Hindawi
Journal of Advanced Transportation
Volume 2023, Article ID 5081016, 18 pages
https://doi.org/10.1155/2023/5081016

Research Article

A Novel Reservation and Allocation Approach of Shared Parking Slots considering the Noncritical Aisle Space

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Received 29 March 2023; Revised 18 May 2023; Accepted 12 June 2023; Published 23 June 2023

Academic Editor: Jaeyoung Lee

Urban areas are experiencing a substantial increase in parking demand, which creates an imbalance due to the limited availability of parking facilities. To address this issue, effective parking management is necessary. Shared parking is an alternative solution that utilizes private plots which are vacant during the day to serve the parking needs of users engaged in nearby activities. While current parking management approaches have achieved high utilization rates for available parking slots, the use of aisle space in parking lots can alleviate oversaturated parking demands, especially during peak periods, for users with relatively short parking durations. This study aims to model the noncritical aisle space with time-space constraints in the shared parking slot allocation problem. We propose a programming model that maximizes the profit of the platform based on a reservation and allocation platform. Numerical experiments demonstrate that utilizing the aisle space can enhance the revenue and enable the platform to accommodate more requests that meet the requirements. Moreover, the presence of aisle parking slots may potentially result in slightly lower turnover rates of normal parking slots.

1. Introduction

Effective parking management is crucial for sustainable development in urban areas. Due to the nonavailability of enough parking resources and high demand for parking, it is inevitable that drivers will cruise in order to find a parking space. This phenomenon of searching for parking spaces generates additional trips, resulting in traffic congestion and air pollution. In a study conducted by Shoup [1] in downtown Los Angeles, cruising for parking during peak periods accounted for 30%–50% of the total traffic in road networks, which consumed 47,000 gallons of gasoline and generated 730 tons of carbon dioxide emissions. Arnott and Rowse [2] investigated the integrated model of parking and congestion in medium-sized US cities and found that cruising for parking accounts for 14% of the cars on the road, resulting in a 50% increase in time loss due to congestion. Furthermore, owing to inadequate supervision, some drivers just park in the aisle for convenience. These uncivilized behaviors require additional concerns from the manager and raise the risks of driving in the parking lot.

There is an urgent need to address these severe problems caused by parking problems. Numerous methods have been proposed to alleviate the disequilibrium between the parking supply and demand. From the perspective of supply and demand, these countermeasures can be categorized into two main kinds. The first type of measures involves using parking facilities as a constraint, which is usually implemented through laws and policies. These measures include prioritizing public transport vehicles and nonmotorized transport modes [3], imposing the "license plate auction" policy to limit the number of private cars [4], levying the road toll on the vehicle [5], and promoting parking expense in urban areas [6, 7].

The other kind aims to adjust the supply to meet the increasing parking demand. This category can be further divided into two subtypes. The first involves expanding the scale of urban parking facilities. However, due to the shortage of available urban land and the much more rapid growth of car ownership, it is not sustainable to construct more facilities simply, particularly in cities like Hong Kong with a high density of traffic, limited road, and parking capacity [8]. In addition, the uncontrolled parking supply encourages car dependency. The second kind, which is widely recognized, emphasizes improving the utilization efficiency of existing parking spaces. As the Internet and communication technology develop by leaps and bounds, more abundant methods help parking management to smooth the demand flow and make high use of existing parking slots through the application of parking guidance systems, parking reservation/allocation systems, etc. As found in a study by Alkheder et al. [9], an intelligent parking reservation and allocation system was proposed and validated for its efficiency and reliability using empirical data.

Similar to other modes of transport demand, the parking demand mode at a particular land use, such as hospital and office buildings, follows a schedule of a high demand during the day and low demand at night. Shared parking, which combines the sharing economy and parking management, has emerged as a new concept to address this issue [10]. It uses unoccupied parking spaces or gaps intended for parking vehicles when owners are not using them. The operation of shared parking is typically based on an e-platform that can be integrated into the fast-growing mobile apps in our daily life. By renting the private residential parking slots to the public, this innovative approach not only enables owners to gain additional revenue from idle resources used to be wasted but also alleviates the shortage of parking spaces.

In this paper, we focus on advanced reservation and allocation of shared parking slots under a platform-based management approach. Suppose that the operators of the e-platform gather information about owners' idle parking spaces, each with an available time window during the day. Requests for parking are also acquired, each with a reserved start and end time. Considering the danger to through-traffic flow caused by the uncivilized parking behavior in the aisle, a novel kind of the parking slot called the "aisle parking slot" (APS) is introduced. To a certain degree, this kind of parking slots makes fuller use of available resources to expand the scale of supply and thence may generate higher profit for relevant operators. However, vehicles in the APS could obstruct the normal parking process of others. Thus, an efficient allocation approach becomes a premise to maintain the order of parking lots. We propose a binary integer linear programming model to allocate parking requests to available parking spaces, intending to maximize revenue under given demand and supply as well as preset the parking fee. With the punishment cost of rejection of customers' requests to the objective function, this model accommodates as many requests as possible under spatial time constraints.

The rest of paper is organized as follows: In the second section, relevant literatures are reviewed. Section 3 gives the problem description, and we formulate a binary integer linear programming model with two cases. Relevant system performance metrics are introduced in Section 4. Section 5 presents numerical results, performs sensitivity analysis of model, and summarizes several management suggestions. Finally, we conclude our work and contributions of this paper with future directions in Section 6.

2. Related Work

For the selection of the relevant literature, we define one criterion of studies that fall outside the scope of this paper. Specifically, we have limited our focus to allocation methods under conditions of parking reservation, and therefore, studies pertaining to conventional parking management are not included. The shared parking mechanism, which is a novel and emerging approach to parking management, is considered a specialized form of parking reservation.

As mixed land use becomes increasingly prevalent in urban areas, commercial activities often take place in residential areas during working hours, resulting in a significant demand for parking. This high demand for parking may lead to haphazard parking in the vicinity. Some studies revealed that the parking sharing policy could meet the parking demand by proper use of available resources in residential buildings [11–13]. Regarding the evaluation of shared parking, Abbott and Bigazzi [14] introduced a relatively small number of off-street stalls from selected residential buildings to the residential parking permit program. They found that this program could reduce on-street parking greatly based on a case study of a high-density residential neighborhood in Canada. Zhao et al. [15] assessed the effectiveness of the proposed model and the algorithm based on the empirical data collected by electronic parking toll collections and questionnaire surveys in Beijing, China. The findings reveal that the implementation of shared parking contributes to the reduction of cruising time, vehicle quantity, and emission.

Several scholars focus on shared parking demand forecasting [16, 17] and willingness of different subjects towards the policy of shared parking [18–20]. Yan et al. [21] constructed a hybrid expected utility-regret model to explore the participation behavior. The results showed that the important factors explaining the engagement included sociodemographic characteristics, social influence, government's role, media attention, platform fee, and revenues.

Similar to the factors of engagement, parkers' choice for a parking space is also influenced by multiple factors such as age, income, parking charge, accessibility, searching time, and availability of a guidance system. Therefore, some researchers have studied the parking choice under the shared parking scheme and found significant factors [12, 22, 23]. Based on the analysis of parking choice, Macea et al. [24] presented a reservation-based parking behavioral hybrid choice model for parking demand management policies in urban areas and indicated the significance of latent variables included in the model. Based on the research of parking choice behavior of hospital parkers, Ji et al. [25] proposed a cumulative prospect theory-based shared parking space allocation model (the CPT-SPSA model) to alleviate the parking difficulties.

In terms of matching problem of shared parking, one line of literature is related to mathematic programming approaches with specified objectives. Some scholars have formulated different kinds of models to optimize the allocation of the shared parking space to maximize the profit of the platform [8, 26, 27], increase utilization of parking spaces

[28, 29], and balance utilization of parking lots [30]. As the scenes and requirements are getting more various, many other factors have been put in the objective function. For example, Ji et al. [31] took the minimum total social cost and the minimum total queue time as the management goals, adding factors of walking time and parking fee. Considering multicandidate adjacent parking slots, Xie et al. [32] established a rolling shared parking allocation model to maximize platform revenue and minimize parking users' travel costs. Kim [33] considered both drivers' preferences and revenues of parking lots and developed an effective algorithm to get a stable set of assignments of parking lots. Simulation results show that the proposed approach provides a reliable solution for drivers to find a parking lot. Zhang et al. [34] allocated shared parking spaces with double objectives of improving utilization and reducing walking distance and evaluated the feasibility of the model based on the data in the central district of Harbin in China. In the light of the deviation from social optimum caused by misreported parking information, Zou et al. [35] introduced mechanism design principles to allocating parking slots to heterogeneous demanding drivers in order to elicit truthful information reporting from drivers.

Another line is related to the uncertainties of conditions such as random supplies [36], random demand cancelations [37], and parking unpunctuality [10, 38]. On the basis of the study by Ni and Sun [39], Zhao et al. [40] developed an intelligent parking management system considering the uncertainties of P-users' and O-users' arrival and departure. The results of simulation showed that shared parking revealed great advantages.

To examine the pricing effects on the shared parking demand, Hao et al. [41] proposed a floating charge method for shared parking, and the floating charge method was proved to improve the utilization rate of idle spaces by more than 60%. Inspired by Kong et al. [42], Xiao and Xu [43] proposed recurrent double Vickrey–Clarke–Groves auctions for matching the supply of parking spaces with demand. To deal with demand disturbance, Shao et al. [44] raised an effective multistage Vickrey–Clarke–Groves (MS-VCG) auction mechanism which can achieve allocative efficiency, incentive compatibility, and individual rationality. In 2020, a novel uniform price strategy (UPS) was introduced to set unique transaction prices for winning participants [45]. Wang and Wang [46] designed a flexible reservation mechanism and a pricing strategy, and the parking price was highly dependent on the drivers' maximum relocation distance and maximum waiting time. Wang et al. [47] proposed a MP-DGS (modified proxy Demange–Gale–Sotomayor) mechanism and a combinatorial system for reservable parking facilities, along with a region-based optimal dynamic parking pricing for unreservable parking facilities. To manage the parking demand, Qian and Rajagopal [48] proposed a dynamic pricing scheme in which parking prices are adjusted in real time based on the knowledge of the demand and traveler heterogeneity.

Previous research studies on parking lot allocation have predominantly focused on assigning parking slots within the designated parking areas, neglecting the potential for temporary parking demand in the aisle spaces. While this approach may appear sufficient to accommodate minimal requests with limited spatial and temporal needs, the benefits of incorporating the aisle space into the parking strategy become more apparent when applied to larger-scale parking facilities. In this context, we introduce a novel reservation and allocation approach that accounts for the efficient use of the aisle space, with the aim of maximizing the revenue.

3. Problem Description and Modelling

Given the heterogeneity of parking slots, this paper presents a modelling approach for a typical parking lot in order to study the allocation of parking slots. As illustrated in Figure 1, the abstract parking area consists of 8 normal parking slots (NPSs) and 2 aisle parking slots (APSs), serving as the basis for this study. The APSs are demarcated orange dashed lines to serve as a warning to parkers. It is imperative that parked vehicles in APSs do not obstruct other normal parking behaviors, and this constraint is taken into account in the subsequent modelling context. It should be noted that the condition of setting this special kind of slots is strict but not limited to the cases such as (a) the aisle must not serve as a main artery road such as the approach towards the entrance or exit of the parking lot; (b) there must be no important gates, such as emergency exits or fire escapes, near the aisle; and (c) the width and length must meet relevant technical standards for a normal parking slot, with the remained gap in width being passable for pedestrian once the vehicle is parked in the APS.

We suppose that both parking slot owners and parking customers submit information of available time windows and reserved arrival time and departure time to the e-parking platform, respectively, at least one day earlier. After collecting the information of supply and demand in the background, the operator could allocate parking slots to users based on global information of reservation. The abovementioned process can be modelled into a binary integer programming problem, taking both revenues from the parking fare and the punishment cost from request rejection into account. The platform sends allocation results to reservation users prior to the requested parking service, including whether the request has been satisfied or not, so that customers whose requests are rejected still have time to find other alternative parking slots in advance. Some further assumptions are provided along with the problem description as follows: (a) parking customers whose requests are satisfied should stick to their requested time (or make sure their actual parking duration shorter than the reserved duration), (b) the satisfied reserved duration contains the time vehicles take to move into the right place, and (c) all of the customers clearly know the available duration of the parking slots and will submit rational parking requests or could be rejected by the platform directly.

The notations used in this paper are defined in Table 1, categorized in terms of parameters and variables. Suppose the platform sorts the customers in the ascending order of their arrival time, so the start time customer i registered is not larger than customer $i+1$. We use three parameters to

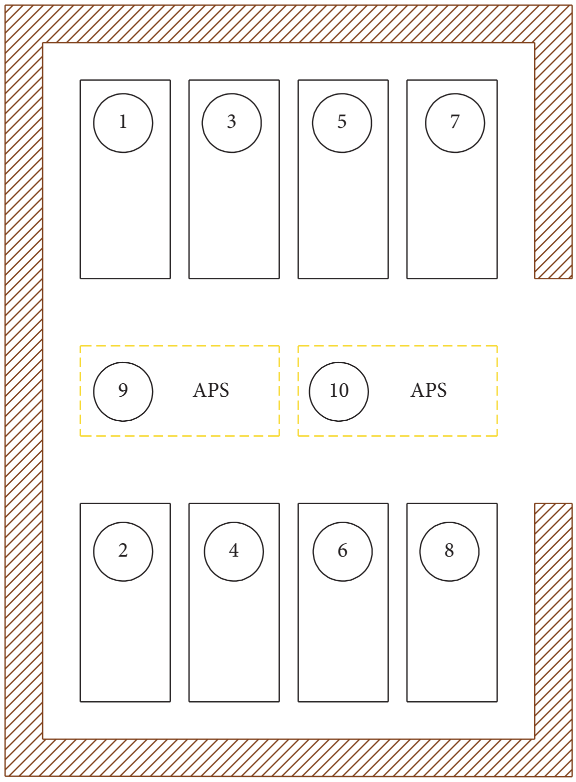


FIGURE 1: Abstract parking lot.

describe the temporal correlation of the demand and spatial correlation of supply. For the supply side, γ_{pq} denotes the interrelation of slots. Unlike normal parking lots where slots are homogeneous and noninterfering, the spatial impact that APSs affect NPSs nearby should not be ignored. It is obvious that the inner APS would affect the inner 4 normal slots and the outer APS would affect all other slots including the inner APS. For the demand side, α_{mn} and β_{mn} are introduced to show the chronological relationship of the arrival and departure time between any two customers.

According to the assumption proposed above in Section 3, the interval of any valid request is not allowed to exceed the available time of each slot. In this case, a binary decision variable x_{ij} is needed to indicate the allocation results, where $x_{ij} = 1$ if request i matches with slot j successfully.

Let α and β denote the unit profit from preset parking charge (the cost rate per hour of parking customer) and the penalty cost of request rejection (the cost rate per customer whose request is rejected), respectively. It is noted that we ignore the purchase cost of parking slots by operators in this paper because the number of rentable parking slots and their available time length are controlled to be identical in different cases. Thus, the purchase cost could be recognized as a constant value which has no impact on the allocation results. Similarly, the relevant cost of operation and infrastructure investment is not considered in the model either.

Then, the reservation and allocation problem based on the e-parking platform can be formulated as the following binary integer linear programming model:

$$\max \sum_{i=1}^{I} \alpha\left(\mathrm{dep}_i - \mathrm{arr}_i\right) \cdot \sum_{j=1}^{J} x_{ij} - \sum_{i=1}^{I} \beta\left(1 - \sum_{j=1}^{J} x_{ij}\right), \tag{1}$$

subject to

$$\sum_{j=1}^{J} x_{ij} \leq 1, \quad \forall i \in I, \tag{2}$$

$$x_{mj} + x_{nj} \leq 1 + M\left(1 - \alpha_{mn}\right), \quad \forall m, n \in I;\ m < n; \forall j \in J, \tag{3}$$

$$x_{mp} + x_{nq} \leq 1 + M\left(3 - \alpha_{mn} - \beta_{mn} - \gamma_{pq} - \gamma_{qp}\right), \quad \forall m, n \in I;\ m < n; \forall p, q \in J;\ p \neq q, \tag{4}$$

$$x_{mp} + x_{nq} \leq 1 + M\left(1 - \alpha_{mn} + \beta_{mn} + 1 - \gamma_{pq}\right), \quad \forall m, n \in I;\ m < n; \forall p, q \in J;\ p \neq q. \tag{5}$$

In this model, the objective function (1) maximizes the profit, which is composed by the revenue from all the customers using the rentable parking slots minus the penalty of rejected ones. In the constraint set, constraint (2) simply means that each request must be allocated to exactly one parking slot; inequality (3) represents that each parking slot can only accommodate one request at a time; constraints (4) and (5) will be discussed in the following.

There are two cases of conflicts in our abstract area when utilizing APSs. The first being examined involves two customers with intersecting registered parking intervals, who are assigned to two parking slots with one of the allocations causing spatial interference. It is noticeable that the cross relation, namely, the intersection, excludes the relation of inclusion. That is to say, one arriving earlier will leave earlier, so is the one arriving later. We suppose one request m is from 8:00 to 11:00, while another one n is from 9:30 to 12:00. If there are two vacant NPSs at that time, these aforementioned two requests will be accepted. However, things might be different in the case of two slots containing one NPS and one APS. From Figure 2(a), it is obvious that temporal and spatial conflicts are unavoidable once both these two requests are accepted. If request m is allocated to the APS while request n is allocated to the inner NPS, the

Table 1: Notations for the parameters and variables.

Notations	Definitions
Parameters	
I	Number of consumers′ demands, indexed by i, $i \in \{1, 2, \ldots, I\}$
J	Number of parking slots, indexed by j, $j \in \{1, 2, \ldots, J\}$
α	Coefficient of the park ing revenue minus rent per shared parking slot per unit time slot, $\alpha > 0$
β	Coefficient of the penalty cost of request rejection due to allocation failure per shared parking slot, $\beta > 0$
arr_i	The registered arrival time of consumer i, $\mathrm{arr}_i < \mathrm{arr}_{i+1}$
dep_i	The registered departure time of consumer i
S_j	The registered start time of available interval of slot j
E_j	The registered end time of available interval of slot j
γ_{pq}	Supply spatial correlation input, binary indicator, to be 1 when parking slot p affects the normal parking in slot q and 0 otherwise
α_{mn}	Demand temporal correlation input, $\alpha_{mn} \in \{0, 1\}$; $\alpha_{mn} = 1$ denotes the departure time of consumer m is more than the arrival time of consumer n and 0 otherwise
β_{mn}	Demand temporal correlation input, $\beta_{mn} \in \{0, 1\}$; $\beta_{mn} = 1$ denotes the departure time of consumer m is less than the arrival time of consumer n and 0 otherwise
M	A sufficiently large positive number
Variables	
x_{ij}	Decision variable, $x_{ij} \in \{0, 1\}$; $x_{ij} = 1$ denotes allocation consumer i to parking slot j, $x_{ij} = 0$ for not allocating

parking route of vehicle n will be obstructed by the vehicle m at 9:30; suppose the operator puts the vehicle m on the inner slot and puts the vehicle n in the APS, then the vehicle cannot leave the parking slot at 11:00. Whichever conflict mentioned above would lead to disastrous disorder to the parking lot and the allocation system. Therefore, only one request could be accepted in this case, and the corresponding condition can be formulated as constraint (4).

The second case is that two customers with overlapping registered parking intervals are assigned to two parking slots, with one of the assignments causing spatial interference. Different from the first case, one situation in this case could be a successful allocation. We suppose one request m is from 8:00 to 12:30, while another one n is from 9:30 to 12:00. We can call that the interval of request m contains that of n. As it can be seen from Figure 2(b), the vehicle n would not reach the given slot at 9:30 if request m was allocated to the APS, while request n was allocated to the inner normal parking slot. However, we suppose the operator puts the vehicle m on the inner slot and puts the vehicle n in the APS, which was able to avoid conflicts. The vehicle m could be successful to arrive at the inner slot because the ASP which may impact m is vacant at 8:00. In the same way, the departure route of the vehicle m will be unobstructed at 12:30, when the vehicle n was already left. Hence, the conflict of the first situation mentioned above in this case can be formulated as constraint (5).

We further notice that both x_{mp} and x_{np} are binary variables, so the left side of inequity (4) or (5) cannot be beyond 2. Thus, big M can be replaced by 1, and inequities (4) and (5) can be transformed to following constraints (6) and (7), respectively:

$$x_{mp} + x_{nq} \le 4 - \alpha_{mn} - \beta_{mn} - \gamma_{pq} - \gamma_{qp}, \quad \forall m, n \in I;\ m < n; \forall p, q \in J;\ p \ne q, \tag{6}$$

$$x_{mq} + x_{np} \le 3 - \alpha_{mn} + \beta_{mn} - \gamma_{pq}, \quad \forall m, n \in I;\ m < n; \forall p, q \in J;\ p \ne q. \tag{7}$$

In the general case of shared parking, the slots vary in rentable periods. As mentioned above, all of the accepted requests must be within the range of the relevant interval. S_j and E_j are introduced to denote the start time and end time of the renting slot j, respectively. Then, the added constraints are presented as follows:

$$S_j \le \mathrm{arr}_i + M\left(1 - x_{ij}\right), \quad \forall i \in I; \forall j \in J, \tag{8}$$

$$E_j \ge \mathrm{dep}_i - M\left(1 - x_{ij}\right), \quad \forall i \in I; \forall j \in J. \tag{9}$$

The final model based on optimal allocation is given as follows:

$$\max \sum_{i=1}^{I} \alpha\left(\mathrm{dep}_i - \mathrm{arr}_i\right) \cdot \sum_{j=1}^{J} x_{ij} - \sum_{i=1}^{I} \beta\left(1 - \sum_{j=1}^{J} x_{ij}\right), \tag{10}$$

subject to constraints (2), (3), and (6)–(9).

In order to make comparison with optical allocation (OA), we also introduce the strategy of allocation based on first-come-first-serve (FCFS). Similarly, customers need to submit their requests to the e-platform in advance. Once the operator has confirmed receipts of all parking requests, he/she will assess whether there are parking spaces available to accommodate the requests in the order of reserved parking time. If a parking space is available, it will be allocated on a priority basis; otherwise, the request will be rejected. If a parking request is accepted, the P-user is expected to arrive on time, pay the parking fee, and leave the parking space as agreed. Otherwise, the customer whose request is rejected is responsible for finding a suitable parking space by cruising around.

For consistence of comparison, we suppose that the number of parking slots and their spatial location relations are the same (Figure 1), so is the request input. In this context, the strategy of FCFS can be divided into 2 solutions, including FCFS-NAA and FCFS-NANA. These two strategies are named based on their own priority levels, with "NAA" and "NANA" representing the order of focus ("N" denotes normal parking slots, and "A" denotes aisle parking slots). For a parking request, procedures of determining whether to accept it are illustrated in Figures 3 and 4. As mentioned above, the prerequisite of parking in APSs is that the departure time of the customer must be earlier than the earliest departure time of customers who might be impacted by APSs (the inner APS will impact the inner four NPSs, and the outer APS will impact any other parking slots). In terms of FCFS-NANA, due to the fact that the outer APS impacts a larger number of users compared to the inner APS, there is no need to focus on the vacant situation of the outer APS if request i cannot be allocated to the inner APS despite its vacancy. In order to determine whether to skip checking the vacancy of outer APS, we differentiate the type of conflicts that arise when the request cannot be allocated to the inner APS, as depicted in Figure 4. Difference between these two solutions lies in the priority of NPSs or APSs. FCFS-NAA gives priority to NPSs, i.e., only when all of the NPS is occupied, vehicles can be allocated to the APS. While in case of FCFS-NANA, all NPSs are cut into two parts by location (inner and outer), and the inner APS is given priority compared with the outer four NPSs. Figure 5 shows the priority levels of these two solutions.

4. System Performance Metrics

To evaluate the performance of models, some common metrics are introduced in this section, including profit, rate of time occupancy, rate of turnover, and rejection rate of requests. Illustrated by the objective function (1) of the model based on OA, the profit is generated from parking charge (according to parking durations) minus the penalty cost of customer rejection:

$$E = \sum_{i=1}^{I} \alpha\left(\text{dep}_i - \text{arr}_i\right) \cdot \sum_{j=1}^{J} x_{ij} - \sum_{i=1}^{I} \beta\left(1 - \sum_{j=1}^{J} x_{ij}\right). \quad (11)$$

The rate of time occupancy describes the utilization of time resource and is given after finishing the allocation:

$$T_{\text{occupancy}} = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J}\left(\text{dep}_i - \text{arr}_i\right) \cdot x_{ij}}{\text{total available time}}. \quad (12)$$

The rate of turnover describes the utilization of space resource and is also given after finishing the allocation:

$$S_{\text{turnover}} = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J} \cdot x_{ij}}{\text{available time} * \text{number of slots}}. \quad (13)$$

In case that the available time of parking slots are different, we introduce "average service time" to describe the "available time" in equation (13), which equals to the total available time divided by the number of slots. Thus, the calculation of turnover rate is presented as follows:

$$S_{\text{turnover}^*} = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J} \cdot x_{ij}}{\text{total available time}}. \quad (14)$$

To a certain extent, the rejection rate represents the service level and the degree of potential satisfaction of customers:

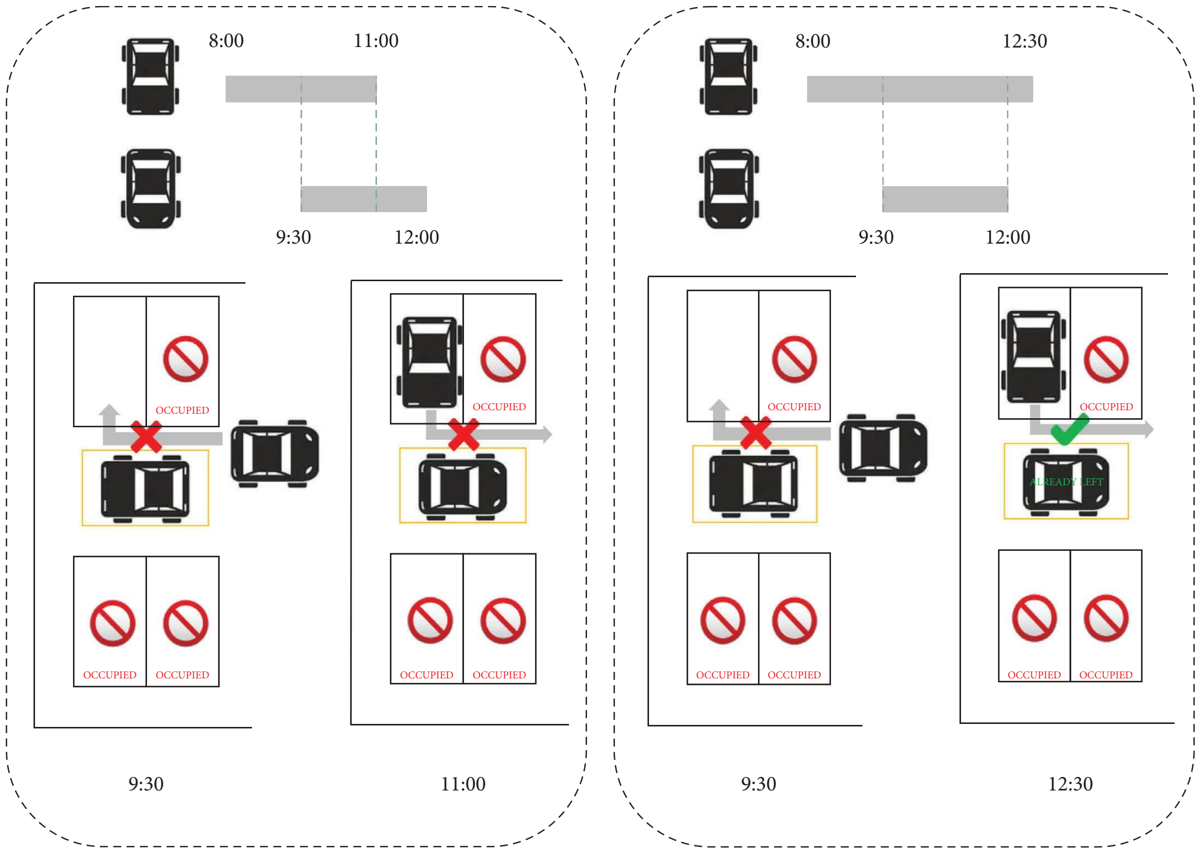
$$\tau_{\text{rejection}} = 1 - \frac{\sum_{i=1}^{I}\sum_{j=1}^{J} \cdot x_{ij}}{\text{number of requests}}. \quad (15)$$

5. Numerical Experiments

In this section, we perform numerical experiments to illustrate the models and results and try to gain some useful insights. The numerical experiments are divided into three parts. We first examine the effects of APSs under three scenes with different utilization levels of the aisle space, by testing how the key performances, including the profit, the rejection rate of requests, the time occupancies, and turnover rates of parking slots, are affected by APSs and the number of requests under these scenes. Finally, we perform an experiment of sensitivity analysis to test how the penalty cost affects the key performance.

We suppose that the platform operates from 9:00 to 17:00 and the available time of each slot ranges from one of the first five intervals (half an hour) to one of the last intervals (half an hour), with the maximum time limit of 8 hours. The available time information for each parking slot is provided in Table 2. The constant values are assumed as follows. The unit parking fare is assumed to be $\alpha = 10$ (CNY/h), and the penalty factor is $\beta = 5$ (CNY). Furthermore, as usually considered in the literature [49–51], the arrival time of parking consumers is Poisson distribution and the parking duration follows a negative exponential distribution in any minute during the whole modelling period. The average parking duration time is assumed to be 3 hours. In addition, Table 3 presents the spatial correlation input. To conduct sensitivity analysis, the number of daily parking requests is varied from 10 to 300 with a step value of 10. Each number is tested with 30 groups, and we select the average value of them.

All experiments are implemented in Gurobi 9.1.2 on an Intel Xeon desktop computer with 32 GB of RAM. All the



(a)

(b)

Figure 2: Two cases of conflicts: (a) two situations in the first case; (b) two situations in the second case.

solution gaps of the numerical results in this paper are less than 4% with the preset time limit of 1200 seconds.

5.1. Effects Evaluation of APSs. To evaluate the effects of APSs, we set three scenes: (i) 8 NPSs, 0 APS; (ii) 8 NPSs, 1 APS; and (iii) 8 NPSs, 2 APSs (Figure 6). In the second scene, one APS is put on the inner place instead of the outer one, where the number of influenced NPSs can be reduced from eight to four.

From Figures 7 and 8, we can see that with an increase in parking demands, the profits initially reach their maximum values in a nonlinear way and then fall down to negative values regardless of setting APSs. In general, the profits of two scenes of setting APSs are both higher than the one without APSs. Correspondingly, the rejection rate of the no-APS scene is always beyond that of others. There is a little difference among the profits among 3 experiment scenes within 20 requests, and the parking platform operator rejects few customers. Then, their maximum profits are simultaneously achieved at an optimal request number of 40, with a value of 306.33, 342.58, and 346.67, respectively. In addition, the difference of the rejection rate is also the largest between scenes with APSs and without APSs. The difference in profits between scenarios with and without APSs gradually widens. When the requests reach 140, the benefits from charge decrease to the same as the penalty cost of rejection in the scene without APSs. However, in two scenes owning APSs, the critical value of requests is increased to 160, which indicates that utilizing the aisle space can meet more demands with the fixed number of NPS to ensure the profitability.

Obviously, the numbers of slots used for parking are not the same under these three scenes, and APSs cannot be treated as NPSs. Therefore, we further focus on the time occupancy and the turnover rate of each slot instead of the whole. Figures 9 and 10 show the relevant metrics of each parking slot in three scenes at the optimal request number of 40. We can see that the time occupancies of eight NPSs in three scenes are similar. However, the turnover rates of the inner four NPSs are significantly lower in parking scenarios with APSs compared to those without APSs. This is attributed to the fact that the NPS in the former case accepts and allocates requests with a longer parking duration to inner parking slots, thereby ensuring sufficient requests to be allocated to the APS. Given a fixed duration of available parking time, an increase in the number of requests for extended parking duration results in a decrease in the turnover rate. Also, for two scenes with APSs, the metrics of the No. 10 slot are clearly different, with the low time occupancy and a high turnover rate, indicating that some requests with very short duration could meet requirements of parking in the No. 10 slot. The turnover rate of the No. 9

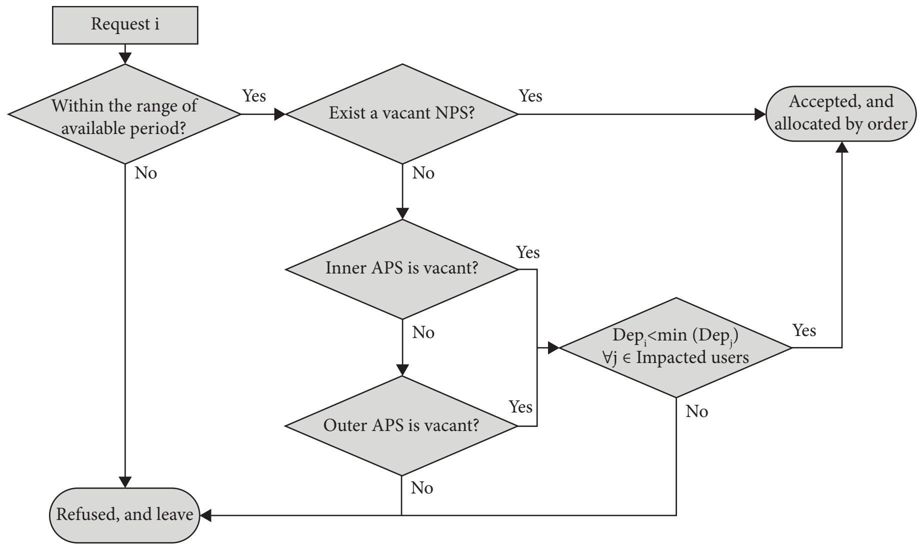


Figure 3: Flowchart of FCFS-NAA.

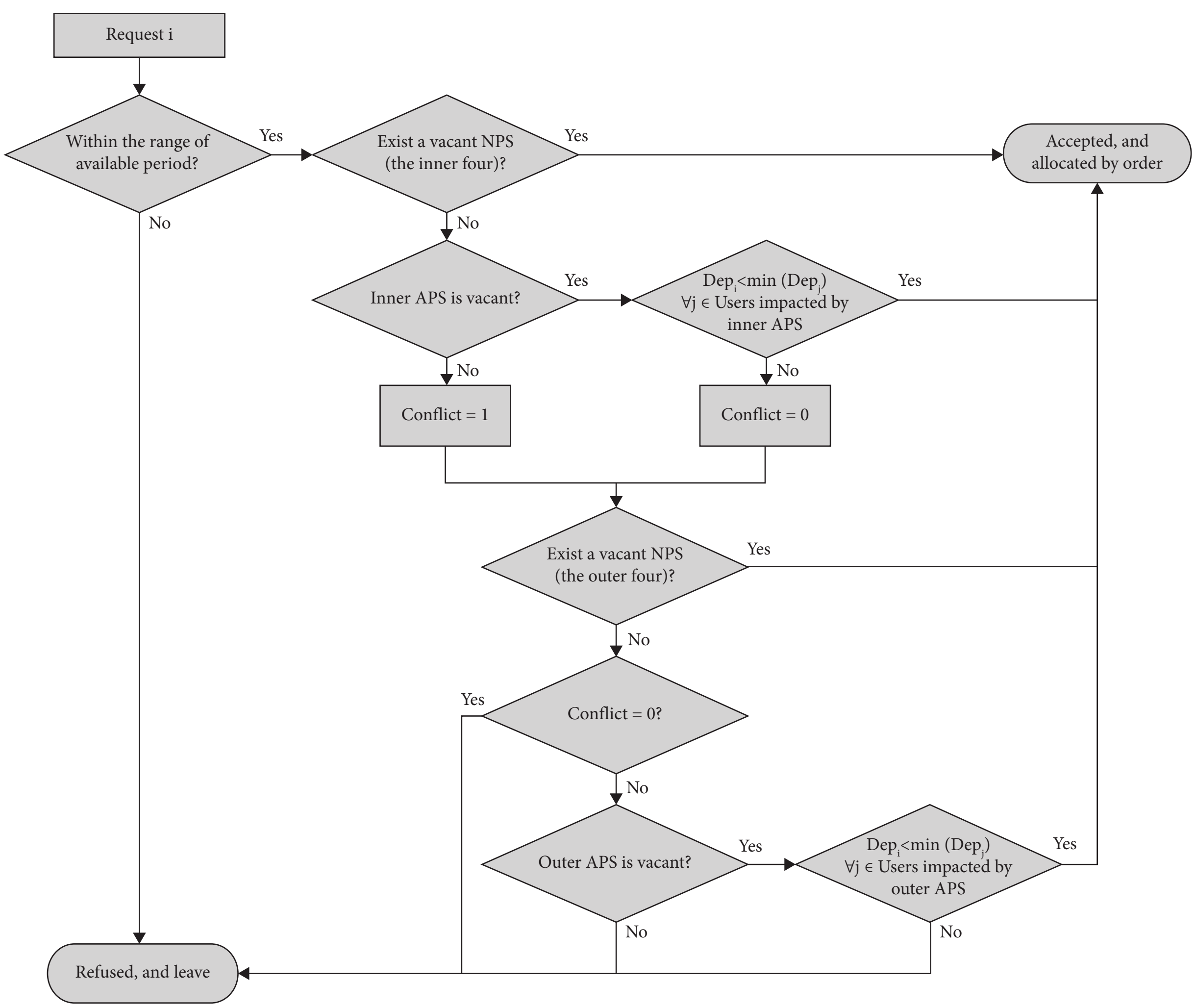


Figure 4: Flowchart of FCFS-NANA.

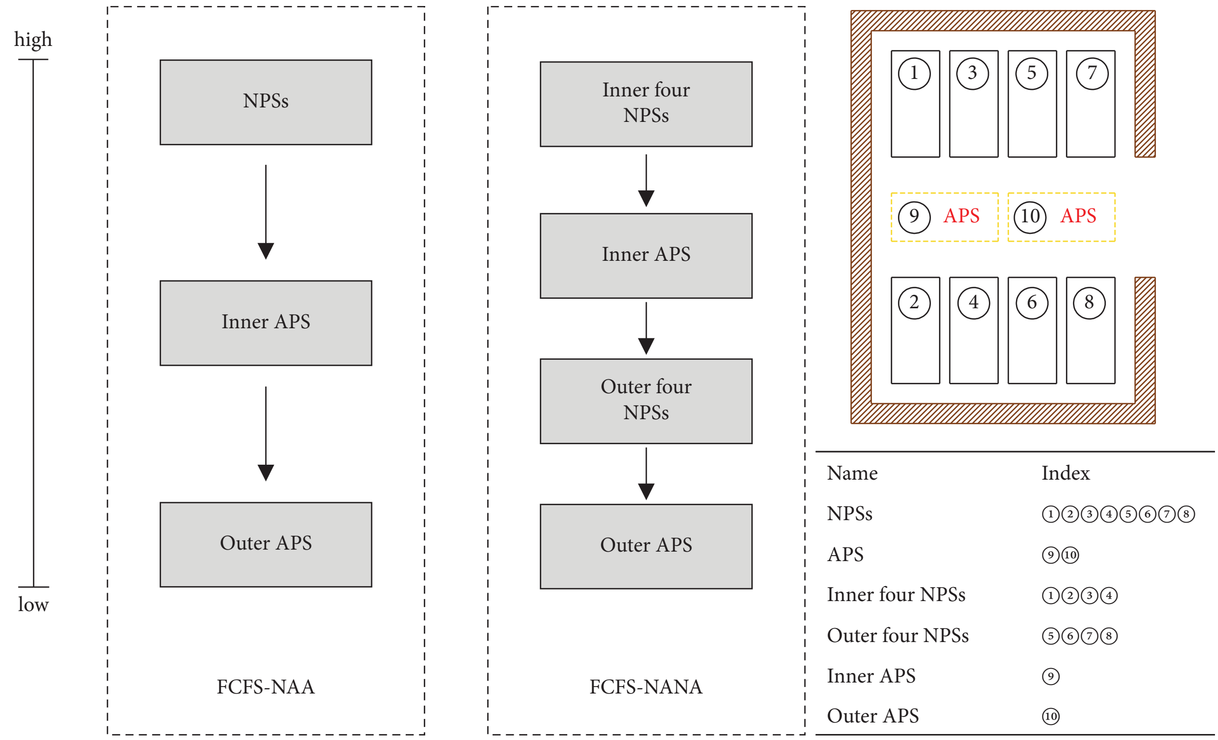


Figure 5: Priority levels of two solutions based on FCFS.

slot is lower in the two-APS scene than that in the one-APS scene, but both time occupancies are close. Similar to the analysis of the inner four APSs, the operator of the e-platform allocates longer-duration requests to the No. 9 slot in the two-APS scene to expand the feasible time window for the No. 10 slot to park as far as possible.

It is clear that the eight NPSs are the same under these three scenes, so we regard the NPSs as a whole and analyze the time occupancy and the turnover rate of the whole with the number of requests. In Figure 11, we can see that there is a little difference in time occupancy among three scenes, indicating that the existence of APSs would not cut down the utilization of NPSs in the temporal dimension. The time occupancies under three scenes increase with the growth of number of requests and remain nearly constant at a value of 0.86. As shown in Figure 12, the turnover rate of NPSs is negatively correlated with the number of APSs. An increase in spatial constraints may lead to the acceptance of more requests for extended parking durations in NPSs, which can potentially be influenced by APSs, contributing to a decline in the turnover rate of the entire NPS. This finding is consistent with the results presented in Figure 10.

From Figures 13 and 14, more requests are possible to be allocated to the APSs with the expansion of the sample size, and the required parking duration of customers allocated to NPSs would be extended correspondingly, which can explain that the gap of the turnover rate between scenes with APSs or not widen as the number of requests grows in Figure 12. In addition, the time occupancy of the No. 10 slot, which influences all other slots, is around 0.1. This observation provides evidence of the effectiveness of the parking system in accommodating temporary parking needs, even if only a small number of requests can be fulfilled under the most stringent constraints.

5.2. Sensitivity Analysis of the Penalty Cost. We select the scene with one APS to evaluate the effects of the penalty cost. The penalty cost is varied across five levels that are evenly distributed around the base value of 5. To a certain extent, the penalty cost can be considered as a measure of the significance of losing customers to the objective. In Figure 15, the profits with lower penalty costs decrease on a milder slope beyond their individual optimal requests. Also, the high penalty cost contributes to the reduction of the optimal number of requests and the maximum value of profit. With the value of 9, the optimal number is 20 and the highest profit is 301.03, while the results with the penalty cost value of 1 are 70 and 426.89, respectively. From Figures 16–18, we can see that the operator prefers to improve time occupancy, and the loss of rejection becomes less important as the penalty cost declines. With a lower penalty cost, requests with longer duration are more possible to be accepted to attempt to fill up the given available time. In consequence, more requests which are "less important" will be rejected, and the turnover rate falls off. Regardless of the penalty cost, three metrics in Figures 16–18 vary little within the request number of 30. As the number of requests increases, the impact of variable penalty costs on the three metrics is demonstrated to be nonlinear, with differences between adjacent values of the penalty cost narrowing. Specifically, although relatively high penalty costs show little difference in time occupancy and the turnover rate, they

Table 2: Available time information for each plot.

Indices	Types	S_j (start time)	E_j (end time)
1	NPS	9:00	16:30
2	NPS	11:00	15:30
3	NPS	11:00	15:00
4	NPS	9:00	16:30
5	NPS	9:00	16:00
6	NPS	10:30	16:00
7	NPS	11:00	16:30
8	NPS	9:00	16:30
9	APS	9:00	17:00
10	APS	9:00	17:00

Table 3: Spatial interrelation input.

γ_{pq}		q									
		1	2	3	4	5	6	7	8	9*	10*
p	1	—	0	0	0	0	0	0	0	0	0
	2	0	—	0	0	0	0	0	0	0	0
	3	0	0	—	0	0	0	0	0	0	0
	4	0	0	0	—	0	0	0	0	0	0
	5	0	0	0	0	—	0	0	0	0	0
	6	0	0	0	0	0	—	0	0	0	0
	7	0	0	0	0	0	0	—	0	0	0
	8	0	0	0	0	0	0	0	—	0	0
	9*	1	1	1	1	0	0	0	0	—	0
	10*	1	1	1	1	1	1	1	1	1	—

Notes: the notation "*" signifies that the parking slot is an APS.

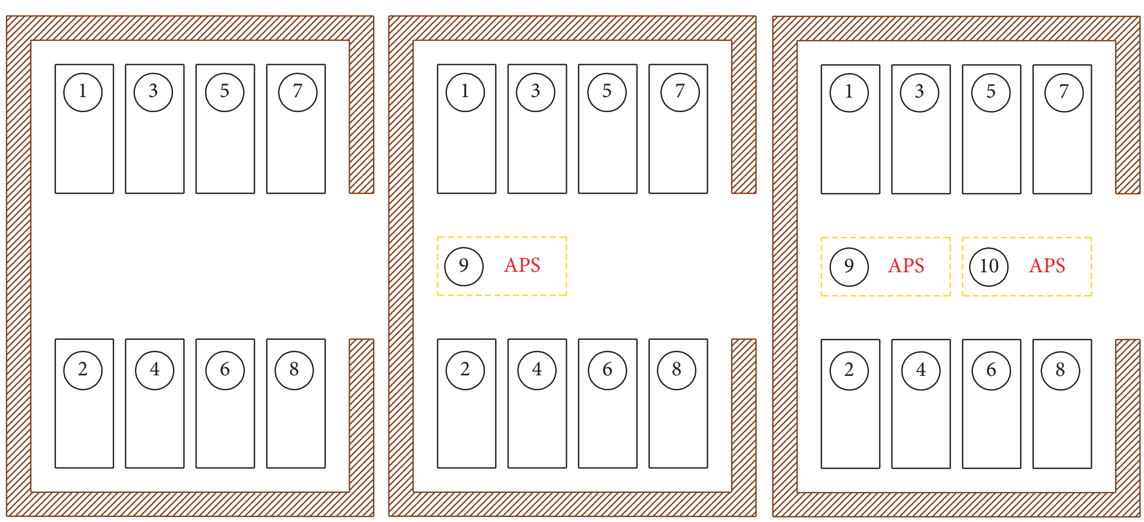


Figure 6: Three experiment scenes.

exhibit significant variations in the optimal request number and profit.

5.3. Comparison between OA and FCFS. We further make a comparison between the optimization-based allocation (OA) with the binary integer linear programming model and two allocation strategies according to the FCFS in Section 3. Because of the different priority levels of APSs in the two strategies, the scene with two APSs is selected to guarantee the completeness of the allocation process. All other parameter values remain unchanged, which are the same as the base setting in the beginning of Section 5.

In Figure 19, it is obvious that the profit under OA is not lower than that under FCFS without optimization, and the advantage of OA is more significant as the number grows. Despite different priorities given to the inner APS in the allocation process through both FCFS-based approaches, the results indicate no noticeable difference. From Figures 20–22, it can be observed that the OA-based

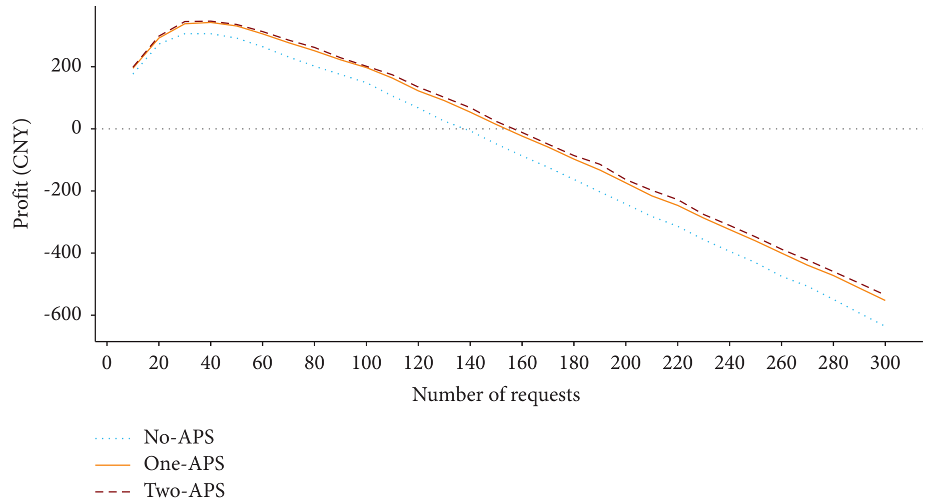


Figure 7: Change in the profit with the number of requests under 3 experiment scenes.

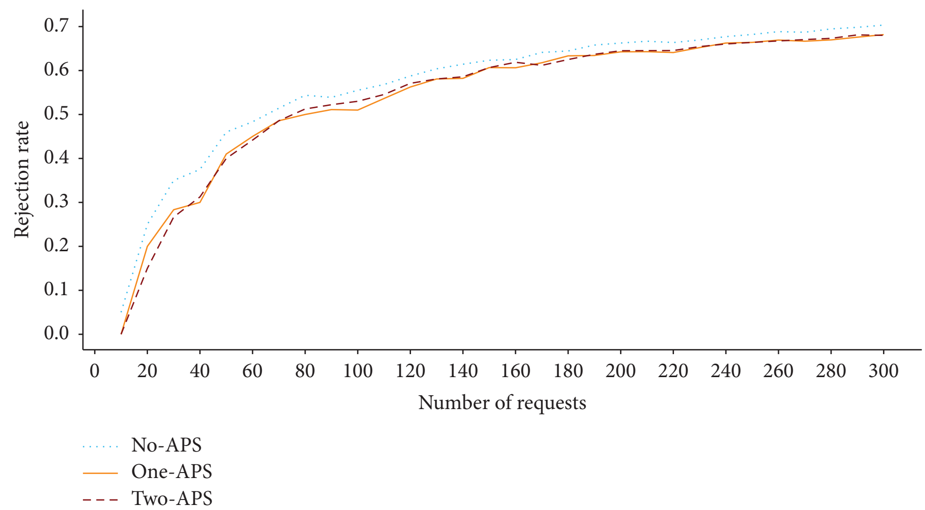


Figure 8: Change in the rejection rate with the number of requests under 3 experiment scenes.

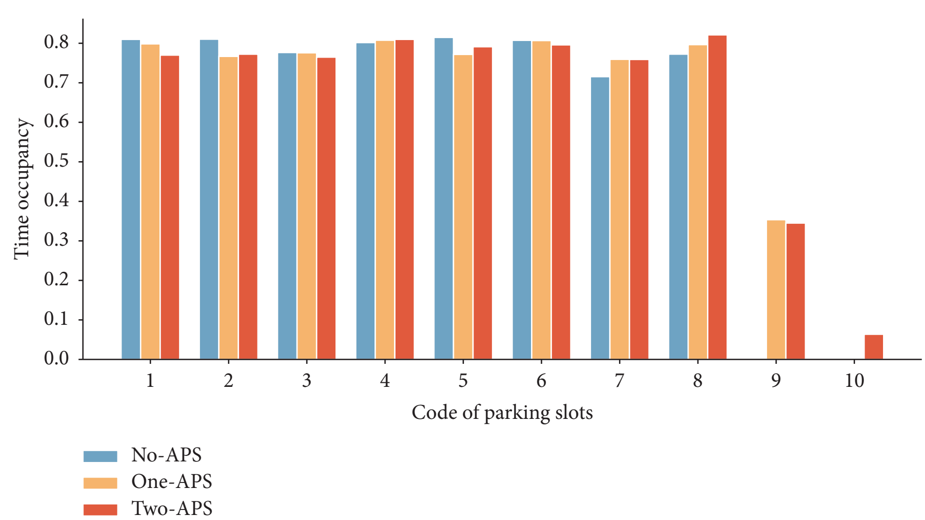


Figure 9: Time occupancy of each parking slot with 40 requests under 3 experiment scenes.

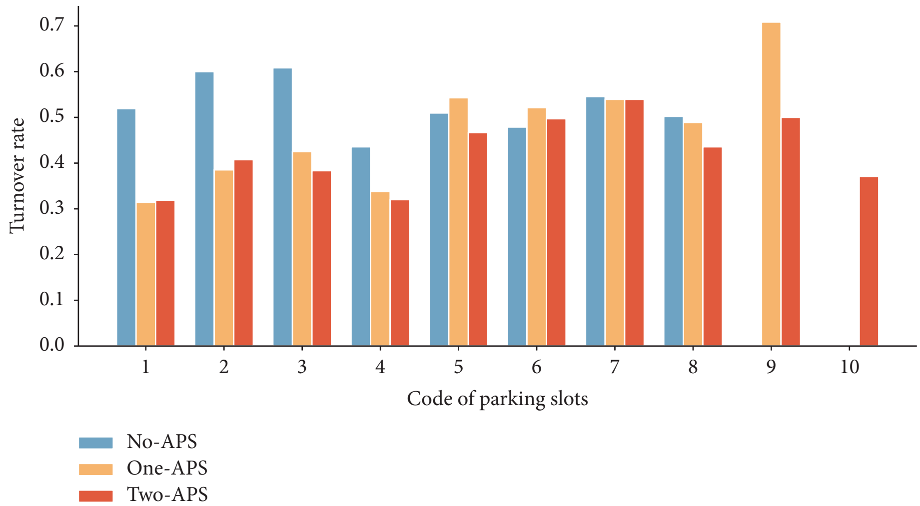


Figure 10: Turnover rate of each parking slot with 40 requests under 3 experiment scenes.

approach leads to fewer rejected requests and a higher turnover rate. Moreover, considering the penalty cost of rejection, blindly increasing the time occupancy may not be the optimal solution since it could result in a loss of profit due to the rejection of requests. The OA-based method prefers to accept more short-term request to reduce negative effects of rejection, as the slower decent curve of profit under OA compared with FCFS.

The results indicate that there is no clear differentiation among the three approaches when the number of parking requests is less than 40. In such scenarios, adopting FCFS-based methods might be more cost-effective without the expenses of implementing a reservation system. As the number of requests increases, employing the OA-based approach would be advantageous, provided that the parking lot is equipped with a reservation system. For traditional parking lots without any reservation system, the FCFS-NAA approach outperforms FCFS-NANA in terms of profit, time occupancy, and turnover rate. Therefore, prior to determining the appropriate approach, it is essential to thoroughly investigate the current and future parking demands, along with considering the expenses of infrastructure construction.

6. Conclusion and Discussion

In this paper, we study the shared parking plot reservation and allocation problem based on the e-parking platform, considering utilizing some aisle space. To address this problem, a binary integer linear programming model is developed to allocate requests to specific parking slots, thereby maximizing the profit of the platform within the constraints of time and space. Numerical experiments are conducted to analyze the effects of APSs and the effectiveness of the optimization-based allocation method compared with the first-come-first-serve method.

The merit of this paper is that we focus on the noncritical aisle space to accept more parking requests and improve the revenue of operators through our allocation method. Our work extends the studies that are limited by merely paying attention to the existing parking slots. In addition, the time constraints in the programming model are continuous time rather than discretizing the daytime into small units as the time slots used in many other literature studies, which are more accurate and personalized.

Through the research, the following important conclusions are derived from the experimental results. First, the introduction of APSs contributes to the enhancement of profits and acceptance of more parking requests. Second, the penalty cost of rejection has a considerable impact on the optimal profit and parking requests. As the penalty cost grows, the disparity of influence narrows in terms of time occupancy, turnover rate, and rejection rate.

Based on these findings, important management insights are summarized as below. Specifically, parking unpunctuality must be taken into consideration as it can harm the robustness of the allocation system, particularly in the case of customers parking in APSs due to their unique location. Thus, two main management methods are recommended. One is to incentivize punctuality or impose penalties for unpunctuality. The other is to maintain the robustness through moving the unpunctual vehicle such as using an AGV (automated guided vehicle). As unmanned vehicles are becoming increasingly common, it is practicable to apply the allocation method to scenes with autonomous vehicles in order to reduce the impact of human unpunctuality. In addition, methods of reducing the penalty cost of rejection, such as release of some coupons, are helpful in improving the revenue and optimal requests.

The future work could be expanded in the following two parts. First, future research can delve into the examination of the impact of unpunctuality on the allocation system in order to develop more robust methods that can handle such challenges. Second, it would be intriguing to explore the effect of dynamic changes in parking charges based on the parking duration and peak/off-peak times, which may further enhance the profitability of the e-parking platform. The potential impact of these extensions on the overall

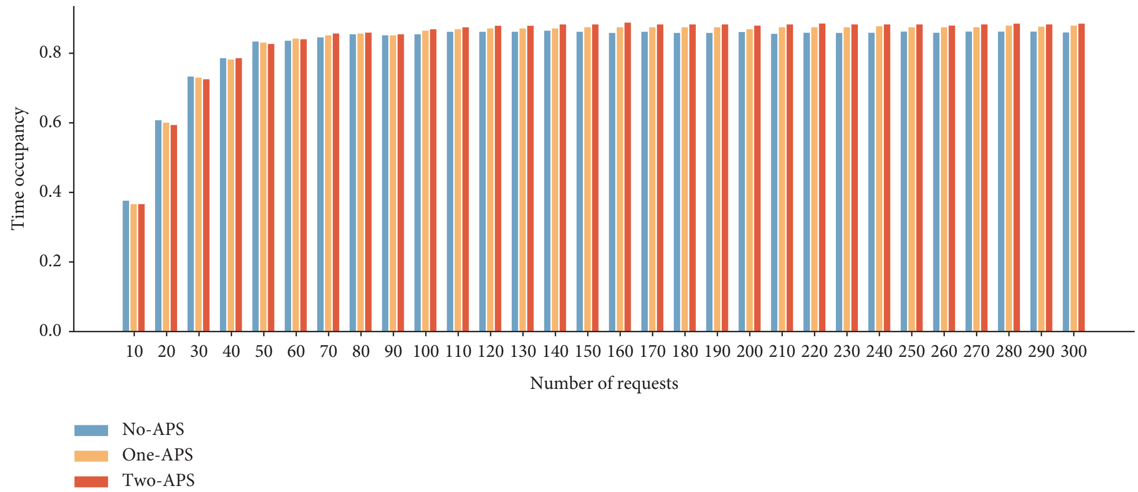


Figure 11: Time occupancy of NPSs with the number of requests under 3 experiment scenes.

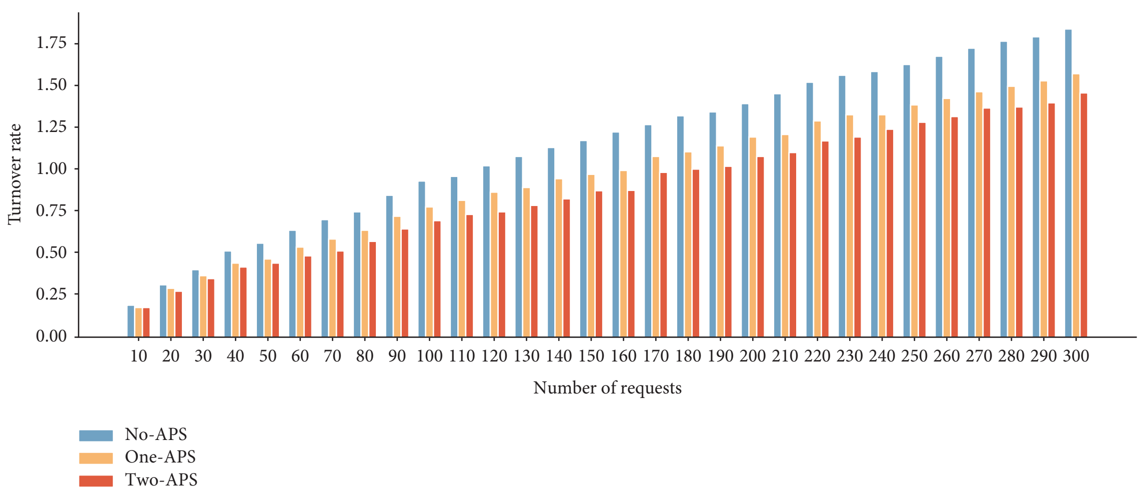


Figure 12: Turnover rate of NPSs with the number of requests under 3 experiment scenes.

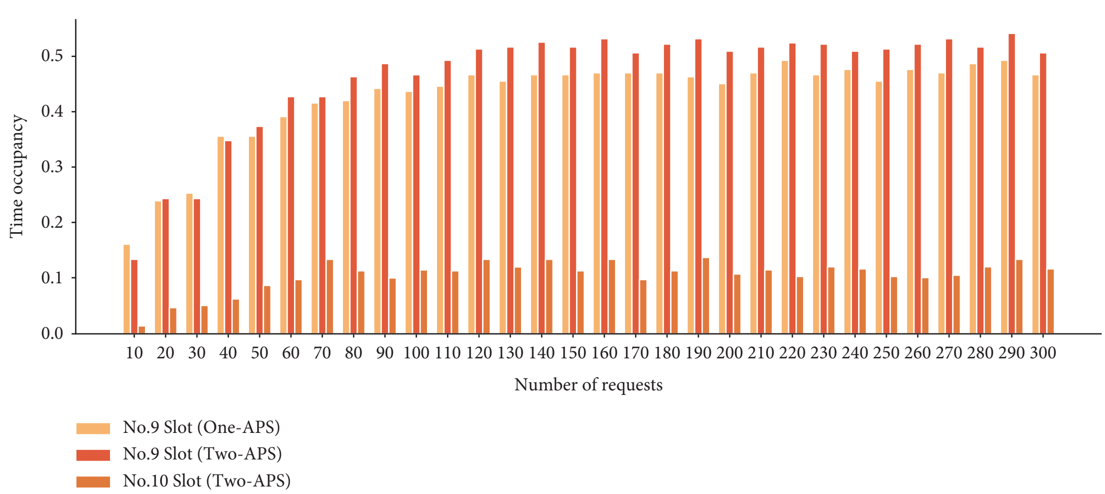


Figure 13: Time occupancy of APSs with the number of requests under scenes with APSs.

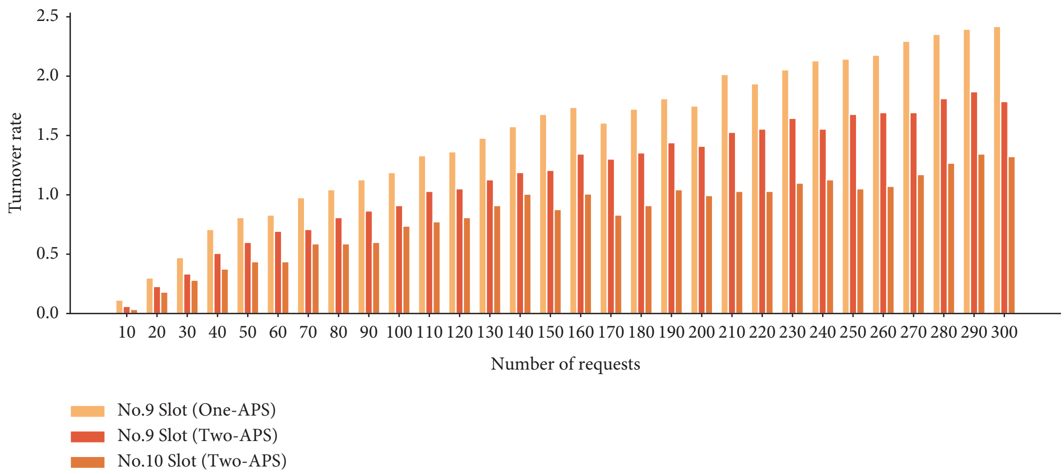


Figure 14: Turnover rate of APSs with the number of requests under scenes with APSs.

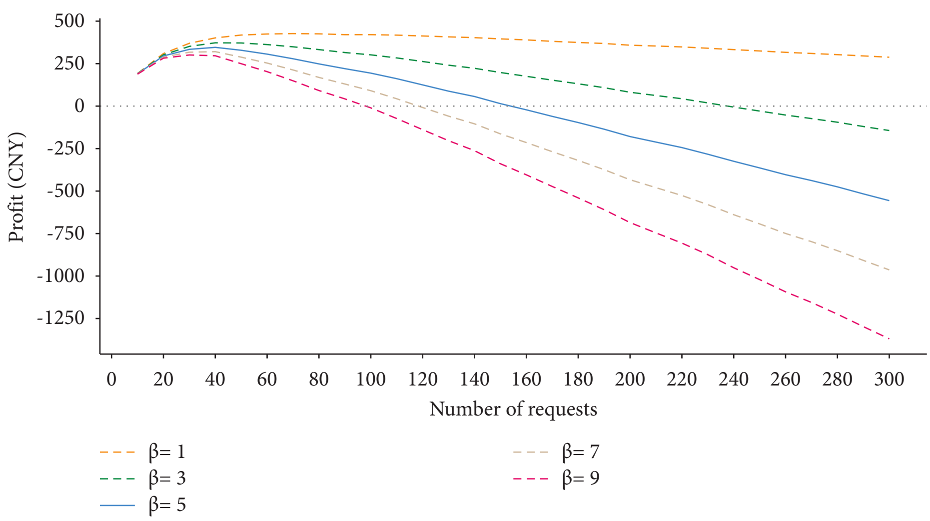


Figure 15: Change in profit with the number of requests under different penalty costs.

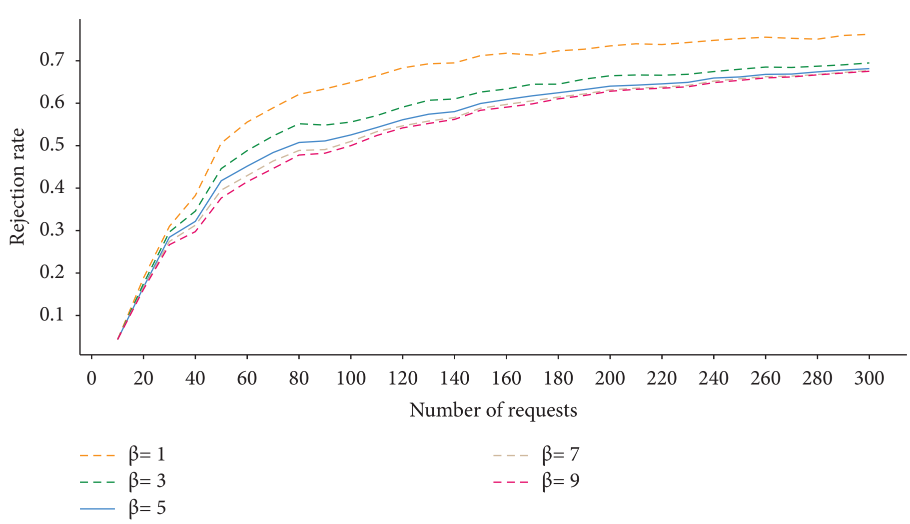


Figure 16: Change in the rejection rate with the number of requests under different penalty costs.

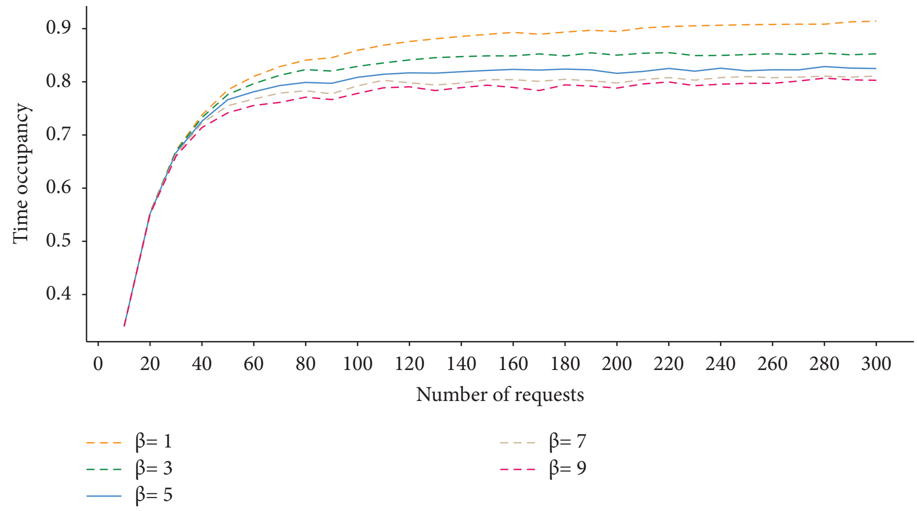


Figure 17: Change in time occupancy with the number of requests under different penalty costs.

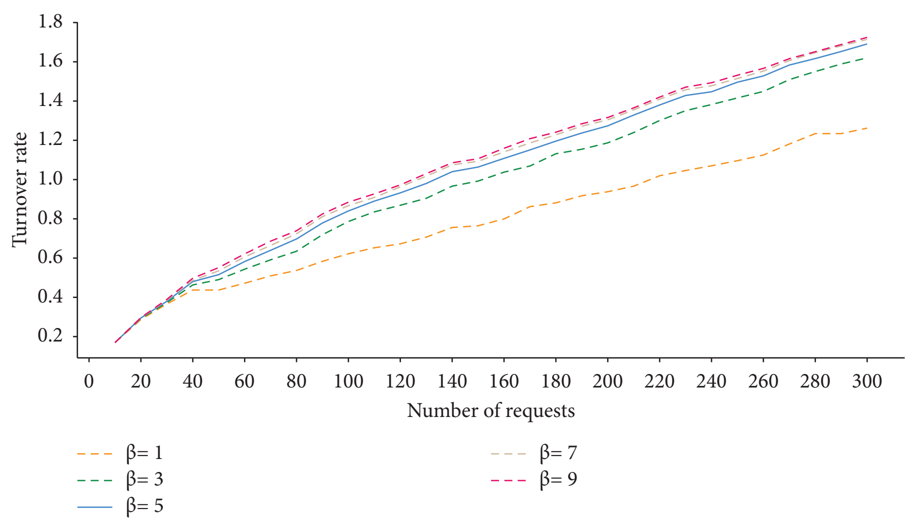


Figure 18: Change in the turnover rate with the number of requests under different penalty costs.

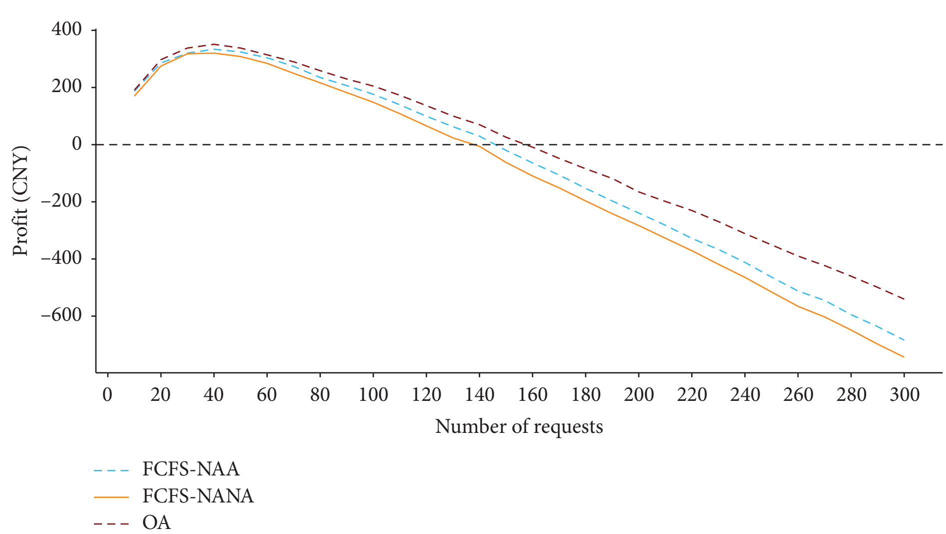


Figure 19: Comparison of profits between OA and FCFS.

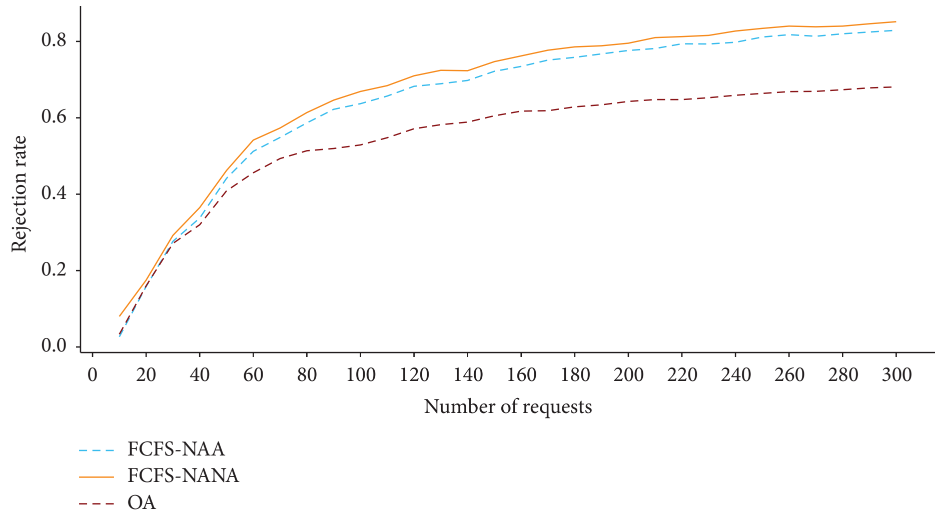


Figure 20: Comparison of rejection rates between OA and FCFS.

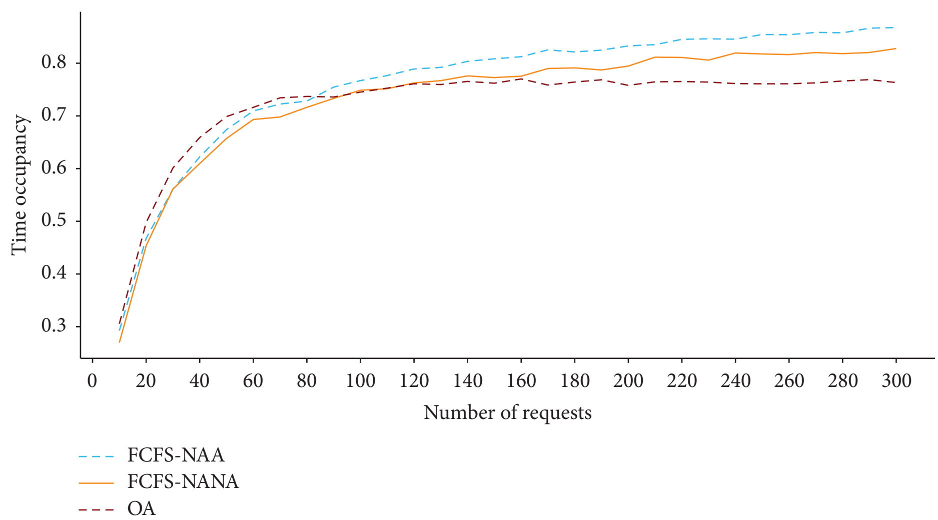


Figure 21: Comparison of time occupancy between OA and FCFS.

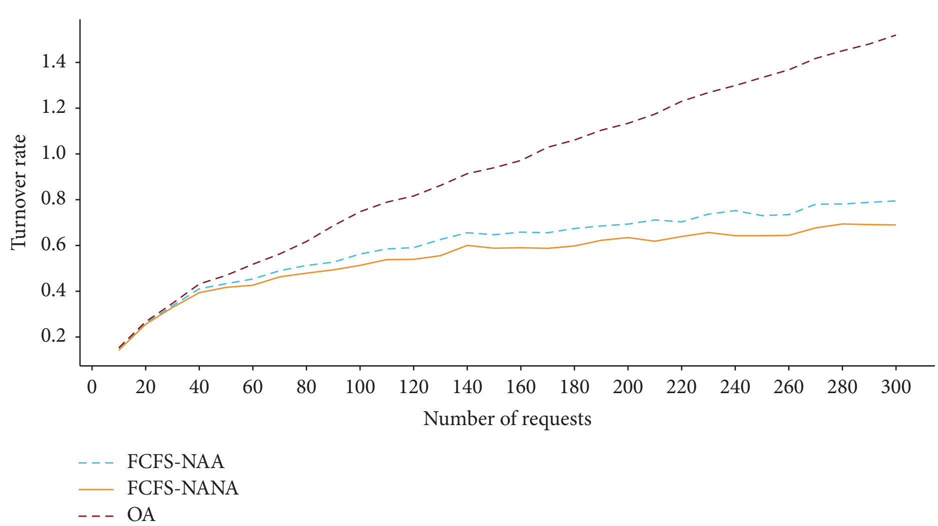


Figure 22: Comparison of turnover rates between OA and FCFS.

performance of the e-parking platform can be evaluated using appropriate experimental methods.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This study was sponsored by Shanghai S&T Development Funding (no. 21DZ1200800) and the Shanghai Collaborative Innovation Research Center for Multi-Network and Multi-Modal Rail Transit.

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