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Research Article

A Battery Electric Vehicle Transportation Network Design Model with Bounded Rational Travelers

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With governments worldwide emphasizing environmental protection and the global focus on carbon reduction, the battery electric vehicle (BEV) industry has developed rapidly. An urban transportation network with BEVs as the main form of transportation will soon become mainstream. Motivated by the abovementioned background, a BEV transportation network design problem is investigated, and a network design model is established. The model aims to minimize the system travel time of BEV transportation networks and optimize the government's lane expansion scheme (the location and number of lanes) under a limited budget. To consider the travel characteristics of BEV drivers, the charging time, range anxiety, and bounded rationality factors are simultaneously incorporated into the model. A heuristic algorithm is designed based on the active set algorithm to obtain the local optimal solution to the actual-scale problem. Moreover, a cutting-plane method is used to convert the original problem into a different form, and a column generation technique is embedded in the abovementioned algorithm to avoid the enumeration of paths. Sensitivity analyses of different levels of rationality of BEV drivers and government investment scales are performed. The experimental results demonstrate that the model and algorithm can effectively solve the problem and provide decision support for the government in formulating transportation infrastructure construction policies.

1. Introduction

As reported by the World Meteorological Organization (WMO), in the past 50 years, more than 11,000 disasters caused by weather and climate have caused 2 million deaths and 3.6 trillion US dollars in economic losses, and nearly 22 million people have become climate refugees [1]. Carbon emissions are becoming a major practical issue affecting human survival and development [2, 3]. Social and economic development processes have simultaneously put considerable pressure on the natural environment. Global carbon dioxide (CO_2) emissions will continue to grow with the future growth of the economy, population, and resource demand. Therefore, mitigating greenhouse gas emissions is among the most urgent and far-reaching issues in the international political economy. The transportation sector is an indispensable and key industry in the process of social production and economic development. Moreover, it is one of the largest contributors to CO_2 emissions. In China, the transportation sector produces a quarter of the total CO_2 emissions [4, 5]. Therefore, this sector has been hindering the achievement of low-carbon goals due to high carbon emissions in energy consumption and transportation infrastructure construction [6]. Determining how to promote carbon emission reductions in high-carbon-emission industries such as the transportation sector has become an important issue of concern to the government and industry.

In recent years, with the strict management and control of carbon emissions by the governments of various countries, the battery electric vehicle (BEV) manufacturing industry has attracted considerable research attention [7].

Substituting BEVs for traditional gasoline vehicles (GVs) is an effective way to reduce greenhouse gas emissions and the dependence of the automobile industry on petroleum [8, 9]. This substitution has two advantages. First, BEVs no longer use fossil fuels and provide more efficient energy use during driving, which can reduce energy consumption in the transportation sector and would be an effective means of curbing fuel depletion and air pollution [10]. Second, when BEVs are powered by renewable methods such as solar and wind energy, CO_2 emissions can be almost completely avoided. According to [11], partial replacement of GVs with BEVs can reduce carbon emissions in the transportation sector by more than 60%. Therefore, various governments have introduced policies to promote the BEV industry to achieve *carbon neutrality* by the middle of the 21st century; consequently, the production of BEVs has increased rapidly. The number of newly registered BEVs nationwide increased from 25,163 in 2014 to 69,010 in 2018, a growth rate of 174% [12]. In China, the number of BEV and plug-in hybrid vehicle (HEV) registrations increased from 1,430 in 2010 to 579,000 in 2017 [5]. Predictably, BEVs will be a major part of future urban transportation networks, and BEV transportation network design will become the focus of both the government and academia because it promotes green transportation [13, 14].

In a BEV transportation network, the behavior of travelers is often different from that in a traditional GV transportation network. On the one hand, *range anxiety* makes the feasible travel path of BEV drivers significantly different from that of GV drivers. Range anxiety reflects travelers' fear of running out of battery energy before reaching their destination (or the charging station) [15, 16]. Since range anxiety is too subjective to measure directly, scholars tend to approximate range anxiety based on the ratio of remaining battery power to maximum battery capacity (i.e., the state of charge, SOC) [17]. In this context, travelers are concerned about their battery SOC, and they will make prudent charging decisions [5, 18]. This is manifested in the fact that travelers often do not allow the SOC of BEVs to be lower than a certain threshold during travel [19] and will choose the most secure travel route that ensures an available power supply [20, 21]. The above phenomenon requires us to establish a different definition for feasible paths in the design of BEV transportation networks.

On the other hand, traveler behaviors are often characterized by bounded rationality. In the traditional transportation network design problem (NDP), it is assumed that travelers choose transportation routes based on the principle of maximum utility; i.e., the traffic flow distribution in the transportation network obeys the perfect rational user equilibrium (PRUE) principle. However, this assumption has been challenged by numerous scholars [22, 23]. Researchers have found that when choosing a departure time [24, 25] or a transportation route [26], travelers' choices are often associated with bounded rationality. In studies of the choice of transportation routes by travelers in different cities, scholars have found that the proportion of those who choose the route with the shortest travel time tends to be between 25% and 75% [27, 28]. This phenomenon is considered to be the result of multiple factors, such as the number of intersections, route complexity, road organization, road style, and the cognitive limitations and psychological biases (people's habits, inertia [29, 30], and myopia [31]) of the traveler [32]. Especially with the popularization of BEVs and the development of traffic technology, road traffic conditions and traffic information will become more complex. Under the influence of the above phenomena, the traffic flow distribution in a BEV transportation network then forms a *bounded rational user equilibrium* (BRUE) state [33, 34]. This is obviously more complicated than the PRUE state of the previous GV transportation network and is worth studying.

Motivated by the two abovementioned behavioral factors, we conduct in-depth research on the design of BEV transportation networks. Our article aims to investigate a BEV transportation NDP considering two types of human choice behaviors (i.e., range anxiety and bounded rationality) (BNDP) to help governments better develop lane expansion schemes for specific regions. The research objective of this article is to answer the following three research questions:

(1) How can the BEV charging time, the range anxiety of BEV drivers, and bounded rationality be modeled?

(2) Considering the abovementioned factors, how can we construct a model for solving the BNDP?

(3) How can an algorithm be designed to efficiently solve the abovementioned model?

To this end, several equilibrium conditions of the transportation network are first developed to describe the BRUE state of the BEV transportation network. Then, a robust network design model is constructed to optimize the government's optimal lane expansion scheme with the goal of minimizing the system travel time. The model optimizes the locations of expansion and the number of lanes. Finally, to solve the abovementioned model, an active set algorithm (ASA)-based heuristic algorithm is designed to identify the local optimal solution. Moreover, a cutting-plane method is used to transform the original problem, and a column generation algorithm (CGA) is embedded in the abovementioned ASA-based heuristic to avoid the enumeration of paths. Overall, the contributions of this article are threefold.

(1) We define a BEV transportation NDP. Compared to traditional studies that consider BEV transportation network design (e.g., He et al. [35] and Cheng et al. [5]), we further introduce the BRUE theory into the model framework. To the best of our knowledge, our article is the first to introduce bounded rational behavior into the BEV transportation NDP.

(2) We establish the relevant equilibrium conditions for the BRUE-based transportation network and extend the theory of bounded rationality in economics to the field of BEV transportation network design. The above conditions can accurately describe the travel behavior of users in the BEV transportation network.

(3) We design an ASA-based heuristic algorithm that explores the local optimal solution of the problem. Several acceleration algorithms are introduced to accelerate the solution efficiency. The above-mentioned algorithm can provide an efficient solution for problems at practical scales and provide effective decision-making support for the government to formulate infrastructure construction policies.

The remainder of this article is organized as follows: In Section 2, the literature related to BEV transportation network equilibrium and NDP is reviewed to identify the current research gaps. Section 3 presents the assumptions, notations, problem descriptions, and mathematical expressions associated with the BRUE state of the BEV transportation network. In Section 4, the BEV transportation NDP (BNDP) is modeled considering the charging time, range anxiety, and BRUE factors. The algorithm for solving the model is presented in Section 5. In Section 6, several numerical experiments are performed to verify the effectiveness of the model and algorithm. Finally, Section 7 concludes the article, and several potential research directions are proposed.

2. Literature Review

Research on the design of BEV transportation networks is inseparable from the discussion of the equilibrium state of the networks. This is because the calculation of the equilibrium state of a transportation network is the basis for evaluating the design scheme of the network. The concept of transportation network equilibrium was first defined by Wardrop in 1952 [36]. Wardrop and Whitehead [36] assumed that all drivers choose their travel route based on the principle of the shortest travel time. Based on the hypothesis of rational man in the economic theory, Wardrop's first principle was proposed and has become the criterion of traffic flow assignment in academia. Early research on the NDP was based on GVs and Wardrop's first principle. Representative research topics include the continuous urban transportation NDP [37, 38], discrete NDP [39, 40], mixed NDP [41], transit NDP [42–44], and toll problem [45, 46]. Representative review articles in different time periods can be found in [46–48]. In recent years, with the growth of BEV ownership, research on the equilibrium state of BEV transportation networks has expanded. In the early stage, He et al. [35] first proposed a BEV transportation network equilibrium model considering charging time and range anxiety. This was the first article to summarize and distinguish the characteristics of BEVs and GVs. The authors noted that the difference between the equilibrium state of a BEV transportation network and that of a GV network is associated with the charging time and phenomenon of range anxiety, and the latter seems unrealistic to eliminate. Consequently, the authors constructed two network equilibrium models, one of which further considered flow-dependent energy consumption.

2.1. Modeling Approach. Considering the characteristics of wireless charging lanes, Chen et al. [49] established a comparatively complicated BEV transportation network equilibrium model. They assumed that BEV drivers could choose to charge wirelessly in a charging lane. To ensure that a BEV has sufficient battery power, the driver can adjust the charge received by changing the travel speed of the BEV. Xu et al. [21] suggested that the conditions proposed by He et al. [35] were incomplete, and they constructed a set of network equilibrium conditions. These conditions allowed both BEVs and GVs to exist in the transportation network. To reflect real-world conditions in the model, the authors further introduced the road grade effect. When a BEV is driving on roads of different grades, the dwell time and swapping cost will vary. However, the authors assumed that only swapping stations existed in the studied transportation network. This assumption limited the impact of dwell time on network equilibrium during the modeling process. Liu and Song [50] developed a BEV transportation network equilibrium model considering flow-dependent electricity consumption. Unlike Xu et al. [21], the authors proposed a solvable mathematical model rather than several equilibrium conditions. Thus, the equilibrium state of the transportation network could be obtained by solving the model directly. Recently, Chen et al. [51] introduced the queuing theory in conjunction with a set of network equilibrium conditions and constructed a BEV transportation network equilibrium model considering the BEV queuing time at charging stations. To solve the proposed mathematical program with complementarity constraints (MPCC), the authors converted the original model into a sequence of relaxed nonlinear programming problems. Generally, when scholars have constructed sets of equilibrium conditions for BEV transportation networks, they have tried to accurately model the relationships among vehicle flows, energy consumption, and queuing behaviors at transportation nodes. However, in characterizing the travel route choice behaviors of travelers, scholars often simply assume that each traveler is perfect rational. However, in practice, bounded rationality is an important characteristic related to travelers in a transportation network, and BEV transportation networks are not exceptions.

According to different definitions of the equilibrium state of BEV transportation networks, scholars have conducted extensive research on NDPs. A typical representative transportation NDP involves the layout of charging facilities (or other infrastructure). For example, Qiu et al. [52] addressed an optimization problem for the layout design of an electrified road for EVs. Based on the classic UE theory proposed by Wardrop [36], the authors established the equilibrium conditions of the transportation network and constructed a network design model. To solve the problem at the actual network scale, the authors designed a modified ASA. Cheng et al. [5] considered the charging time of BEVs and range anxiety factors and constructed a network design model based on the equilibrium conditions proposed by He et al. [35]. The authors aimed to minimize the system travel time and optimize the government's lane expansion scheme to limit traffic congestion. Furthermore, a robust optimization model was constructed considering the uncertainty of the transportation demand. Chen et al. [49] assumed that BEV drivers can adjust their driving speeds on electrified roads to increase the charge they receive. Based on the equilibrium conditions proposed, the authors discussed the government's optimal link reconstruction scheme (in which links are converted to electrified roads) under a limited budget. Liu and Song [50] further considered the abovementioned factors and constructed a congestion toll model. The authors verified the nonuniqueness of the vehicle flow distribution, and the robust optimization theory was introduced into the research framework to improve the worst-case scenario for the traffic flow distribution. Unfortunately, the authors assumed that travelers' choices of transportation routes obey the PRUE principle, which is equivalent to ignoring the impact of the irrational behavior of travelers on travel time in the transportation network.

2.2. Solution Approach. Since the introduction of bounded rationality into the transportation field, scholars have extended its application to transportation NDPs, transportation network equilibrium problems, traffic assignment problems, and transportation planning problems [53]. Various algorithms have been gradually developed and refined to obtain locally (or globally) optimal solutions to these problems [54, 55].

For example, Di et al. [56] characterized the topological structure of the BRUE set, decomposing the transportation network pricing problem into multiple subproblems. A heuristic algorithm called ASA was developed to achieve a suboptimal toll scheme for transportation networks considering bounded rationality and risk aversion. However, the limitation of the algorithm is that it is not applicable to large networks. Eikenbroek et al. [34] considered the best-/worst-case BRUE relative to the total travel time and solved the problem using a branch-and-price approach and a two-layer programming model to explore the difficulties of the application of the algorithm in BRUE. The results showed that both of these algorithms can only obtain local optimal solutions and that global solutions are difficult to achieve. Wang et al. [57] developed a tolerance-based column generation algorithm (TBCG) for the bounded rational dynamic user equilibrium (BR-DUE) model of transportation networks, which extends the traditional CGA by adopting four strategies in the spatial and temporal dimensions to take advantage of circumventing path enumeration without causing degradation in solution quality. Batista and Leclercq [58] extended the transportation NDP to regional transportation networks considering the bounded rationality of drivers and developed an integrated traffic model. Monte Carlo simulations and the method of successive averages (MSA) were applied simultaneously to solve the network equilibrium, and the impact of drivers' preferences for reliable travel times on the regional traffic network was discussed. Very recently, Jiang and Ceder [59] considered the effect of bounded rational factors on the continuity of the mapping function based on the traditional traffic assignment model. A heuristic decision algorithm based on the MSA method was designed to obtain approximate fixed-point expressions in the discontinuous case to model public transportation route choice behavior.

2.3. Research Gaps. Based on the abovementioned literature, two research gaps can be identified. First, the existing research has not broadly focused on network equilibrium or BEV transportation network design considering the bounded rationality of travelers. In almost all previous studies, it was assumed that drivers are completely rational when discussing network flow equilibrium. However, with the development of transportation technology and information technology in recent years, the uncertainty and complexity of the transportation environment have increased. Travelers often do not have access to all the information they need, making it difficult for them to be completely rational when making decisions. Thus, studies of BRUE in BEV transportation networks make more sense in the context of our article. Second, algorithm development for network design models with bounded rationality has been limited, especially for BEV transportation networks. Existing studies have proposed several local or global optimal algorithms for solving perfect rational NDPs. However, the objective function of robust optimization involved in our article is different from that in existing research. Currently, research on methods for solving robust optimization models in the context of bounded rationality is still insufficient. Thus, determining how to solve robust optimization models with network equilibrium conditions remains a challenge. To fill these two research gaps, we first define the bounded rationality of travelers in a BEV transportation network and then provide a mathematical expression of the BRUE state in the network. Next, a model is constructed to solve the transportation NDP. Finally, considering the characteristics of the model, a heuristic algorithm based on the framework of ASA is designed to obtain a local optimal solution. Overall, the above model and algorithm can provide the necessary decision support for the government to rationally construct the transportation infrastructure and achieve sustainable urban transportation development.

3. Problem Description

3.1. Assumptions and Notations. To facilitate modeling, we introduce several assumptions for the BNDP. Most of these assumptions are drawn from the established literature to ensure that we do not oversimplify the problem.

(1) Behavior of BEV drivers: When a BEV driver chooses a transportation path, it is assumed that he or she will follow the bounded rationality principle. Under the influence of this principle, when the difference between the travel times along different paths is below a given threshold (e.g., 10 minutes or 20 minutes), all these paths are considered feasible paths (see Section 4.1 for the definition) for BEV drivers [55]. The BEV transportation network equilibrium state based on this principle is further called the BRUE state. Due to the nonuniqueness of feasible paths, the vehicle flow distribution in the BRUE state also becomes nonunique.

(2) Charging time of BEVs: Unlike traditional GVs, BEVs require charging times that generally take

several hours [5]. Therefore, if a BEV needs to be charged during travel, the charging time cannot be ignored and needs to be added to the travel time. We assume that BEV drivers can charge their BEV only at a charging station in the BEV transportation network. The charging time is proportional to the amount of battery that needs to be charged, as noted by He et al. [35]. It is assumed that each charging station has only one type of charging pile and that the number of charging piles at a charging station is sufficient. In other words, we do not consider the case of BEV queuing at charging stations (following Chen et al. [49]). Some might argue that queuing is a factor to consider. If we add a virtual link before the charging station, then a BEV can drive on the virtual link before arriving at the charging station. After we reasonably adjust the performance function of this virtual link, we can consider the queuing of BEVs at the charging station. This modification does not affect the solution difficulty of the model.

(3) BEV energy consumption and the range anxiety of travelers: Each BEV has a fixed battery storage capacity; thus, we assume that each BEV has the same initial battery power (i.e., the SOC of the battery before the trip). Note that this assumption can be relaxed by introducing different types of BEVs, and different types of BEVs have different battery capacities and SOCs (same as Chen et al. [49]). The battery power decreases linearly as the transportation distance increases. Due to the existence of range anxiety, BEV drivers will not allow the SOC of the battery to fall below a predetermined value at any time (e.g., 5 kWh or 10 kWh, following He et al. [35]).

(4) Link performance function: The relationship between the traffic flow volume and the travel time along each link can be abstracted as a function called the *performance function* [5]. We assume that in a BEV transportation network, the performance function for each link follows the form of the Bureau of Public Roads (BPR) function (following He et al. [35]). Moreover, the travel times on different links are assumed to be additive. This means that the travel time along a path is equal to the sum of the travel times on all links it contains (following Cheng et al. [5]).

(5) The government's decision: The government aims to establish a reasonable network design scheme (specifically, a lane expansion scheme) with the goal of minimizing the system travel time under a limited budget. We assume that the government will not construct additional links and will only expand lanes based on existing links. In addition, for modeling convenience, we assume that the government can expand each link by 3 lanes at most and that a new lane associated with the same link has the same capacity as old lanes. The above assumptions can also be relaxed by introducing additional binary variables (same as Cheng et al. [5]).

For convenience, the notations frequently used in this article are given as follows:

Sets:

N_1: set of nodes with charging stations
N_2: set of nodes without charging stations
N: set of nodes, where $N = N_1 \cup N_2$
A: set of links
W: set of O-D (origin-destination) pairs
P_w: set of *usable paths* for O-D pair w
P_w^+: set of *usable paths* with traffic flows greater than 0, where $P_w^+ := \{p: f_p^w > 0, p \in P_w\}$
$A(p)$: set of links along path p

Parameters:

a or (i, j): link $a = (i, j)$
w or (r, s): O-D pair $w = (r, s)$
p: path p
$o(w)$: origin of O–D pair w
δ_{ap}^w: the path-link incidence, which equals 1 if path $p \in P_w$ includes link a and 0 otherwise

Decision variables:

x_a: vehicle flow on link a
f_p^w: vehicle flow on usable path p for O–D pair w
$u_{(a,k)}$: a binary variable; 1 when expanding link a by k lanes and 0 otherwise

3.2. BEV Transportation Network Formulation. A typical BEV transportation network can be abstracted as $G := (N, A)$. Here, G is defined to consist of a node set N and a link set A. Different from a traditional urban transportation network, there are charging stations at some nodes in the BEV transportation network. To describe the abovementioned situation, we let $N := N_1 \cup N_2$, where N_1 and N_2 represent the sets of nodes with and without stations, respectively. Moreover, equation (1) is the performance function used to describe the travel time of BEVs along each link. This equation has the same form as the BPR function according to Assumption (4):

$$t_a\left(v_a, l_{(a,k)}\right) = t_a^0\left[1 + \alpha_1\left(\frac{v_a}{C_a^{\mathrm{cap}} + C_a^{\mathrm{unit}}\left(l_{(a,1)} + 2l_{(a,2)}\right)}\right)^{\alpha_2}\right]. \tag{1}$$

In the above equation, we let $t_a(v_a, u_{(a,k)})$ denote the travel time on link a with v_a and $u_{(a,k)}$ and $a := (i, j)$ represent a link, where j and i are the head node and tail node of link a, respectively. Notations v_a and $\mathbf{l} = (l_{(a,k)})$ denote the vehicle flow volume and the lane expansion scheme for link a. Here, $l_{(a,k)}$ is a binary variable, and when it is equal to 1, the government expands link a by k lanes. Notably, we only consider $k = 1$ and $k = 2$ in equation (1) to match Assumption (5). Equation (1) could easily be applied to add any number of lanes. t_a^0, C_a^{cap}, and C_a^{unit} denote the free-flow travel time, current capacity, and increase in capacity after expanding link a by one lane, respectively. Finally, α_1 and α_2 are two positive parameters that are predetermined.

4. Model Establishment

4.1. Formulation of the BRUE State. In this section, the BRUE state in the BEV transportation network is discussed. We assume that all vehicles in the transportation network are BEVs. This is because with strict environmental policies being established by various governments, BEVs will come to dominate transportation networks in the foreseeable future [5]. We can also relax the abovementioned assumption by introducing different types of vehicles (e.g., GVs, HEVs, and BEVs). According to the bounded rationality principle, each BEV driver can choose his or her travel path freely from origin to destination when the travel times along several paths do not exceed a threshold value (e.g., 10 minutes or 20 minutes). However, due to limited battery capacity and range anxiety phenomena, some paths are not feasible for a BEV driver. To describe the different properties of a path in a BEV transportation network, we first present several definitions as follows.

Definition 1. (**useable path**). A path is useable if and only if the BEV drivers can complete the journey by selecting this path and the SOC of the battery will not cause the drivers to feel range anxiety at any time during their trip [35].

Definition 1 indicates that, in a BEV transportation network, the SOC at any point along a usable path must be greater than or equal to the minimum battery power that causes range anxiety. We use Figure 1 to further illustrate Definition 1.

A BEV transportation network with 4 nodes and 5 links is shown in Figure 1, and the length of each link is marked above it. We mark several nodes with shading to indicate that they are the nodes with charging stations (i.e., Node 2 and Node 3 in Figure 1). We assume that a BEV driver needs to travel from Node 1 to Node 4. Therefore, there are three options for the driver: 1-2-4 (Path 1), 1-4 (Path 2), and 1-3-4 (Path 3). We set the battery capacity to 15 kWh and the BEV electricity consumption to 1 kWh per kilometer. The battery of the BEV is fully charged at Node 1 (i.e., the SOC of the BEV equals 15 kWh). Based on the above information, we find that Path 2 is unusable (even without considering range anxiety) because the BEV battery power consumed on link $(1, 4)$ is equal to 20 kWh. If we define the range anxiety of the BEV driver as 2 kWh, then Path 1 further becomes unusable. This is because after the BEV driver travels 13 kilometers on link $(2, 4)$, the SOC of the BEV will be less than 2 kWh. Similarly, if their range anxiety is further increased to 3 kWh, the BEV driver will not be able to reach Node 4 from Node 1 because all paths are unusable. Based on Definition 1, we further present the following definitions of a feasible path and the network BRUE state to describe travel in a BEV transportation network:

Definition 2. (**feasible path**). For each origin-destination (O-D) pair, a path is **feasible** if and only if this path is **usable** (satisfies Definition 1), and the difference between the travel time along this path and the travel time along the path with the shortest travel time is no larger than a threshold value [5].

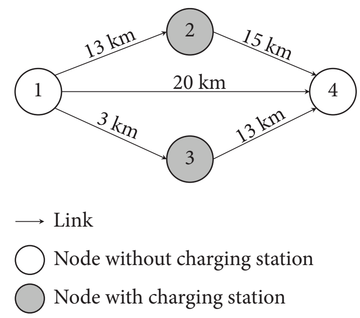

Figure 1: An example BEV transportation network.

Definition 3. (**network BRUE state**). The traffic flow distribution in the BEV transportation network reaches the BRUE state if and only if all transportation demands are satisfied and each BEV travels along a *feasible* path defined in Definition 2 [55].

To model the BRUE state in the BEV transportation network, we let W represent the O-D pair set, with corresponding element w (e.g., Node A to Node B). P_w denotes the set of all usable paths in the BEV transportation network defined in Definition 1, and the corresponding element is p. We set f_p^w and c_p^w as the traffic volume and shortest charging time for the BEV driver on path $p \in P_w$, respectively. The abovementioned charging time c_p^w should ensure that there is no range anxiety on path p. δ_{ap}^w is a binary variable. If path $p \in P_w$ consists of link $a \in A$, then δ_{ap}^w equals 1; otherwise, it equals 0. The O-D demand of pair w is denoted as q_w. Finally, we assume that BEV drivers with the same transportation demand (i.e., same origin and destination) have the same bounded rationality threshold value, i.e., $\overline{\varepsilon}^w$. This assumption can also be relaxed by introducing different types of BEV drivers. Thus, we can establish the following proposition:

Proposition 1. *Vector $(\mathbf{v}, \mathbf{f})^{\mathrm{T}} = (v_a, f_p^w)^{\mathrm{T}}$ denotes a BRUE traffic flow distribution for the BEV transportation network if and only if for each w, a λ^w solution exists that satisfies the following equilibrium conditions:*

$$\overline{c}_p^w \geq \lambda^w \forall p \in P_w, \tag{2}$$

$$\overline{c}_p^w \leq \lambda^w + \overline{\varepsilon}^w \forall p \in P_w^+ := \left\{p\colon f_p^w > 0, p \in P_w\right\}, \tag{3}$$

$$\sum_{p \in P_w} f_p^w = q_w, \tag{4}$$

$$v_a = \sum_{w \in W} \sum_{p \in P_w} f_p^w \delta_{ap}^w \quad \forall a \in A, \tag{5}$$

$$\overline{c}_p^w = \sum_{a \in A(p)} t_a\left(v_a, l_{(a,k)}\right) + c_p^w \forall p \in P_w, \tag{6}$$

where $\mathbf{f} = (f_p^w) \geq 0$.

Proof. See the proof in Section 2.1 of Lou et al. [55] for details (Q.E.D.).

However, when incorporating these equilibrium conditions into a model, all paths in the BEV transportation network need to be enumerated due to the existence of constraint (3) since it requires $f_p^w > 0$. To facilitate the model solution process, we further introduce a slack variable ε_p^w to transform the BRUE conditions (1) to (6) as follows:

$$\bar{c}_p^w - \varepsilon_p^w - \lambda^w = 0 \forall p \in P_w, w \in W, \tag{7}$$

$$f_p^w\left(\bar{\varepsilon}^w - \varepsilon_p^w\right) \geq 0 \forall p \in P_w, w \in W, \tag{8}$$

$$\sum_{p \in P_w} f_p^w = q_w \forall w \in W, \tag{9}$$

$$\bar{c}_p^w = \sum_{a \in A(p)} t_a\left(v_a, u_{(a,k)}\right) + c_p^w \forall p \in P_w, w \in W, \tag{10}$$

where $\varepsilon = (\varepsilon_p^w)$ and $(f, \varepsilon) \geq 0$.

Remark 1. We can easily verify that when $\bar{\varepsilon}^w$ equals 0, the variable λ^w represents the equilibrium travel time for O-D pair w in the PRUE state, as noted by He et al. [35]. When $\bar{\varepsilon}^w > 0$, the traffic flow distribution that satisfies constraints (1), (5), and (7)–(10) is a BRUE distribution according to Definition 3. Currently, λ^w represents the shortest travel time for O-D pair w, and ε_p^w denotes the difference between the travel time along path $p \in P_w$ and the shortest travel time $\lambda^w \in W$.

4.2. Robust BEV Network Design Model. Before constructing the network design model, we first focus on the solution of the network equilibrium state (i.e., the BRUE state). In the BRUE state, the traffic flow along each link is not unique because BEV derivers can choose arbitrarily from several feasible paths [55]. This uncertainty poses a challenge for the government in BEV transportation network design. However, among the numerous choices of travelers, there are two extreme situations: the shortest system travel time case (the best case) and the longest system travel time case (the worst case). In this section, we first focus on the abovementioned two extreme cases in BEV transportation networks because the traffic flow distributions in these two situations can provide estimates of the performance of the BEV transportation network design. Referring to the modeling method proposed by Liu and Song [50], we construct a network equilibrium model as an MPCC to facilitate the design of the corresponding solution algorithm. To transform constraint (8) into an equilibrium constraint, we first introduce a slack variable, say y_p^w. Thus, the model that can provide the best-/worst-case solution for the BEV transportation network is established as follows: BC/WC-BRUE with vector $(\mathbf{v}, \mathbf{f}, \mathbf{y}, \varepsilon, \lambda)^{\mathrm{T}}$ as decision variables. In BC/WC-BRUE, constraint (8) is converted into constraints (12) to (14).

BC/WC-BRUE:

$$\min_{(\mathbf{v},\mathbf{f},\mathbf{y},\boldsymbol{\varepsilon},\boldsymbol{\lambda})} / \max \sum_{a \in A} \sum_{k=1,2} t_a\left(v_a, l_{(a,k)}\right) v_a + \sum_{w \in W} \sum_{p \in P_w} c_p^w f_p^w, \tag{11}$$

s.t. (1), (5), (7), (9), and (10),

$$f_p^w y_p^w = 0 \quad \forall w \in W, p \in P_w, \tag{12}$$

$$y_p^w \geq \varepsilon_p^w - \bar{\varepsilon}^w \quad \forall w \in W, p \in P_w, \tag{13}$$

$$y_p^w \geq 0 \forall w \in W, p \in P_w, \tag{14}$$

where $(\mathbf{f}, \varepsilon, \mathbf{o}) \geq 0$, $\mathbf{v} = (v_a)$, $\mathbf{y} = (y_p^w)$, and $\lambda = (\lambda^w)$.

Then, we construct a robust network design model based on the above BC/WC-BRUE model. For a real-world decision-making problem, if the solution scheme to the problem can improve upon the worst-casesolution, then we consider the scheme robust [55]. In the urban transportation NDP, the government is often cautious of the worst case when formulating policies. This motivates us to provide a robust optimization solution as a powerful decision-making reference for the government. Based on the equilibrium model, a robust optimization model (i.e., a min-max programming model) for solving the BNDP (short as BNDPM) is given as follows. Before introducing the model, we define the following notations. Let $I_{\max}$ denote the upper limit of government investment and ξ_a represent the investment required to expand link a by one lane. Then, the BNDPM can be expressed as follows.

BNDPM:

$$\min_{\mathbf{l}} \max_{(\mathbf{v},\mathbf{f},\mathbf{y},\boldsymbol{\varepsilon},\boldsymbol{\lambda})} \sum_{a \in A} \sum_{k=1,2} t_a\left(v_a, l_{(a,k)}\right) v_a + \sum_{w \in W} \sum_{p \in P_w} c_p^w f_p^w, \tag{15}$$

s.t. (1), (5), (7), (8), (9), (10), (12), (13), and (14),

$$\sum_{a \in A} \xi_a\left(l_{(a,1)} + 2 l_{(a,2)}\right) \leq I_{\max}, \tag{16}$$

$$0 \leq l_{(a,k)} \leq 1 \forall a \in A, k = 1, 2, \tag{17}$$

$$l_{(a,k)}\left(1 - l_{(a,k)}\right) = 0 \forall a \in A, k = 1, 2, \tag{18}$$

where $\mathbf{l} = (l_{(a,k)})$.

Equation (15) is the objective function, and the lane expansion scheme $\mathbf{l} = (l_{(a,k)})$ minimizes the worst-case scenario (i.e., the flow distribution associated with the longest system travel time) in the BEV transportation network. Constraint (16) restricts the investment from exceeding the ceiling $I_{\max}$. Constraints (17) and (18) ensure that the variable $l_{(a,k)}$ is binary.

To facilitate the description of this problem, we divide the objective function of the BNDPM into two parts: the inner problem and the outer problem. A similar division approach was used by Lou et al. [55]. The inner problem refers to the maximized part of objective function (15), i.e.,

$\max_{(\mathbf{v},\mathbf{f},\mathbf{y},\varepsilon,\lambda)} \sum_{a\in A}\sum_{k=1,2} t_a(v_a, l_{(a,k)})v_a + \sum_{w\in W}\sum_{p\in P_w} c_p^w f_p^w$. We let $\Gamma(\mathbf{l})$, Θ, and $\psi(\Theta)$ represent the feasible region, decision variable vector, and objective function of the inner problem of the BNPDM, respectively. Then, the inner problem of the BNPDM can be equivalently expressed as BNDPM-IN:

$$\text{BNDPM} - \text{IN}: \ \Omega(\mathbf{l}) = \arg\max_{\Theta}\{\psi(\Theta): \ \Theta \in \Gamma(\mathbf{l})\}. \quad (19)$$

Based on the expression proposed above, we can further convert the original BNDPM into BNDPM-R, which is shown as follows.

BNDPM-R:

$$\min_{\mathbf{l}} \Omega(\mathbf{l}), \quad (20)$$

subject to $\Omega(\mathbf{l}) = \arg\max_{\Theta}\{\psi(\Theta): \ \Theta \in \Gamma(\mathbf{l})\}$ and constraints (16) to (18).

By analyzing the BNDPM-R, we can observe a certain connection between the inner problem and the outer problem; specifically, the feasible region of the inner problem $\Gamma(\mathbf{u})$ is determined based on the decision variable (i.e., vector $\mathbf{l}$) of the outer problem. The abovementioned relationship can be used to transform the BNDPM (or BNDPM-R) into a generalized semi-infinite min-max problem [60]. However, solving such problems has always been a challenge [55]. To date, only a few studies (e.g., Royset et al. [61] and Polak and Royset [60]) have discussed the solution to this kind of problem. Moreover, the inner problem is an MPCC because of the existence of constraints (12) to (14). This characteristic further causes the BNDPM to violate the Mangasarian–Fromovitz constraint qualification at any feasible point in set $\Gamma(\mathbf{l})$. Therefore, the algorithms proposed by previous scholars may not be suitable for solving the BNDPM or BNDPM-R. To obtain an effective solution to the abovementioned model, we modify the method proposed by Lou et al. [55] and design a heuristic algorithm, as described in Section 5.

5. Solution Approach

In this section, we solve the robust network design model with an ASA-based heuristic method. The steps of this algorithm are briefly summarized as follows: First, a series of active sets is created according to the equilibrium conditions of the network equilibrium model. Then, we obtain the BRUE state of the BEV transportation network by solving the BC/WC-BRUE model based on the ASA. To avoid the enumeration of paths, we need to use the CGA to generate feasible paths before solving the model. Finally, the robust network design model is converted into an ordinary semi-infinite optimization problem and solved with an ASA-based cutting-plane method. The method used to construct the active set and the solution procedure of the BC/WC-BRUE model is introduced in Section 5.1. CGA for path enumeration is also introduced in this section. Section 5.2 presents the method used to transform the robust network design model and the ASA-based cutting-plane method, which is used to solve the transformed model. See He et al. [35] for a detailed introduction to the proof of convergence of the algorithm.

5.1. Solution Procedure for the BC/WC-BRUE Model. The core concept of the ASA is to rewrite the two parts of one equilibrium condition into two different constraints. For example, in BC/WC-BRUE, the mathematical expression of the equilibrium condition (12) can be rewritten as the following equation:

$$0 \le f_p^w \perp y_p^w \ge 0 \forall w \in W, a \in A. \quad (21)$$

Notably, this condition consists of two parts: variables f_p^w and y_p^w. Therefore, we develop two active sets, namely, $\Phi_P: = \{(w,p): f_p^w = 0, \forall p \in P_w, w \in W\}$ and $\Phi_{\overline{P}}: = \{(w,p): y_p^w = 0, \forall p \in P_w, w \in W\}$, according to constraint (19). Based on sets Φ_P and $\Phi_{\overline{P}}$, BC/WC-BRUE can be reformulated as R-BC/WC-BRUE, which is shown as follows.

R-BC/WC-BRUE:

$$\min/\max_{(\mathbf{v},\mathbf{f},\mathbf{y},\boldsymbol{\varepsilon},\boldsymbol{\lambda})} \sum_{a\in A}\sum_{k=1,2} t_a\left(v_a, l_{(a,k)}\right)v_a + \sum_{w\in W}\sum_{p\in P_w} c_p^w f_p^w, \quad (22)$$

s.t. (1), (5), (7), (9), (10), (13), and (14),

$$f_p^w = 0 \quad \forall (w,p) \in \Phi_P^{\varpi}, \quad (23)$$

$$y_p^w \ge 0 \forall (w,p) \in \Phi_{\mathrm{P}}^{\varpi}, \quad (24)$$

$$f_p^w \ge 0 \forall (w,p) \in \Phi_{\overline{\mathrm{P}}}^{\varpi}, \quad (25)$$

$$y_p^w = 0 \forall (w,p) \in \Phi_{\overline{\mathrm{P}}}^{\varpi}, \quad (26)$$

where $\varepsilon \ge 0$.

We use the solution of WC-BRUE as an example and introduce the ASA as follows:

Step 1: Let $\varpi = 1$ and solve for the PRUE state of the BEV network with $\mathbf{l} = \mathbf{0}$ by applying the algorithm proposed by Xin et al. [62] or the classic Frank–Wolfe algorithm. To avoid path enumeration, we also develop a CGA to accelerate the algorithm computation process. Moreover, to obtain good strongly stationary solutions, one may also execute this step with multiple initial solutions [55]. Based on the solution $(\mathbf{v}, \mathbf{f}, \mathbf{y}, \varepsilon, \lambda)^{\mathrm{T}}$, we set Φ_P^1 and $\Phi_{\overline{P}}^1$ and proceed to Step 2.

Step 2: Let vector $(\mathbf{v}^{\varpi}, \mathbf{f}^{\varpi}, \mathbf{y}^{\varpi}, \varepsilon^{\varpi}, \lambda^{\varpi})^{\mathrm{T}}$ solve the R-BC-BRUE problem, and go to Step 3.

Step 3: Set $\Pi_P^{\varpi}: = \left\{(w,p) \in \Phi_P^{\varpi} \cap \Phi_{\overline{P}}^{\varpi}: \nu_{\varpi}^{(w,p)} > 0\right\}$, where $\nu_{\varpi}^{(w,p)}$ represents the multipliers associated with constraint (21). If $\Pi_P^{\varpi} = \varnothing$, then stop. At this point, the vector $(\mathbf{v}^{\varpi}, \mathbf{f}^{\varpi}, \mathbf{y}^{\varpi}, \varepsilon^{\varpi}, \lambda^{\varpi})^{\mathrm{T}}$ reaches the local optimum of the R-BC-BRUE problem; otherwise, go to Step 4.

Step 4: Set $\Phi_P^{\varpi+1} = \Phi_P^{\varpi} - \Pi_P^{\varpi}$ and $\Phi_{\overline{P}}^{\varpi+1} = \{(w, p)\colon y_p^w = 0, \forall p \in P_w, w \in W\}$, and go to Step 5.

Step 5: Set $\varpi = \varpi + 1$, and go to Step 2.

Here, we introduce the CGA in Step 1 in detail. Recall that Definition 1 indicates that not all paths are usable in the BEV transportation network. This requires us to calculate the usable paths before solving for the BRUE state of the BEV transportation network to avoid enumerating paths for each O-D pair $w \in W$. This approach motivated us to modify the CGA proposed by Cheng et al. [5] to accelerate the algorithm calculation process. Before proposing the algorithm calculation process, we need to develop a model and introduce the relevant notations for each O-D pair w. Let $\widetilde{v}_a$ denote the BEV flow volume on link a. Since v is a variable in the model, a wavy line is added above it to distinguish the variable v_a. Similarly, we use $\widetilde{l}_{(a,k)}$ to represent the parameter associated with the lane expansion scheme. m_a^w is a binary variable. When the CGA obtains a new usable path $p \in P_w$ containing link $a \in A$, the value of the binary variable is 1; otherwise, it equals 0. e_i and g_i denote the charging operation time (i.e., the time required by the driver to prepare before the BEV starts charging) and the charging time at node $i \in N_1$, respectively. C_i^w is a binary variable indicating whether the BEV charges at node $i \in N_1$. H_i^w represents the increase in SOC per unit time at node $i \in N_1$. Λ is a node-link incidence matrix. R_w is a vector of length $|N|$, where the notation $|\cdot|$ denotes the cardinality of a set. Only two nonzero components are included in R_w: the value of one component is 1 and that of the other is -1. These components are associated with the origin (denoted as $o(w)$) and destination (denoted as $d(w)$) of O-D pair w, respectively. In terms of battery-related information for BEVs, we use S_0 and S_i^w to denote the initial SOC of the BEV and the SOC at node $i \in N_1$, respectively. $S_{\max}$ represents the maximum SOC of the BEV. u_a and r denote the length of link a and the BEV energy consumption per unit distance, respectively. θ_a^w is a binary variable and equals 1 only when $m_a^w = 1$. RA_w indicates the lower limit of the SOC allowed by the driver, which is associated with range anxiety. O_i represents the amount of charge that the charging station at node $i \in N$ can provide. For node $i \in N_1$, O_i is a sufficiently large number; for node $i \in N_2$, O_i equals 0. Finally, Q and T are two sufficiently large positive numbers.

CGA: For each O-D pair w,

$$\min_{(\mathscr{M},\mathbf{C},\mathbf{H})} \sum_{a\in A} \sum_{k=1,2} t_a\left(\widetilde{v}_a, \widetilde{l}_{(a,k)}\right) m_a^w + \sum_{i\in N_1} \left(e_i C_i^w + g_i H_i^w\right), \tag{27}$$

$$\text{s.t. } \Lambda \mathscr{M}_w = R_w, \tag{28}$$

$$S_j^w - S_i^w + u_a r - H_j^w = \theta_a^w \forall (i, j) = a \in A, \tag{29}$$

$$S_i^w - u_a r \geq -Q\left(1 - m_a^w\right) + RA_w \quad \forall (i, j) = a \in A, \tag{30}$$

$$-T\left(1 - m_a^w\right) \leq \theta_a^w \leq T\left(1 - m_a^w\right) \quad \forall a \in A, \tag{31}$$

$$0 \leq H_i^w \leq O_i \quad \forall i \in N, \tag{32}$$

$$0 \leq S_i^w \leq S_{\max} \quad \forall i \in N, \tag{33}$$

$$S_{o(w)}^w = S_0, \tag{34}$$

$$m_a^w \in \{0, 1\} \quad \forall a \in A, \tag{35}$$

$$C_i^w \geq \frac{H_i^w}{O_{\max}} \quad \forall i \in N, \tag{36}$$

$$O_{\max} = \max_{i\in N_1} \{O_i\}, \tag{37}$$

$$C_i^w \in \{0, 1\} \quad \forall i \in N, \tag{38}$$

where $\mathscr{M}_w = (m_a^w)$, $\mathbf{C} = (C_i^w)$, and $\mathbf{H} = (H_i^w)$.

Based on the abovementioned model, we propose the following method to explore the usable paths for each O-D pair by applying the CGA:

Step 1: For each $w \in W$, we set $\widetilde{\mathbf{v}} = \mathbf{0}$ and solve the model. A set $P_w = \{p_w\}$ is created based on the metric $\mathscr{M}$, and c_p^w is calculated accordingly.

Step 2: We solve for the BRUE state of the network based on P_w (see the algorithm proposed in Section 5.1 for details) and obtain vector $(\widetilde{\mathbf{v}}, \widetilde{\lambda}^w)^{\mathrm{T}}$.

Step 3: For each $w \in W$, we solve the CGA again and obtain a new usable path p_w. For all O-D pairs, if $\sum_{a\in A} \sum_{k=1,2} t_a(\widetilde{v}_a, \widetilde{l}_{(a,k)}) m_a^w + \sum_{i\in N_1} (e_i C_i^w + g_i H_i^w) \geq \widetilde{\lambda}^w + \overline{\varepsilon}^w$ always holds, then vector $(\widetilde{\mathbf{v}}, \widetilde{\lambda}^w)^{\mathrm{T}}$ represents the BRUE flow distribution in the BEV transportation network; otherwise, go to Step 2.

5.2. Solution Procedure for the Robust Optimization Model. In this section, an ASA-based heuristic algorithm is designed to solve the robust optimization model, i.e., the BDNPM. Since the inner problem and outer problem of the original model affect each other, we first consider separating these two problems. Here, we convert BNDPM-IN into P-BNDPM-IN by adding a penalty term. With this approach, an ordinary semi-infinite optimization problem is obtained, and this problem can be solved with the cutting-plane method proposed by Lawphongpanich and Hearn [63].

P-BDNPM-IN($\mathbf{l}$):

$$\max_{(\mathbf{v},\mathbf{f},\mathbf{y},\boldsymbol{\varepsilon},\boldsymbol{\lambda})} \sum_{a\in A}\sum_{k=1,2} t_a\left(v_a, l_{(a,k)}\right)v_a + \sum_{w\in W}\sum_{p\in P_w} c_p^w f_p^w - B\sum_{w\in W}\sum_{p\in P_w}\left(\overline{c}_p^w - \sum_{a\,\in\,A(p)} t_a\left(v_a, l_{(a,k)}\right) - c_p^w\right)^2, \tag{39}$$

s.t. (1), (5), (7), (9), (12), (13), and (14).

Let $\widehat{\psi}(\Theta,\mathbf{l})$ denote function (39) and Ξ denote the feasible region of the above model. If the penalty is infinite, then $\widehat{\psi}(\Theta,\mathbf{l})$ is an upper bound of $\psi(\Theta)$. For a feasible BEV transportation network lane expansion scheme $\mathbf{l}$, we let $\overline{\Theta}$ denote the solution to BNDPM-IN. $\widehat{\Theta}$ represents the solution to P-BDNPM-IN($\mathbf{l}$) for some $B>0$. Then, we can obtain the following relationship:

$$\begin{cases} \psi(\overline{\Theta}) = \widehat{\psi}(\overline{\Theta},\mathbf{l}), \\ \widehat{\psi}(\overline{\Theta},\mathbf{l}) \le \widehat{\psi}(\widehat{\Theta},\mathbf{l}). \end{cases} \tag{40}$$

When $\widehat{\psi}(\overline{\Theta},\mathbf{l})$ is minimized, $B\sum_{w\in W}\sum_{p\in P_w}(\overline{c}_p^w - \sum_{a\,\in\,A(p)} t_a(v_a, l_{(a,k)}) - c_p^w)^2$ in the objective function equals zero, so the equation in equation (40) holds. Moreover, since $\overline{\Theta}$ is a feasible solution to P-BDNPM-IN($\mathbf{l}$), the inequality in equation (40) also holds. Based on these notations, we develop a penalized BDNPM (short as BDNPM-P) as follows.

BDNPM-P:

$$\min_{\mathbf{u}} \max_{\Theta\in\Xi} \widehat{\psi}(\Theta,\mathbf{l}), \tag{41}$$

subject to constraints (16), (17), and (18).

Note that BDNPM-P is an ordinary semi-infinite optimization problem because the feasible region of the inner problem (i.e., Ξ) is not related to the variable$\mathbf{l}$. To apply the cutting-plane method, we introduce an auxiliary variable ς and convert BDNPM-P into BDNPM-P-1.

BDNPM-P-1:

$$\min_{(\mathbf{l},\varsigma)} \varsigma, \tag{42}$$

subject to $\varsigma \ge \widehat{\psi}(\Theta,\mathbf{l})$ for all $\Theta\in\Xi$ and constraints (16), (17), and (18).

Then, we use $\Theta^1, \Theta^2, \ldots, \Theta^n$ to represent the elements of set Ξ and propose a relaxed version of BDNPM-P-1, namely, R-BDNPM-P-1.

R-BDNPM-P-1:

$$\min_{(\mathbf{l},\varsigma)} \varsigma, \tag{43}$$

subject to $\varsigma \ge \widehat{\psi}(\Theta^i,\mathbf{l})$ for all $i = 1,2,\ldots,n$ and constraints (16), (17), and (18).

It should be noted that, in R-BDNPM-P-1, we introduce a set called $\overline{\Xi} \coloneqq \{\Theta^1, \Theta^2, \ldots, \Theta^n\}$ to approximate set Ξ. Let $(\widetilde{\mathbf{l}},\widetilde{\varsigma})$ denote a global optimal solution to R-BDNPM-P-1. If $(\widetilde{\mathbf{l}},\widetilde{\varsigma})$ is feasible for BDNPM-P-1, then it is also an optimal solution to R-BDNPM-P-1. The feasibility of $(\widetilde{\mathbf{l}},\widetilde{\varsigma})$ can be verified by comparing $\widehat{\psi}(\widetilde{\Theta},\widetilde{\mathbf{l}})$ and $\widetilde{\varsigma}$. Specifically, if the solution $\widetilde{\Theta}$ to P-BDNPM-IN($\widetilde{\mathbf{l}}$) satisfies $\widehat{\psi}(\widetilde{\Theta},\widetilde{\mathbf{l}}) \le \widetilde{\varsigma}$, then $(\widetilde{\mathbf{l}},\widetilde{\varsigma})$ is a feasible (also optimal) solution to BDNPM-P-1. Otherwise, $(\widetilde{\mathbf{l}},\widetilde{\varsigma})$ is infeasible for BDNPM-P-1. At this time, R-BDNPM-P-1 should be solved again to obtain an improved solution with $\Xi \cup \{\widetilde{\Theta}\}$. Based on the above descriptions, we give the solution procedure for the BNDPM as follows:

Step 1: Set $\mathbf{l} = \mathbf{0}$, and solve WC-BRUE with ASA to obtain Θ^1. Set the parameter $\tau = 1$, and let $\overline{\Xi}^{\tau} = \{\Theta^1\}$.

Step 2: Solve R-BDNPM-P-1 with $\overline{\Xi}^{\tau}$, and obtain $(\mathbf{l}^{\tau}, \varsigma^{\tau})^T$.

Step 3: Solve P-BDNPM-IN $(\mathbf{l}^{\tau})$ with $\mathbf{l}^{\tau}$, and obtain $\Theta^{(\tau+1)} = (\mathbf{v}^{(\tau+1)}, \mathbf{f}^{(\tau+1)}, \mathbf{y}^{(\tau+1)}, \varepsilon^{(\tau+1)}, \lambda^{(\tau+1)})$.

Step 4: If $\widehat{\psi}(\Theta^{(\tau+1)}, \mathbf{l}^{\tau}) \le \varsigma^{\tau}$, stop; currently, vector $\mathbf{l}^{\tau}$ is a locally optimal robust lane expansion scheme for the BEV transportation network. Otherwise, go to Step 5.

Step 5: Set $\overline{\Xi}^{(\tau+1)} = \overline{\Xi}^{\tau} \cup \{\Theta^{(\tau+1)}\}$ and $\tau = \tau + 1$, and go to Step 2.

6. Numerical Example

In this section, numerical experiments are performed based on the transportation network in Sioux Falls [64, 65]. As shown in Figure 2, 24 nodes, 76 links, and 90 O-D pairs exist in this network according to Cheng et al. [5].

6.1. Parameter Setting. For the performance parameters of BEVs, we set the BEV power limit to 40 kWh and the initial SOC of all BEVs to 25 kWh according to Cheng et al. [5]. The power consumption per kilometer of all BEVs is set to 0.29 kWh according to the data presented by He et al. [35]. For the parameters in the performance function (i.e., equation (1)), we let $\alpha_1 = 0.15$ and $\alpha_2 = 3$ [5]. Moreover, t_a^{free} and CA_a were set as recommended by He et al. [35]. The link distances are assumed to be 2.5 times the link free-flow travel times. For all O-D pairs, the range anxiety for BEV drivers is set to 0.1 kWh. For the parameters in the CGA model, we let $e_i = 0.3$ for all charging stations, $g_i = \begin{cases} 0.7, & \text{Level A station} \\ 10.0, & \text{Level B station} \\ 40.0, & \text{Level C station} \end{cases}$ min/kWh, and $S_0 = 0.25 S_{\max}$ based on the research of Chen et al. [49]. The bounded rationality parameter for traveler $\overline{\varepsilon}^w$ is set to α times the path travel time in the PRUE state, and we conduct a sensitivity analysis of α. Finally, we set 5 nodes in the transportation network with charging piles. Node 11

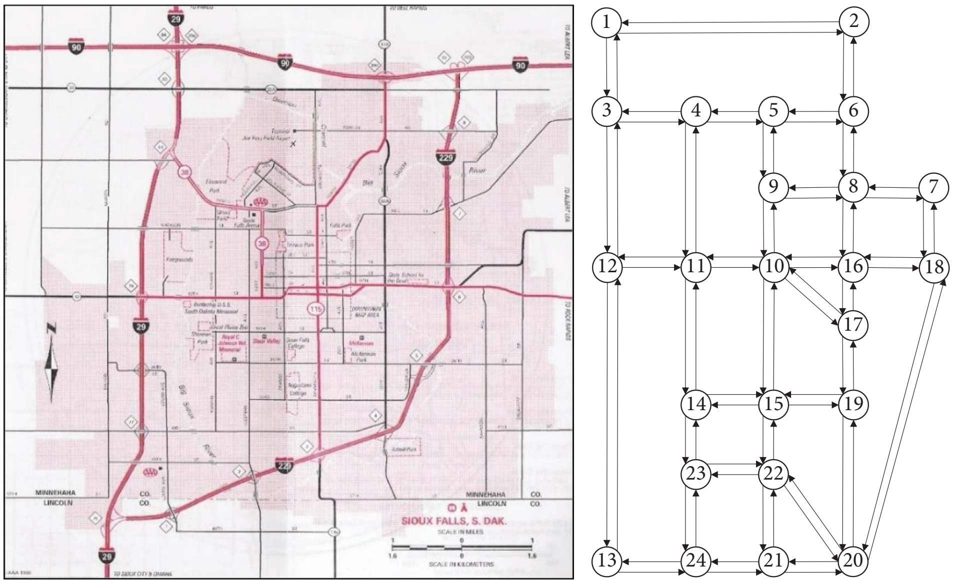

Figure 2: The Sioux Falls network (source: He et al. [66]).

and Node 15 are equipped with Level A charging piles. Node 5 and Node 16 are equipped with Level B charging piles. At Node 12, several Level C charging piles are installed.

All the codes were run on a computer with an Intel (R) Core (TM) i7-1260P CPU and 16 GB of RAM. This computer was manufactured by Lenovo (in China). Among the various models constructed in this article, the nonlinear programming models were solved with CONOPT, the mixed-integer linear programming models were solved with CPLEX 12.4 [67], and the mixed-integer nonlinear programming models were solved with DICOPT [68].

6.2. Sensitivity Analysis of the Degree of Bounded Rationality. In this section, we adjust parameter α associated with bounded rationality to observe the impact of different degrees of rational behavior on the vehicle flow distribution in the BEV transportation network. We set the range of parameter α from 0.00 to 0.20 and perform 20 experiments with 0.01 as the step size. The convergence of the algorithm is shown in Figure 3 (worst case, $\alpha = 0.20$). The best-/worst-case system total travel times in different situations are shown in Figures 4 and 5, respectively.

As parameter α gradually increased, the BEV transportation network total travel time in the best case gradually decreased and tended to stabilize. In contrast, the system travel time in the worst case gradually increased. The above situation is in line with our expectations. As parameter α increases, the choice behaviors of BEV drivers in the transportation network become increasingly irrational. Additionally, a BEV driver can make concessions to facilitate the travel of other individuals. Therefore, the total travel time in the BEV transportation network in the best case will gradually tend to the *system optimal* (SO) state. Conversely, the opposite trend occurs in the worst case. With increasing irrationality (increase in parameter α), the behaviors of BEV drivers may hinder the travel of other individuals in extreme cases, thus increasing the total travel time of the BEV transportation network. Therefore, the difference between the best- and worst-case BRUE states increases. Specifically, in our experiments, the difference between the two increased from 100.26 minutes when $\alpha = 0.01$ to 35,444 minutes when $\alpha = 0.20$. These two values differ by a factor of more than 350.

We further determine the BEV flow distribution for the best and worst cases with $\alpha = 0.05$ and $\alpha = 0.20$. Notably, as parameter α increases, the BEV flow distribution in the PRUE state will not change; however, the difference between the best- and worst-case BRUE states becomes more obvious. For example, when parameter $\alpha = 0.05$, the flow difference for link (1, 2) in the best and worst cases is only 0.41. However, when parameter α increases to 0.20, this flow difference increases to 6.67. Another noteworthy phenomenon is that the flows along some links are 0. Specifically, BEVs have limited SOC capacities, and the range anxiety of travelers may limit vehicle use. Moreover, some links may not be included in a usable path due to the particularities of the corresponding geographical locations.

6.3. Sensitivity Analysis of the Government Investment Scale. In this section, we fix the parameter α and explore the changes in the lane expansion scheme for the BEV transportation network by adjusting the scale of government investment. We set $\alpha = 0.10$, and the investment scale is increased from 0 to 100 in steps of 20. Therefore, six scenarios (C0 to C5) are explored. The optimal lane expansion schemes and system travel times in different scenarios are shown in Table 1. The system travel times in different scenarios are also shown in Figure 6.

Notably, with increased government investment, the number of lane expansions significantly increases. The number of lane expansions increases from 4 in Scenario C1 to 24 in Scenario C5, an increase of 600%. The system travel

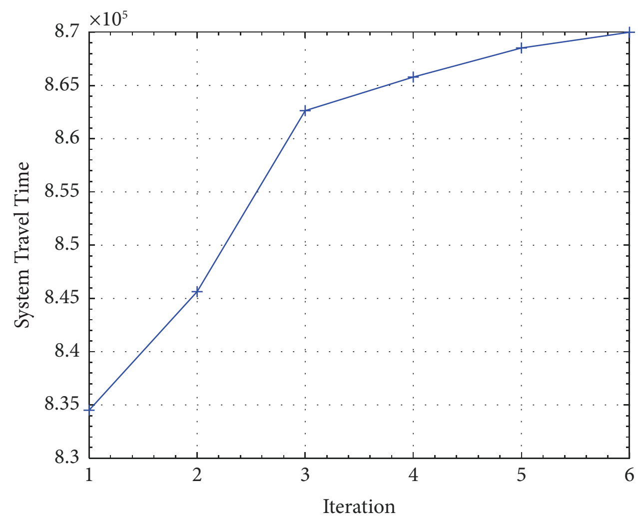

Figure 3: Convergence of the algorithm (worst case, $\alpha = 0.20$).

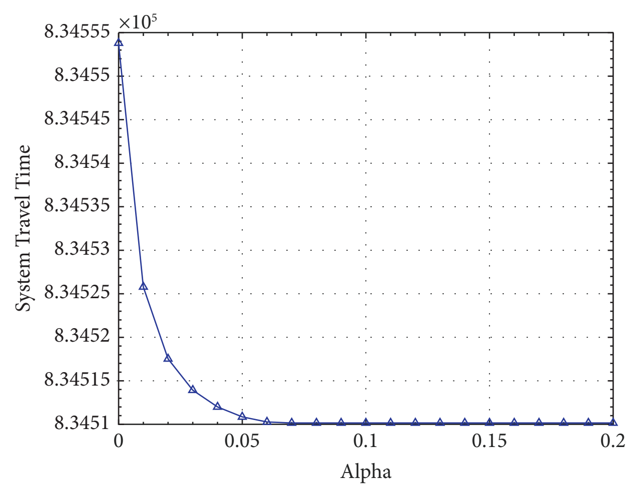

Figure 4: Comparison of best-case total travel times in the network for different α values.

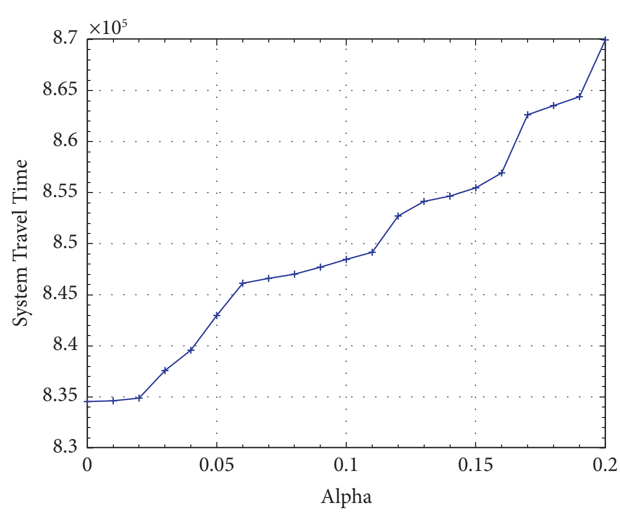

Figure 5: Comparison of worst-case total travel times in the network for different α values.

Table 1: The optimal lane expansion schemes in different scenarios.

Scenario	Investment scale	Optimal lane expansion scheme	System travel time for the worst case (minutes)	Program run time (s)
C0	0	None	848468.903	346
C1	20	15–19 (one lane) and 23-22 (three lanes)	846289.057	387
C2	40	2–6 (three lanes) and 9–5(there lanes)	840678.151	356
C3	60	2–6 (three lanes), 4–11 (one lane), 6-5 (three lanes), 9–5 (there lanes), and 17–19 (one lane)	838213.004	324
C4	80	3-4 (three lanes), 3–12 (three lanes), 4-5 (three lanes), 4–11 (one lane), 6–8 (three lanes), 8-7 (two lanes), 8-9 (one lane), 12-11 (one lane), and 12-13 (three lanes)	836600.272	374
C5	100	3-4 (three lanes), 3–12 (three lanes), 4-5 (three lanes), 4–11 (one lane), 6–5 (one lane), 9–5 (one lane), 12-13 (three lanes), 19–17 (one lane), 22–15 (two lanes), 23-22 (three lanes), and 24-23 (three lanes)	834553.847	365

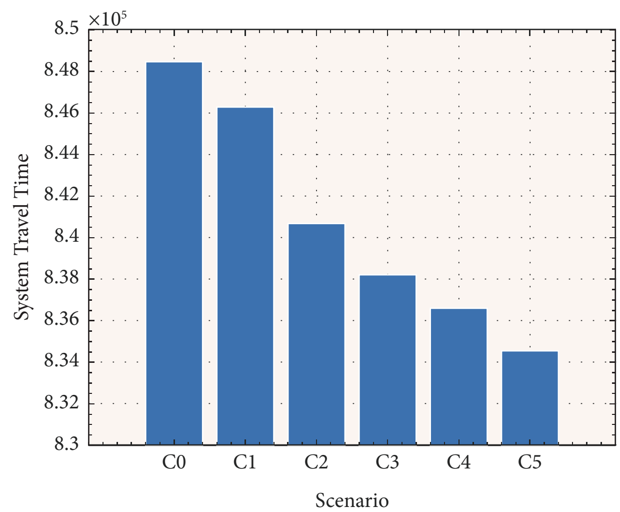

Figure 6: System travel times under different scenarios.

time is reduced by 13915.06 minutes; however, this decreasing trend is not significant. Specifically, the limited travel distance of BEVs and traveler anxiety limit the number of usable paths available in the BEV transportation network. Thus, the government could further reduce the travel time of vehicles in the system by simultaneously establishing charging stations and implementing lane expansion.

In addition, Figure 6 shows that the system travel time insignificantly decreases when the investment amount increases from 20 to 40. Subsequently, the effect of investment is not as obvious as that in the previous scenarios. This result suggests that government investment is characterized by diminishing marginal returns. In other words, with increasing government investment, the corresponding rate of decrease in system travel time will first increase and then decrease. Based on the current parameters, the government's optimal investment scale should be 40. The above phenomenon indicates that for different investment goals, the government should scientifically set the investment scale to maximize social welfare.

On the one hand, as the scale of government investment increases, the number of lanes included in expansion gradually increases. This finding is consistent with the results in the table. On the other hand, the links where the expanded lanes are located are almost all connected to nodes with charging stations (e.g., Node 5, Node 11, Node 12, and Node 16). This situation is in line with expectations. The links connected to nodes with charging stations are more likely to become components of usable paths. Moreover, as the scale of government investment increases, some links repeatedly appear in lane expansion schemes (e.g., links 2–6, 4–11, and 5–9). These links are bottlenecks in the current transportation network. The government should focus on such bottleneck sections in the system and expand them first to effectively alleviate congestion in the entire network.

6.4. Managerial Insights. Based on the above calculation results, we can obtain the following managerial insights:

(1) The bounded rational behavior of BEV drivers significantly affects the total system travel time. This suggests that the government must consider the diverse travel behaviors of travelers when developing the transportation infrastructure. Due to the irrational behavior of travelers, there may be deviations between the distribution of traffic flow and the government's forecast, which in turn could result in a large number of ineffective investment policies. The government must fully investigate the travel behaviors of users in a region and then implement appropriate decisions regarding BEV transportation network design or other transportation infrastructure construction. Moreover, as the bounded rationality threshold increases, the system travel time in the best case in the BEV transportation network develops toward the travel time in the SO state. This finding indicates that as the rationality of BEV drivers declines, the optimal solution in the best case tends to revolve around the overall interest of users in the system rather than personal interests. This suggests that the government must take effective measures to compensate BEV drivers and achieve system-level optimization. For example, the government can provide different forms of subsidies to BEV drivers who give up their personal benefits and adhere to certain transportation policies. In addition, some links have low traffic volumes due to being part of unusable links. This is due to the BEV range characteristics. The government should implement timely infrastructure development (e.g., lane expansion) based on the range of BEVs to enable BEV drivers to take full advantage of the urban infrastructure.

(2) The government's investment in the BEV transportation network exhibits diminishing marginal returns and key link effects. On the one hand, as the amount of investment increases, the effect of investment in the construction of the transportation network infrastructure does not increase

proportionally. When the investment reaches a certain scale, the travel time in the BEV transportation network system will not be proportionally reduced. This is the classic principle of diminishing marginal returns in economics. The above results suggest that the government must reasonably determine the scale of investment when implementing large investment projects related to BEV transportation network design. The government must find a balance between the investment scale and investment effect to avoid wasting financial resources. On the other hand, there are some key links in BEV transportation networks. Regardless of the scale of government investment, some links are always included in the optimal investment scheme identified with the model. This finding further suggests that the government must pay attention to these special links. When the investment scale is relatively small, priority should be given to the expansion of these key links. Thus, the effect of government investment can be maximized, and the total travel time of drivers can be effectively reduced.

7. Conclusions

In this article, a BEV transportation NDP is investigated. To address issues related to the mileage limitations of BEVs, traveler range anxiety, and bounded rationality, the problem is formulated as an MPCC. The model aims to minimize the system travel time in BEV transportation networks and establish an optimal lane expansion scheme based on the available investment budget. An ASA-based heuristic algorithm is proposed to solve the real-world-scale problem.

Numerical experiments are performed to assess the impacts of different traveler behaviors on travel times. Moreover, a sensitivity analysis of the investment scale is performed to explore the optimal level of government investment. The experimental results show that different levels of rationality among travelers influence the flow distribution in the best- and worst-case scenarios to various degrees. The greater the degree of irrationality is, the closer the best case is to the SO state of the BEV transportation network. Government investment is characterized by the law of diminishing marginal returns. Most of the links included in the investment scheme are connected to nodes with charging stations. Therefore, the experimental results verify that there are some bottleneck links in the BEV transportation network. Mitigating these bottlenecks is the key to improving network efficiency. In addition, the solution approach can be efficiently used to support the proposed network design scheme and can potentially be applied to even larger networks.

Overall, the model and algorithms we propose can effectively provide decision-making support for the government in infrastructure construction in BEV transportation networks. We extend the bounded rationality model proposed by [55] for BEV transportation network design. Through the ongoing adjustment of the model, the robust joint optimization of congestion tolls and network design could be achieved.

Future work could focus on the collaborative optimization of different transportation network design schemes based on BEV characteristics. For example, the collaborative optimization of construction schemes for charging facilities (e.g., wireless charging roads and charging stations) considering bounded rationality is an interesting research direction. In addition, the energy consumption of BEVs varies for different traffic flows. Determining how to incorporate these variations into the proposed model framework is also a potential research direction. In addition, uncertainties are not considered in this study (e.g., see the article considering uncertainty factor [69]), and it will be an interesting research direction to incorporate uncertainties into the research.

Data Availability

The data used to support the findings of this study are included within the article and are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] World Meteorological, *The Atlas of Mortality and Economic Losses from Weather, Climate and Water Extremes*, p. 2021, 1970–2019.

[2] Z. Chu, M. Cheng, and N. N. Yu, “A smart city is a less polluted city,” *Technological Forecasting and Social Change*, vol. 172, Article ID 121037, 2021.

[3] X. Zhao, Y. Shang, and M. Song, “Industrial structure distortion and urban ecological efficiency from the perspective of green entrepreneurial ecosystems,” *Socio-Economic Planning Sciences*, vol. 72, Article ID 100757, 2020.

[4] Y. Wang, Y. Zhou, L. Zhu, F. Zhang, and Y. Zhang, “Influencing factors and decoupling elasticity of China’s transportation carbon emissions,” *Energies*, vol. 11, no. 5, p. 1157, 2018.

[5] K. Cheng, Y. Zou, X. Xin, and S. Gong, “Optimal lane expansion model for a battery electric vehicle transportation network considering range anxiety and demand uncertainty,” *Journal of Cleaner Production*, vol. 276, Article ID 124198, 2020.

[6] R. Xie, J. Fang, and C. Liu, "The effects of transportation infrastructure on urban carbon emissions," *Applied Energy*, vol. 196, pp. 199–207, 2017.
[7] M. Xu, T. Wu, and Z. Tan, "Electric vehicle fleet size for carsharing services considering on-demand charging strategy and battery degradation," *Transportation Research Part C: Emerging Technologies*, vol. 127, Article ID 103146, 2021.
[8] L. Hu, J. Dong, and Z. Lin, "Modeling charging behavior of battery electric vehicle drivers: a cumulative prospect theory based approach," *Transportation Research Part C: Emerging Technologies*, vol. 102, pp. 474–489, 2019.
[9] R. A. Waraich, M. D. Galus, C. Dobler, M. Balmer, G. Andersson, and K. W. Axhausen, "Plug-in hybrid electric vehicles and smart grids: investigations based on a microsimulation," *Transportation Research Part C: Emerging Technologies*, vol. 28, pp. 74–86, 2013.
[10] X. Zhao, Y. Ye, J. Ma, P. Shi, and H. Chen, "Construction of electric vehicle driving cycle for studying electric vehicle energy consumption and equivalent emissions," *Environmental Science and Pollution Research*, vol. 27, no. 30, pp. 37395–37409, 2020.
[11] A. Colmenar-Santos, A.-M. Muñoz-Gómez, E. Rosales-Asensio, and Á. López-Rey, "Electric vehicle charging strategy to support renewable energy sources in Europe 2050 low-carbon scenario," *Energy*, vol. 183, pp. 61–74, 2019.
[12] B. Lin, B. Ghaddar, and J. Nathwani, "Electric vehicle routing with charging/discharging under time-variant electricity prices," *Transportation Research Part C: Emerging Technologies*, vol. 130, Article ID 103285, 2021.
[13] A. Montoya, C. Guéret, J. E. Mendoza, and J. G. Villegas, "A multi-space sampling heuristic for the green vehicle routing problem," *Transportation Research Part C: Emerging Technologies*, vol. 70, pp. 113–128, 2016.
[14] M. Schiffer and G. Walther, "The electric location routing problem with time windows and partial recharging," *European Journal of Operational Research*, vol. 260, no. 3, pp. 995–1013, 2017.
[15] M. Xu, H. Yang, and S. Wang, "Mitigate the range anxiety: siting battery charging stations for electric vehicle drivers," *Transportation Research Part C: Emerging Technologies*, vol. 114, pp. 164–188, 2020.
[16] Q. Yuan, W. Hao, H. Su, G. Bing, X. Gui, and A. Safikhani, "Investigation on range anxiety and safety buffer of battery electric vehicle drivers," *Journal of Advanced Transportation*, vol. 2018, pp. 1–11, 2018.
[17] H. Wang, G. Zhou, R. Xue, Y. Lu, and J. A. McCann, "A driving-behavior-based SOC prediction method for light urban vehicles powered by supercapacitors," *IEEE Transactions on Intelligent Transportation Systems*, vol. 21, no. 5, pp. 2090–2099, 2020.
[18] T. Miwa, H. Sato, and T. Morikawa, "Range and battery depletion concerns with electric vehicles," *Journal of Advanced Transportation*, vol. 2017, Article ID 7491234, 12 pages, 2017.
[19] J. Neubauer and E. Wood, "The impact of range anxiety and home, workplace, and public charging infrastructure on simulated battery electric vehicle lifetime utility," *Journal of Power Sources*, vol. 257, pp. 12–20, 2014.
[20] Y. Yang, E. Yao, Z. Yang, and R. Zhang, "Modeling the charging and route choice behavior of BEV drivers," *Transportation Research Part C: Emerging Technologies*, vol. 65, pp. 190–204, 2016.
[21] M. Xu, Q. Meng, and K. Liu, "Network user equilibrium problems for the mixed battery electric vehicles and gasoline vehicles subject to battery swapping stations and road grade constraints," *Transportation Research Part B: Methodological*, vol. 99, pp. 138–166, 2017.
[22] D. Kai-Ineman and A. Tversky, "Prospect theory: an analysis of decision under risk," *Econometrica*, vol. 47, no. 2, pp. 363–391, 1979.
[23] R. Ilin and G. L. Rogova, "Decision-making involving low probability high consequence events under risk and uncertainty," in *Proceedings of the 2017 IEEE Conference on Cognitive and Computational Aspects of Situation Management (CogSIMA)*, IEEE, Savannah, GA, USA, March 2017.
[24] H. S. Mahmassani and G.-L. Chang, "Experiments with departure time choice dynamics of urban commuters," *Transportation Research Part B: Methodological*, vol. 20, no. 4, pp. 297–320, 1986.
[25] T. Schwanen and D. Ettema, "Coping with unreliable transportation when collecting children: examining parents' behavior with cumulative prospect theory," *Transportation Research Part A: Policy and Practice*, vol. 43, no. 5, pp. 511–525, 2009.
[26] L. Zhou, S. Zhong, S. Ma, and N. Jia, "Prospect theory based estimation of drivers' risk attitudes in route choice behaviors," *Accident Analysis & Prevention*, vol. 73, pp. 1–11, 2014.
[27] S. N. Hadjidimitriou, M. Dell'Amico, G. Cantelmo, and F. Viti, "Assessing the consistency between observed and modelled route choices through GPS data," in *Proceedings of the 2015 International Conference on Models and Technologies for Intelligent Transportation Systems (MT-ITS)*, IEEE, Budapest, Hungary, June 2015.
[28] S. Zhu and D. Levinson, "Do people use the shortest path? An empirical test of Wardrop's first principle," *PLoS One*, vol. 10, no. 8, Article ID e0134322, 2015.
[29] W. Samuelson and R. Zeckhauser, "Status quo bias in decision making," *Journal of Risk and Uncertainty*, vol. 1, no. 1, pp. 7–59, 1988.
[30] T. Lotan, "Effects of familiarity on route choice behavior in the presence of information," *Transportation Research Part C: Emerging Technologies*, vol. 5, no. 3-4, pp. 225–243, 1997.
[31] E. A. Bogers, F. Viti, and S. P. Hoogendoorn, "Joint modeling of advanced travel information service, habit, and learning impacts on route choice by laboratory simulator experiments," *Transportation Research Record*, vol. 1926, no. 1, pp. 189–197, 2005.
[32] H. González Ramírez, L. Leclercq, N. Chiabaut, C. Becarie, and J. Krug, "Travel time and bounded rationality in travellers' route choice behaviour: a computer route choice experiment," *Travel Behaviour and Society*, vol. 22, pp. 59–83, 2021.
[33] H. S. Mahmassani and G.-L. Chang, "On boundedly rational user equilibrium in transportation systems," *Transportation Science*, vol. 21, no. 2, pp. 89–99, 1987.
[34] O. A. Eikenbroek, G. J. Still, E. C. van Berkum, and W. Kern, "The boundedly rational user equilibrium: a parametric analysis with application to the network design problem," *Transportation Research Part B: Methodological*, vol. 107, pp. 1–17, 2018.
[35] F. He, Y. Yin, and S. Lawphongpanich, "Network equilibrium models with battery electric vehicles," *Transportation Research Part B: Methodological*, vol. 67, pp. 306–319, 2014.
[36] J. G. Wardrop and J. I. Whitehead, "Correspondence. some theoretical aspects of road traffic research," *Proceedings - Institution of Civil Engineers*, vol. 1, no. 5, pp. 767-768, 1952.
[37] D. Z. Wang and H. K. Lo, "Global optimum of the linearized network design problem with equilibrium flows,"

Transportation Research Part B: Methodological, vol. 44, no. 4, pp. 482–492, 2010.
[38] W. Y. Szeto, Y. Jiang, D. Wang, and A. Sumalee, "A sustainable road network design problem with land use transportation interaction over time," *Networks and Spatial Economics*, vol. 15, no. 3, pp. 791–822, 2015.
[39] S. Wang, Q. Meng, and H. Yang, "Global optimization methods for the discrete network design problem," *Transportation Research Part B: Methodological*, vol. 50, pp. 42–60, 2013.
[40] H. Liu and D. Z. Wang, "Modeling and solving discrete network design problem with stochastic user equilibrium," *Journal of Advanced Transportation*, vol. 50, no. 7, pp. 1295–1313, 2016.
[41] P. Luathep, A. Sumalee, W. H. Lam, Z.-C. Li, and H. K. Lo, "Global optimization method for mixed transportation network design problem: a mixed-integer linear programming approach," *Transportation Research Part B: Methodological*, vol. 45, no. 5, pp. 808–827, 2011.
[42] E. Miandoabchi, R. Z. Farahani, W. Dullaert, and W. Y. Szeto, "Hybrid evolutionary metaheuristics for concurrent multi-objective design of urban road and public transit networks," *Networks and Spatial Economics*, vol. 12, no. 3, pp. 441–480, 2012.
[43] K. An and H. K. Lo, "Robust transit network design with stochastic demand considering development density," *Transportation Research Procedia*, vol. 7, pp. 300–319, 2015.
[44] A. Nayan and D. Z. Wang, "Optimal bus transit route packaging in a privatized contracting regime," *Transportation Research Part A: Policy and Practice*, vol. 97, pp. 146–157, 2017.
[45] J. Ekström, A. Sumalee, and H. K. Lo, "Optimizing toll locations and levels using a mixed integer linear approximation approach," *Transportation Research Part B: Methodological*, vol. 46, no. 7, pp. 834–854, 2012.
[46] X. He, X. Chen, C. Xiong, Z. Zhu, and L. Zhang, "Optimal time-varying pricing for toll roads under multiple objectives: a simulation-based optimization approach," *Transportation Science*, vol. 51, no. 2, pp. 412–426, 2017.
[47] H. Yang and H.-J. Huang, *Mathematical and Economic Theory of Road Pricing*, 2005.
[48] R. Z. Farahani, E. Miandoabchi, W. Y. Szeto, and H. Rashidi, "A review of urban transportation network design problems," *European Journal of Operational Research*, vol. 229, no. 2, pp. 281–302, 2013.
[49] Z. Chen, F. He, and Y. Yin, "Optimal deployment of charging lanes for electric vehicles in transportation networks," *Transportation Research Part B: Methodological*, vol. 91, pp. 344–365, 2016.
[50] Z. Liu and Z. Song, "Network user equilibrium of battery electric vehicles considering flow-dependent electricity consumption," *Transportation Research Part C: Emerging Technologies*, vol. 95, pp. 516–544, 2018.
[51] R. Chen, X. Qian, L. Miao, and S. V. Ukkusuri, "Optimal charging facility location and capacity for electric vehicles considering route choice and charging time equilibrium," *Computers & Operations Research*, vol. 113, Article ID 104776, 2020.
[52] Y. Qiu, Y. Du, S. Yu, and S. Jiang, "Location design of electrification road in transportation networks for on-way charging," *Journal of Advanced Transportation*, vol. 2020, Article ID 7096767, 13 pages, 2020.
[53] E. Siri, S. Siri, and S. Sacone, "Day-to-day discrete-time traffic assignment model for transport networks affected by disruptive events," *European Journal of Control*, vol. 68, Article ID 100684, 2022.
[54] X. Di, H. X. Liu, J.-S. Pang, and X. J. Ban, "Boundedly rational user equilibria (BRUE): mathematical formulation and solution sets," *Transportation Research Part B: Methodological*, vol. 57, pp. 300–313, 2013.
[55] Y. Lou, Y. Yin, and S. Lawphongpanich, "Robust congestion pricing under boundedly rational user equilibrium," *Transportation Research Part B: Methodological*, vol. 44, no. 1, pp. 15–28, 2010.
[56] X. Di, H. X. Liu, and X. J. Ban, "Second best toll pricing within the framework of bounded rationality," *Transportation Research Part B: Methodological*, vol. 83, pp. 74–90, 2016.
[57] D. Wang, F. Liao, Z. Gao, and H. Timmermans, "Tolerance-based strategies for extending the column generation algorithm to the bounded rational dynamic user equilibrium problem," *Transportation Research Part B: Methodological*, vol. 119, pp. 102–121, 2019.
[58] S. Batista and L. Leclercq, "Regional dynamic traffic assignment with bounded rational drivers as a tool for assessing the emissions in large metropolitan areas," *Transportation Research Interdisciplinary Perspectives*, vol. 8, Article ID 100248, 2020.
[59] Y. Jiang and A. A. Ceder, "Incorporating personalization and bounded rationality into stochastic transit assignment model," *Transportation Research Part C: Emerging Technologies*, vol. 127, Article ID 103127, 2021.
[60] E. Polak and J. O. Royset, "On the use of augmented Lagrangians in the solution of generalized semi-infinite min-max problems," *Computational Optimization and Applications*, vol. 31, no. 2, pp. 173–192, 2005.
[61] J. O. Royset, E. Polak, and A. Kiureghian, "Adaptive approximations and exact penalization for the solution of generalized semi-infinite min-max problems," *SIAM Journal on Optimization*, vol. 14, no. 1, pp. 1–34, 2003.
[62] X. Xin, X. Wang, L. Ma, K. Chen, and M. Ye, *Shipping Network Design–Infrastructure Investment Joint Optimization Model: A Case Study of West Africa*, Maritime Policy & Management, 2021.
[63] S. Lawphongpanich and D. W. Hearn, "An MPEC approach to second-best toll pricing," *Mathematical Programming*, vol. 101, no. 1, pp. 33–55, 2004.
[64] L. J. Leblanc, "An algorithm for the discrete network design problem," *Transportation Science*, vol. 9, no. 3, pp. 183–199, 1975.
[65] J. Long, Z. Gao, H. Zhang, and W. Y. Szeto, "A turning restriction design problem in urban road networks," *European Journal of Operational Research*, vol. 206, no. 3, pp. 569–578, 2010.
[66] J. He, H. Yang, T.-Q. Tang, and H.-J. Huang, "Optimal deployment of wireless charging lanes considering their adverse effect on road capacity," *Transportation Research Part C: Emerging Technologies*, vol. 111, pp. 171–184, 2020.
[67] I. Cplex, *11.0 Reference Manual*, ILOG CPLEX Division, Gentilly, Paris, 2008.
[68] G. R. Kocis and I. E. Grossmann, "Computational experience with DICOPT solving MINLP problems in process systems engineering," *Computers & Chemical Engineering*, vol. 13, no. 3, pp. 307–315, 1989.
[69] W. Wang and Y. Wu, "Is uncertainty always bad for the performance of transportation systems?" *Communications in transportation research*, vol. 1, Article ID 100021, 2021.