Hindawi
Journal of Advanced Transportation
Volume 2023, Article ID 5230483, 15 pages
https://doi.org/10.1155/2023/5230483

Research Article

Joint Optimal Train Rescheduling and Passenger Flow Control for Speed Limit and High-Demand Scenarios of Urban Rail Transits

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Received 12 August 2022; Revised 12 September 2023; Accepted 20 September 2023; Published 28 September 2023

Academic Editor: Long Truong

In the operation of urban rail transits, train delays occur frequently, and emergency management is one of the key factors to ensure the service quality. For speed limit scenarios with high demand, this paper takes the train running in the restricted manual (RM) mode in the speed limit zone as an example and discusses a coping method by jointing train rescheduling and passenger flow control. With the goal of maximizing the number of passengers served and minimizing total train delay, a nonlinear optimization model is constructed by taking the train operation-related requirements and passenger flow control-related indicators as constraints, and the model is reformulated to a mixed-integer programming (MIP) model with quadratic constraints, which can be solved by the Gurobi solver. In order to obtain effective solutions faster, a two-stage approach is discussed, which first obtains a rescheduling timetable and then dynamically adjusts the requirement of boarding equalization to obtain the passenger flow control scheme. The validity of the model and the solution approach are discussed with the help of numerical experiments. The results suggest that the model and solution approach are feasible. When the number of trains is fixed, the reasonable implementation of passenger flow control will help to increase the number of passengers served, and the pursuit of a higher equalization of boarding is not conducive to the number of passengers served and the full utilization of transport capacity. The two-stage approach has certain advantages over the direct computing method. The methods in this paper have a guiding value for emergency decision-making in similar delay scenarios.

1. Introduction

Urban rail transits (URTs) focus on providing safe and reliable services for citizens to travel. However, a large number of uncertain factors affect its normal operation and induce train delays. Due to the characteristics of the short station spacing, the high departure frequency, the huge number of passengers, and the simple layout of tracks, the tolerance of delays for URTs is small. When the initial delay is minor (i.e., disturbance), small adjustments such as reducing dwell time or running time can be used to enable the punctuality as soon as possible, and the impact on passengers is minor. When the initial delay is large (i.e., disruption) to ensure the safety and service of passengers, not only the train timetable needs to be greatly rescheduled, but also the passenger management may need to be strengthened to relieve the pressure caused by the imbalance of low supply and high demand. Emergency management of URTs is a kind of problem with research value [1–3], and operators of URTs are also very interested in this problem.

Quickly obtaining an effective rescheduling timetable is one of the key factors to ensure the service level during failure periods. For lines with a large passenger flow (i.e., high demand), when the degree of the delay is serious, it is necessary to implement passenger flow control due to the decline of transport capacity [4, 5]. However, the co-optimization of the two (train rescheduling and passenger flow control) has not been fully discussed. At the same time, it is worth noting that the attributes of failure scenarios are very diverse, and there is no unified response strategy. This paper focuses on a kind of train delay scenario in which trains can only pass through the speed limit zone at a limited speed. To the best of our knowledge, the mathematical description of train rescheduling and passenger flow control in the scenario has not been discussed. In this regard, we construct a cooperative optimization model of the two and discuss the corresponding solution approach, so as to provide a reference for the research and practice of similar scenarios.

1.1. Literature Review. The research on train rescheduling and passenger flow control is helpful to ensure the passenger service and operation safety. Related problems have received extensive attention. The study reviews the literature from three aspects, train rescheduling, passenger flow control, and their joint optimization.

For train rescheduling in disturbance scenarios, Yin et al. [6] considered the passenger service and the energy consumption and discussed a stochastic programming problem for train rescheduling. Xu and Chen [7] constructed a stochastic programming model with the goal of minimizing the interval deviation of the train departure and the train arrival. Hou et al. [8] proposed a method that takes into account the train delay, the number of stranded passengers, and the energy consumption. Xia and Hu [9] proposed an optimization method with the goal of minimizing the train delay and the recovery time. Hu et al. [10] proposed an integrated rescheduling strategy for train operation with the goal of minimizing the total travel time of passengers, including ways to extend train dwelling time and skip-stopping. For operation management in disruption scenarios, Wang et al. [11] designed an iterative optimization framework considering the number of train cancellations, trains' delay time, passengers' waiting time, and riding time. Huang et al. [12] discussed an optimization model including the rescheduling strategy of the bidirectional operation and the part route and combined a two-stage approach to carry out the real-time optimization. For the train rescheduling after a disruption, Gao et al. [1] constructed an optimization model considering the train operation efficiency and the number of passengers served by combining the skip-stopping strategy. Yin et al. [13] discussed a rescheduling method considering flexible addition of backup trains and the train operation efficiency. Liao et al. [14] proposed a deep learning-based real-time rescheduling method considering energy saving. Yin et al. [15] established an optimization model aiming at saving energy and passenger waiting time. Xu et al. [16] discussed a train rescheduling model aiming at minimizing the average generalized delay of passengers and solved it via the genetic algorithm (GA). Zhen and Jing [17] considered passenger selection behaviors and constructed an optimization model with the goal of minimizing the negative impact of train delays on passengers. For a single-track fault scenario of dual-track lines combined with the crossover application strategy, Xu et al. [18] established an optimization model with the goal of minimizing the total delay and designed an efficient solution approach based on the discrete events. In addition, Xu et al. [19, 20] and Kang et al. [21] discussed the rescheduling problem of last trains.

Passenger flow control is a consideration for saturated or oversaturated lines. Huang et al. [22] discussed the formulation method of the cooperative passenger flow control strategy. Shi et al. [23] established a multistation coordination control model oriented to safety (i.e., reducing the risk of passenger aggregation). Yin et al. [24] constructed an equilibrium control model for passenger flows with the goal of minimizing the total passenger delay. Li et al. [25] established a cooperative control model with the goal of minimizing the total waiting time of passengers and maximizing the passenger turnover. Zhao et al. [26] suggested a cooperative control model for passenger flow considering different stations and different periods with the goal of minimizing the loss of passenger delays and maximizing the passenger turnover. Cao and Ma [27] proposed a multistation coordination control method based on the calculation of passenger flow in sections. Zhang et al. [28] proposed a flow control model that can dynamically determine the number of passengers boarding each train at each station, and solved it by the dynamic programming approach. For high-frequency lines, Liu et al. [29] constructed a mixed-integer linear programming (MILP) model for the co-optimization of train timetable and passenger flow control and used Lagrangian relaxation methods to quickly obtain quasi-optimal solutions. Aiming at the oversaturated state and undesired risks of stations, Shi et al. [30] constructed an optimization model for the cooperative control of network passenger flow with the goal of minimizing the total waiting time of passengers and the risk of passenger aggregation. At the same time, Shi et al. [31] also proposed a cooperative optimization method for train timetable and passenger flow control aiming at minimizing the passenger waiting time.

Few studies have focused on the cooperative optimization of train rescheduling and passenger flow control under train delay scenarios. As shown in Table 1, Bešinović et al. [4] believed that it is meaningful to consider the two synergistically to cope with the huge demand and propose a management framework for disruption scenarios with the goal of restoring the train operation to the original timetable as soon as possible and minimizing the waiting time of passengers outside stations. Yang et al. [5] proposed a bilevel programming model for the cooperative optimization of train rescheduling and passenger flow control with the minimum total delay as the upper-level objective and the maximum number of passengers served as the lower-level objective. Focusing on the challenge of large passenger flow in interchange stations during peak periods, Li et al. [32] constructed a collaborative optimization model of passenger flow control and train rescheduling with the goal of minimizing the weighted waiting time of passengers outside and inside stations. For large passenger flow scenarios, Hao et al. [33] proposed a management strategy that can regulate the train operation time and control the number of passengers boarding a train with the help of the Markov decision process. Focusing on train delays of the high-demand and high-frequency line, Li et al. [34] suggested a model predictive control-based method by taking the minimum of total delay as objective and the passenger control as constraints. Furthermore, for delays affecting large-scale URT networks, Yuan et al. [35] proposed a mixed-integer nonlinear programming model considering train rescheduling and passenger flow control.

1.2. Main Contributions. At present, in order to ensure the operation order and service quality, more and more studies have paid attention to train rescheduling and passenger flow control, and various models and methods have been put forward. It is worth noting that train delay scenarios are diverse, which leads to differences in mathematical descriptions. However, previous studies focused on typical scenarios, such as initial delays caused by a single-point failure, impassability of a single section, or station due to a disruption. Some atypical scenarios, such as multiple stations or sections that fail to pass properly, are yet to be addressed. Specifically, models or methods in typical scenarios cannot be directly applied to atypical scenarios, and their applicability needs to be further discussed. Different from previous studies, this paper focuses on a scenario not discussed, in which the train can only pass the affected sections (i.e., speed limit zone) at a limited speed, especially the problem of train rescheduling and passenger flow control for the scenario, and gives the mathematical formulations and solution approaches. Specifically, the contributions of this paper are as follows.

(1) Existing research on train rescheduling usually focuses on typical scenarios, while some scenarios have not been discussed. This paper focuses on a kind of scenario in which trains can only pass through the speed limit zone at a limited speed and passenger demands are high. We formulate a complex model with multi-nonlinear items and reconstruct it as a mixed-integer programming problem with only quadratic items. The model considers the safety interval protection of train running under the condition of speed limit, the high passenger demand, and the coupling of passenger flow and train operation.

(2) There is an obvious real-time requirement for operation management under high-demand conditions, that is, to obtain an emergency program as soon as possible. Since train rescheduling and passenger flow control are considered at the same time, the optimization model constructed in this paper is relatively complex, and it takes much time to solve the model directly. Therewith, firstly, it is reconstructed into a mixed-integer programming model with only quadratic terms by using three lemmas. Then, combined with the background of high demand, the model is decomposed into two submodels for solving problems, respectively. At the same time, combining with the adjustment of the requirement of boarding equalization, a two-stage method is designed to solve the problem. The results show that the method can obtain a quasi-optimal solution quickly and meet the real-time requirement of emergency adjustment.

The remainder is organized as follows. Section 2 further describes the problem of the study and constructs the initial mathematical model. Furthermore, Section 3 gives some lemmas of transforming the initial model into a mixed-integer programming (MIP) model and proposes a two-stage approach for solving the model. In Section 4, a numerical experiment is carried out. Section 5 presents the conclusions of this paper.

2. Mathematical Formulations

2.1. Problem Description. Speed limit scenarios do exist in URTs, and both trackside equipment failures and on-board equipment failures may cause train running at a limited speed. Taking a speed limit scenario caused by a zone controller (ZC) failure as an example, as shown in Figure 1, a ZC failure occurs suddenly. As a result, train T1 cannot pass through the corresponding zone (S2 to S4) normally. In order to ensure the service, under the protection of the automatic train protection (ATP) system, the train can switch to the restricted manual driving mode (i.e., RM mode) to pass through the speed limit zone at a low speed and return to the normal driving mode at the non-speed-limit section or station. Specifically, train T1 can switch to the RM mode at station S2 and then switch back to the normal mode after arriving at station S4.

The scenario in Figure 1 mainly involves two issues. The first is the problem of train rescheduling. Due to the failure, the planned timetable is no longer applicable, and the timetable needs to be rescheduled in time. For the RM mode, trains with a speed limit must be at least one station and one section apart from trains in front of them. This is a significant difference from the normal scenario of train rescheduling.

The second is the problem of passenger flow control. The service capacity of the affected line will be greatly reduced due to the larger train interval. As shown in Figure 2, many passengers tend to be stranded due to the reduced service ability, especially in a rush hour. The excessive aggregation of passengers can easily lead to accidents, and the timely implementation of passenger flow control is crucial to ensuring the safety of passengers at a station, as shown in Figure 3. Specifically, the problem considered in this paper is how to coordinate train timetable and passenger flow control scheme to ensure the operation order and passenger service for a speed limit scenario.

2.2. Assumptions. For the proposed problem, we make the following three assumptions.

Assumption 1. Trains pass through the speed limit zone at a low speed, and the sequence of trains are known.

Assumption 2. The demand is known, the arrival of passengers conforms to a uniform distribution [10], and the destination rate of arrival passengers at each station is fixed.

Table 1: Summary of some closely related studies on operation management.

Publication	Scenario	Type of model	Objective	Solution approach
[4]	Line, disruption, and huge passenger demand	Nonlinear programming	Train operation returns to original timetable and waiting time of passengers outside stations	Iterative metaheuristic approach
[5]	Line, single-point disturbance, overloaded passengers, and train skip-stopping rescheduling	Linear bilevel programming	Total train delay (upper-level model) and number of passengers served (lower-level model)	Sensitivity analysis-based algorithm
[32]	Rush hours, platform jam, inbound passenger flow control, and timetable regulation at transfer station	Nonlinear programming	Average waiting time of both inbound and transfer passengers	Improved genetic algorithm
[33]	Line, single-point disturbance, and crowded situations	Markov decision process	Total delay of all the disturbed trains with minimal impact on both operation costs and service quality	Approximation dynamic programming
[34]	High-frequency line, single-point disturbance, and overloaded passengers	Quadratic programming	Total train delay	Model predictive control and MATLAB optimization tool box
[35]	Network, single-point disturbance, and high demand	Mixed-integer nonlinear programming	Punctuality and regularity in train operations, the passenger waiting time, the passenger flow burden of platforms, and passenger flow control costs	Iterative nonlinear programming approach and Gurobi solver
This paper	Line, high demand, and train speed limit (multisection)	Mixed-integer quadratic programming	Total train delay and number of passengers served	Two-stage approach and Gurobi solver

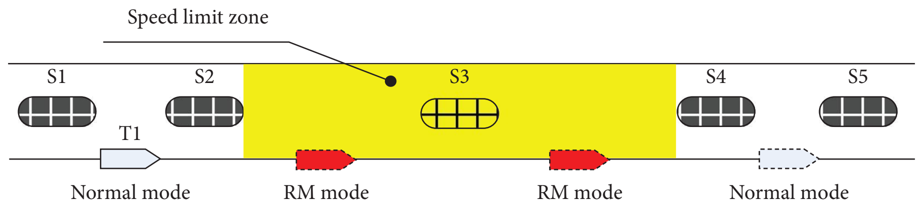


Figure 1: Train passes through speed limit zone at a limited speed.

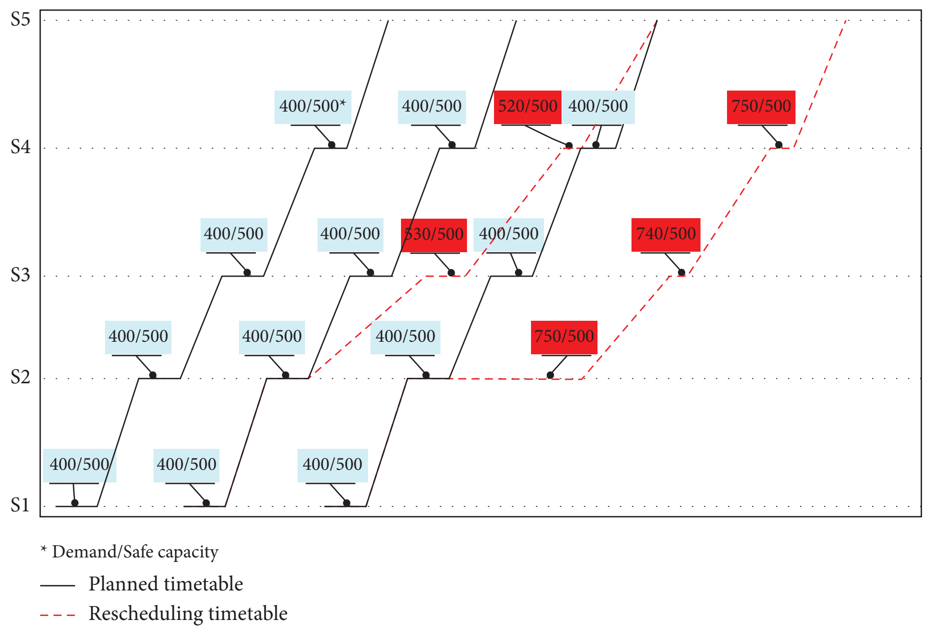


Figure 2: Passenger service under train delays.

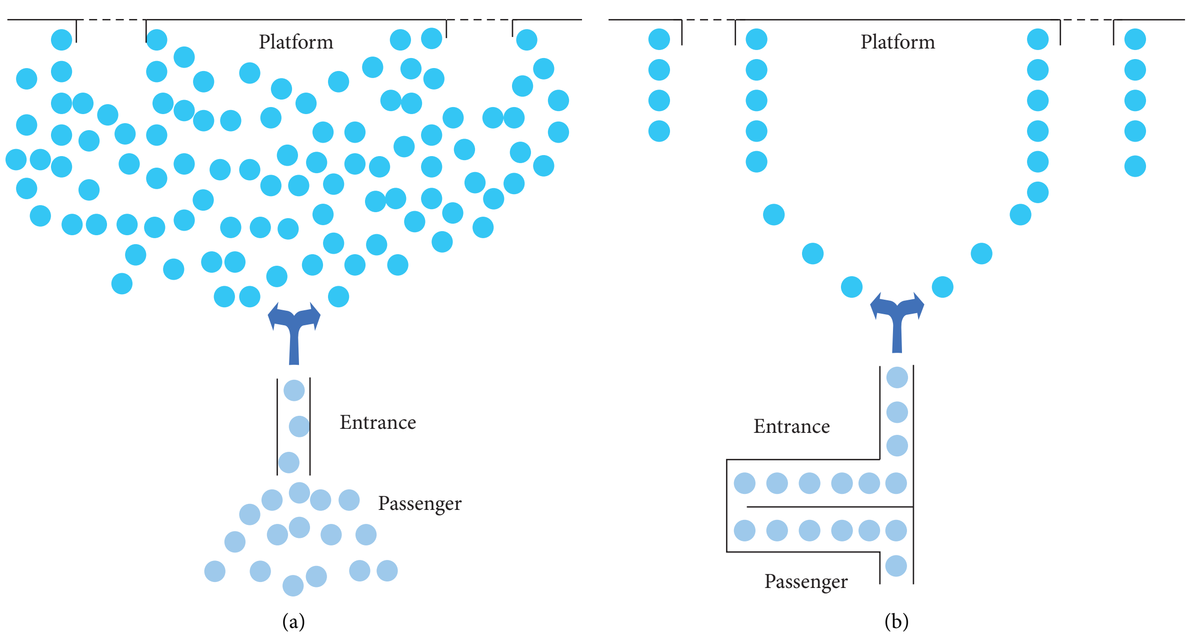


Figure 3: Effects of passenger flow control at a station: (a) passenger distribution without flow control and (b) passenger distribution with flow control.

Assumption 3. The walking time of passengers from the entrance facilities to the platform is neglected [22, 23]. That is, when they are allowed to enter, the passengers can reach the platform immediately.

In addition, to reduce the difficulty, only one direction (e.g., up-direction) is considered in the modeling process, and the same principle can be used for the other direction.

2.3. Notations. According to the problem background and modeling needs, we define the following sets, parameters, and variables.

Sets:

N: set of trains, $i \in \mathbf{N}$
S: set of stations, $j, k \in \mathbf{S}$
$\mathbf{S}^d$: set of stations in the speed limit zone
T: set of time units, $m \in \mathbf{T}$

Parameters:

$A_{i,j}/D_{i,j}$: planned arrival/departure time of train i at station j
$r_{j,j+1}/R_{j,j+1}$: minimum running time between stations j and station $j+1$ under the normal/speed limit scenario
I^h/I^p: minimum tracking interval/minimum departure-arrival interval
w^n/w^d: minimum dwell time of trains at station j under the normal/speed limit scenario
t_i^{ss}/t_i^{st}: additional time at sections/stations due to the degraded driving mode
t^s/t^c: failure occurrence time/failure duration
$T^{\min -D}$: minimum value of total delay
ε: the tolerance coefficient of delay degree; the smaller the delay tolerance coefficient, the smaller the requirement on total delay
t_m: the mth timestamp
v_j^{ini}: the number of initially stranded passengers on the platform of station j
$L^{\max}$: the maximum capacity of a train
$C_j^{\max}$: the safe capacity for the platform of station j
$p_{k,j}$: the proportion of passengers boarding at station k who are going to station j
δ_j: the number of passengers arriving per second for station j
$O^{\max}/O^{\min}$: the maximum/minimum for the controlled number of entering passengers (i.e., controlled entering volume) in any time unit
B: the maximum deviation of controlled entering volume in adjacent time unit
μ: the coefficient of boarding equalization rate; the more equal the number of passengers served at different stations, the smaller the equalization rate of boarding
$\varsigma_{i,j}$: clearance time of train i at station j
τ_u: the step length for adjusting the coefficient of boarding equalization rate
De_j: the expected total demand for station j during the involved periods
M: a suitably large constant.

Intermediate variables:

$\lambda_{i,j,m}$: 0-1 variable set for the comparison between departure time $d_{i,j}$ and timestamp t_m, and $\lambda_{i,j,m} = 1$ if $d_{i,j} \geq t_m$
$x_{i,j}/y_{i,j}$: auxiliary variables set for the linearization of IF-THEN/MIN functions
$v_{i,j}^r$: the number of stranded passengers on the platform after train i departing from station j
$v_{i,j}^{\mathrm{add}}$: the increased volume of entering passengers in station j between $d_{i,j}$ and $d_{i,j-1}$
$L_{i,j}^c$: the number of passengers in train i when it arrives at station j
$L_{i,j}^n$: the number of passengers boarding train i at station j
$L_{i,j}^f$: the number of passengers alighting from train i at station j

Decision variables:

$a_{i,j}/d_{i,j}$: arrival/departure time of train i at station j
$o_{j,m}$: the controlled entering volume for station j during time unit m

2.4. Constraints

2.4.1. Constraints Related to Train Operation. The constraints related to train operation involve tracking interval, dwell time, running time, and so on, which are discussed as follows:

(1) Tracking Interval. For trains in normal operation, with the help of advanced operation control systems, a small interval (i.e., headway) can be realized. The interval requirement for train operation involves the tracking interval and the departure-arrival interval, and the two cannot be less than the corresponding minimum value. Specifically, for $i, i-1 \in \mathbf{N}$ and $j \in \mathbf{S}$, we have

$$\begin{aligned} a_{i,j} - a_{i-1,j} &\geq I^h, \\ d_{i,j} - d_{i-1,j} &\geq I^h, \\ a_{i,j} - d_{i-1,j} &\geq I^p. \end{aligned} \tag{1}$$

For communication-based train control (CBTC) systems, as shown in Figure 4(a), train running intervals are small in moving block or quasi-moving block, but the safety protection mode may change if the train running at a speed limit. Specifically, as shown in Figure 4(b), there are two trains operation in the RM mode, and train 2 can leave station A only after train 1 leaves station C.

Two conditions need to be met if a train operation is carried out in the RM mode. One is that the ZC failure has occurred and has not been recovered, and the other is that the train is in the speed limit zone. Specifically, for $i \in \mathbf{N}$ and $j \in \mathbf{S}^d$, if $d_{i,j}$ is less than $t^s + t^c$, we have

$$d_{i,j} \geq d_{i-1,j+2} + \varsigma_{i-1,j+2}. \tag{2}$$

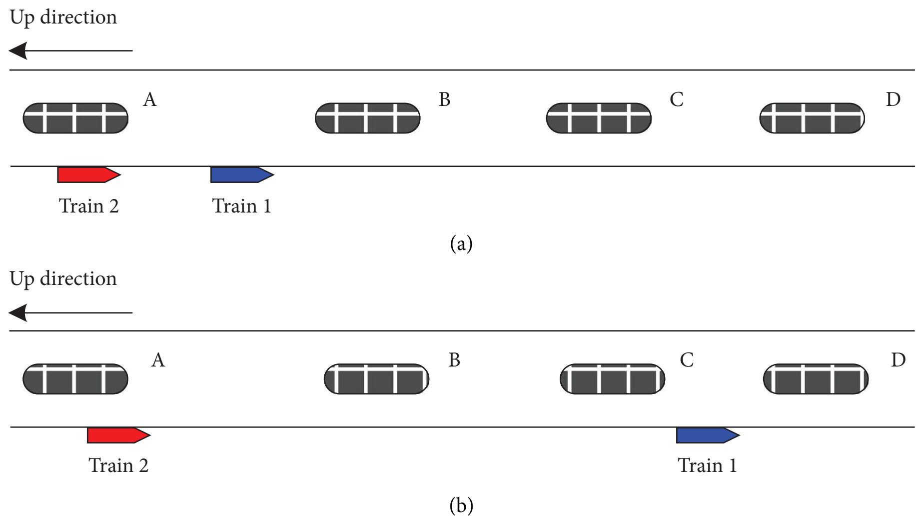


FIGURE 4: Safety interval of train operation in RM mode.

(2) Dwell Time. For trains in normal operation, the dwell time at a station shall not be less than the minimum dwell time. Correspondingly, for $i \in \mathbf{N}$ and $j \in \mathbf{S}$, we have

$$d_{i,j} - a_{i,j} \geq w^n. \tag{3}$$

At the same time, when a train is in the RM mode, the station operating time of the train is longer than normal. Then, for $i \in \mathbf{N}$ and $j \in \mathbf{S}^d$, if $d_{i,j}$ is less than $t^s + t^c$, we have

$$d_{i,j} - a_{i,j} \geq w^d. \tag{4}$$

(3) Running Time. The running speed for trains is affected by various factors such as passenger comfort requirements, power systems, and track conditions, and the running time in a section will not be less than the corresponding minimum time. Normally, for $i \in \mathbf{N}$ and $j, j+1 \in \mathbf{S}$, we have

$$a_{i,j+1} - d_{i,j} \geq r_{j,j+1}. \tag{5}$$

For trains in the RM mode, the running time in the same section is usually larger than normal. Specifically, for $i \in \mathbf{N}$ and $j, j+1 \in \mathbf{S}^d$, if $d_{i,j}$ is less than $t^s + t^c$, we have

$$a_{i,j+1} - d_{i,j} \geq R_{j,j+1}. \tag{6}$$

(4) Driving Mode Switching. Since the downgrading process of a driving mode usually requires a certain operation and confirmation process, a certain additional time is required. The upgrading process can be completed quickly and automatically, and the time consumption is usually negligible. As for the time requirement of the downgrading process for driving mode switching, this paper discusses it in two cases. In the first case, the train downgrades the driving mode in a section. For a train in the speed limit section when the failure occurs, it needs to stop first and switches the driving mode. Correspondingly, for $i \in \mathbf{N}$ and $j, j-1 \in \mathbf{S}^d$, if the planned time $A_{i,j}$ of train i arriving at station j ahead is greater than t^s, the train has left the station $j-1$. Then, we have

$$a_{i,j} \geq A_{i,j} + t_i^{\mathrm{ss}}. \tag{7}$$

In the second case, the train downgrades the driving mode at a station. If the train is at a station when the failure occurs and the section in front of the station is in the speed limit zone, the train needs to switch the driving mode at the station. The constraints of the corresponding departure time can be expressed as follows: for $i \in \mathbf{N}$ and $j \in \mathbf{S}^d$, if the planned time $A_{i,j}$ of train i arriving at station j is no more than t^s, the planned time $D_{i,j}$ of the train leaving the station is greater than t^s. Then, we have

$$d_{i,j} \geq D_{i,j} + t_i^{\mathrm{st}}. \tag{8}$$

(5) Other Operation-Related Constraints. In order to further ensure the order of trains, it is required that the arrival and departure times to be adjusted should not be earlier than the corresponding planed arrival and departure times. Specifically, for $i \in \mathbf{N}$ and $j \in \mathbf{S}$, we have

$$\begin{aligned} a_{i,j} &\geq A_{i,j}, \\ d_{i,j} &\geq D_{i,j}. \end{aligned} \tag{9}$$

2.4.2. Constraints Related to Passenger Flow Control

(1) Stranded Passengers. When train i leaves station j, the stranded passengers at the station are the stranded passengers when the preceding train (i.e., train $i-1$) leaves the station plus the passengers arriving within the interval between train i and train $i-1$ leaving the station and minus the boarding passengers of train i at the station. Then, for $i, i-1 \in \mathbf{N}$ and $j \in \mathbf{S}$, we have

$$v_{i,j}^r = v_{i-1,j}^r + v_{i,j}^{\mathrm{add}} - L_{i,j}^n. \tag{10}$$

Meanwhile, as shown in Figures 2 and 3, there is a risk if too many stranded passengers on platforms. Specifically, for $i, i-1 \in \mathbf{N}$ and $j \in \mathbf{S}$, we have

$$v^r_{i,j} \le C^{\max}_j. \tag{11}$$

(2) Alighting Passengers. When train i stops at station j, the number of alighting passengers is the sum of the passengers on board who end at the station. For any direction, we can define an OD distribution matrix $\mathbf{P}$:

$$\mathbf{P} = \begin{bmatrix} 0 & p_{1,2} & p_{1,3} & \cdots & p_{1,|\mathbf{S}|-1} & p_{1,|\mathbf{S}|} \\ 0 & 0 & p_{2,3} & \cdots & p_{2,|\mathbf{S}|-1} & p_{2,|\mathbf{S}|} \\ 0 & 0 & 0 & \cdots & p_{3,|\mathbf{S}|-1} & p_{3,|\mathbf{S}|} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & p_{|\mathbf{S}|-1,|\mathbf{S}|} \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}. \tag{12}$$

Then, for $i, i-1 \in \mathbf{N}$ and $j \in \mathbf{S}$, the number of alighting passengers can be calculated as follows:

$$L^f_{i,j} = \sum_{k=1}^{j} \left(p_{k,j} \times L^n_{i,k} \right). \tag{13}$$

(3) Boarding Passengers. The number of passengers boarding train i at station j needs to be discussed in two cases. The first case is that the current demand is small and the remaining capacity of the train is sufficient, and the demand can all enter the train. Then, for $i, i-1 \in \mathbf{N}$ and $j \in \mathbf{S}$, if $v^r_{i-1,j} + v^{\text{add}}_{i,j} + L^c_{i,j} - L^f_{i,j} \le L^{\max}$, we have

$$L^n_{i,j} = v^r_{i-1,j} + v^{\text{add}}_{i,j}. \tag{14}$$

The second case is that the current demand is large and the remaining capacity of the train cannot meet the demand, and the number of boarding passengers is the remaining capacity. Then, for $i, i-1 \in \mathbf{N}$, $j \in \mathbf{S}$, if $v^r_{i-1,j} + v^{\text{add}}_{i,j} + L^c_{i,j} - L^f_{i,j} > L^{\max}$, we have

$$L^n_{i,j} = L^{\max} - L^c_{i,j} + L^f_{i,j}. \tag{15}$$

According to the above two cases and the relationship of supply and demand, for $i, i-1 \in \mathbf{N}$, $j \in \mathbf{S}$, the number of boarding passengers can be further formulated as follows:

$$L^n_{i,j} = \min\left(L^{\max} - L^c_{i,j} + L^f_{i,j}, v^r_{i-1,j} + v^{\text{add}}_{i,j} \right). \tag{16}$$

At the same time, in order to avoid the uneven distribution of passengers served at different stations and ensure the balance [5] and fairness [36] of passenger flow control for all passengers (i.e., the requirement of boarding equalization), for $j, k \in \mathbf{S}$ and $j \ne k$, we have

$$-\mu \le \frac{\sum_{i\in\mathbf{N}} L^n_{i,j}}{\mathrm{De}_j} - \frac{\sum_{i\in\mathbf{N}} L^n_{i,k}}{\mathrm{De}_k} \le \mu. \tag{17}$$

(4) Passenger on Board. When a train arrives at the first station, it should be empty. When the train arrives at another station, the number of passengers on board at the station is the number of passengers on board when the train arrives at the previous station plus the number of boarding passengers at the previous station and minus the number of alighting passengers at the previous station. Then, for $i \in \mathbf{N}$, we have

$$L^c_{i,j} = \begin{cases} 0, & j = 1, \\ L^c_{i,j-1} + L^n_{i,j-1} - L^f_{i,j-1}, & j \in \dfrac{\mathbf{S}}{\{1\}}. \end{cases} \tag{18}$$

(5) Passenger Control. If the arrival of passengers conforms to the uniform distribution and the flow control is not considered, the number of passengers arriving at the platform within the interval between the two trains successively leaving station j is as follows:

$$v^{\text{add}}_{i,j} = \left(d_{i,j} - d_{i-1,j} \right) \times \delta_j. \tag{19}$$

For passenger flow control, we use the time-indexed method to divide the involved period into several time units and form the time unit set $\mathbf{T}$. Then, the boundaries of each time unit can be used as timestamps, which can be represented as $\left\{ t_0, t_1, t_2, t_3, t_4, \cdots, t_{|\mathbf{T}|-2}, t_{|\mathbf{T}|-1}, t_{|\mathbf{T}|} \right\}$, where $t_{\text{start}} = t_0$ and $t_{\text{end}} = t_{|\mathbf{T}|}$. Based on the time units, the controlled number of entering passengers in each time unit can be optimized. Among them, for station j and time unit m, there is a corresponding controlled entering volume $o_{j,m}$.

Since the controlled number of entering passengers is related to the platform capacity, the controlled entering volume should be within a certain range. Then, for $j \in \mathbf{S}$ and $m \in \mathbf{T}$, we have

$$0 \le o_{j,m} \le O^{\max}. \tag{20}$$

At the same time, the controlled entering volume should not be greater than the actual demand, and the accumulation phenomenon needs to be considered. Specifically, for $j \in \mathbf{S}$, we have

$$o_{j,m} \le \begin{cases} \left(t_m - t_0 \right) \times \delta_j + v^{\text{ini}}_j, & m = 1, \\ \left(t_m - t_0 \right) \times \delta_j + v^{\text{ini}}_j - \displaystyle\sum_{e=1}^{m-1} o_{j,e}, & m \in \dfrac{\mathbf{T}}{\{1\}}. \end{cases} \tag{21}$$

Furthermore, considering the fairness and operability of passenger flow control for a single station, Shi et al. [23, 30, 31] suggested that the fluctuation of the controlled entering volume should be stable, and the difference in controlled entering volumes between two adjacent time units should not be too large. Based on this, for $j \in \mathbf{S}$ and $m, m-1 \in \mathbf{T}$, we have

$$-B \le o_{j,m} - o_{j,m-1} \le B. \tag{22}$$

(6) Entering Passengers. To calculate the new entering passengers within the departure interval of adjacent trains at the same station, the controlled entering volumes and train operation need to be coupled. A Boolean variable $\lambda_{i,j,m}$ is

introduced to represent the relationship between departure time $d_{i,j}$ and timestamp t_m. Meanwhile, we specify that if $d_{i,j}$ is not less than t_m, $\lambda_{i,j,m}$ is equal to 1, otherwise it is equal to 0. Then, for $i \in \mathbf{N}$, $j \in \mathbf{S}$, and $m \in \mathbf{T}$, we have

$$\lambda_{i,j,m} = \begin{cases} 1, & d_{i,j} \geq t_m, \\ 0, & d_{i,j} < t_m. \end{cases} \tag{23}$$

The entering passengers of adjacent trains in the departure interval of the same station are related to the time unit included in the interval. Thus, the number of new entering passengers can be calculated based on the controlled entering volume of each time unit in the interval between the arrival times of two adjacent trains. Specifically, for $i \in \mathbf{N}$ and $j \in \mathbf{S}$, the new entering passengers between two adjacent trains can be expressed as follows:

$$v_{i,j}^{\text{add}} = \sum_{m=1}^{Q} \left(\lambda_{i,j,m} - \lambda_{i-1,j,m}\right) \times o_{j,m}. \tag{24}$$

2.5. Objective Function. Minimizing train delays is a common objective for train rescheduling [5, 9, 34]. Meanwhile, the purpose of implementing passenger flow control is to ensure operation safety and service quality and to make full utilization of transport capacity. Kang et al. [22] suggested that when implementing passenger control, priority should be given to the utilization of transport capacity. Li et al. [25] and Zhao et al. [26] emphasized passenger turnover in the context of passenger flow control, and Yang et al. [5] and Zhang et al. [28] also focused on maximizing boarding passengers. In view of the balance of boarding passengers at different stations and the operation safety of each station in the constraints, this paper takes the minimum total delay of trains and the maximum number of boarding passengers (i.e., passengers served) as the optimization goal, as shown in the following equation:

$$\min f = \sum_{i\in\mathbf{N}} \sum_{j\in\mathbf{S}_i} \left(a_{i,j} - A_{i,j} + d_{i,j} - D_{i,j}\right) - \sum_{i\in\mathbf{N}} \sum_{j\in\mathbf{S}_i} L_{i,j}^{n}. \tag{25}$$

As mentioned above, based on the above constraints and the objective function, the optimization model shown in the following equation can be obtained (Model 1, for short):

$$\begin{aligned} \min f &= \sum_{i\in\mathbf{N}} \sum_{j\in\mathbf{S}_i} \left(a_{i,j} - A_{i,j} + d_{i,j} - D_{i,j}\right) - \sum_{i\in\mathbf{N}} \sum_{j\in\mathbf{S}_i} L_{i,j}^{n} \\ s.t. &\begin{cases} \text{Constaints } (1)-(15), (18)-(20), (22)-(26), \\ a_{i,j}, d_{i,j} \in \mathbf{R}^{+}, \quad \forall i \in \mathbf{N}, j \in \mathbf{S}, \\ o_{j,m}^{\text{st}} \in \mathbf{Z}^{+}, \quad \forall j \in \mathbf{S}, m \in \mathbf{T}. \end{cases} \end{aligned} \tag{26}$$

3. Model Reformulation and Solution Approach

3.1. Linearization of Nonlinear Constraints. In Model 1, constraints (2), (4), (6), and (23) are nonlinear constraints with IF-THEN terms, and constraint (16) includes a MIN term. Therefore, to obtain the optimal solution with the help of a solver that can solve quadratic programming problems (e.g., Gurobi), the above problem needs to be reformulated as a mixed-integer programming (MIP) problem. Then, the following three lemmas are proposed.

Lemma 4. *For $i \in \mathbf{N}$ and $j, j+1 \in \mathbf{S}^d$, the nonlinear constraints (2), (4), and (6) can be transformed into the linear form shown in equation (29).*

$$\begin{cases} d_{i,j} \geq t^s + t^c - \mathrm{M} \times \left(1 - x_{i,j}\right), \\ d_{i,j} \geq d_{i-1,j+2} + \varsigma_{i-1,j+2} - \mathrm{M} \times x_{i,j}, \\ d_{i,j} - a_{i,j} \geq w^d - \mathrm{M} \times x_{i,j}, \\ a_{i,j+1} - d_{i,j} \geq R_{j,j+1} - \mathrm{M} \times x_{i,j}, \\ x_{i,j} \in \{0, 1\}, \end{cases} \tag{27}$$

where if $d_{i,j} < t^s + t^c$, due to the existence of larger number M, *then $x_{i,j}$ in $d_{i,j} \geq t^s + t^c - M \times (1 - x_{i,j})$ is equal to 0. And $x_{i,j}$ is equal to 0 or 1 if $d_{i,j} \geq t^s + t^c$.*

Lemma 5. *For $i \in \mathbf{N}$ and $j \in \mathbf{S}$, the nonlinear constraint (18) can be transformed into the linear form shown in equation (30).*

$$\begin{cases} L_{i,j}^{n} \geq L^{\max} - L_{i,j}^{c} + L_{i,j}^{f} - \mathrm{M} \times \left(1 - y_{i,j}\right), \\ L_{i,j}^{n} \geq v_{i-1,j}^{r} + v_{i,j}^{\text{add}} - \mathrm{M} \times y_{i,j}, \\ L_{i,j}^{n} \leq L^{\max} - L_{i,j}^{c} + L_{i,j}^{f}, \\ L_{i,j}^{n} \leq v_{i-1,j}^{r} + v_{i,j}^{\text{add}}, \\ y_{i,j} \in \{0, 1\}, \end{cases} \tag{28}$$

where if $L^{\max} - L_{i,j}^{c} + L_{i,j}^{f} < v_{i-1,j}^{r} + v_{i,j}^{add}$, $y_{i,j}$ is equal to 1. If $L^{\max} - L_{i,j}^{c} + L_{i,j}^{f} > v_{i-1,j}^{r} + v_{i,j}^{add}$, $y_{i,j}$ is equal to 0. If $L_{\max} - L_{i,j}^{c} + L_{i,j}^{f} = v_{i-1,j}^{r} + v_{i,j}^{add}$, $y_{i,j}$ is equal to 1 or 0. Then, we have $L_{i,j}^{n} = \min(L^{\max} - L_{i,j}^{c} + L_{i,j}^{f}, v_{i-1,j}^{r} + v_{i,j}^{add})$.

Lemma 6. *For $i \in \mathbf{N}$, $j \in \mathbf{S}$, and $m \in \mathbf{T}$, the nonlinear constraint (25) can be transformed into the linear form shown in equation (31).*

$$\begin{cases} t_m - d_{i,j} \leq \mathrm{M} \times \left(1 - \lambda_{i,j,m}\right), \\ d_{i,j} - t_m \leq \mathrm{M} \times \lambda_{i,j,m} - 1, \\ \lambda_{i,j,m} \in \{0, 1\}, \end{cases} \tag{29}$$

where if the relation between the two integers is $d_{i,j} \geq t_m$, then $t_m - d_{i,j} \leq 0$ and $d_{i,j} - t_m \geq 0$, and $\lambda_{i,j,m}$ can only be equal to 1. At the same time, if $d_{i,j} < t_m$, then $t_m - d_{i,j} > 0$ and $d_{i,j} - t_m < 0$, and $\lambda_{i,j,m}$ can only be equal to 0.

As mentioned above, Model 1 can be further expressed in the linear form shown in the following equation (Model 2, for short):

$$\min f = \sum_{i\in \mathbf{N}}\sum_{j\in \mathbf{S}_i}\left(a_{i,j} - A_{i,j} + d_{i,j} - D_{i,j}\right) - \sum_{i\in \mathbf{N}}\sum_{j\in \mathbf{S}_i} L^n_{i,j},$$

$$s.t.\begin{cases} d_{i,j} \ge t^s + t^c - \mathrm{M}\times\left(1 - x_{i,j}\right), & \forall i \in \mathbf{N}, j \in \mathbf{S},\\ d_{i,j} \ge d_{i-1,j+2} + \varsigma_{i-1,j+2} - \mathrm{M}\times x_{i,j}, & \forall i, i-1 \in \mathbf{N}, j, j+1 \in \mathbf{S},\\ d_{i,j} - a_{i,j} \ge w^d - \mathrm{M}\times x_{i,j}, & \forall i \in \mathbf{N}, j \in \mathbf{S},\\ a_{i,j+1} - d_{i,j} \ge R_{j,j+1} - \mathrm{M}\times x_{i,j}, & \forall i \in \mathbf{N}, j, j+1 \in \mathbf{S},\\ L^n_{i,j} \ge L^{\max} - L^c_{i,j} + L^f_{i,j} - \mathrm{M}\times\left(1 - y_{i,j}\right), & \forall i \in \mathbf{N}, j \in \mathbf{S},\\ L^n_{i,j} \ge v^r_{i-1,j} + v^{\mathrm{add}}_{i,j} - \mathrm{M}\times y_{i,j}, & \forall i, i-1 \in \mathbf{N}, j \in \mathbf{S},\\ L^n_{i,j} \le L^{\max} - L^c_{i,j} + L^f_{i,j}, & \forall i \in \mathbf{N}, j \in \mathbf{S},\\ L^n_{i,j} \le v^r_{i-1,j} + v^{\mathrm{add}}_{i,j}, & \forall i, i-1 \in \mathbf{N}, j \in \mathbf{S},\\ t_m - d_{i,j} \le \mathrm{M}\times\left(1 - \lambda_{i,j,m}\right), & \forall i \in \mathbf{N}, j \in \mathbf{S}, m \in \mathbf{T},\\ d_{i,j} - t_m \le \mathrm{M}\times \lambda_{i,j,m} - 1, & \forall i \in \mathbf{N}, j \in \mathbf{S}, m \in \mathbf{T},\\ \text{Constraints (1) – (3), (5), (7), (9) – (12)},\\ \text{Constraints (13 – 17), (19), (20), (22) – (24), (26)},\\ x_{i,j}, y_{i,j}, \lambda_{i,j,m} \in \{0,1\}, & \forall i \in \mathbf{N}, j \in \mathbf{S}, m \in \mathbf{T},\\ a_{i,j}, d_{i,j} \in \mathbf{R}^+, & \forall j \in \mathbf{S}, m \in \mathbf{T},\\ o^{\mathrm{st}}_{j,m} \in \mathbf{Z}^+, & \forall j \in \mathbf{S}, m \in \mathbf{T}. \end{cases} \tag{30}$$

3.2. Solution: A Two-Stage Approach. The model above (Model 2) is a typical MIP model, but the computing time of the model may be long. In this regard, taking into account emergency requirements, this paper proposes a two-stage approach to solve the model, as follows.

The idea of the two-stage approach is obtaining the rescheduling timetable with minimum total delay first and then optimizing the scheme of passenger flow control. For a sudden failure, a feasible emergency scheme is often given priority. Since passenger flow control is related to the time unit and train operation, the problem can be divided into two stages, the rescheduling timetable optimization stage and passenger flow control scheme optimization stage. For the rescheduling timetable optimization stage, according to the constraints related to train operation in Model 2, this paper constructs an optimization model (Model 3, for short) with the goal of minimizing the total delay (equation (31)). The model is also an MIP model, which can be solved by the Gurobi solver.

$$\min f_1 = \sum_{i\in \mathbf{N}}\sum_{j\in \mathbf{S}_i}\left(a_{i,j} - A_{i,j} + d_{i,j} - D_{i,j}\right),$$

$$s.t.\begin{cases} d_{i,j} \ge t^s + t^c - \mathrm{M}\times\left(1 - x_{i,j}\right), & \forall i \in \mathbf{N}, j \in \mathbf{S},\\ d_{i,j} \ge d_{i-1,j+2} + \varsigma_{i-1,j+2} - \mathrm{M}\times x_{i,j}, & \forall i, i-1 \in \mathbf{N}, j, j+1 \in \mathbf{S},\\ d_{i,j} - a_{i,j} \ge w^d - \mathrm{M}\times x_{i,j}, & \forall i \in \mathbf{N}, j \in \mathbf{S},\\ a_{i,j+1} - d_{i,j} \ge R_{j,j+1} - \mathrm{M}\times x_{i,j}, & \forall i \in \mathbf{N}, j, j+1 \in \mathbf{S},\\ \text{Constraints (1) – (3), (5), (7), (9) – (12)},\\ x_{i,j} \in \{0,1\}, & \forall i \in \mathbf{N}, j \in \mathbf{S},\\ a_{i,j}, d_{i,j} \in \mathbf{R}^+, & \forall i \in \mathbf{N}, j \in \mathbf{S}. \end{cases} \tag{31}$$

Furthermore, for the optimization stage of the passenger flow control scheme, since the rescheduling timetable is obtained in the first stage, according to the related constraints of passenger flow control in Model 2, an MIP model (Model 4, for short) can be established with the goal of maximizing the number of passengers served, as shown in the following equation:

$$\max f_2 = \sum_{i\in \mathbf{N}}\sum_{j\in \mathbf{S}} L^n_{i,j},$$

$$s.t.\begin{cases} L^n_{i,j} \ge L^{\max} - L^c_{i,j} + L^f_{i,j} - \mathrm{M}\times\left(1 - y_{i,j}\right), & \forall i \in \mathbf{N}, j \in \mathbf{S},\\ L^n_{i,j} \ge v^r_{i-1,j} + v^{\mathrm{add}}_{i,j} - \mathrm{M}\times y_{i,j}, & \forall i, i-1 \in \mathbf{N}, j \in \mathbf{S},\\ L^n_{i,j} \le L^{\max} - L^c_{i,j} + L^f_{i,j}, & \forall i \in \mathbf{N}, j \in \mathbf{S},\\ L^n_{i,j} \le v^r_{i-1,j} + v^{\mathrm{add}}_{i,j}, & \forall i, i-1 \in \mathbf{N}, j \in \mathbf{S},\\ t_m - d_{i,j} \le \mathrm{M}\times\left(1 - \lambda_{i,j,m}\right), & \forall i \in \mathbf{N}, j \in \mathbf{S}, m \in \mathbf{T},\\ d_{i,j} - t_m \le \mathrm{M}\times \lambda_{i,j,m} - 1, & \forall i \in \mathbf{N}, j \in \mathbf{S}, m \in \mathbf{T},\\ \text{Constraints (13) – (17), (19), (20), (22) – (24), (26)},\\ y_{i,j}, \lambda_{i,j,m} \in \{0,1\}, & \forall i \in \mathbf{N}, j \in \mathbf{S}, m \in \mathbf{T},\\ o^{st}_{j,m} \in \mathbf{Z}^+, & \forall j \in \mathbf{S}, m \in \mathbf{T}. \end{cases} \tag{32}$$

In view of the need to determine a suitable coefficient of boarding equalization rate for the optimization of passenger flow control, combined with that decision makers usually expect to obtain multiple options for comparison, this paper considers further dynamically setting the acceptable coefficient of boarding equilibrium rate to obtain emergency schemes as much as possible. Specifically, as shown in Figure 5, this paper designs a solution flow of the two-stage approach based on Model 3, Model 4, and Gurobi solver. Since Model 3 and Model 4 are obtained by decomposing Model 2 and all the constraints of the model are involved, the obtained solutions based on the approach can be regarded as feasible solutions of Model 2.

For the end condition in Figure 5, we can require that the computing time not exceed a certain value, or the deviation degree between the current optimal value and the existing optimal value is smaller than a certain value.

4. Numerical Experiment

4.1. Experiment Description. The simulation line with 8 stations is the first phase of a line in Shanghai City, and the up-direction of the line is from station HT to station YQ, as shown in Figure 6. The minimum running time of the sections under normal and speed limit scenario are 260, 150, 160, 130, 120, 165, and 200 s and 500, 300, 310, 220, 255, 330, and 410 s, respectively. The planned dwell times of the stations are 50, 30, 30, 30, 30, 30, 30, and 50 s, respectively. The minimum dwell time at different stations under normal and speed limit scenario are 25 and 30 s, respectively. At the same time, the minimum tracking interval and minimum departure-arrival interval are 120 and 90 s, respectively. For the failure scenario, we assume a zone controller fails at 07 :

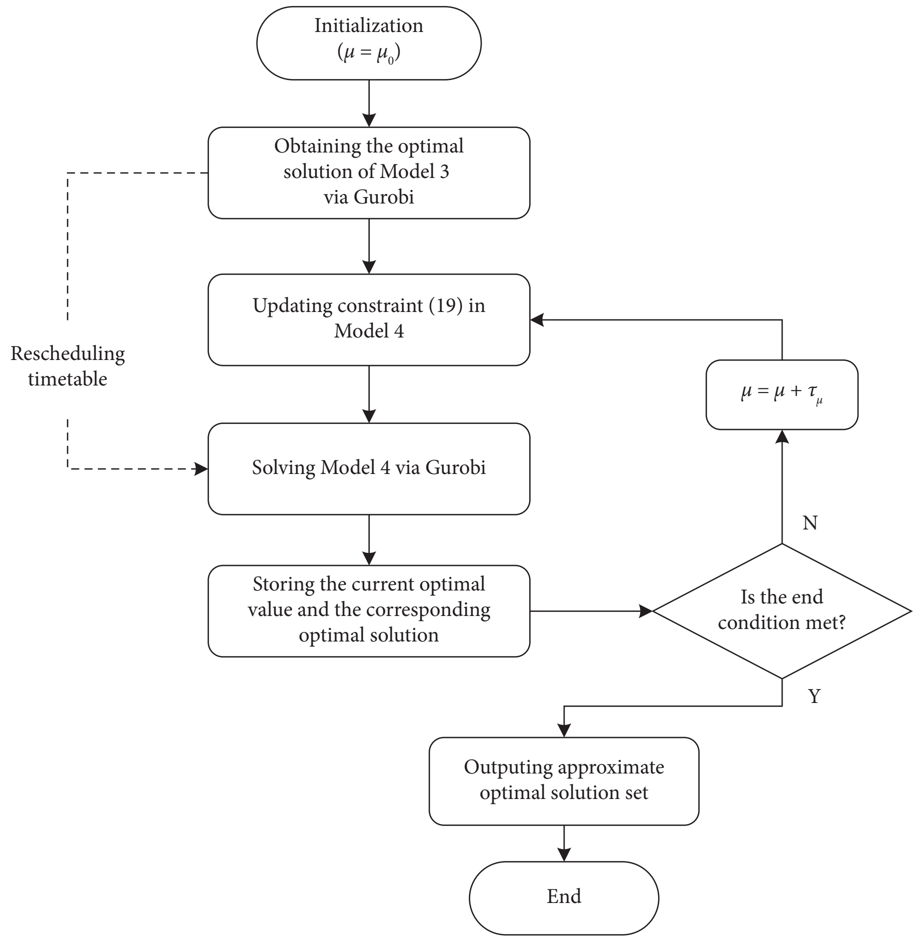


Figure 5: Solution flow of the two-stage approach.

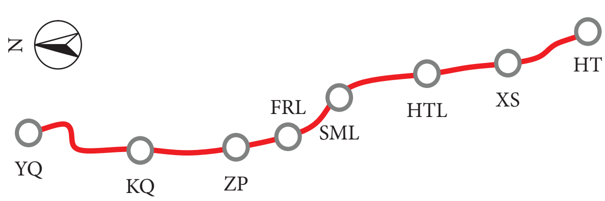


Figure 6: Simulation line.

55, and the expected failure duration is 15 min (i.e., recovery at 08 : 10). During the failure period, the speed limit zone involving in stations HTL, SML, FRL and the two section of them, and trains need to operate in RM mode in the zone.

Only the upward direction is considered, and the arrival rates of passengers are set to 4.5, 5.4, 2.4, 2.4, 3, 3.9, 3, and 0 person/s, respectively. The corresponding destination rates of arrival passenger are showed in Table 2. At the same time, for the safe capacity, the upper limit of each station is set to 1160, 1080, 1040, 1000, 960, 920, 900, and 0, respectively.

For the simulation, 20 trains are considered, the train capacity is set to 1800 persons, and the planned timetable for the first and last three trains is fixed. The length of a time unit is set to 60 s, the maximum controlled entering volume per unit is set to 300 persons, and the deviation of the controlled entering volume of adjacent time units is set to less than 50 persons [24]. At the same time, the starting time of passenger flow control is 07 : 55.

Furthermore, the larger integer M is set to 50000. For the algorithm shown in Figure 5, the initial coefficient of boarding equalization rate is 10%, and the step length is 5%. During the iterative process, if the computing time exceeds 10 seconds or the coefficient of boarding equalization rate is greater than 50%, the process ends. In order to compare the difference between the two-stage approach and the approach based on solving Model 2 via the Gurobi solver (Model 2-based approach, for short), the coefficient of boarding equalization rate in Model 2 is set to 15%. The solution of the model is to call the Gurobi solver via Python and rely on a laptop configured as a Windows 10 system and an i7-8750H CPU and 16 GB RAM. Among them, the termination condition of the Gurobi solver takes the duality gap as 0.1%.

4.2. Results and Analysis. Table 3 shows the results of the Model 2-based approach and the two-stage approach. The Model 2-based approach takes a long time (32.763 s) to solve, the number of passengers served is 72 586, and the rescheduling timetable obtained is shown in Figure 7. Meanwhile, according to Model 3, the minimum value of the delay time is 26 800 s, and the computing time is 0.132 s. The corresponding total delay from Model 2 is also 26 800 s, and

Table 2: Destination rates of arrival passenger.

Station	HT	XS	HTL	SML	FRL	ZP	KQ	YQ
HT	—	0.100 30	0.112 78	0.082 71	0.103 23	0.132 71	0.235 41	0.232 86
XS	—	—	0.161 68	0.196 68	0.143 76	0.165 00	0.167 46	0.165 42
HTL	—	—	—	0.259 09	0.269 09	0.177 28	0.139 09	0.155 45
SML	—	—	—	—	0.341 43	0.225 71	0.212 86	0.200 00
FRL	—	—	—	—	—	0.523 16	0.215 79	0.261 05
ZP	—	—	—	—	—	—	0.421 43	0.578 57
KQ	—	—	—	—	—	—	—	1
YQ	—	—	—	—	—	—	—	—

Table 3: Results of the two approaches.

Method	Number of passengers served	Coefficient of boarding equalization rate	Total delay (s)	Computing time (s)
Model 2-based approach	72 586	15%	26 800	32.763
Two-stage approach	{71 880, 72 571, 73 134, 73 701}	{10%, 15%, 20%, 25%}	26 800	10.132

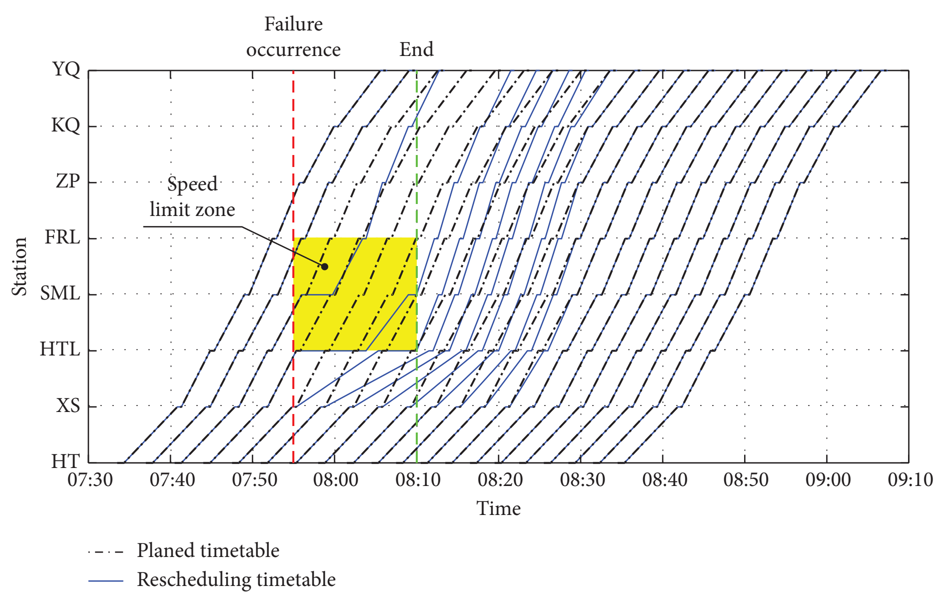


Figure 7: Result of train rescheduling.

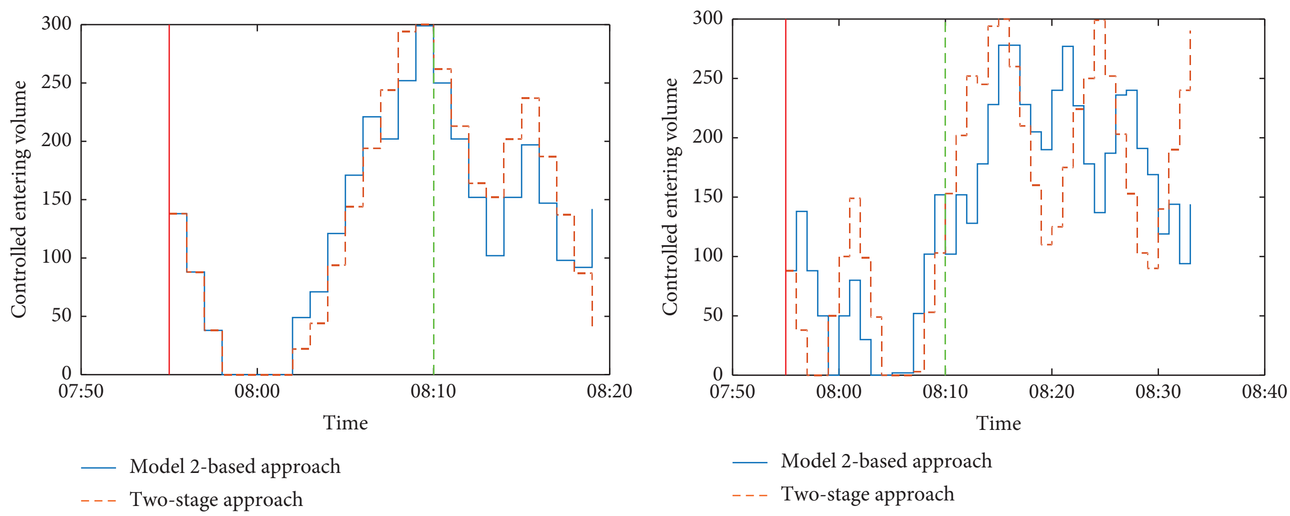
the rescheduling timetable obtained by the two methods is consistent. In contrast, the two-stage approach obtains 4 solutions in 10.132 s with different coefficients of boarding equalization rate. When the coefficient of boarding equalization rate is both taken as 15%, the optimal values obtained by the two approaches are close, and the deviation is 15. This should be related to the large demand, a certain number of trains, and the same requirement of boarding equalization.

Figure 7 shows the planed timetable and the rescheduling timetable. It can be seen that the train running time of the section in the speed limit zone has increased significantly, the intervals of the trains in the zone are relatively large, and the two are related to the RM mode discussed. For example, the departure time of the train at the HTL station is more than the departure time of the preceding train at the FRL station during such a failure. In addition, after trains leave the speed limit zone, the running time in the sections is relatively shorter. After the failure is over, the train operation intervals become smaller. For the consistency of the two timetables, it may be caused by the requirement of minimizing train delay in the objective function of the two optimization models (Model 2 and Model 3) involved, and it is also related to the high passenger demand.

Specific to the stations, Table 4 shows the number of passengers served at different stations and their proportion (i.e., ratio of passengers served to total demand). For the Model 2-based approach, the maximum proportion of passengers served is 92.2%, and the minimum proportion is 77.2%. For the two-stage approach, the maximum value is 100%, and the minimum value is 77.1%. Both meet the corresponding requirements of boarding equalization (15% and 25%). Combining Tables 3 and 4, it can be seen that the coefficient of boarding equalization rate has an impact on the number of passengers served. When the requirement for boarding equalization is high (i.e., the coefficient of boarding equalization rate is small), the number of passengers served

Table 4: Number of passengers served and its proportion (%).

Method	Station						
	HT	XS	HTL	SML	FRL	ZP	KQ
Model 2-based approach	12710/77.2	15045/89.2	6972/90.0	6644/77.2	8626/77.2	12329/85.4	10260/92.2
Two-phase approach	12675/77.1	14749/87.5	7268/93.8	6636/77.1	8662/77.5	12360/85.6	11351/100



(a) (b)

Figure 8: Passenger flow control scheme. (a) Station HTL. (b) Station FRL.

will be limited. As shown in Table 3, compared with the value of 10%, when the coefficient of boarding equalization rate is 20%, the corresponding number of passengers served is 1154 more.

Figure 8 shows two schemes of passenger flow control for stations HTL and FRL. During the failure period, the controlled entering volume of the two schemes is small, which matches the reduced transport capacity during the period and benefits to the operation safety of the stations. After the failure, the controlled entering volume increases. The change of the volume represents a dynamic passenger flow control process. The difference between the two schemes also reflects the idea of cooperative passenger flow control between different stations. Combined with the requirements for passenger flow control parameters in the case, both schemes are feasible.

The results indicate the validity of the models and the solution approaches. The two-stage approach can obtain multiple groups of quasi-optimal solutions, and the Model 2-based approach can obtain the optimal solution. However, the Model 2-based approach is inefficient, and it is necessary to determine an appropriate coefficient of boarding equalization rate in advance. In contrast, the two-stage approach first obtains the train rescheduling timetable and then does not need to optimize the timetable during the iteration process. Thus, the efficiency of the two-stage approach is higher. For example, in the above case, the approach completed four iterations within 10 s (i.e., the coefficient of boarding equalization rate is changed from 0.10 to 0.25 with the step size of 0.05). Combined with the above results, it can be considered that the two-stage approach is an alternative method in the situation where both train rescheduling and passenger flow control need to be optimized at the same time.

In addition, for the implementation of a train rescheduling scheme and a passenger flow control scheme, on the one hand, the dispatcher responsible for train operation can manage trains by issuing adjustment instructions according to the rescheduling scheme and can also directly transmit the confirmed rescheduling scheme to the train operation control system to control train's operation. On the other hand, the dispatcher responsible for passenger flow organization can send the passenger flow control plan to the station staff, and the field staff can organize the passenger flow according to the control scheme. At the same time, since passengers need to enter the station through the automated fare collection (AFC) gates, the working state of gates can be adjusted according to the passenger flow control scheme to achieve the effect of passenger flow control [37].

5. Conclusion

There are many train delay scenarios in an urban rail transit system, and the discussion of coping strategies helps to improve their management. Aiming at a kind of speed limit scenario, this paper studies the cooperative optimization models and solution approaches of train rescheduling and passenger flow control. The results of the numerical experiment reflect the train running characteristics in the scenario and show that the constructed MIP model is effective. At the same time, when such a failure scenario occurs during rush periods, the implementation of the cooperative passenger flow control helps to maintain the passenger volume on trains and ensure the operation safety of stations. However, the pursuit of higher boarding equalization is not conducive to the number of passengers served and the full utilization of transport capacity. Compared with setting the coefficient of boarding equalization rate to 10%, the coefficient of 25% can increase the number of passengers served by about 2 000. In addition, based on the rescheduling timetable with minimum total delay, obtaining a quasi-optimal passenger control scheme by adjusting the coefficient of boarding equalization rate dynamically is an effective approach for such scenarios.

In the study, the differences between two train driving modes in a speed limit zone are considered in the process of modeling train rescheduling problem. Combined with the passenger flow control problem, an optimal management method for dealing with speed limit scenarios is given, which has certain implications for the emergency decision-making of the relevant subjects. For further research, we can focus on the cooperative regulation of the train routing and rolling stock circulation in such scenarios with a large failure range.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Acknowledgments

This work was supported and sponsored by Shanghai Collaborative Innovation Research Center for Multi-Network & Multi-Modal Rail Transit and the research on key technologies of Operation Control and Environment and Disaster Monitoring of 600 km/h high-speed maglev train (grant no. 2022YFG0085).

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