





## Research Article

# A Cooperative Lane-Changing Strategy for Weaving Sections of Urban Expressway under the Connected Autonomous Vehicle Environment

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To alleviate the lane-changing conflicts between weaving vehicles and enhance the traffic efficiency in the weaving section of urban expressway under the connected autonomous vehicle (CAV) environment, a cooperative lane-changing strategy for CAVs is proposed. The strategy consists of an upper layer of decision making, which determines the lane-changing sequences of weaving vehicles based on their lane-changing advantages quantified by a set of utility functions, and a lower layer of control, which generates detailed instructions of speed adjustments and lane-changing manoeuvres for weaving vehicles. To verify the effectiveness of the proposed strategy under different traffic demand settings, a numerical simulation, including a base case and a control case, is conducted. Then, to further verify the effectiveness of the proposed strategy for the mixed traffic state and compare its performance with the existing CAV lane-changing method, benchmark and comparison tests with six different market penetration rates (MPRs) of CAVs are carried out under the congested demand setting. In addition, the delay improvement ratio, inverse time-to-collision, and ratio of large deceleration time are selected as performance indicators to investigate the effect of the proposed strategy on enhancing the operational efficiency, traffic safety, and passenger's comfort within the weaving section. According to the simulation results, the overall efficiency, safety and comfort in the weaving section under the CAV environment, are all improved, when the proposed strategy is applied to weaving vehicles. The proposed strategy is also superior to the existing CAV lane-changing method on maintaining traffic efficiency and safety. Therefore, the proposed cooperative lane-changing strategy, based on CAV technologies, shows good potential in solving the problem of lane-changing conflicts within the weaving section and facilitating the traffic management and traffic control of urban expressway.

## 1. Introduction

Urban expressway, which has the characteristics of large capacity, high speed, and high safety, can facilitate long-distance travel for residents among different urban areas. However, with the fast progress of urbanization, the amount of private car travels has significantly increased, resulting in frequent traffic congestion in urban expressway during rush hours. In particular, due to the restriction of urban land use and the high density of traffic demand, an expressway off-ramp often closely follows an on-ramp, and sometimes, the on-ramp and the off-ramp are directly connected by a short

auxiliary lane. Such kind of layout forms a typical weaving section. A research study [1] shows that within the short range of weaving section, there are a large number of accelerating, decelerating, and mandatory lane-changing behaviours, which make the traffic conditions in the weaving section relatively complicated and highly indeterminate. Therefore, during the rush hours with the high flow rate, the weaving section can easily become a major congestion spot in the expressway.

In a typical weaving section of urban expressway, there are two major types of mandatory lane-changing vehicles, namely, the diverging vehicles and the merging vehicles,

where the former come from the mainline of expressway and change to the auxiliary lane in the weaving section in order to exit the expressway and the latter enter the auxiliary lane and change lane in order to merge into the mainline of expressway. The opposite origins and destinations of these two types of lane-changing manoeuvres can cause a considerable amount of traffic conflicts. Specifically, in a weaving lane-changing scenario, a diverging vehicle and a merging vehicle, which are close to each other, might intend to change to each other's lane at the same time. Such conflict in lane-changing intention will make these two vehicles to "compete" against each other in order to change lane first and move into the acceptable gap in front of the "opponent" vehicles. Usually, the vehicle with advantages in relative position or relative speed will force the other vehicle to yield in speed or even give up the lane-changing intention temporarily to give way. Such outcome of the "competition" between two weaving vehicles seems rather intuitive, but sometimes, it is highly sensitive to driver's subjective judgment. In the worst-case scenario, the disadvantageous vehicle refuses to give way to the advantageous vehicle, causing a dead lock between two weaving vehicles. The accumulation of such kind of dead locks will eventually lead to the delay in journey time, reducing the operational efficiency of weaving section.

Therefore, it is necessary to thoroughly understand the lane-changing behaviour in the weaving section and design a comprehensive lane-changing strategy for the weaving section [2], as to facilitate traffic management and traffic control of the urban expressway, thus improving the traffic efficiency.

In retrospect, there have been numerous literature studies on the study of characteristics of lane-changing behaviour in the weaving section. Hidas [3] developed a lane-changing model for the weaving section by adopting the concept of intelligent agent and classified the lane-changing behaviour into three types, namely, free, forced, and cooperative. Jin [4] established a LWR lane-changing model for the weaving section, based on whether the vehicles were weaving or nonweaving vehicles. Marczak et al. [5] systematically analysed the lane-changing behaviour in the weaving section at a microscopic level. Chen et al. [6] developed a probability model of car-following and lane-changing binary choice for vehicles in the weaving section. Wan et al. [7] analysed a variety of merging strategies using the NGSIM data set and constructed a combined sequential decision-making model for the weaving section that can dynamically simulate vehicles' choice of target gap and lane-changing strategy. Kusuma et al. [8] found out the group lane-changing behaviour in the weaving section and proposed a random utility formula to characterize different lane-changing mechanisms. Peng et al. [9] proposed a multistage lane-changing decision-making model based on the refined cellular automata for the weaving section.

Other researchers focused on alleviating the traffic conflicts caused by weaving lane-changing behaviour and improving the efficiency of the weaving section. Sun et al. [10] used multiagents modelling technology and established a cooperative lane-changing model to solve the problem of

lane-changing conflicts in the weaving section. Bham [11] proposed a simple and efficient weaving model using microscopic simulation, so as to guide lane-changing behaviour in the weaving section. Mai et al. [12] adopted the concept of the cooperative intelligent transport system and proposed a lane-changing advisory for the weaving section. Sulejic et al. [13] developed an algorithm based on particle swarm optimization to optimize the lane-changing distribution and alleviate the problem of lane-changing concentration in the weaving section. Tilg et al. [14] applied the automated vehicle technology and proposed a multiclass hybrid model to optimize the lane-changing distribution and increase the capacity of the weaving section.

With the development of CAV technologies, the share of information among vehicles and infrastructures can be realized through V2V and V2I communications [15]. In addition, the optimized instruction generated by roadside control units can be sent to vehicles in real time, effectively improving the travel efficiency and safety of vehicles. Thus, inspired by the previous research studies on the optimization of lane-changing behaviour in the weaving section and the concept of CAVs, this paper proposes a novel cooperative lane-changing strategy for CAVs in weaving sections of urban expressway at a microscopic level. In this strategy, an upper layer of decision making is built to give advice on the lane-changing sequences for weaving vehicles and a lower layer of control is built to provide guidelines for vehicles' speed adjustments and lane-changing manoeuvres. Also, a numerical simulation is set to verify the effectiveness of the proposed strategy.

The main contributions of this study are as follows:

- (1) Solving the problem of lane-changing conflicts in the weaving lane-changing scenario by applying CAV technologies to provide weaving vehicles with the decision of lane-changing sequences and detailed control instructions of speed adjustments and lane-changing manoeuvres.
- (2) Effectively improving the operational efficiency, traffic safety, and passenger's comfort within the weaving section of urban expressway under the CAV environment.

The rest of the paper is organized as follows. The design of the cooperative lane-changing strategy is introduced in the next section. After that, the setups and performance indicators of the numerical simulation are thoroughly described, followed by the analysis and discussion of simulation results. At last, some concluding remarks are mentioned to end this paper.

## 2. Cooperative Lane-Changing Strategy

In the general case of the lane-changing scenario (see Figure 1), the lane-changing vehicle ( $SV$ ) takes into consideration its relative position and speed with respect to the leading vehicle ( $\widehat{LV}$ ) and following the vehicle ( $\widehat{FV}$ ) in the target lane, as well as the space of acceptable gap ( $d$ ) between  $\widehat{LV}$  and  $\widehat{FV}$ .

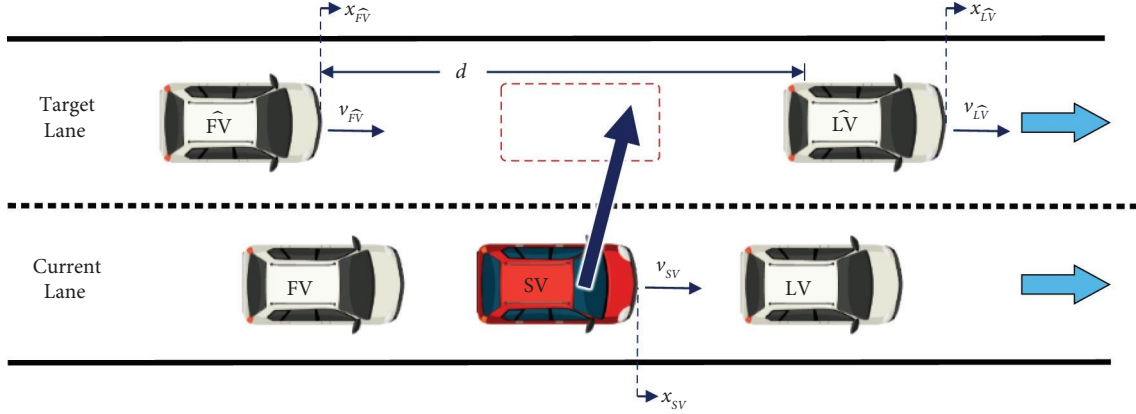


FIGURE 1: Schematic diagram of a general lane-changing scenario.

$$\begin{cases} v_{SV}(t + \Delta t) = b_{\max} \Delta t + \sqrt{b_{\max}^2 \Delta t^2 - b_{\max} \left\{ 2 \left[ x_{LV}(t) - l - x_{SV}(t) \right] - v_{SV}(t) \Delta t - \frac{v_{LV}^2(t)}{b_{\max}} \right\}}, \\ v_{FV}(t + \Delta t) = b_{\max} \Delta t + \sqrt{b_{\max}^2 \Delta t^2 - b_{\max} \left\{ 2 \left[ x_{SV}(t) - l - x_{FV}(t) \right] - v_{FV}(t) \Delta t - \frac{v_{SV}^2(t)}{b_{\max}} \right\}}. \end{cases} \quad (1)$$

For example, in the classical Gipps lane-changing model [16] (see equation (1)), the lane-changing manoeuvre is considered safe and can be carried out if  $d$  is longer than the length of  $SV$  and the condition in equation (2) is fulfilled. Specifically, the required decelerating rates for  $SV$  and  $FV$  to maintain safe car-following speeds with respect to  $LV$  and  $SV$  and avoid collision after the lane change cannot exceed the maximum decelerating rate.

$$\begin{cases} \frac{v_{SV}(t + \Delta t) - v_{SV}(t)}{\Delta t} \geq b_{\max}, \\ \frac{v_{FV}(t + \Delta t) - v_{FV}(t)}{\Delta t} \geq b_{\max}. \end{cases} \quad (2)$$

where  $x_{SV}(t)$  and  $v_{SV}(t)$  are the position and speed of  $SV$  at time  $t$ ;  $x_{LV}(t)$  and  $v_{LV}(t)$  are the position and speed of  $LV$  at time  $t$ ;  $x_{FV}(t)$  and  $v_{FV}(t)$  are the position and speed of  $FV$  at time  $t$ ;  $v_{SV}(t + \Delta t)$  and  $v_{FV}(t + \Delta t)$  are speeds of  $SV$  and  $FV$  at time  $(t + \Delta t)$ ;  $b_{\max}$  is the maximum decelerating rate of the vehicle; and  $l$  is the length of the vehicle.

However, in the case of weaving lane-changing scenario (see Figure 2), both the weaving vehicles ( $SV1$  and  $SV2$ ) have to simultaneously consider (I) the relative position and speed with respect to the leading vehicles ( $LV2$  and  $LV1$ ) and following vehicles ( $FV2$  and  $FV1$ ) in their target lanes, (II) the relative position and speed with respect to each other, and (III) the space of acceptable gaps in front of ( $d_{2,front}$  and  $d_{1,front}$ ) and behind ( $d_{2,rear}$  and  $d_{1,rear}$ ) each other. Therefore, it is improper to simply apply the general microscopic lane-changing model to the weaving lane-changing scenario because it might cause confusion in the decision making of vehicles' lane-changing sequences.

Alternatively, it is more practical to consider the weaving vehicles as a group, namely, the weaving group, and transform the problem of two weaving vehicles' conflicting lane-changing manoeuvres into the problem of rearrangement of lane-changing sequences within the weaving group. Due to the vast deployments of the emerging CAV technologies in the near future, it is expected that the information of vehicle states (position and speed) can be easily shared within the weaving group if both the weaving vehicles are CAVs, and optimized instructions regarding driving decision and vehicle control generated by roadside control units can be transferred to the weaving group in real time. In this case, a cooperative lane-change strategy at the microscopic level is proposed for CAVs in the weaving section, which consists of an upper layer of decision making for lane-changing sequences of two weaving vehicles and a lower layer of control for vehicles' speed adjustments and lane-changing manoeuvres.

### 2.1. Decision-Making Layer for Lane-Changing Sequences.

The main purpose of the decision-making layer is to determine the lane-changing sequences of the two weaving vehicles, that is, whether  $SV1$  or  $SV2$  should change lane first and move into the acceptable gap in front of the other vehicle ( $d_{2,front}$  or  $d_{1,front}$ ). The prerequisite for the decision-making layer is shown in equation (3), which ensures that the relative position of two weaving vehicles in the longitudinal direction is less than one vehicle length. In addition, all vehicles in this study are assumed to be small passenger cars with identical attributes.

$$|x_{SV1} - x_{SV2}| < l, \quad (3)$$

where  $x_{SV1}$  and  $x_{SV2}$  are positions of  $SV1$  and  $SV2$  and  $l$  is the length of vehicle.

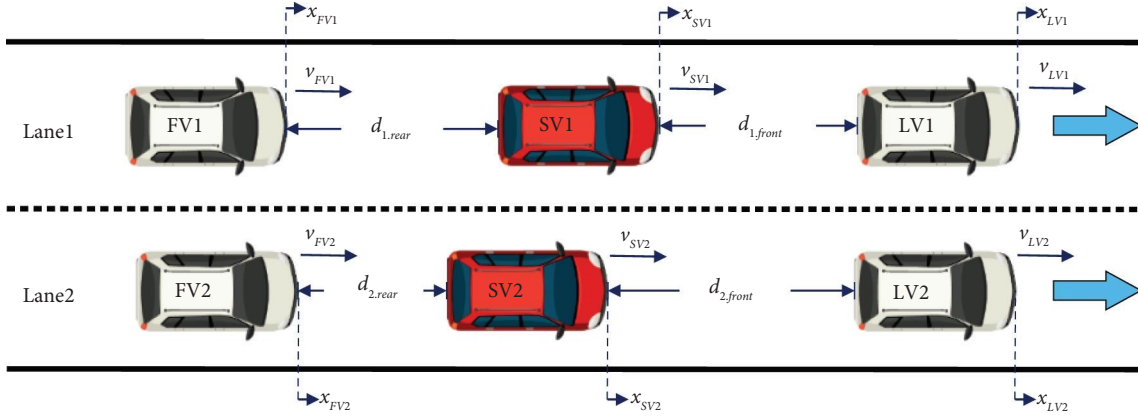


FIGURE 2: Schematic diagram of a weaving lane-changing scenario.

The most important part of the decision-making process is to quantify how much advantage each weaving vehicle has over the other to change lane first. Hence, a set of lane-changing utility functions are designed to quantify the lane-changing advantage in the following aspects.

**2.1.1. Space of Acceptable Gaps.** The first aspect of the lane-changing advantage is related to the space of acceptable gap in front of and behind each weaving vehicle (see equation (4)).

$$\begin{cases} U_1^1 = \omega_{\text{front}} \frac{d_{2,\text{front}}}{d_{1,\text{front}} + d_{2,\text{front}}} + \omega_{\text{rear}} \frac{d_{1,\text{rear}}}{d_{1,\text{rear}} + d_{2,\text{rear}}} \in [0, 1], \\ U_2^1 = \omega_{\text{front}} \frac{d_{1,\text{front}}}{d_{1,\text{front}} + d_{2,\text{front}}} + \omega_{\text{rear}} \frac{d_{2,\text{rear}}}{d_{1,\text{rear}} + d_{2,\text{rear}}} \in [0, 1], \end{cases} \quad (4)$$

where  $U_1^1$  and  $U_2^1$  are utility values of SV1 and SV2, respectively, regarding the first aspect of advantage;  $\omega_{\text{front}}$  and

$\omega_{\text{rear}}$  are importance factors regarding the space of acceptable gaps in front of and behind the weaving vehicles ( $\omega_{\text{front}} + \omega_{\text{rear}} = 1$ );  $d_{1,\text{front}}$  and  $d_{2,\text{front}}$  are the space of acceptable gaps in front of SV1 and SV2; and  $d_{1,\text{rear}}$  and  $d_{2,\text{rear}}$  are the acceptable gaps behind SV1 and SV2.

For example, if  $d_{2,\text{front}}$  is larger than  $d_{1,\text{front}}$ , it will be more favorable for SV1 to move into the acceptable gap in front of SV2. And, if  $d_{1,\text{rear}}$  is larger than  $d_{2,\text{rear}}$ , it will be more favorable for SV2 to move into the acceptable gap behind SV1. So,  $U_1^1$  will receive a greater value than  $U_2^1$ , and SV1 has the advantage over SV2 to change the lane first and vice versa.

**2.1.2. Relative Speeds with respect to Leading and Following Vehicles in Target Lanes.** The second aspect of the lane-changing advantage is related to weaving vehicles' relative speeds with respect to the leading and following vehicles in their target lanes (see equation (5)).

$$\begin{cases} U_2^1 = \omega_{\text{lead}} \frac{\max(v_{SV2} - v_{LV1}, 0)}{\max(v_{SV1} - v_{LV2}, 0) + \max(v_{SV2} - v_{LV1}, 0) + \varepsilon} + \omega_{\text{follow}} \frac{\max(v_{FV2} - v_{SV1}, 0)}{\max(v_{FV2} - v_{SV1}, 0) + \max(v_{FV1} - v_{SV2}, 0) + \varepsilon} \in [0, 1], \\ U_2^2 = \omega_{\text{lead}} \frac{\max(v_{SV1} - v_{LV2}, 0)}{\max(v_{SV1} - v_{LV2}, 0) + \max(v_{SV2} - v_{LV1}, 0) + \varepsilon} + \omega_{\text{follow}} \frac{\max(v_{FV1} - v_{SV2}, 0)}{\max(v_{FV2} - v_{SV1}, 0) + \max(v_{FV1} - v_{SV2}, 0) + \varepsilon} \in [0, 1], \end{cases} \quad (5)$$

where  $U_2^1$  and  $U_2^2$  are the utility values of SV1 and SV2, respectively, regarding the second aspect of advantage;  $\omega_{\text{lead}}$  and  $\omega_{\text{follow}}$  are the importance factors regarding weaving vehicles' relative speeds with respect to the leading vehicles and the following vehicles in their target lanes ( $\omega_{\text{lead}} + \omega_{\text{follow}} = 1$ );  $v_{SV1}$  and  $v_{SV2}$  are the speeds of SV1 and SV2;  $v_{LV1}$ ,  $v_{LV2}$ ,  $v_{FV1}$ , and  $v_{FV2}$  are speeds of leading vehicles and following vehicles; and  $\varepsilon = 10^{-7} \text{ m/s}$  is an extremely small value set to prevent zero denominator.

For example, when  $(v_{SV2} - v_{LV1})$  is larger than  $(v_{SV1} - v_{LV2})$ , it will be easier for SV1 to match up the speed of LV2 if SV1 moves into the acceptable gap in front of SV2. When  $(v_{FV2} - v_{SV1})$  is larger than  $(v_{FV1} - v_{SV2})$ , it will be easier for FV1 to match up the speed of SV2 if SV2 moves into the acceptable behind SV1. So,  $U_2^1$  will receive a greater value than  $U_2^2$ , and SV1 has the advantage over SV2 to change the lane first and vice versa.

**2.1.3. Relative Position between Weaving Vehicles.** The third aspect of the lane-changing advantage is related to weaving vehicles' relative position with respect to each other (see equation (6)).

$$\begin{cases} U_3^1 = \frac{\max(x_{SV1} - x_{SV2}, 0)}{l} \in [0, 1], \\ U_3^2 = \frac{\max(x_{SV2} - x_{SV1}, 0)}{l} \in [0, 1], \end{cases} \quad (6)$$

where  $U_3^1$  and  $U_3^2$  are the utility values of SV1 and SV2, respectively, regarding the third aspect of advantage.

For example, if  $x_{SV1}$  is larger than  $x_{SV2}$ , it will be more favorable for SV1 to change the lane first, and thus,  $U_3^1$  will receive a greater value than  $U_3^2$  and vice versa.

**2.1.4. Relative Speed between Weaving Vehicles.** The fourth aspect of the lane-changing advantage is related to weaving

vehicles' relative speed with respect to each other (see equation (7)).

$$\begin{cases} U_4^1 = \frac{v_{SV1}}{v_{SV1} + v_{SV2}} \in [0, 1], \\ U_4^2 = \frac{v_{SV2}}{v_{SV1} + v_{SV2}} \in [0, 1], \end{cases} \quad (7)$$

where  $U_4^1$  and  $U_4^2$  are the utility values of SV1 and SV2, respectively, regarding the fourth aspect of advantage.

For example, if  $v_{SV1}$  is larger than  $v_{SV2}$ , it will be more favorable for SV1 to change lane first, and thus,  $U_4^1$  will receive a greater value than  $U_4^2$  and vice versa.

**2.1.5. Total Lane-Changing Advantage.** The total lane-changing advantage for each vehicle in the weaving group (see equation (8)) is the summation of four aspects of advantages defined previously, multiplied by a couple of coefficient factors.

$$\begin{cases} U_{\text{Total}}^1 = \lambda_1 \exp\left[\frac{-\max(L - L_{lc1} - L_{SV1}, 0)}{L - L_{lc}}\right] (U_1^1 + U_2^1 + U_3^1 + U_4^1), \\ U_{\text{Total}}^2 = \lambda_2 \exp\left[\frac{-\max(L - L_{lc2} - L_{SV2}, 0)}{L - L_{lc2}}\right] (U_1^2 + U_2^2 + U_3^2 + U_4^2), \end{cases} \quad (8)$$

where  $U_{\text{Total}}^1$  and  $U_{\text{Total}}^2$  are the total utility values of SV1 and SV2, respectively, regarding the total advantage; and  $\lambda_1$  and  $\lambda_2$  are the coefficient factors describing the driving styles of SV1 and SV2 (an aggressive driver is expected to have more advantage than a cautious driver to change lane first);

$$\lambda_1 \text{ or } \lambda_2 = \begin{cases} > 1 \text{ for aggressive driving style} \\ = 1 \text{ for neutral driving style, } \lambda_1 \text{ and } \lambda_2 > 0, \\ < 1 \text{ for cautious driving style,} \end{cases} \quad (9)$$

$\exp[(-\max(L - L_{lc1} - L_{SV1}, 0))/(L - L_{lc1})]$  and  $\exp[(-\max(L - L_{lc2} - L_{SV2}, 0))/(L - L_{lc2})]$  are the coefficient factors describing the urgency for the mandatory lane change of SV1 and SV2;  $L$  is the total length of the weaving section, measured from merging gore to diverging gore;  $L_{lc1}$  and  $L_{lc2}$  are defined as the required longitudinal distances for lane-changing manoeuvres of SV1 and SV2; and  $L_{SV1}$  and  $L_{SV2}$  are relative distances of SV1 and SV2 with respect to merging gore. As the vehicle drives downstream and approaches the end of the weaving section, its degree of urgency for mandatory lane change increases exponentially.

Since the utility values for four aspects of the lane-changing advantage mentioned previously are normalized to the values within the interval  $[0, 1]$  (see equations (4)–(7)), it is considered that all four aspects contribute equally to the total lane-changing advantage.

Overall, the total advantage of SV1 and SV2 is compared against each other to determine their lane-changing sequences. For instance, if  $U_{\text{Total}}^1$  is greater than  $U_{\text{Total}}^2$ , the lane-changing sequences are decided as SV1 changing the lane first and moving into the acceptable gap in front of SV2 while SV2 changing the lane later and moving into the acceptable gap behind SV1 and vice versa.

**2.2. Control Layer for Speed Adjustments and Lane-Changing Manoeuvres.** The upper layer of decision making gives the advice on the lane-changing sequences for weaving vehicles. Next, the lower layer of control can guide weaving vehicles to adjust their speed and execute the lane-changing manoeuvres simultaneously. In addition, the prerequisite that has to be fulfilled for the control layer is shown in equation (10), which ensures enough space for the execution of weaving lane-changing behaviour.

$$\begin{cases} x_{LV2} - l - x_{FV1} > 2l, & \text{if SV1 is decided to change lane first,} \\ x_{LV1} - l - x_{FV2} > 2l, & \text{if SV2 is decided to change lane first.} \end{cases} \quad (10)$$

Take the weaving group in Figure 2 as an example, if SV1 is decided to change the lane first, SV1 will be instructed to accelerate with the maximum accelerating rate ( $a_{\text{max}}$ ) and SV2 will be instructed to decelerate with a comfortable decelerating rate ( $b_{\text{comfort}}$ ), until the clearance between SV1 and SV2 fulfills the condition in equation (11), which

prevents the collision between  $SV1$  and  $SV2$  during the lane-changing manoeuvres (see Figure 3(a)).

$$x_{SV1} - l - x_{SV2} > 0. \quad (11)$$

In addition, during the acceleration process of  $SV1$ , it has to keep its speed under the maximum car-following speed with respect to  $LV2$  so that it can safely follow  $LV2$  after changing the lane. On the other hand, if  $SV2$  is decided to change the lane first, the speed adjustment processes for  $SV1$  and  $SV2$  are quite opposite.

Once the condition in equation (10) is fulfilled, two weaving vehicles can execute lane-changing manoeuvres and move horizontally into their desired acceptable gaps simultaneously (see Figure 3(b)).

### 3. Numerical Simulation

**3.1. Simulation Setup.** To verify the effectiveness of the proposed cooperative lane-changing strategy, a numerical simulation is conducted using Python 3.8. The simulation scenario (see Figure 4) is a hypothetical weaving section in the urban expressway, consisting of a mainline and an auxiliary lane. The length of the weaving section ( $L$ ) is 150 m, and the lane width is 3.75 m. The traffic flow of the mainline includes diverging vehicles ( $MD$ ) and pass-through vehicles ( $MP$ ) which drive through the weaving section in the mainline without lane change. Also, the traffic flow of the auxiliary lane includes merging vehicles ( $AM$ ) and pass-through vehicles ( $AP$ ) which drive through the weaving section in the auxiliary lane without lane change. The vehicle attributes and other calibrated parameters in the simulation are listed in Table 1. In specific, values of  $\omega_{\text{front}}$  and  $\omega_{\text{lead}}$  are taken as 0.6 and values of  $\omega_{\text{rear}}$  and  $\omega_{\text{follow}}$  are taken as 0.4. It is mainly due to the common sense in the real-world driving scenario that when a vehicle manages to change lane, it usually concerns more about the space of the front acceptable gap than that of the rear acceptable gap in the target lane and concerns more about the relative speed to the leading vehicle than that to the following vehicle in the target lane.

In order to simulate operational condition of urban expressway under different traffic demands, six demand settings of different traffic flow rates are applied to the simulation scenario (see Table 2), where demand setting #1 is corresponded to the free-flow state and demand setting #6 is corresponded to the congested state.

Here,  $q_{\text{main}}$  and  $q_{\text{aux}}$  are total flow rates in the mainline and auxiliary lane, respectively;  $q_{MD}$ ,  $q_{MP}$ ,  $q_{AM}$ , and  $q_{AP}$  are flow rates corresponding to diverging vehicle in the mainline, pass-through vehicle in the mainline, merging vehicle in the auxiliary lane, and pass-through vehicle in the auxiliary lane, respectively; and  $v_{\text{des}}$  is the desired operating speed of expressway.

Two test cases, base case and control case, are conducted to examine the effectiveness of the proposed strategy under different demand settings. Meanwhile, the car-following behaviours in both cases are depicted by the classical Gipps car-following model [17].

**3.1.1. Base Case.** For base case, all vehicles are assumed to be conventional human driven vehicles (HDVs). Hence, the test is carried out without applying the proposed cooperative lane-changing strategy. Both diverging vehicles and merging vehicles enter the weaving section and perform lane-changing behaviours in accordance with the Gipps lane-changing model (see equation (1)), which means that the lane-changing vehicle only executes lane-changing manoeuvre when the acceptable gap in the target lane is adequate. In addition, to prevent the situation that the lane-changing vehicle cannot find any adequate acceptable gap and fails to perform lane-changing behaviour while reaching the end of the weaving section, a forced lane-changing mechanism is introduced. In specific, when the distance between a lane-changing vehicle and the end of the weaving section (diverging gore) is less than  $v_{\text{des}}T$ , the forced lane-changing mechanism is initiated, and the road segment within the distance  $v_{\text{des}}T$  away from the end of weaving section is defined as the forced lane-changing segment. In order to perform the forced lane change, the lane-changing vehicle will first reduce its speed with a decelerating rate of  $-1 \text{ m/s}^2$  until it finds a gap, which is longer than the vehicle length, in the target lane. Then, the lane-changing vehicle will directly execute lane-changing manoeuvre regardless of the relative speed with respect to the following vehicle in the target lane while simultaneously adjusting the speed to match up the safe car-following speed with respect to the leading vehicle in the target lane. As a result, the following vehicle in the target lane will be forced to adjust speed in order to give way to the lane-changing vehicle. The flow chart of the base case is illustrated in Figure 5(a).

**3.1.2. Control Case.** For the control case, all vehicles are assumed to be CAVs. In this case, all lane-changing mechanisms are the same as those in the base case, except that the proposed cooperative lane-changing strategy can be applied to the eligible weaving group of CAVs. Before any lane-changing manoeuvre, the lane-changing vehicle always checks its current condition in the first place and figures out if there is a weaving vehicle in the adjacent lane, which intends to change lane at the same time. If a weaving vehicle exists and the relative position of two vehicles fulfills the prerequisite for the decision-making layer of the proposed strategy (see equation (3)), the decision regarding the lane-changing sequences of two weaving vehicles will be automatically generated by the roadside control unit. Meanwhile, if the prerequisite for the control layer of the proposed strategy (see equation (10)) is fulfilled, the control instructions of speed adjustments and lane-changing manoeuvres will be sent to the vehicles in real time. Then, the weaving vehicles can perform lane-changing behaviour safely and efficiently. If weaving lane-changing behaviour is unable to be carried out, the vehicle will still be able to perform lane-changing behaviour according to the Gipps lane-changing model. Also, the forced lane-changing mechanism is still applicable if necessary. The flow chart of the control case is illustrated in Figure 5(b).

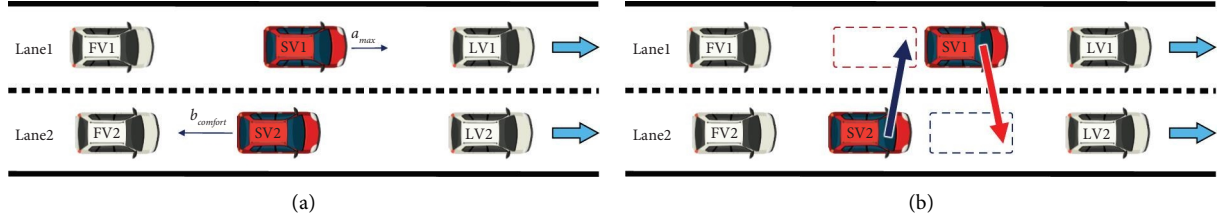


FIGURE 3: Schematic diagram of the control layer. (a) Speed adjustment. (b) Lane-changing manoeuvres.



FIGURE 4: Schematic diagram of the simulation scenario.

TABLE 1: Simulation parameters.

Parameters	Descriptions	Values
$l$	Vehicle length	5 m
$w$	Vehicle width	2 m
$M$	Vehicle weight	1500 kg
$T$	Lane-changing duration	4 s
$\tau$	Reaction time	1 s
$a_{\max}$	Maximum accelerating rate	3 m/s <sup>2</sup>
$b_{\max}$	Maximum decelerating rate	-8 m/s <sup>2</sup>
$b_{\text{comfort}}$	Comfortable decelerating rate	-3 m/s <sup>2</sup>
$\omega_{\text{front}}$	Importance factor regarding the space of acceptable gaps in front of the weaving vehicles	0.6
$\omega_{\text{rear}}$	Importance factor regarding the space of acceptable gaps behind the weaving vehicles	0.4
$\omega_{\text{lead}}$	Importance factor regarding weaving vehicles' relative speeds with respect to leading vehicles in target lanes	0.6
$\omega_{\text{follow}}$	Importance factor regarding weaving vehicles' relative speeds with respect to following vehicles in target lanes	0.4
$\lambda$	Driving style coefficient factor	0.9 ~ 1.1 (normally distributed)
$\Delta t$	Simulation time-step	0.1s

TABLE 2: Demand settings.

Nos.	$q_{\text{main}}$ (vphpl)	$q_{MD}$ (vphpl)	$q_{MP}$ (vphpl)	$q_{aux}$ (vphpl)	$q_{AM}$ (vphpl)	$q_{AP}$ (vphpl)	$v_{\text{des}}$ (kph)
1	1000	400	600	500	400	100	80
2	1200	480	720	600	480	120	70
3	1400	560	840	700	560	140	60
4	1600	640	960	800	640	160	50
5	1800	720	1080	900	720	180	40
6	2000	800	1200	1000	800	200	30

3.1.3. *Tests for the Mixed Traffic State.* Due to the gradual increase of MPR of CAVs from 0% (base case) to 100% (control case) in the real world, it is expected in the near future that the mixed traffic flow, which consists of HDVs and CAVs at the same time, will dominate the roadway traffic for a relatively long period of time before the complete deployment of CAVs. Therefore, in order to further investigate the effectiveness of the proposed strategy under the mixed traffic state and compare its performance with the existing CAV lane-changing method, two sets of tests (benchmark and comparison tests) with six different MPRs

(see Table 3) are carried out, under demand setting #6 (congested state). In the benchmark test, weaving CAVs perform the lane-changing behaviour in accordance with the lane-changing mechanisms of the control case in Figure 5(b), for which the proposed cooperative lane-changing strategy is applicable. In the comparison test, weaving CAVs perform lane-changing behaviour according to the cooperative lane-changing strategy proposed by Xue et al. [18]. In addition, HDVs in both tests perform the lane-changing behaviour according to the lane-changing mechanisms of the base case in Figure 5(a).

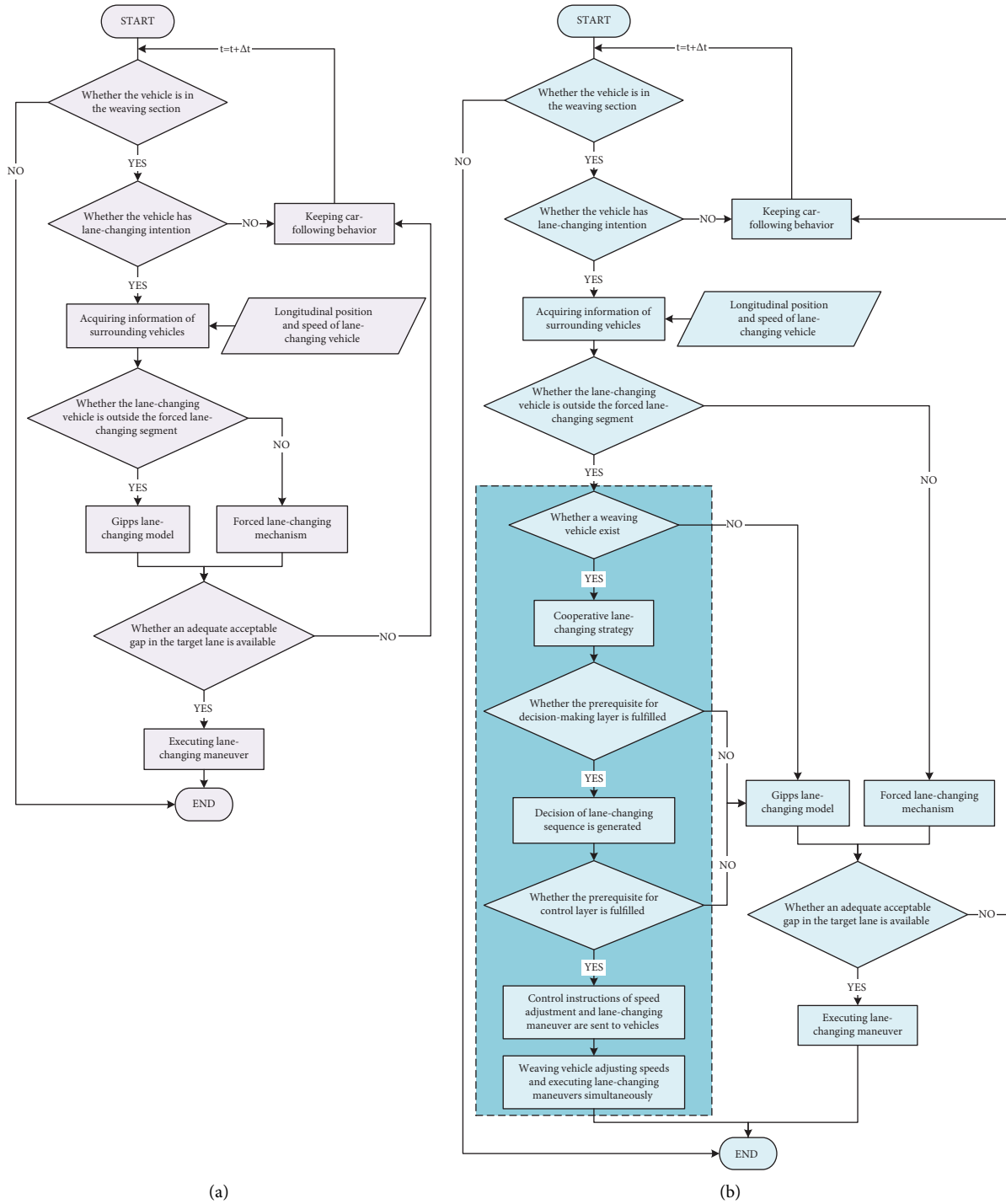


FIGURE 5: Flow charts. (a) Base case. (b) Control case.

TABLE 3: MPRs of CAVs in tests for the mixed traffic state.

Test nos.	CAV MPR (%)
1	0
2	20
3	40
4	60
5	80
6	100

### 3.2. Performance Indicators

3.2.1. *Operational Efficiency.* To assess the operational efficiency of the entire weaving section, the average delay in journey time per vehicle for the base case and the control case under six different demand settings is calculated using the following equation. Journey time is defined as the time an individual vehicle spends driving through the weaving section.



TABLE 4: Simulation results of efficiency indicators for the base case and the control case.

Demand setting no.	$T_{j,\text{exp}}$ (s/veh)	Base case		Control case		$\zeta(\%)$
		$\overline{T_j^{\text{base}}}$ (s/veh)	$\overline{\delta_j^{\text{base}}}$ (s/veh)	$\overline{T_j^{\text{control}}}$ (s/veh)	$\overline{\delta_j^{\text{control}}}$ (s/veh)	
1	6.975	7.089	0.114	7.086	0.111	2.45
2	7.971	8.187	0.216	8.181	0.209	3.06
3	9.300	9.712	0.412	9.693	0.393	4.68
4	11.16	11.91	0.752	11.87	0.712	5.42
5	13.95	15.67	1.72	15.45	1.50	12.8
6	18.60	23.01	4.41	21.92	3.32	24.8

$$\overline{\delta_j} = \overline{T_j} - T_{j,\text{exp}}, \quad (12)$$

where  $\overline{\delta_j}$  is the average delay in journey time per vehicle;  $\overline{T_j}$  is the average journey time per vehicle; and  $T_{j,\text{exp}}$  is the expected journey time per vehicle (see equation (13)).

$$T_{j,\text{exp}} = \frac{L + l}{v_{\text{des}}}. \quad (13)$$

Then, the delay improvement ratio (see equation (14)) is used to examine the reduction in average delay in journey time per vehicle of the control case relative to that of the base case under different demand settings. A larger delay improvement ratio indicates more effectiveness of the proposed strategy in improving the overall efficiency of the weaving section.

$$\zeta = \frac{\overline{\delta_j^{\text{base}}} - \overline{\delta_j^{\text{control}}}}{\overline{\delta_j^{\text{base}}}} \times 100\%, \quad (14)$$

where  $\overline{\delta_j^{\text{base}}}$  and  $\overline{\delta_j^{\text{control}}}$  are the average delays in journey time per vehicle for the base case and the control case, respectively.

**3.2.2. Traffic Safety.** The traffic safety in the weaving section is evaluated by the inverse time-to-collision [19] (see equation (15)), which indicates the risk of collision between the weaving vehicle and its current leading vehicle. When  $\text{TTC}^{-1}$  is equal to 0, there is no collision risk. When  $\text{TTC}^{-1}$  is greater than 0, the risk of collision increases as  $\text{TTC}^{-1}$  increases. Since  $\text{TTC}^{-1}$  of the individual vehicle may vary as the vehicle drives downstream in the weaving section, the maximum value of  $\text{TTC}^{-1}$  of each vehicle within the time period that it spends driving in the weaving section is selected and used for the safety evaluation.

$$\text{TTC}^{-1} = \max\left(\frac{v_{SV} - v_{LV}}{x_{LV} - x_{SV}}, 0\right), \quad (15)$$

where  $\text{TTC}^{-1}$  is the inverse time-to-collision;  $v_{SV}$  and  $x_{SV}$  are the speed and position of the weaving vehicle; and  $v_{LV}$  and  $x_{LV}$  are the speed and position of the leading vehicle.

**3.2.3. Passenger's Comfort.** As seen in Table 1, the comfortable decelerating rate is set to be  $-3 \text{ m/s}^2$ , which means that a large decelerating rate surpassing  $-3 \text{ m/s}^2$  will make passenger uncomfortable. In this case, passenger's comfort can be evaluated by the ratio of large deceleration time (see

equation (16)), which is defined as the total large deceleration time of an individual vehicle divided by its actual journey time. A smaller ratio of large deceleration time indicates a higher degree of passenger's comfort.

$$\varphi_{\text{LDT}} = \frac{T_{LD}}{T_j}, \quad (16)$$

where  $\varphi_{\text{LDT}}$  is the ratio of large deceleration time of the individual vehicle;  $T_{LD}$  is the total large deceleration time of the individual vehicle; and  $T_j$  is the actual journey time of the individual vehicle.

## 4. Analysis and Discussion of Simulation Results

The simulation results of performance indicators of operational efficiency for the base case and the control case are listed in Table 4, and the average delays in journey time per vehicle for the base case and the control case, respectively, as well as the delay improvement ratios, under six different demand settings are plotted in Figure 6. It can be seen that compared with those in the base case, the average delays in journey time per vehicle in the control case are reduced under all six demand settings. Meanwhile, the delay improvement ratios increase as overall traffic flow rates in the weaving section increase from the free-flow state (demand setting #1) to the congested state (demand setting #6). This means that the application of the proposed cooperative lane-changing strategy can alleviate the delay in journey time and improve the traffic efficiency in the weaving section in general. Also, the effectiveness of the proposed strategy in efficiency improvement becomes greater as the traffic flow gets more congested.

In addition, as shown in Figure 6, the incremental trend of the delay improvement ratio is nonlinear. At low flow rates near the free flow, the delay improvement ratios stay low (under 5%) and increase slowly, whereas at high flow rates near congestion, the increments of delay improvement ratios are tremendous (24.8% at congestion), which means that the proposed strategy works much better at the high flow rate than the low flow rate.

Figure 7 shows percentages of vehicles that make forced lane changes, against all lane-changing vehicles in the weaving section, under different demand settings, for the base case and the control case, respectively. It can be seen that compared with those in the base case, percentages of forced lane-changing vehicles in the control case are reduced

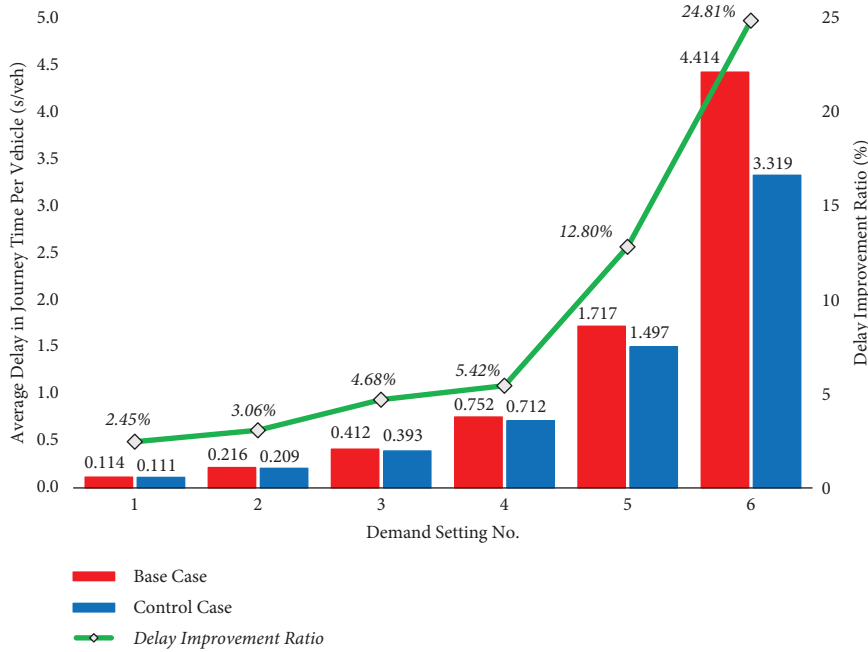


FIGURE 6: Performance indicator of operational efficiency for the base case and the control case.

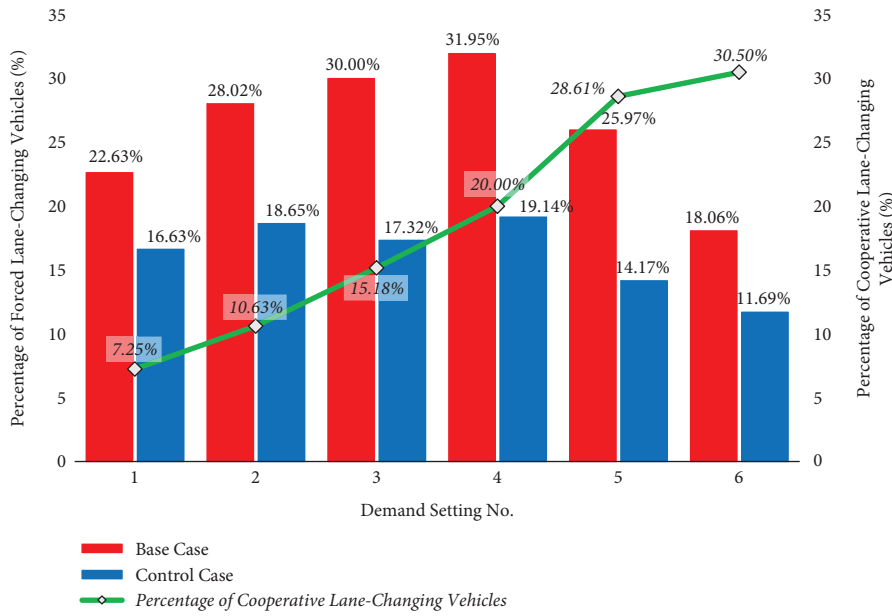


FIGURE 7: Percentage of forced lane-changing vehicles and cooperative lane-changing vehicles.

TABLE 5: Simulation results of benchmark tests with different CAV MPRs under demand setting #6.

Test nos.	CAV MPR (%)	$T_{j,exp}$ (s/veh)	Operational efficiency			Traffic safety		Comfort $\phi_{LDT}$ (%)
			$\bar{T}_j$ (s/veh)	$\bar{\delta}_j$ (s/veh)	$\zeta$ (%)	$\overline{TTC^{-1}}$ ( $s^{-1}/veh$ )	$\overline{TTC}$ (s/veh)	
1	0	18.6	23.01	4.414	0	0.153	6.52	1.81
2	20	18.6	22.62	4.022	8.88	0.150	6.66	1.79
3	40	18.6	22.57	3.974	9.96	0.146	6.86	1.78
4	60	18.6	22.45	3.849	12.8	0.145	6.88	1.73
5	80	18.6	22.35	3.747	15.1	0.144	6.96	1.60
6	100	18.6	21.92	3.319	24.8	0.140	7.12	1.55

TABLE 6: Simulation results of comparison tests with different CAV MPRs under demand setting #6.

Test nos.	CAV MPR (%)	$T_{j,exp}$ (s/veh)	Operational efficiency			Traffic safety		Comfort $\phi_{LDT}$ (%)
			$\bar{T}_j$ (s/veh)	$\bar{\delta}_j$ (s/veh)	$\zeta$ (%)	$\overline{TTC^{-1}}$ (s <sup>-1</sup> /veh)	$\overline{TTC}$ (s/veh)	
1	0	18.6	23.01	4.414	0	0.153	6.52	1.81
2	20	18.6	22.79	4.392	0.50	0.153	6.54	1.78
3	40	18.6	22.97	4.373	0.93	0.155	6.45	1.75
4	60	18.6	22.75	4.149	5.99	0.155	6.46	1.69
5	80	18.6	22.36	3.763	14.7	0.158	6.34	1.62
6	100	18.6	22.32	3.720	15.7	0.160	6.25	1.47

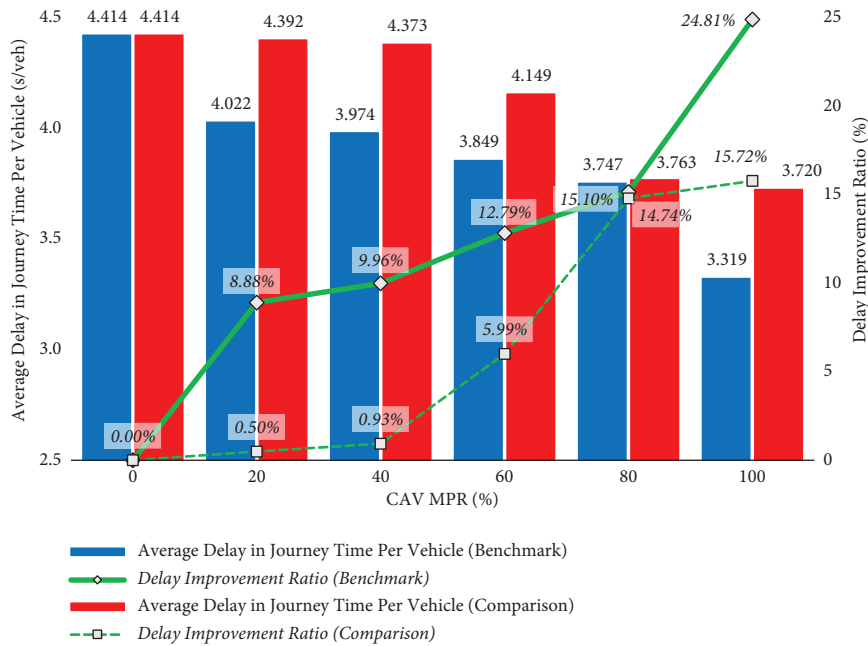


FIGURE 8: Performance indicator of operational efficiency of benchmark and comparison tests with different CAV MPRs under demand setting #6.

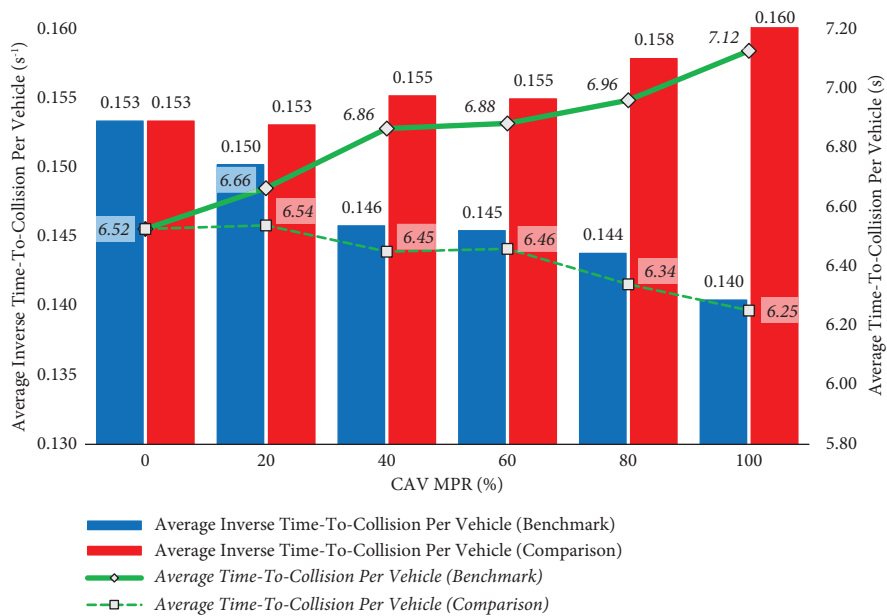


FIGURE 9: Performance indicator of traffic safety of benchmark and comparison tests with different CAV MPRs under demand setting #6.

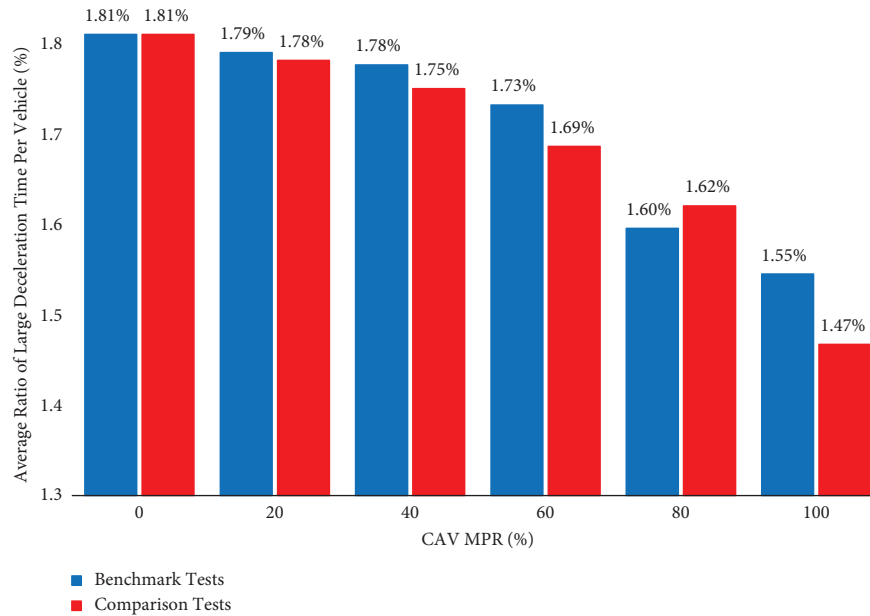


FIGURE 10: Performance indicator of passenger's comfort of benchmark and comparison tests with different CAV MPRs under demand setting #6.

under all six demand settings. This means that the application of the proposed cooperative lane-changing strategy can effectively prevent vehicles from making undesirable forced lane changes, as to enhance the driving smoothness and passenger's comfort within the weaving section. Meantime for the control case, it is also plotted in Figure 7 about the trend line of percentages of vehicles, which execute lane-changing behaviours utilizing the proposed cooperative lane-changing strategy, against all lane-changing vehicles in the weaving section, under different demand settings. It can be seen that the percentages of cooperative lane-changing vehicles increase as the traffic flow rate increases, which means that more lane-changing vehicles prefer to adopt the proposed strategy as the traffic flow gets more congested. This also demonstrates why the proposed strategy is more effective near the congested state. Therefore, the proposed cooperative lane-changing strategy can be used as a good tool for the traffic flow optimization of weaving sections under congestion.

Next, Tables 5 and 6 exhibit the simulation results of three categories of performance indicators of benchmark tests and comparison tests, respectively, for the mixed traffic state, under demand setting #6. Besides, the simulation results regarding performance indicators of operational efficiency, traffic safety, and passenger's comfort for benchmark tests and comparison tests with different CAV MPRs are plotted in Figures 8–10, respectively.

First, it can be seen in Figure 8 that for both the benchmark and comparison tests, the average delay in journey time per vehicle gradually decreases as the CAV MPR increases, which leads to the gradual increase of the delay improvement ratio. However, it is noticed that under any CAV MPR, the average delay in journey time per vehicle of the benchmark test is smaller than that of the comparison

test and the delay improvement ratio of the benchmark test is higher than that of the comparison test.

Second, it can be seen in Figure 9 that for the benchmark test, the average inverse time-to-collision per vehicle gradually decreases as the CAV MPR increases, which leads to the gradual increase of the average time-to-collision per vehicle. Quite oppositely, for the comparison test, the average inverse time-to-collision per vehicle gradually increases and the average time-to-collision per vehicle gradually decreases, as the CAV MPR increases. So, under any CAV MPR, the average inverse time-to-collision per vehicle of the benchmark test is smaller than that of the comparison test and the average time-to-collision per vehicle of the benchmark test is larger than that of the comparison test.

Third, it can be seen in Figure 10 that for both the benchmark and comparison tests, the average ratio of large deceleration time per vehicle gradually decreases as the CAV MPR increases. In addition, it is noticed that under most CAV MPRs, the average ratio of large deceleration time per vehicle of the benchmark test is slightly higher than that of the comparison test, but the difference is quite insignificant.

The trends of simulation results of three categories of performance indicators illustrate that with the increasing MPR of CAVs, the application of the proposed cooperative lane-changing strategy can better alleviate the delay in journey time of vehicles, better reduce the collision risk of vehicles by increasing their collision time, and better reduce the average percentage of time, during which each vehicle spends undergoing large deceleration of its entire journey time. Therefore, for the mixed traffic state, the proposed cooperative lane-changing strategy shows decent potential in improving the operational efficiency, traffic safety, and passenger's comfort in the weaving section. Besides,

compared with Xue's method, the proposed lane-changing strategy shows considerable superiority on maintaining operational efficiency and traffic safety. However, the performance of the proposed strategy on facilitating passenger's comfort is a bit inferior to Xue's method.

## 5. Conclusion

By adopting the concept of CAV technologies, a cooperative lane-changing strategy at the microscopic level is proposed, which consists of an upper layer of decision making and a lower layer of control, in order to alleviate the lane-changing conflicts between weaving vehicles in the weaving section of urban expressway. The decision-making layer determines the lane-changing sequences of weaving vehicles based on their lane-changing advantages quantified by a set of utility functions. The control layer generates detailed instructions of speed adjustments and lane-changing manoeuvres for weaving vehicles.

A numerical simulation, which includes a base case and a control case, is conducted to verify the effectiveness of the proposed strategy under six different traffic demand settings, from the free-flow state to the congested state. In addition, benchmark and comparison tests with six different CAV MPRs are carried out under the congested demand setting to further verify the effectiveness of the proposed strategy for the mixed traffic state and compare its performance with the existing CAV lane-changing method. Three categories of indicators, namely, operational efficiency (delay improvement ratio), traffic safety (inverse time-to-collision), and passenger's comfort (ratio of large deceleration time), are used to examine the performance of the proposed strategy. The simulation results show that the proposed strategy can effectively reduce the delay in journey time of vehicle driving through the weaving section, thus improving the operational efficiency of the weaving section. Moreover, as the traffic flow gets more congested, the percentage of vehicles preferring the cooperative lane-changing behaviours becomes higher, which makes the effectiveness of the proposed strategy in efficiency improvement become greater. The simulation results of the benchmark and comparison tests further show that as the CAV MPRs increase, the proposed cooperative lane-changing strategy can better improve the operational efficiency, traffic safety, and passenger's comfort in the weaving section for mixed traffic states, and its performance on maintaining operational efficiency and traffic safety is superior than the existing CAV lane-changing method.

Overall, it is promising that under the CAV environment in the near future, the proposed cooperative lane-changing strategy can serve as a good method to deal with the problem of lane-changing conflicts within the weaving section, thus enhancing the traffic management and control of urban expressway. For future works, it is suggested to validate the effectiveness of the proposed strategy in a more realistic environment, for which the discretionary lane-changing behaviour should be considered, and vehicles' information collected from the real world can be used for simulation as well.

## Data Availability

The Python data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Acknowledgments

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