. Ma et al. [16] proposed a route planning model using origin-destination (OD) route matching and customized bus network design, considering the social benefits and travel costs. This research investigates on regional segmentation and route planning based on customer

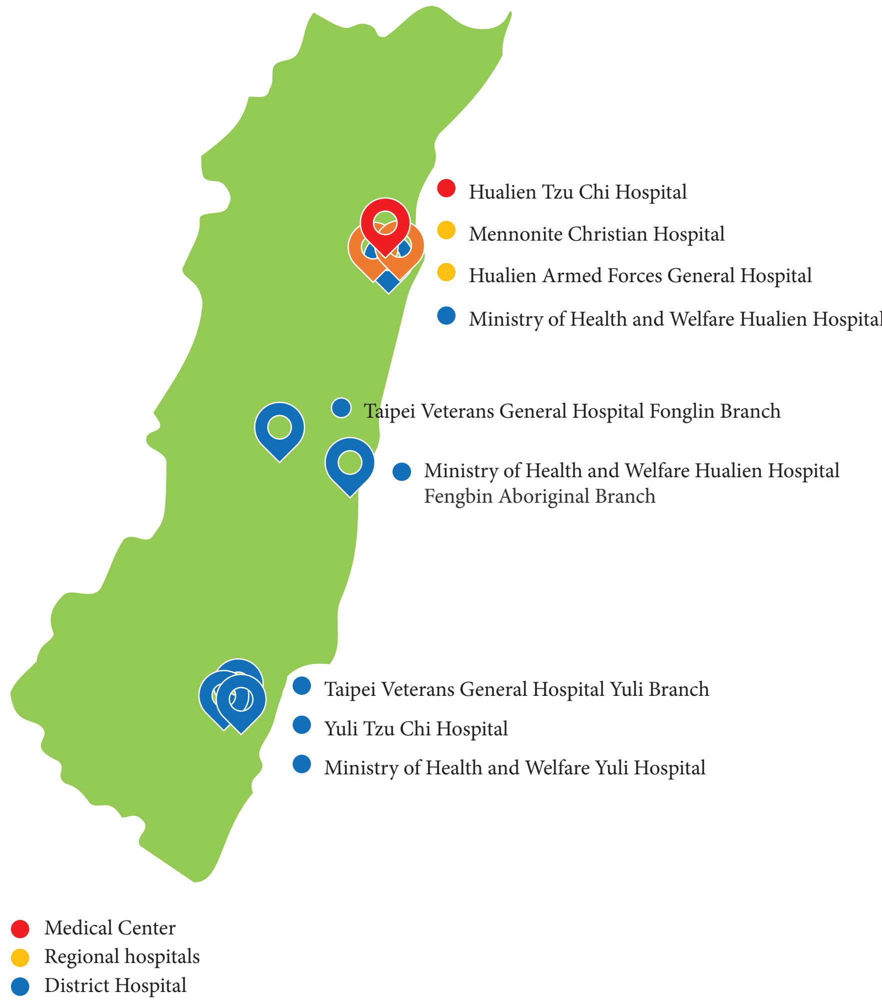

Figure 3: Allocation of medical resources in Hualien County.

demands, to enhance customer satisfaction and loyalty, while also improving operational efficiency and service quality. Furthermore, it identifies necessary routes to eliminate unnecessary operational costs. Taking Beijing as a case study, the number of routes, total route length, fleet size, station coverage rate, average passenger load, and service rate all outperform the current situation. Al-Hawari et al. [17] analyzed itinerary information and customer feedback to improve service quality and passenger satisfaction while reducing operating costs.

With the enactment of the Americans with Disabilities Act (ADA), it is required that public transportation services including demand responsive transportation (DRT) system should adhere to ADA regulations. Amirgholy and Gonzales [18] developed a dynamic model for DRT dispatching and utilized dynamic pricing based on demand, as well as adjusted off-peak vehicle utilization to enhance the system efficiency. Daganzo and Ouyang [19] introduced an analytical framework applicable to DRT, focusing on ridesharing mechanisms. If multiple passengers are geographically and temporally adjacent, the study suggested system-based ridesharing to increase vehicle utilization and reduce passengers' travel costs. Angelelli et al. [20] explored cooperation among companies providing DRT services. They established upper and lower limits for workload exchange among companies based on passenger volume and travel time, leading to operational cost savings. Chandakas [21] predicted demand fluctuation by using the ARIMA model and validated the results with data from Toulouse's DRTS in France. The findings highlighted the potential to enhance transport efficiency and service quality and demonstrated that reservation rates and service types would significantly influence demand fluctuations. In accordance with the findings of the above studies, the operational efficiency of DRT services involved planning for vehicle utilization, ridesharing adjustments, passenger volume, and travel time, while also need to fulfill customer demands.

The Hualien County rehabilitation bus officially launched a reservation system in June 2022. It uses telephone and online reservations in parallel but still adopts the first-come, first-served method. The problem is that patients with higher levels of disabilities, such as those who are bedridden or in wheelchairs, may not be able to make an appointment for a rehabilitation bus. Through the market design method, the research considers the needs of patients and operating units, to enable patients with a higher level of disability to receive services reasonably, and allow the organization undertaking the service to reduce operating costs.

From the examples of school admissions and kidney exchange, economists believe these differ from a currency trading market. Nevertheless, they can extend the experience from the currency market and propose a matching algorithm so that those who need it most can obtain resources and which also allows the effectiveness of resources to be maximized. The three most commonly discussed algorithms in this field are the Boston algorithm, the deferred acceptance algorithm, and the top trading cycle. The Boston algorithm originated from the Boston school choice, it adopts a first-come, first-served method. If students put star schools as their first choice, they will easily fail the rankings with high scores. They will tend to choose the second or third choice without directly following their true preferences, which are not satisfied with strategy-proofness principles. Here, strategy-proofness means that the pair will not hide users' true preferences. In other words, students or their parents can quickly obtain schools with better rankings by hiding their true preferences. Abdulkadiroğlu et al. [22, 23] solved the problem of school admissions with the delayed acceptance algorithm and the top trading cycle and later expanded it to many fields to solve problems practically.

Gale and Shapley [24] used the deferred acceptance algorithm to solve the problem of matching men and women, and proposed that one side of the bilateral matching, representing schools, could be increased from a single entity to multiple entities, as quotas that the school can offer for matching, while the other side remains unchanged, representing students. This can be extended to the problem of university admissions. Roth [25] believed that the matching problem for men and women is a one-to-one match, while the matching problem for school admissions is a one-to-many match, and the results of these two matches are different. He believed that the matching pattern must have a stable outcome. In the case of the male-female matching problem, if the male initiates the confession, then there will be a male-optimal stable outcome, and the male prefers a male-optimal stable match. The same holds true when the female initiates the confession. Therefore, both parties will express their true preferences to achieve the most stable outcome. However, the school admissions problem has a student-optimal stable match rather than a college-optimal stable match. Students will express their true preferences as the dominant strategy, which is the main difference between the one-to-one and one-to-many matching patterns. Based on this characteristic, this study uses a one-to-many matching pattern for the desired matching between patients and seats on rehabilitation buses, as the seats are offered in multiple quotas. Patients apply to the rehabilitation buses, resulting in the most optimal stable outcome for the passengers.

Sheply and Scarf [26] introduced the top trading cycle algorithm and proved this method could generate stable results. Roth et al. [27] applied the top trading cycle to kidney transplant matching. Kojima et al. [28] divided the matching mechanism design into two categories: one-sided and two-sided matchings. Two-sided matching is mainly based on the deferred acceptance algorithm, namely, the bilateral pairing matching problem. Let's assume that both sides have their own preferences, and one-sided matching also has two groups, but one side has no preference for the other side. A typical example of bilateral pairing is school admission and company job assignment. Taking school admission as an example, the school hopes to select outstanding students, and the students also wish to get accepted to schools with richer resources or higher rankings. Another famous case of one-sided matching is the example of kidney exchange. As long as the patient can exchange the kidney, he does not care who gets it; but only cares about the similarity of blood type and human leukocyte antigen or antibody.

Preferences are significant factors influenced matching results, especially for those problems without currency involved. For example, the government aims that education can be popularized to all citizens; thus, tuition fees should be as low as possible, and some student loans may be provided. The school targets to recruit suitable students to study and the students also favor particular schools. In this case, the price is not the priority, but the students' preference for the school and the school's attitude towards the students are key factors determining the matching results.

Consumers' preferences mentioned in individual economics are generally divided into three categories: strict preference, weak preference, and no difference. Huang [29] defined that strict preference means consumers indicating A is better than B, which is represented by symbols as $A \succ B$; weak preference means that consumers think A may be better than B, or that A may be at least as good as B, which is expressed as $A \succsim B$; while consumers express that A is as good as B, which means that there is no difference between A and B, represented by symbols, that is, $A \sim B$.

Druckman and Lupia [30] addressed the fact that preferences do not suddenly occur but arise from the interaction between individuals and environment. Dhar et al. [31] mentioned that consumers' similarities, differences, and preferences for the same option would change based on different tasks and environments. The comparison processes also affect the priority. Hanson [32] believed that preference is not static, and the type of preference may have four schemes: modification, restriction, addition, and subtraction. Öztürké et al. [33] proposed that the type of preference can be divided into the basic structure $[P, I]$, extended structure $[P, Q, I]$, and valence structure. In this context, P, I, and Q represent preference structure, indifference order, and situations where the two elements are the same or uncertain, respectively. These preferences can be further sorted using four distinct options: certain/uncertain and strong/weak. Bailey [34] pointed out that performance and preferences are not necessarily correlated. In addition, most previous studies assumed that preferences would not change over time. However, Kanade et al. [35] examined the impacts of dynamic preference lists affecting stable matching results. Aziz et al. [36] explored that stable matching would be influenced due to the uncertainty of preferences with limited information.

Abdulkadiroglu and Andersson [37] mentioned that Pareto efficiency, stability, and strategy-proofness could be used as indicators to measure the goodness of the matching algorithms. Pareto efficiency refers to improving the pair's welfare without harming others' rights and interests. For example, if there is a matching that allows students to be paired with a more preferred school without compromising the pairing of other students, the pairing reaches Pareto efficient. Stability means that none of the players feel that the results after matching are worse than before, which is also known as being individually rational. It should be noted that stability represents that none of the other matching results are more attractive to pair members, also called pairwise stability. The authors also mentioned that if any tiebreaker is needed in ranking students, the deferred acceptance algorithm has stability and strategy-proofness for students.

In recent years, matching design has been applied in various fields. Peng et al. [38] used the deferred acceptance algorithm to propose a ridesharing mechanism through the car-sharing platform to reduce travel costs. Elhenawy and Rakha [39] explored the configuration of shared bicycles and rental stations with a deferred acceptance algorithm and used a 2-opt local search algorithm for dynamic programming. Schummer and Abizada [40] adjusted for weather-induced airport congestion, aircraft arrival times, and cancellations with deferred acceptance algorithms; Pathak et al. [41] used the deferred acceptance algorithm to discuss medical matching between ventilators and patients and delved into the matching rules. Chu and Lan [42] discussed resource allocation between charities and care cases with a deferred acceptance algorithm. Xu et al. [43] discussed private parking space sharing using a modified top trading cycle. Kong et al. [44] examined a top trading cycle as a parking space sharing on the applications of the IOTs.

This study aims to incorporate market design algorithms into the reservation system of rehabilitation buses based on the theoretical foundation of market design and relevant research on practical applications. The preference for choosing a rehabilitation bus will be based on the patient's medical treatment schedule. The rehabilitation bus will prioritize patients based on their disability level, willingness to ride with others, number of successful reservations, and mileage. The algorithm's stability, strategy-proofness, and Pareto efficiency are expected to satisfy the needs of both patients and rehabilitation bus contractors.

3. Modeling Framework and Methodologies

This study extends from our previous work [45], patients have time slot preferences, and rehabilitation buses have demand preferences for patients, which would be a typical two-sided matching problem. The deferred acceptance algorithm and the top trading cycle are implemented to obtain the best allocation of resources.

The deferred acceptance algorithm and the top trading cycle have two types: two-side matching and one-side matching. In two-sided matching, such as matching students with schools, students have a choice of schools, and schools also select suitable students; both parties have preferences. In one-side matching, such as respirator allocation, patients have no particular preference on which respirator they are to be assigned. At the same time, the hospital side hopes to distribute respirators to users in urgent need. This study uses bilateral pairing as a model. Rehabilitation patients prefer the boarding time, while the rehabilitation organizer hopes to prioritize critically ill patients. In addition, the transportation service providers expect the patients to share rides, so both parties' preferences can be accommodated.

This study aims to address the rehabilitation bus reservation system. In the past, phone reservation or the current online registration system has been adopted, where registering in advance secures a seat on the rehabilitation bus. Although the administering unit has conducted a home visit to confirm the patient's disability level prior to granting registration qualification, the reservation process does not consider parameters such as disability level or the number of reservations made within the same month, making it impossible to evaluate whether the patients with the greatest needs can be accommodated. This study incorporates two algorithms from market design, the deferred acceptance algorithm and top trading cycle, to take into account parameters such as disability level to prioritize the order of rehabilitation bus schedules for patients and provide seats to patients with higher priorities.

The following are the relevant settings of this study.

(i) A set of patients $D = \{d_1, d_2 \ldots, d_{|D|}\}$
(ii) A set of rehabus shifts $S = \{s_1, \ldots, s_{|S|}\}$
(iii) Each rehabus shift $s \in S$ has a capacity of q_s
(iv) Each patient $d \in D$ has a strict preference relation P_d over $S \cup \{0\}$, $\{0\}$ signifies an unmatch
(v) Each rehabus shift has a strict preference $\gtrsim_s$ over $D \cup \{0\}$, $\{0\}$ signifies an unmatch

Assuming that μ represents the matching function between passengers and rehabilitation bus shifts: μ: $D \longrightarrow S$ or each $s \in S$, then define $\mu(s) = \{d \in D | \mu(d) = s\}$, when $|\mu_s| \leq q_s$ and all $s \in S$, then the matching function μ is feasible. Assume that $q = (q_s)_{s \in S}$ represents the total number of seats of all rehabilitation bus shifts; $P = (P_d)_{d \in D}$ represents the total preference of all patients; and $\gtrsim = (\gtrsim_s)_{s \in S}$ represents the total preference of all rehabilitation buses for patients, and the mechanism φ is a matching problem defined by parameters such as $\{D, S, q, P, \gtrsim\}$.

Our model is revised from the assignment game model of Shimada, Yamazaki, and Takano (2020), and the objective function is evaluated by the minimal system matching scores.

We introduce the binary decision variable $x = (x_{\text{ds}})_{(d,s) \in D \times S}$ such that

$$x_{\text{ds}} = \begin{cases} 1, & \text{if the patient } d \text{ is assigned to rehabilitation bus sift } s, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

for all $(r, h) \in D \times S$

This study aims to find the maximum total utility of the matching and the minimum number of blocking pairs.

(i) u_{ds} represents the utility obtained by the patient and the rehabilitation bus shift, where $u_d(s)$ is the utility obtained by patients, prioritizing shifts obtained with higher priority, and $u_s(d)$ is the utility obtained by rehabilitation bus shifts, assigned to higher priority positions. The total utility is $u_{ds} = u_d(s) + u_s(d)$, given a matching $x \in \{0,1\}^{|D|\times|S|}$.

(ii) ω_{ds} represents the presence of blocking pairs, where $\omega_{ds} = 0$ indicates the absence of blocking pairs, and $\omega_{ds} = 1$ indicates the presence of blocking pairs. A stable matching algorithm will not result in a blocking pair. The binary decision variable is denoted as follows: $\omega = (\omega_{ds})_{(d.s)\in D\times S}$.

Maximizing matching efficiency involves effectively utilizing the available time slots, reducing the number of unassigned seats, and allocating them to patients with the highest needs, such as those with higher disability levels. It should be noted that there are no other matching outcomes more appealing than the results obtained in this study, without any blocking pairs.

The formulas are as follows:

$$\text{Max} \sum_{d\in D}\sum_{s\in S} u_{ds} x_{ds}. \tag{2}$$

Subject to

$$\sum_{s\in S} x_{ds} \le 1\,(\forall d \in D), \tag{3}$$

$$\sum_{d\in D} x_{ds} \le q_s\,(\forall s \in S), \tag{4}$$

$$q_s x_{ds} + q_h \sum_{j\succ d^s} x_{dj} + \sum_{i\succ s^d} x_{is} \ge q_s\left(1-\omega_{ds}\right)(\forall d \in D, \forall s \in S), \tag{5}$$

$$\begin{aligned} &\sum_{d\in D}\sum_{s\in S} \omega_{ds} = 0,\\ &x_{ds} \in \{0,1\}\,(\forall d \in D, \forall s \in S),\\ &\omega_{ds} \in \{0,1\}\,(\forall d \in D, \forall s \in S). \end{aligned} \tag{6}$$

The constraints are described as follows. Equation (3) assumes that each patient should be assigned to at most one rehabilitation bus seat. Equation (4) ensures that the number of matches would be less or equal to the total capacity provided by the rehabilitation bus schedule. Equation (5) refers to Shimada et al. [46] proof of disruptions in the restricted model regarding the matching of resident physicians and hospitals. Where $j \succ d^s$, it means that, after the matching process, patient d prefers shift j more than the one assigned to them, shift s. Also, $i \succ s^d$ implies that, after the matching process, the rehabilitation bus prefers patient i more than the one assigned to them, patient d. When $\omega_{ds} = 0$, it indicates no disruptions, and the matching is stable. If $\omega_{ds} = 1$, it means there are disruptions in the matching.

In this study, relevant parameter settings are modified to meet the requirements of the current research.

Here, we assume that patients who want to take the rehabilitation bus and the rehabilitation bus itself are treated as the two sides of the matching. There are two sets; one is the set of patients $D = \left\{d_1, d_2 \ldots, d_{|D|}\right\}$, and the other set is the shift schedule $S = \left\{s_1, s_2, \ldots, s_{|S|}\right\}$ of the rehabilitation bus with q_s representing the seat quota of each shift schedule s; for example, one rehabilitation bus can accommodate two seats for wheelchairs and there are two rehabilitation buses for one shift; that is, there are four places for this shift, and a patient can only be allocated one seat at most. Each patient d has a strict preference P_d over $S \cup \{0\}$, 0 means that the patient is not assigned to a seat; for example, $sP_d s'$ means that patient d prefers shift s to s', and R_d indicates the "at least as good" relation, since patient d prefers shift s to s', but s and s' are at least as good:

$$sR_d s' \leftrightarrow sP_d s' \text{ or } s = s'. \tag{7}$$

Each rehabilitation bus shift s has a strict preference $\succ_s$ over the set of patient 2^D, $d \succ_s d'$ means that the rehabus shift s prefers patient d more than d', and $\succsim_s$ means "at least as good," with the relation between the two shown as follows:

$$d \succsim_s d' \leftrightarrow d \succ_s d' \text{ or } d = d'. \tag{8}$$

Kojima et al. [28] explained preferences by stating that each college's preference for selecting students is "responsive," meaning that each college has its own preferences and standards, so when selecting students, they prioritize based on the student's qualifications. In this study, we also assume that if the following formula holds true, then the preference of the rehabilitation bus for patient selection, denoted by $\succ_s$, is also "responsive":

(1) For any $C \subset D$ with $|C| < q_s$ and any $d \in C\backslash D$, $(C \cup d) \succ_s C \Longleftrightarrow d \succ_s 0$

(2) For any $C \subset D$ with $|C| < q_s$ and any $d, c \in C\backslash D$, $(C \cup d) \succ_s (C \cup c) \Longleftrightarrow d \succ_s c$

Then, the set of all strict responsive preferences can be expressed as follows:

$$\mathscr{R} = \left\{ (\succ_l)_{l\in D\cup S} \middle| \succ_s \text{ is responsive}, \succ_s \in S \right\}. \tag{9}$$

Abdulkadiroglu and Andersson [37] mentioned that using exogenous and quantifiable criteria as a sorting basis is called nonstrategic, while the preference order is based on personal preference and cannot be verified, which is called strategic. In this study, the preference for the rehabilitation bus schedule for patients is nonstrategic, while the preference for the rehabilitation bus schedule is strategic.

However, Abdulkadiroglu and Andersson [37] also addressed that a strict preference could easily verify whether it produces Pareto efficiency, but confirming when the preference is indifferent is not easy. Therefore, in the process of matching, when a tie is encountered, it must be broken through a tiebreaker. However, if there are multiple tiebreakers, the agent's proposal may not be optimal and there will be inefficiencies; thus, if there is a single tiebreaker, the following results hold. For students, the deferred acceptance algorithm is stable and strategy-proof. Moreover, the top trading cycle has Pareto efficiency and strategy-proofness.

Abdulkadiroğlu et al. [47] claimed that the tiebreaker is bijection t: $N \longrightarrow \mathbb{N}$, and the tie associated between schools $s \in S$ can be broken by $\succcurlyeq_s$ and $\succ_s^t$. The term "bijection" here refers to a function that maps each element of a set of natural numbers N to a unique element in another set of natural numbers $\mathbb{N}$, and for each element in the set of natural numbers N, there exists a unique corresponding element in the set of natural numbers $\mathbb{N}$, to ensure that each applicant can receive a unique position. $\succ_s^t$ can be expressed as: $i \succ_s^t j$ if and only if $i \succ_s j$ or when i and j at the same ranking and $t(i) > t(j)$; if each school has different rules for the tiebreaker, use $\tau = (t_{s_1}, \ldots, t_{s_{|s|}})$ as the sum of the tiebreakers.

Kojima et al. [28] believe that a stable match μ is Pareto-efficient and envy-free, while Roth [48] believes that in some cases, the deferred acceptance algorithm cannot satisfy both stability and Pareto efficiency. In addition, Roth [25] mentioned that if the students make the application, there will be optimal and stable results for the students. If the school proposes it, there will be no optimal and stable results for the school; that is, there is a one-way strategy-proofness. In this study, following the characteristic of deferred acceptance algorithm for one-to-many matching, students initiate the application to achieve the optimal stable outcome for themselves, while schools do not have the optimal stable outcome. It is assumed that patients applying to rehabilitation buses can obtain the optimal stable outcome.

The following are the matching steps of the deferred acceptance algorithm and the top trading cycle:

(1) Deferred acceptance algorithm matching steps

Step 1: Each rehabilitation bus schedule has a passenger quota limit. Each passenger proposes their most favorite shift according to the order of preference, and each shift temporarily provides a passenger with a seat according to the order of preference of the passengers. If the quota of the shift is full, the application for the shift is rejected by other passengers.

In general, at

Step m: Passengers accepted in the previous step can propose the next favorite shift according to their preference. Each shift will be compared with the passengers temporarily accepted in the order of passenger preference and the passenger quota limit. Currently, each rehabilitation bus in Taiwan allows to take up to 2 passengers with their wheelchairs simultaneously. A passenger with higher priority will be accepted while other passengers who apply for this shift will be rejected.

When there is no new application for passengers, the matching will be terminated, and the temporarily accepted passengers will be converted to formal acceptance for each shift. The passengers will get the shift at that time. This step is referred to as Step m.

(2) The top trading cycle matching steps

Step 1: Place a counter on each shift to record the remaining places for each shift. Each passenger points to their favorite shift according to his/her preference, and each shift points to the passenger with the highest priority, forming at least one cycle. The passenger in each cycle obtains a seat on the shift he/she points to and removes it. Then, the shift's counter is reduced by one, and if the counter reaches zero, the shift is removed.

In general, at

Step n: Each remaining passenger points to their favorite among the remaining shifts, and each shift points to the passenger with the highest priority among the remaining passengers, forming at least one cycle. In each cycle, the passenger gets a seat on the shift he points to and is removed, and the shift counter is decreased by one. If the counter is reduced to zero, the shift is removed.

When every passenger has been given a seat, or when all preferences of the passenger have been considered, the matching process is terminated. This step is referred to as Step n.

Here, we provide an example to illustrate the deferred acceptance algorithm and the top trading cycle procedure:

Example 1. There are rehabilitation bus shifts s_1 and s_2, and passengers t_1, t_2, t_3, t_4, t_5. Each shift has two available seats.

The following represents passengers' preferences for the rehabilitation bus shifts:

$$\begin{aligned} &t_1\colon s_1 P_{t_1} s_2,\\ &t_2\colon s_2 P_{t_2} s_1,\\ &t_3\colon s_1 P_{t_3} s_2,\\ &t_4\colon s_1 P_{t_4} s_2,\\ &t_5\colon s_2 P_{t_5} s_1. \end{aligned} \tag{10}$$

Below are the preferences of the rehabilitation bus shifts for the patients:

$$\begin{aligned} &s_1\colon t_1 \succsim_{s_1} t_2 \succsim_{s_1} t_3 \succsim_{s_1} t_4 \succsim_{s_1} t_5,\\ &s_2\colon t_4 \succsim_{s_2} t_1 \succsim_{s_2} t_3 \succsim_{s_2} t_5 \succsim_{s_2} t_2. \end{aligned} \tag{11}$$

The steps of the deferred acceptance algorithm matching are as follows:

Step 1: Patients t_1, t_3, and t_4 apply to shift s_1. The priority order of shift s_1 for these patients is $t_1 \succsim_{s_1} t_2 \succsim_{s_1} t_3 \succsim_{s_1} t_4 \succsim_{s_1} t_5$. Therefore, t_1 and t_3 are provisionally accepted, and t_4 is rejected. Simultaneously, patients t_2 and t_5 apply to shift s_2. The priority order of shift s_2 for these patients is $t_4 \succsim_{s_2} t_1 \succsim_{s_2} t_3 \succsim_{s_2} t_5 \succsim_{s_2} t_2$. As a result, t_2 and t_5 are provisionally accepted.

Step 2: Patient t_4 applies to shift s_2. In the previous step, shift s_2 provisionally accepted patients t_2 and t_5. The priority order of shift s_2 for these patients is

$t_4 \succsim_{s_2} t_1 \succsim_{s_2} t_3 \succsim_{s_2} t_5 \succsim_{s_2} t_2$. Consequently, shift s_2 retains patient t_5, rejects patient t_2, and provisionally accepts patient t_4. Patient t_2, who was rejected in the earlier step, applies to shift s_1. In the previous steps, shift s_1 provisionally accepted patients t_1 and t_3. The priority order of shift s_1 for these patients is $t_1 \succsim_{s_1} t_2 \succsim_{s_1} t_3 \succsim_{s_1} t_4 \succsim_{s_1} t_5$. Hence, shift s_1 retains patient t_1, rejects patient t_3, and provisionally accepts patient t_2.

Step 3: Patient t_3 applies to shift s_2. In the previous steps, shift s_2 provisionally accepted patients t_4 and t_5. The priority order of shift s_2 for these patients is $t_4 \succsim_{s_2} t_1 \succsim_{s_2} t_3 \succsim_{s_2} t_5 \succsim_{s_2} t_2$. Therefore, shift s_2 retains patient t_4, rejects patient t_5, and provisionally accepts patient t_3. Patient t_5, who was rejected earlier, applies to shift s_1. In the previous steps, shift s_1 provisionally accepted patients t_1 and t_2. The priority order of shift s_1 for these patients is $t_1 \succsim_{s_1} t_2 \succsim_{s_1} t_3 \succsim_{s_1} t_4 \succsim_{s_1} t_5$. Consequently, shift s_1 temporarily retains patients t_1 and t_2 and rejects patient t_5.

Step 4: With all patients having submitted their applications and no new applications being received, shifts s_1 and s_2 convert provisionally accepted patients to formal acceptances. This means that patients t_1 and t_2 secure seats on shift s_1, and patients t_3 and t_4 secure seats on shift s_2. The matching process concludes.

The steps of the top trading cycle matching are as follows:

Step 1: Patients point to their most preferred shifts, and shifts point to their highest-priority patients. Specifically, patients t_1, t_3, and t_4 point to shift s_1. The priority order of shift s_1 for these patients is $t_1 \succsim_{s_1} t_2 \succsim_{s_1} t_3 \succsim_{s_1} t_4 \succsim_{s_1} t_5$. Shift s_1 points to patient t_1, forming a cycle. Patients t_2 and t_5 point to shift s_2. The priority order of shift s_2 for these patients is $t_4 \succsim_{s_2} t_1 \succsim_{s_2} t_3 \succsim_{s_2} t_5 \succsim_{s_2} t_2$. However, shift s_2 doesn't form a cycle. Consequently, patient t_1 secures a seat on shift s_1 and exits the matching process. Shift s_1 has one remaining seat, and shift s_2 has two remaining seats.

Step 2: Patients t_4 point to shift s_1. The priority order of shift s_1 for these patients is $t_1 \succsim_{s_1} t_2 \succsim_{s_1} t_3 \succsim_{s_1} t_4 \succsim_{s_1} t_5$. Since patient t_1 has already left the matching process, shift s_1 points to patient t_2. Patients t_2 and t_5 point to shift s_2. The priority order of shift s_2 for these patients is $t_4 \succsim_{s_2} t_1 \succsim_{s_2} t_3 \succsim_{s_2} t_5 \succsim_{s_2} t_2$, so shift s_2 points to patient t_4. A cycle forms where shift s_2 points to patient t_4, patient t_4 points to shift s_1, shift s_1 points to patient t_2, and patient t_2 points to shift s_2. Therefore, patient t_2 secures a seat on shift s_2, and patient t_4 secures a seat on shift s_1, both leaving the matching process. Shift s_1 has no remaining seats, and shift s_2 has one remaining seat.

Step 3: Patients t_3 and t_5 point to shift s_2, and shift s_2 points to patient t_3, forming a cycle. As a result, patient t_3 secures a seat on shift s_2, and shift s_2 has no remaining seats. The matching process concludes.

Table 1 presents the results of the example using the deferred acceptance algorithm and the top trading cycle matching:

Table 1: The results of the example using the deferred acceptance algorithm and the top trading cycle pairing are as follows.

	Deferred acceptance algorithm		Top trading cycle	
	Shift s_1	Shift s_2	Shift s_1	Shift s_2
Patients	t_1, t_2	t_3, t_4	t_1, t_4	t_2, t_3

4. Application of Models and Computational Results

The numerical analysis in this study is based on the rehabilitation bus dispatching data provided by the Hualien Mennonite Foundation from June to December in 2016. This data serves as a reference for model planning; however, patient-specific information is not recorded in the dataset. The disability level, ranking shift, ride sharing, and number of appointments of patients are simulated based on the interview information with our industrial partners. There are a total of 9,533 records, served by 25 rehabilitation buses. On average, a vehicle operates 5.2 round trips per day. This study uses patients and rehabilitation bus schedules as the two sides of the matching. To simplify the model, here we first take the one-way journey with limited fleet size from the patient's designated place to the medical institution as an example, and set the appointment time from 8:00 to 11:30 a.m. and 1:00 to 4:30 p.m. The rehabilitation bus could be reserved every half hour, and a total of 16 time slots are available in this case. Two rehabilitation buses could be dispatched in each time slot, and one rehabilitation vehicle can take two wheelchairs and their accompanying family members.

Taking the Hualien Tzu Chi Medical Center as an example, the patient completes the appointment, and then the hospital reservation system would suggest a consultation time based on the corresponding date and consultation number. The patient can request the rehabilitation bus pick-up service according to the recommended consultation time. This study sets the service reservation request rule that patients can make an appointment for four vehicle shifts, which must be prioritized. The appendix provides additional relevant information.

The selection of patients by the rehabilitation bus service provider is based on the needs of patients and operational considerations. The main difference between our proposed approach and the current reservation method is to consider mutual preference ranking among the rehabilitation bus service providers and the patients, as well as the reservation process. In terms of preference ranking, patients can prioritize their preferred vehicle time slots based on their medical appointments. On the other hand, for the rehabilitation bus service, it is considered a social welfare service while resources are limited, so they must be allocated to patients in greatest need. Patients who do not require bed rest or wheelchair assistance may have other transportation options, such as taxis or public buses. However, patients who are bedridden or heavily reliant on wheelchairs need special vehicles for transportation. In addition, according to the information on the Hualien County Government's social welfare web page, each service user can use the rehabilitation bus for a maximum of eight trips per week. To ensure fair distribution of resources, the number of fulfilled rides is also taken into consideration. Furthermore, carpooling and the distance for pick-up are considered to evaluate the operational efficiency of the rehabilitation bus operators. Carpooling can increase vehicle utilization, while the distance of travel can maximize the number of passengers carried.

As for changes in the reservation process, during the matching process, the preferences of both the rehabilitation bus and each applicant need to be taken into account. Thus, a certain number of applicants are required for the matching process. Back to the current mechanism, reservations are opened seven days prior to the travel date and then would be arranged based on first-come, first-serve rules. In this study, the same period is considered for reservation. Once the matching is completed, patients are notified if their reservation is successful. It should be noted that last-minute reservations are not considered in this study.

In addition, the level of disability of patients is used as their need, and the three items of shared rides, number of rides, and destination arrival distance are used as operational considerations. In Taiwan, disability qualifications are divided into four levels: extremely severe, severe, moderate, and mild. However, patients who do not need wheelchairs have more vehicle choices, so this study did not use disability qualifications as the evaluation standard. According to investigations, the qualifications for boarding rehabilitation buses in counties and cities in Taiwan are determined by the responsible units of the county government. Still, most of them cannot clearly define the degree of disability. However, Taipei City's "Taipei City Passenger Service Notice for Small Air-Conditioned Vehicles for the Disabled" uses specific A, A1, A2, and B, four levels as the priority order for booking vehicles. Among them, "Special A" refers to those who are in a vegetative state with severe disabilities needed the assistance of crutches or wheelchairs; "A1" means people with severe visual impairment; "A2" means people with severe disabilities except for A and A1 and "B" means people with moderate and mild disabilities. Therefore, this research is graded according to the definition of the Taipei City Government and represented by 1 to 4. 4 indicates A with the most severe disability, 3 represents A1, 2 represents A2, and 1 represents B, as shown in Table 2.

From the perspective of rehabilitation bus operations, sharing rides can reduce operating and vehicle maintenance costs. This study divides the willingness to share rides into five levels. The score of 5 means that they agree with carpooling, while a score of 1 means they disagree entirely, as the second reference value when the degree of disability is the same. In addition, the number of times taking the rehabilitation bus is mainly counted as one round trip per month, with a total of 8 times. In this study, the number of rides in the current month is used as the reference value when the degree of disability and willingness to ride is the same. Priority is given to those with fewer rides. Finally, the deadheading miles without transporting patients are also a waste of operating costs for rehabilitation bus contractors. This study takes the North District of Hualien County as an example. A medical center and three regional hospitals in Hualien are all near Hualien City. It is about 40 kilometers from Heping Village, which is the most northerly part of the North District. As shown in Table 3, the degree of disability, willingness to ride and the number of rides are all the same, and the shorter mileage will be used as the tiebreaker. To avoid multiple tie-breakers, the stable results generated by the deferred acceptance algorithm may not be the agent's proposed optimal results, so each rehabilitation bus schedule has a strict preference for each patient according to the four priority orders. For example, patients A and B choose the same shift s, and the disability levels of the two patients are both 4 points. In contrast, patient A's willingness to share a ride is 5 points, while patient B's willingness to share a ride is 4 points; thus, the priority order of shift s for the two patients is $A \succ_s B$.

Regarding the reservation process of the rehabilitation bus, it is generally open at a fixed time. For example, in Hualien County, it is seven days before the boarding date. If there are counties and cities with precise classifications, they are separated by opening hours. In Taipei City, patients with specific A-level disabilities can make reservations at 9:00 am five days before the ride, and A1-level patients can make reservations at 1:30 pm five days before the ride. Reservations are made by telephone or online. This method will likely cause congestion during system opening hours, and those who make reservations first will get seats first. In this study, the level of disability has been considered during the matching process. There is no need to make an appointment by time slot. Patients only need to log on to the Internet during the opening hours to make an appointment. After the system is paired, the patient will be notified of whether the appointment is successfully made? If the appointment is completed, the patient is informed of the appointment time. If the appointment is unsuccessful, the contact information of the rehabilitation taxi is provided, or the rehabilitation taxi can be included in the system for matching in the future. Figure 4 illustrates the appointment processes in this study.

This study aims to apply the top trading cycle and patient proposing deferred acceptance algorithms for matching, to analyze identical numerical cases and find the better fitness algorithm in real-world operations. After the two algorithms are matched, the same result is obtained in Table 4. With 72 patients and 64 seats on the rehabilitation bus, 63 patients were successfully matched, with one seat left over. In the case of the remaining vacancy in S12, the result shows that no one selected this shift due to lack of preference. In this study, among 63 patients, 57 patients obtained the first priority time slot, 1 patient obtained the second priority time slot, 3 patients obtained the third priority time slot, and 2 patients obtained the fourth priority time slot. In addition, since the rehabilitation bus aims to prioritize patients with higher disability levels for its services, this study will sum up the disability level scores of the patients who receive services to represent the value of the rehabilitation bus's investment in providing services. Based on the simulation results, the total disability level score of the patients receiving services is 170 points.

Table 2: Classification of disability levels in service notices for passengers of small air-conditioned cars for people with disabilities in Taipei City.

Disability class	Grading of this study	Definition
Specific A	4	Vegetative; above severe, with lower limb impairment, needing crutches, or wheelchairs
A1	3	Severely visually impaired
A2	2	People with severe disabilities except for specific A and A1
B	1	Moderately and mildly handicapped

Table 3: Priority order for matching patient and rehabilitation bus.

Set	First priority	Second priority	Third priority	Fourth priority
Rehabilitation bus	Disability level	Willingness to share a ride	The number of rides	The shorter mileage
Patient	According to the recommended consultation time, choose four shifts			

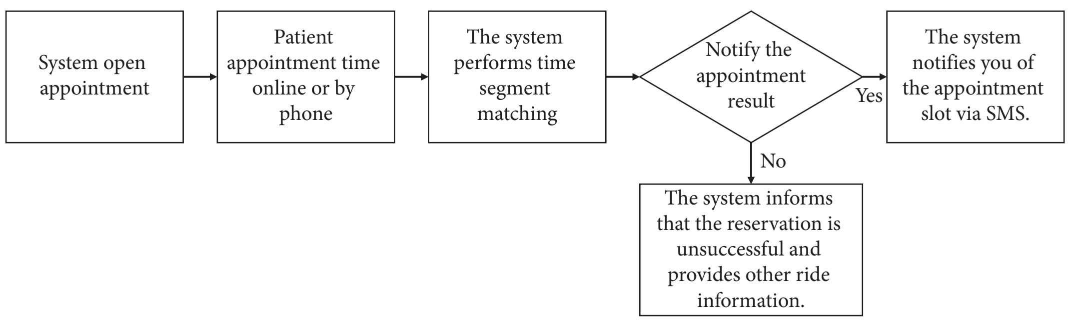

Figure 4: Rehabilitation bus matching booking use flowchart.

Table 4: Using the deferred acceptance algorithm and top trading cycle as stable matching mechanisms, the simulation results of assigning patients and rehabilitation buses get the same result.

Booking shift		Patients who have made an appointment			
s_1	Patient number	d_{48}	d_{59}	d_{41}	d_{28}
	Disability level	4	4	3	2
	Preference level	1	1	1	1
s_2	Patient number	d_{68}	d_{06}	d_{25}	d_{36}
	Disability level	4	3	3	2
	Preference level	1	1	2	1
s_3	Patient number	d_{60}	d_{58}	d_{11}	d_{54}
	Disability level	4	4	3	3
	Preference level	1	1	1	1
s_4	Patient number	d_{05}	d_{31}	d_{53}	d_{56}
	Disability level	4	3	1	1
	Preference level	1	1	3	3
s_5	Patient number	d_{14}	d_{40}	d_{71}	d_{55}
	Disability level	2	1	1	1
	Preference level	1	1	1	1
s_6	Patient number	d_{33}	d_{30}	d_{07}	d_{22}
	Disability level	4	3	2	1
	Preference level	1	1	1	1
s_7	Patient number	d_{51}	d_{15}	d_{35}	d_{44}
	Disability level	3	3	3	1
	Preference level	1	1	1	1

Table 4: Continued.

Booking shift		Patients who have made an appointment			
s_8	Patient number	d_{13}	d_{49}	d_{45}	d_{09}
	Disability level	4	4	3	2
	Preference level	1	1	1	1
s_9	Patient number	d_{20}	d_{03}	d_{24}	d_{08}
	Disability level	4	3	2	1
	Preference level	1	1	1	1
s_{10}	Patient number	d_{19}	d_{67}	d_{12}	d_{16}
	Disability level	4	3	3	1
	Preference level	1	1	1	1
s_{11}	Patient number	d_{43}	d_{26}	d_{38}	d_{50}
	Disability level	4	4	3	1
	Preference level	1	1	1	4
s_{12}	Patient number	d_{10}	d_{04}	d_{21}	
	Disability level	4	3	1	
	Preference level	1	1	1	
s_{13}	Patient number	d_{61}	d_{70}	d_{17}	d_{23}
	Disability level	3	2	1	1
	Preference level	1	1	1	4
s_{14}	Patient number	d_{72}	d_{42}	d_{02}	d_{29}
	Disability level	4	4	2	1
	Preference level	1	1	1	3
s_{15}	Patient number	d_{64}	d_{65}	d_{34}	d_{63}
	Disability level	4	3	2	1
	Preference level	1	1	1	1
s_{16}	Patient number	d_{62}	d_{32}	d_{57}	d_{01}
	Disability level	4	4	4	3
	Preference level	1	1	1	1

Unmatch: $d_{18}, d_{27}, d_{37}, d_{39}, d_{46}, d_{47}, d_{52}, d_{66}, d_{69}$.

Table 5: Overall simulate results of first-come, first-served for the rehabilitation bus seats.

Booking shift		Patients who have made an appointment			
s_1	Patient number	d_{18}	d_{28}	d_{37}	d_{41}
	Disability level	1	2	1	3
	Preference level	1	1	1	1
s_2	Patient number	d_{06}	d_{36}	d_{46}	d_{47}
	Disability level	3	2	1	1
	Preference level	1	1	1	1
s_3	Patient number	d_{11}	d_{25}	d_{48}	d_{52}
	Disability level	3	3	4	1
	Preference level	1	1	3	3
s_4	Patient number	d_{05}	d_{31}	d_{49}	d_{53}
	Disability level	4	3	4	1
	Preference level	1	1	4	3
s_5	Patient number	d_{14}	d_{40}	d_{51}	d_{55}
	Disability level	2	1	3	1
	Preference level	1	1	4	1
s_6	Patient number	d_{07}	d_{22}	d_{30}	d_{33}
	Disability level	2	1	3	4
	Preference level	1	1	1	1
s_7	Patient number	d_{15}	d_{35}	d_{44}	d_{45}
	Disability level	3	3	1	3
	Preference level	1	1	1	2
s_8	Patient number	d_{09}	d_{13}	d_{27}	d_{39}
	Disability level	2	4	1	1
	Preference level	1	1	1	1

Table 5: Continued.

Booking shift		Patients who have made an appointment			
s_9	Patient number	d_{03}	d_{08}	d_{20}	d_{24}
	Disability level	3	1	4	2
	Preference level	1	1	1	1
s_{10}	Patient number	d_{12}	d_{16}	d_{19}	d_{50}
	Disability level	3	1	4	1
	Preference level	1	1	1	3
s_{11}	Patient number	d_{26}	d_{38}	d_{43}	d_{67}
	Disability level	4	3	4	3
	Preference level	1	1	1	3
s_{12}	Patient number	d_{04}	d_{10}	d_{21}	d_{72}
	Disability level	3	4	1	4
	Preference level	1	1	1	4
s_{13}	Patient number	d_{17}	d_{61}	d_{69}	d_{70}
	Disability level	1	3	1	2
	Preference level	1	1	4	1
s_{14}	Patient number	d_{02}	d_{42}	d_{64}	d_{65}
	Disability level	2	4	4	3
	Preference level	1	1	2	2
s_{15}	Patient number	d_{34}	d_{57}	d_{62}	d_{63}
	Disability level	2	4	3	1
	Preference level	1	2	2	1
s_{16}	Patient number	d_{01}	d_{23}	d_{29}	d_{32}
	Disability level	3	1	1	4
	Preference level	1	1	1	1

Unmatch: $d_{54}, d_{56}, d_{58}, d_{59}, d_{60}, d_{66}, d_{68}, d_{71}$.

Table 6: Comparison of "first-come, first-served" and "matching" algorithms.

	First-ranking shift	Second-ranking shift	Third-ranking shift	Fourth-ranking shift	Total disability level score
First-come, first-served	49	6	5	4	155
Matching algorithms	57	1	3	2	170

Table 7: The research model setting.

Patient	Disability level	Ranking shift				Sharing rides	Appointments	Distance
		R_1	R_2	R_3	R_4			
*d*01	3	16	15	14	13	5	5	35
*d*02	2	14	13	12	11	3	6	10
*d*03	3	9	8	10	11	3	7	42
*d*04	3	12	11	10	13	2	2	31
*d*05	4	4	3	2	1	4	2	40
*d*06	3	2	1	3	4	5	3	17
*d*07	2	6	5	4	3	1	5	15
*d*08	1	9	8	10	11	2	6	29
*d*09	2	8	7	6	5	5	4	31
*d*10	4	12	11	10	13	5	2	33
*d*11	3	3	2	1	4	4	5	14
*d*12	3	10	9	8	7	2	4	16
*d*13	4	8	7	6	5	5	3	37
*d*14	2	5	4	6	3	2	1	35
*d*15	3	7	6	8	5	2	2	28
*d*16	1	10	9	11	12	4	4	14
*d*17	1	13	12	11	10	4	1	20
*d*18	1	1	2	3	4	4	4	10
*d*19	4	10	9	11	12	3	6	8
*d*20	4	9	8	10	11	1	4	25
*d*21	1	12	11	13	10	5	2	32

Table 7: Continued.

Patient	Disability level	Ranking shift R_1	R_2	R_3	R_4	Sharing rides	Appointments	Distance
d22	1	6	5	7	4	4	5	21
d23	1	16	15	14	13	3	4	41
d24	2	9	8	10	11	2	3	10
d25	3	3	2	4	1	1	4	35
d26	4	11	10	9	12	2	1	36
d27	1	8	7	6	5	5	1	39
d28	2	1	2	3	4	5	6	11
d29	1	16	15	14	13	5	2	30
d30	3	6	5	7	4	3	2	32
d31	3	4	3	5	2	2	4	29
d32	4	16	15	14	13	2	2	22
d33	4	6	5	7	4	4	7	19
d34	2	15	14	13	12	1	3	28
d35	3	7	6	8	5	1	7	37
d36	2	2	1	3	4	2	7	16
d37	1	1	2	3	4	5	1	40
d38	3	11	10	12	9	2	2	34
d39	1	8	7	6	5	4	5	25
d40	1	5	4	6	3	5	1	21
d41	3	1	2	3	4	4	7	13
d42	4	14	13	12	11	1	6	7
d43	4	11	10	9	12	3	2	33
d44	1	7	6	5	4	4	4	22
d45	3	8	7	6	5	1	5	38
d46	1	2	1	3	4	4	4	18
d47	1	1	2	3	4	5	5	12
d48	4	1	2	3	4	4	4	16
d49	4	8	7	6	5	2	2	26
d50	1	9	8	10	11	2	7	19
d51	3	7	6	8	5	2	2	14
d52	1	2	1	3	4	3	1	27
d53	1	3	2	4	1	4	6	7
d54	3	3	2	1	4	2	5	37
d55	1	5	4	6	3	1	4	17
d56	1	6	5	4	3	3	4	13
d57	4	16	15	14	13	2	5	25
d58	4	3	2	4	1	1	5	21
d59	4	1	2	3	4	1	1	42
d60	4	3	2	1	4	3	2	19
d61	3	13	12	11	10	1	3	40
d62	4	16	15	14	13	3	1	34
d63	1	15	14	16	13	4	5	39
d64	4	15	14	13	16	1	6	19
d65	3	15	14	13	12	4	4	42
d66	1	2	1	3	4	4	7	6
d67	3	10	9	11	12	5	3	40
d68	4	2	1	3	4	4	2	42
d69	1	16	15	14	13	2	3	12
d70	2	13	12	11	10	1	1	28
d71	1	5	4	3	2	1	1	16
d72	4	14	13	15	12	5	5	27

This study employed some numerical studies based on the real dispatching data, to arrange appointments by patient IDs. The base case follows a first-come, first-served rule, to prioritize patients with earlier reservations, as the existing operational mechanism in practice. In this section, d_{01} denotes to the earliest appointment, whereas d_{72} corresponds to the latest reservation. As shown in Table 5, a total of 64 out of 72 patients were able to secure a seat on the rehabilitation bus. Among them, 49 patients were given the first priority time slot, 6 patients were given the second priority time slot, 5 patients were given the third priority time slot, and 4 patients were given the fourth priority time slot. The total disability level score of the patients receiving services was 155 points.

Table 8: Rehabilitation bus shift allocation.

Shift	Time	Quota
s_1	8:00	4
s_2	8:30	4
s_3	9:00	4
s_4	9:30	4
s_5	10:00	4
s_6	10:30	4
s_7	11:00	4
s_8	11:30	4
s_9	13:00	4
s_{10}	13:30	4
s_{11}	14:00	4
s_{12}	14:30	4
s_{13}	15:00	4
s_{14}	15:30	4
s_{15}	16:00	4
s_{16}	16:30	4

Table 9: Disability level and willingness to share rides.

Disability level		Sharing rides	
4	A+	5	Willing
3	A1	4	↑
2	A2	3	
1	B	2	↓
		1	Unwilling

Table 10: Each rehabilitation bus shift's priority for patients.

	The shift's priority for patients																			
S_1	P_{48}	P_{59}	P_{68}	P_{60}	P_{5}	P_{58}	P_{41}	P_{6}	P_{11}	P_{54}	P_{25}	P_{28}	P_{36}	P_{37}	P_{47}	P_{18}	P_{46}	P_{66}	P_{52}	P_{53}
S_2	P_{68}	P_{48}	P_{60}	P_{59}	P_{58}	P_{5}	P_{6}	P_{11}	P_{41}	P_{54}	P_{25}	P_{31}	P_{36}	P_{28}	P_{46}	P_{66}	P_{52}	P_{37}	P_{47}	P_{18}
	P_{53}	P_{71}																		
S_3	P_{60}	P_{58}	P_{5}	P_{68}	P_{48}	P_{59}	P_{11}	P_{54}	P_{25}	P_{31}	P_{6}	P_{41}	P_{28}	P_{36}	P_{14}	P_{7}	P_{53}	P_{37}	P_{47}	P_{18}
	P_{46}	P_{66}	P_{52}	P_{71}	P_{40}	P_{56}	P_{55}													
S_4	P_{5}	P_{58}	P_{68}	P_{48}	P_{33}	P_{60}	P_{59}	P_{31}	P_{25}	P_{6}	P_{11}	P_{41}	P_{30}	P_{54}	P_{14}	P_{7}	P_{28}	P_{36}	P_{40}	P_{71}
	P_{55}	P_{53}	P_{56}	P_{37}	P_{47}	P_{18}	P_{46}	P_{44}	P_{22}	P_{66}	P_{52}									
S_5	P_{33}	P_{13}	P_{49}	P_{30}	P_{31}	P_{51}	P_{15}	P_{45}	P_{35}	P_{14}	P_{7}	P_{9}	P_{40}	P_{71}	P_{55}	P_{22}	P_{56}	P_{44}	P_{27}	P_{39}
S_6	P_{33}	P_{13}	P_{49}	P_{30}	P_{51}	P_{15}	P_{35}	P_{45}	P_{7}	P_{9}	P_{14}	P_{22}	P_{56}	P_{44}	P_{40}	P_{27}	P_{39}	P_{55}		
S_7	P_{13}	P_{49}	P_{33}	P_{51}	P_{15}	P_{35}	P_{45}	P_{30}	P_{12}	P_{9}	P_{44}	P_{27}	P_{39}	P_{22}						
S_8	P_{13}	P_{49}	P_{20}	P_{45}	P_{3}	P_{51}	P_{15}	P_{12}	P_{35}	P_{9}	P_{24}	P_{27}	P_{39}	P_{8}	P_{50}					
S_9	P_{20}	P_{19}	P_{43}	P_{26}	P_{3}	P_{67}	P_{12}	P_{38}	P_{24}	P_{8}	P_{50}	P_{16}								
S_{10}	P_{19}	P_{43}	P_{26}	P_{10}	P_{20}	P_{67}	P_{12}	P_{38}	P_{3}	P_{4}	P_{61}	P_{24}	P_{70}	P_{16}	P_{8}	P_{50}	P_{21}	P_{17}		
S_{11}	P_{43}	P_{26}	P_{10}	P_{19}	P_{20}	P_{42}	P_{38}	P_{4}	P_{67}	P_{61}	P_{3}	P_{70}	P_{2}	P_{24}	P_{21}	P_{17}	P_{16}	P_{8}	P_{50}	
S_{12}	P_{10}	P_{42}	P_{72}	P_{43}	P_{19}	P_{26}	P_{4}	P_{61}	P_{38}	P_{67}	P_{65}	P_{70}	P_{2}	P_{34}	P_{21}	P_{17}	P_{16}			
S_{13}	P_{72}	P_{42}	P_{64}	P_{10}	P_{62}	P_{32}	P_{57}	P_{61}	P_{65}	P_{1}	P_{4}	P_{70}	P_{2}	P_{34}	P_{17}	P_{21}	P_{29}	P_{63}	P_{23}	P_{69}
S_{14}	P_{72}	P_{42}	P_{64}	P_{62}	P_{32}	P_{57}	P_{65}	P_{1}	P_{2}	P_{34}	P_{63}	P_{29}	P_{23}	P_{69}						
S_{15}	P_{64}	P_{62}	P_{32}	P_{57}	P_{72}	P_{65}	P_{1}	P_{34}	P_{63}	P_{29}	P_{23}	P_{69}								
S_{16}	P_{62}	P_{32}	P_{57}	P_{64}	P_{1}	P_{29}	P_{23}	P_{69}	P_{63}											

5. Conclusion and Discussion

In order to improve the demand and supply matching problem of the rehabilitation transportation service in eastern Taiwan, this study examined the deferred acceptance algorithm and the top trading cycle method to match patients and service vehicles. In accordance with patients' priorities and desired time slots of rehabilitation transportation service, service providers could dispatch the vehicle to serve those requests based on the degree of disability of the patient, the willingness to share rides, the number of appointments, and the distance from the designated place of the patient to the medical institution.

This study examined the data provided by the Hualien Mennonite Foundation from June to December 2016. The simulation results showed that out of the 72 patients, 63 were successfully matched. Among them, 57 patients got their first-ranking shift, 1 got the second-ranking shift, 3 got the third-ranking shift, and 2 got the fourth-ranking shift. The total score of the disability level for these serviced patients was 170 points. Compared with the current "first-come, first-served" mode, using the same group of simulated data, 48 patients got the first-ranking shift, 6 got the second-ranking shift, 4 got the third-ranking shift, and 5 got the fourth-ranking shift, with a total score of 155 points for their disability level. Results listed in Table 6 shows our proposed matching algorithms could significantly satisfy patients' desired time slots.

For the consistent results obtained by the two algorithms in this study, we conducted a verification by simulating with 10 people and two shifts. If the two shifts have consistent selection criteria for patients' preferences, such as first comparing disability levels and then ridesharing willingness if tied, then the matching results of the two algorithms will be consistent. However, if the two shifts have different selection criteria for patients, such as one shift prioritizing disability levels and the other shift prioritizing ridesharing willingness, then the matching results of the two algorithms will be different. In the future, we can use this finding to adjust the preference selection rules for some rehabilitation bus shifts and further explore the differences between the two algorithms, such as more complex matchings with partitions of time slots, and to compare the timing of using the two algorithms or hybrid them. Several future research lines are listed as follows. Detailed parameter settings could be viewed in appendix Tables 7–10.

(1) Considering a round trip or a series of trip chain process with multiple demand requests

(2) Considering the feasibility of combing passengers' locations during the phase of route planning, with time-space network viewpoints

(3) Considering an effective zoning and clustering area, to reduce overall service distance

(4) Considering a dynamic and stochastic matching mechanism during a changing environment

Data Availability

Data are attached in the appendix.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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