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Research Article

Safety Evaluation for Highway Geometric Design Based on Spatial Path Properties

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Safety is an important aspect of road design. In highway geometric design, road engineers use a two-stage design method due to its convenience, but the available two-dimensional design tools on the market are believed to impose limitations. Horizontal and vertical alignment parameters are considered and determined at the designer's discretion, potentially downplaying the three-dimensional characters of spatial curves. This study focuses on the three-dimensional characteristics of highway alignments and investigates a safety evaluation method to establish the relationship between crash rate and spatial curve properties. This will necessitate an analytical investigation into the effects of higher-order properties, such as curvature and torsion, on the geometry of spatial Cartesian curves. First, these combinations of horizontal and vertical alignments were categorized into six classifications, each with its own spatial expression in mathematical form. After manipulating the curvature and torsion of the spatial curve algebraically, the correlation between geometric design variables and crash rate was ultimately established. A few cases involving geometric design data and crash facts were utilized for verification. The results revealed a considerable positive correlation between curvature or torsion variance and crashes per million vehicles kilometers, as a slight difference between curvature and torsion could also be spotted. And curvature distribution is correlated with collision frequency more closely than torsion spatial variation.

1. Introduction

Highway geometric design should not only meet the needs of traffic but also consider social, economic, environmental, and other factors. The composition of a road's spatial alignment is a comprehensive product of an integrated part of an integrated transportation process. Since the 1930s, national road design standards, policy studies, and guidelines have been implemented to address economic, environmental, and other social issues [1, 2]. However, more and more people are affected by man-made road disasters, making road risk control crucial [3]. Among them, the safety of highway alignment design is the primary consideration, and safety is also a vital factor in road design that may have an impact on other social compositions [4]. According to the data of the World Health Organization in 2022, approximately 1.3 million people around the world have their lives terminated due to road traffic accidents every year. The concept of flexibility design believes that there are many road accidents related to the unreasonable design of highway alignment [5, 6]. This makes it necessary for designers to re-examine the importance of road design. It is likely that the economy and safety of the highway construction are in conflict, but a reasonable safety design could be the best compromise [6]. For the purpose of sustainability, research should place a greater emphasis on the consistency of dynamic route design from the perspective of improved personal and property safety. However, the traditional two-dimensional evaluation methods may not truly reflect the real situation of road alignment design.

Horizontal alignment is of paramount significance to location in highway geometric design, while vertical alignment is restricted by altitude and has great impacts on project costs, including earthwork, construction, land use, and user costs [5]. Due to the complexity of three-dimensional (3D) spatial curve design, a two-stage method is typically adopted to facilitate the combinations of horizontal and vertical alignments (spatial curve) by completing the horizontal location first and then the vertical design [7, 8]. In the two stages, quantitative reference values are assigned to design elements as control standards to meet fundamental safety and comfort requirements. However, for horizontal alignment and profile combinations, previous studies appeared to be limited to qualitative analysis.

3D highway alignment design and safety evaluation are a persistent problem for which there are no always-satisfactory implementation methods. How to comprehensively consider the coordination of horizontal and vertical alignments is challenging. Several studies have begun to contemplate and investigate combined highway alignment design [9–11]. To optimise highway design, Wang et al. [7] and Zhou et al. [12] created a method for constructing a 3D model of highway alignments using parameter programming. The use of visualization technology was an alternative way of creating a 3D model of highway alignments. It was believed to provide an opportunity for designers to calculate virtual perspective views and control specific parameters with visualization tools to verify the 3D alignment [13]. In addition, spatial curve fitting to highway alignments was supposed to be beneficial to the better design of the next generation of highways and future research on vehicle infrastructure integration, route guidance, and congestion management.

Although a route is a spatial curve with randomness in design, it has certain regularity consisting primarily of six types of horizontal and vertical elements combinations (including tangent segment, circular curve, and spiral curve of horizontal alignment and tangent segment and parabola curve of vertical alignment). This paper aims to build a parametric representation of highway geometric alignments based on the mathematical properties of curves in order to evaluate the interrelations between crash rate and spatial path planning.

The remaining sections of the paper are structured as follows. Literature review is presented in Section 2, discussing evaluation methods for highway geometric, limitations of 2D methods, and contributions of 3D models. In Section 3, spatial curve properties used in highway design and 3D geometric models of spatial combinations are introduced. Meanwhile, the third section provides an evaluation method for design indexes. In Section 4, the application and discussion of the new approach are illustrated through case studies, and analysis results are provided. Conclusions and significant findings are summarized in the final section.

2. Literature Review

Road design research has made significant progress in recent decades. Advanced computer simulations and new materials have enabled improvements in road quality and safety features. In the early stages of highway geometric design, it may be practical and feasible to measure and evaluate safety by comparing computations to design policy. When the design parameters are fulfilled, the road is considered to be safe and reliable. Interactive highway safety design model (IHSDM) is a typical application that provides a policy review module to compare roadway segment geometry to pertinent design policy [14]. It was discovered that the evaluation of highway geometric design consistency checks is an important issue in design and traffic operation evaluation [15]. Because only considering separated indexes that meet design standards does not guarantee the alignment fluidity and safety of the highway, the coordination research on horizontal and vertical alignments is a critical procedure that should commence with a preliminary design in order to be easily adaptable. IHSDM also supports the evaluation module for design consistency [16]. The review shows that operating speed consistency [17, 18], traffic volume and crash consistency [19, 20], vehicle stability and driver workload consistency [21], and design consistency of highway elements as well as sight distance consistency [22, 23] were considered to be the research focus.

However, there are still key issues that need to be addressed. Road accidents remain a significant global public health concern. Existing road standards and design practices do not adequately account for the complex physics. Previous studies focused more on qualitative analysis by external means, such as engineering experience, a two-dimensional (2D) analysis model, and polynomial or spline fitting [24]. In essence, highway alignments are 3D spatial curves but horizontal and vertical alignments are treated differently in 2D projections. Thus, the evaluation of geometric continuity is calculated in two planes, respectively. It cannot precisely reflect the characteristics of spatial alignment continuity in 3D Euclidean space [25]. Moreover, research studies in 2D lack explicit quantitative coordination of combined alignments [25].

3D alignment models provide a more comprehensive understanding of the driving experience. They enable designers to visualize clear-sight triangles, simulate curves and grades, identify concealed dangers, and analyze collisions from multiple angles [26]. Studies have demonstrated that 3D alignment design significantly improves safety over 2D methods [23]. For example, 3D models optimise sight distance on hills and around curves based on the eye height of the driver more effectively [27]. The occurrence of collisions can be reduced by incorporating appropriate gradients and curves on crucial sections of roadway [28]. Alternately, 3D alignment design aids in reducing the travel time of motorists and releasing the loss caused by traffic congestion [29].

3. Methodology

3.1. Basic Properties of a Spatial Curve. In highway geometric design, five line-type elements including tangent segment, transition curves (Euler spiral), circular curves in the horizontal plane (3 types), and tangent and parabola segments in the vertical plane (2 types), are used to build the complete path by permutations and combinations in the space coordinate system. Different combinations of horizontal and vertical elements will generate different parametric equations. Six types of combinations are shown in Table 1, and their space shapes are descripted [30].

Table 1: Combination of horizontal and vertical alignments.

Spatial position	Vertical alignment	Shape description
Horizontal alignment	Tangent & †tangent	Spatial straight line
	Tangent & parabola	Crest or sag curve
	Euler spiral & tangent	Conical helix
	Euler spiral & parabola	Conical helix
	Circular curve & tangent	Cylindrical helix
	Circular curve & parabola	Cylindrical helix

Note. Sign & †denotes superposition of two line elements.

The coordinate points of the spatial curve (x, y, z) (see Figure 1) can be represented by the following parametric equation:

$$r(x, y, z) = \left[F_x(l), F_y(l), F_z(l)\right], \quad (1)$$

where l represents arc length (corresponding to stake number of highway).

3.1.1. Tangent. The parametric equation of a straight line in two-dimensional Cartesian space is shown in the following equation:

$$\begin{cases} x = x_0 + l\cos\alpha, \\ y = y_0 + l\sin\alpha, \end{cases} \quad (2)$$

where (x_0, y_0) is a starting point for calculating in the straight line; l is the distance between the calculated point and the point (x_0, y_0), m; α is an angle between tangent and X-axis, °.

If the line is a combination of horizontal and vertical tangent that locates in 3D Cartesian space ($\mathbb{R}^3$), the parametric equation can be written in the following equation:

$$\begin{cases} x = x_0 + l\cos\alpha, \\ y = y_0 + l\sin\alpha, \\ z = z_0 + li, \end{cases} \quad (3)$$

where (x_0, y_0, z_0) is a starting point for calculating in $\mathbb{R}^3$ and i denotes the slope of spatial straight line.

3.1.2. Spiral Transition Curve. Generally, the Euler spiral is used in the design of spiral transition curves [31]. The curvature radius varies from infinite at the tangent end of the spiral to the radius of the circular arc at the end that adjoins the circular arc ($\infty \longrightarrow R$). The radius of curvature at any point on an Euler spiral varies inversely with the distance measured along the spiral.

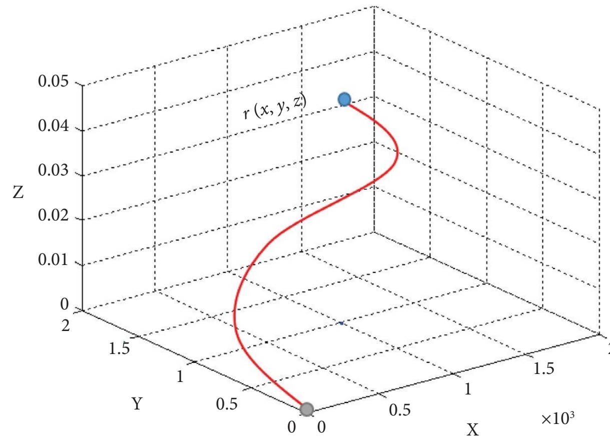

Figure 1: Presentation of the highway spatial curve.

Defining l as the length of a spiral curve beginning with a straight segment, R as the corresponding radius, and β as the deflection angle of this segment in radians, Euler's spiral is formulated as follows:

$$A^2 = lR = \frac{l^2}{2\beta} = 2\beta R^2, \qquad \beta = \frac{l^2}{2A^2}, \quad (4)$$

where A is the spiral parameter; l is the segment length of spiral curve; and β is a parameter of the spiral equation that denotes the azimuth angle of the located point along the tangent direction.

Referring to the fundamental theorem of calculus, the geometrical relationship of Euler spiral parameters can be expressed as follows:

$$\begin{cases} dl = \rho d\beta, \\ dx = dl\cos\beta, \\ dy = dl\sin\beta. \end{cases} \quad (5)$$

The parametric equation of the spiral curve is shown in equation (6) through definite integral to l in the interval $[0, l]$.

$$\begin{cases} x = \int_0^l \cos\beta dl = \int_0^l \cos\frac{l^2}{2A^2}dl, \\ y = \int_0^l \sin\beta dl = \int_0^l \sin\frac{l^2}{2A^2}dl. \end{cases} \quad (6)$$

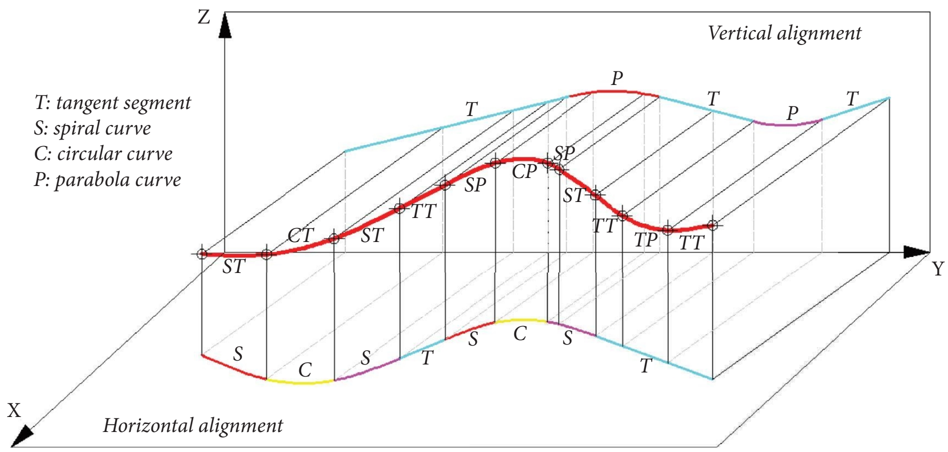

FIGURE 2: Spatial curves combinations model.

3.1.3. Circular Curve. The most common type of horizontal curves used to connect tangent segments of highways is the circular curve. The parametric equation of circular curve is written in the following equation:

$$\begin{cases} x = m + R\cos\theta = m + R\cos\dfrac{l}{R}, \\ \\ y = n + R\sin\theta = n + R\sin\dfrac{l}{R}, \end{cases} \quad (7)$$

where (m, n) denotes the center of a circle; R represents the radius of circular curve; and θ is a parameter of circle equation that denotes the azimuth angle.

3.1.4. Parabola. The vertical curve is the transition section between the deflection points connecting adjacent slopes. Building vertical curves can ensure the smooth connection of geometric alignments and adequate visibility for driving safety. Generally, a quadratic parabola is used to complete the transition.

The equation of a parabola curve can be expressed as $z = al^2 + bl$, where l represents the curve length of a point from the basic point on the parabola. The tangent line of a parabola can be written as $z' = 2al + b$. Supposing the starting point of the parabola curve is (l_0, z_0), i_1 and i_2 denote the slope of tangents before and after the parabola, respectively. S represents the total length of the parabola. Thus, two parameters of the equation can be solved to gain $b = i_1$ and $a = i_2 - i_1/2S$. The equation of parabola curve is shown in the following equation:

$$z = z_0 + \frac{i_2 - i_1}{2S} l^2 + i_1 l. \quad (8)$$

3.2. 3D Highway Geometric Model. According to the selection and explanation of vertical alignments and shape descriptions in Section 3.1, six different combinations are defined in this study (Table 2), where TT represents a spatial combination alignment consisting of straight line in the horizontal projection plane and straight line in the vertical projection plan (Figure 2), TP denotes a spatial combination alignment consisting of straight line in the horizontal projection plane and parabola in the vertical projection plane, ST means a spatial combination alignment consisting of Euler spiral in the horizontal projection plane and straight line in the vertical projection plane, SP means a spatial combination alignment consisting of Euler spiral in the horizontal projection plane and parabola in the vertical projection plane, CT signifies a spatial combination alignment consisting of circular curve in the horizontal projection plane and straight line in the vertical projection plane, and CP signifies a spatial combination alignment consisting of circular curve in the horizontal projection plane and parabola in the vertical projection plane.

Figure 2 shows 6 type combinations in 3D, as already stated in Table 2. Based on the 6 types of line element combinations, any point coordinate in the space along the highway center line can be determined by algebraic representation. For example, the equations of a horizontal spiral or circular curve with a vertical parabola (type SP or CP) can be expressed as formula (9). Besides, the different manifestations of line segments on different planes (xy and zy planes) are defined into four types (T, S, C, and P) to describe the changes in the curvature of a line segment. For example, two S curves were connected by a C curve, which could be considered as a part of the circle. Then, a curve segment could translate smoothly into a straight segment like the T segment shown.

Table 2: Combined forms of highway geometric alignments.

No.	Type	Horizontal alignment	Vertical alignment	Parametric equation
1	TT	Tangent	Tangent	$(x_0 + l\cos\alpha, y_0 + l\sin\alpha, z_0 + li)$
2	TP		Parabola	$(x_0 + l\cos\alpha, y_0 + l\sin\alpha, z_0 + i_2 - i_1/2Sl^2 + i_1 l)$
3	ST	Euler spiral	Tangent	$(\int_0^l \cos l^2/2A^2\,dl, \int_0^l \sin l^2/2A^2\,dl, z_0 + li)$
4	SP		Parabola	$(\int_0^l \cos l^2/2A^2\,dl, \int_0^l \sin l^2/2A^2\,dl, z_0 + i_2 - i_1/2Sl^2 + i_1 l)$
5	CT	Circular curve	Tangent	$(m + R\cos l/R, n + R\sin l/R, z_0 + li)$
6	CP		Parabola	$(m + R\cos l/R, n + R\sin l/R, z_0 + i_2 - i_1/2Sl^2 + i_1 l)$

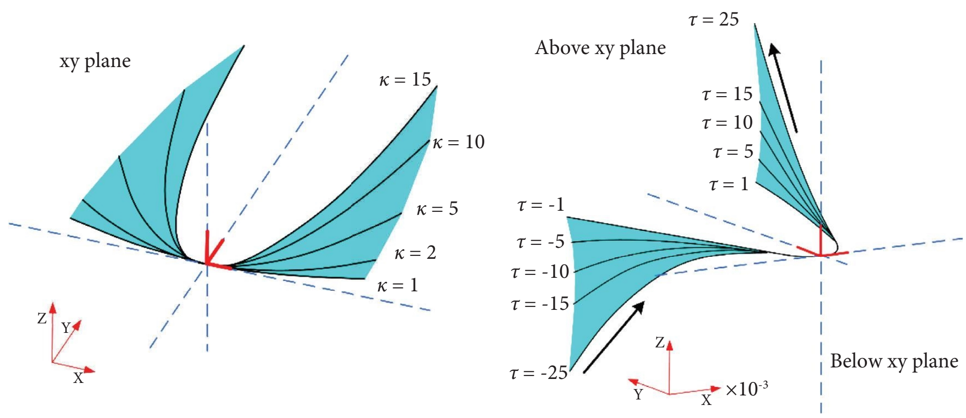

Figure 3: Space relationship of curvature and torsion.

$$r_{SP}(l) = \begin{cases} x = m + R\cos\dfrac{l}{R}, \\ y = n + R\sin\dfrac{l}{R}, \\ z = z_0 + \dfrac{i_2 - i_1}{2S}l^2 + i_1 l, \end{cases} \quad \text{or } r_{CP}(l) = \begin{cases} x = \displaystyle\int_0^l \cos\dfrac{l^2}{2A^2}dl, \\ y = \displaystyle\int_0^l \sin\dfrac{l^2}{2A^2}dl, \\ z = z_0 + \dfrac{i_2 - i_1}{2S}l^2 + i_1 l. \end{cases} \tag{9}$$

3.3. Design Indexes

3.3.1. Curvature. In a spatial parametric curve, the computational formula of curvature κ can be written as shown in equation (10) [32]. The z terms in the equation represent that the computing result is a spatial curvature. If the z terms were removed, the equation would be reduced to planar parametric curves. In a planar curve, the computing value denotes the local reciprocal of curvature and generates an indication of the bending in the curve. Thus, a zero value represents a straight line.

$$\kappa(l) = \frac{\left|r'(l) \times r''(l)\right|}{\left|r'(l)\right|^3} = \frac{\sqrt{\left(y'z'' - y''z'\right)^2 + \left(z'x'' - z''x'\right)^2 + \left(x'y'' - x''y'\right)^2}}{\left(x'^2 + y'^2 + z'^2\right)^{3/2}}, \tag{10}$$

where $x' = dx/dl$ and $x'' = d^2x/dl^2$.

Algebraic operation shows that, for a parametric curve, the curvature value in the equation will always be positive as opposed to being a signed value as in the planar case. It is believed that a spatial curve can technically bend in an infinite number of directions, as opposed to just two directions in a planar curve [33]. As the curve is traversed, the tangent vector can either move in the clockwise or counterclockwise directions, and these two directions respectively correspond to positive and negative curvature. However, in a spatial curve, the tangent vector can rotate in any direction. Thus, curvature is defined as the magnitude of the bending without a direction, and an inflection point in a spatial curve cannot be described as a point where the sign of curvature changes.

Geometrically, curvature is a measure of how quickly the unit tangent is moving with respect to distance along the curve direction. Physically, it represents "bending" in the space of a curve. The reciprocal of curvature is called the radius of curvature denoted by

$$\rho = \frac{1}{\kappa}. \tag{11}$$

March [32] argued that the curvature is the magnitude of the change in direction of the tangent along the curve. When the curvature is zero, the tangent vector will not change, and the curve will continue in a straight line. On the other hand, a high value of curvature will result in the rapid direction changing of the tangent vector. An infinite curvature, as in the case of a cusp, represents the discontinuity in the tangent vector along the curve.

3.3.2. Torsion. Torsion τ is another property of spatial curves that is used to measure the tendency of a curve to twist out of the plane. For a curve in parametric, the computational formula is shown in equation (12) [32], where $x''' = d^3x/dl^3$. Obviously, a curve in the planar will have a zero value torsion. Because the coordinate value of the z terms is zero, thus there will be a zero from first-order derivative to three-order derivative in the z direction. One thing to notice is that, unlike curvature, the torsion of a curve in space has a signed value (see Figure 3).

$$\begin{aligned}\tau(l) &= \frac{\left(r'(l)\times r''(l)\right)\cdot r'''(l)}{\left|r'(l)\times r''(l)\right|^2}\\ &= \frac{\left(y'z''x''' - y''z'x'''\right) + \left(z'x''y''' - z''x'y'''\right) + \left(x'y''z''' - x''y'z'''\right)}{\left(y'z'' - y''z'\right)^2 + \left(z'x'' - z''x'\right)^2 + \left(x'y'' - x''y'\right)^2}.\end{aligned} \tag{12}$$

Physically, torsion is a measurement of the changing rate of the osculating plane (XY plane) relative to the governing parameter l (corresponding to stake number location). The osculating plane is defined as the plane spanned by the curve tangent and normal vectors, namely, $\overrightarrow{T} = [dx/dl\, dy/dl\, dz/dl]$ and $\overrightarrow{N} = d\overrightarrow{T}dl/|d\overrightarrow{T}/dl|$. Therefore, a constant zero torsion denotes a planar curve lying in the osculating plane.

The parametric representation of each spatial curve segment is built in Table 3, and their properties are calculated based on curvature and torsion. This representation provides valuable geometric meaning to the spatial shape of the curve, as the physical meaning of curvature and torsion is well understood.

3.4. Evaluation Model. In order to determine the interrelations of highway geometric alignments and crash rates, the selected highway is divided into several segments in terms of spatial curve combinations. Related characteristic functions revealing the influence of curvature/torsion differences on crashes per million vehicles-kilometers are used to evaluate the quality of highway geometric design.

3.4.1. Local Evaluation. The curvature/torsion difference of adjacent segments is shown in the following equation:

$$\Delta x_i = \left|\overline{x_i} - \overline{x_{i-1}}\right| \%\overset{\propto}{\rightleftharpoons} CR_i, \quad i \in [1, +\infty), \tag{13}$$

where Δx_i denotes the curvature/torsion difference of adjacent combination segments i and $i-1$; $\overline{x_i}$ represents the average value of curvature/torsion in section i; CR_i denotes crash rate of the section i; and $\%\overset{\propto}{\rightleftharpoons}$ means that there is a certain correlation between Δx_i and CR_i.

Crashes per million vehicles-kilometers are calculated to represent the crash rate. The results show each combination segment has a corresponding value of the crash rate ($\Delta\kappa_i/\Delta\tau_i$ to CR_i, where $\Delta\kappa_i/\Delta\tau_i$ denotes curvature/torsion difference of adjacent section i and $i-1$). Thus, the rationality of alignment coordination in design can be determined by the contrast analysis.

3.4.2. Global Evaluation. For the whole highway, the arithmetic mean values of spatial curvature/torsion are calculated as follows:

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}. \tag{14}$$

Then, the standard deviation of spatial curvature/torsion is shown in the following equation:

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}. \tag{15}$$

To evaluate the dispersion of a probability distribution of each spatial curve property (curvature/torsion), the coefficient of variation is introduced, as shown in the following equation:

$$CV = \frac{\sigma}{\overline{x}}. \tag{16}$$

The coordination of horizontal and vertical alignments, through the above calculation, can be described and adjusted to cut down the crash rate.

4. Results and Discussion

The design documents and nearly five years of crash data from 21 roads were collected for study analysis, including 6 expressways and 15 ordinary highways in China. Each road

Table 3: Curvature and torsion computation of geometric alignments.

No.	Type	Parametric equation	Curvature	Torsion
1	TT	$\begin{pmatrix} x_0 + l\cos\alpha \\ y_0 + l\sin\alpha \\ z_0 + li \end{pmatrix}$	0	0
2	TP	$\begin{pmatrix} x_0 + l\cos\alpha \\ y_0 + l\sin\alpha \\ z_0 + i_2 - i_1/2Sl^2 + i_1 l \end{pmatrix}$	$\gamma/[1 + (\gamma l + i_1)^2]^{3/2}$	0
3	ST	$\begin{pmatrix} \int_0^l \cos l^2/2A^2 dl \\ \int_0^l \sin l^2/2A^2 dl \\ z_0 + li \end{pmatrix}$	$l/A^2(1+i^2)$	$li/A^2(1+i^2)$
4	SP	$\begin{pmatrix} \int_0^l \cos l^2/2A^2\, dl \\ \int_0^l \sin l^2/2A^2\, dl \\ z_0 + i_2 - i_1/2Sl^2 + i_1 l \end{pmatrix}$	$\sqrt{(l/A^2)^2[1 + (\gamma l + i_1)^2] + \gamma^2}/[1 + (\gamma l + i_1)^2]^{3/2}$	$(l/A^2)^3(\gamma l + i_1) - l/A^2\gamma/(l/A^2)^2[1 + (\gamma l + i_1)^2] + \gamma^2$
5	CT	$\begin{pmatrix} m + R\cos l/R \\ n + R\sin l/R \\ z_0 + li \end{pmatrix}$	$1/R(1+i^2)$	$i/R(1+i^2)$
6	CP	$\begin{pmatrix} m + R\cos l/R \\ n + R\sin l/R \\ z_0 + i_2 - i_1/2Sl^2 + i_1 l \end{pmatrix}$	$\sqrt{1/R^2[1 + (\gamma l + i_1)^2] + \gamma^2}/[1 + (\gamma l + i_1)^2]^{3/2}$	$(\gamma l + i_1)/R[1 + (\gamma l + i_1)^2 + R^2\gamma^2]$

Note. $\gamma = i_2 - i_1/s$ denotes changing rate of grade; S = length of vertical curve; i_1, i_2 = grades of the former and latter tangents, respectively; and $(i_2 - i_1)$ = algebraic difference in grades.

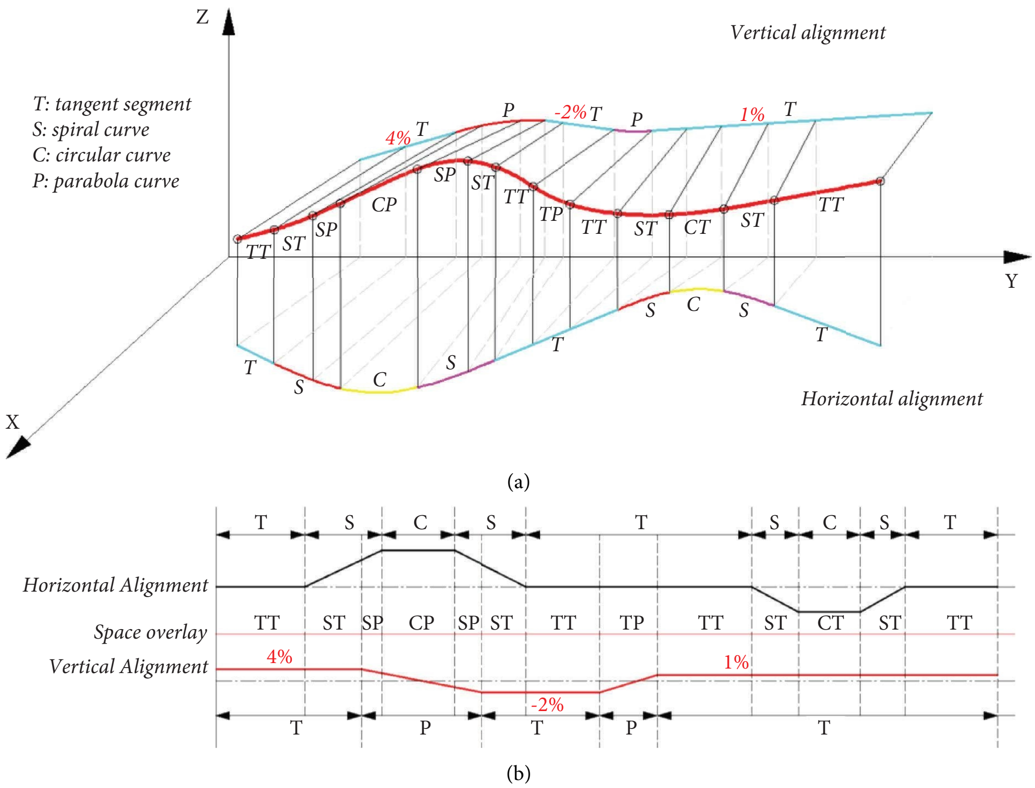

Figure 4: The combination of space alignments. (a) Three-dimensional space state. (b) Two-dimensional decomposition.

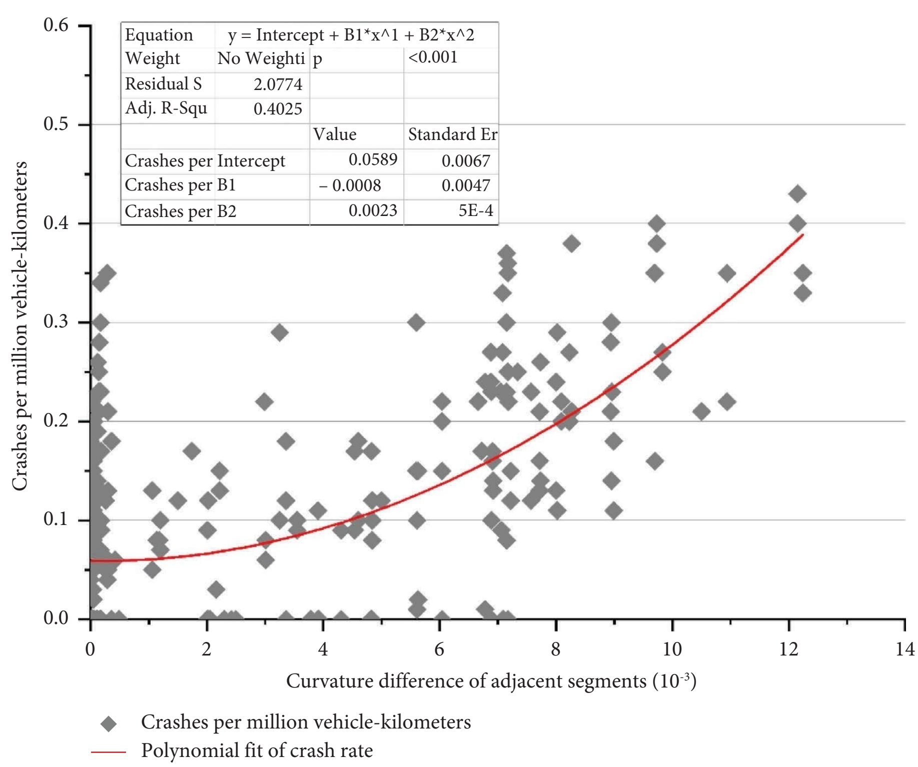

Figure 5: Effect analysis of crash rate and curvature difference.

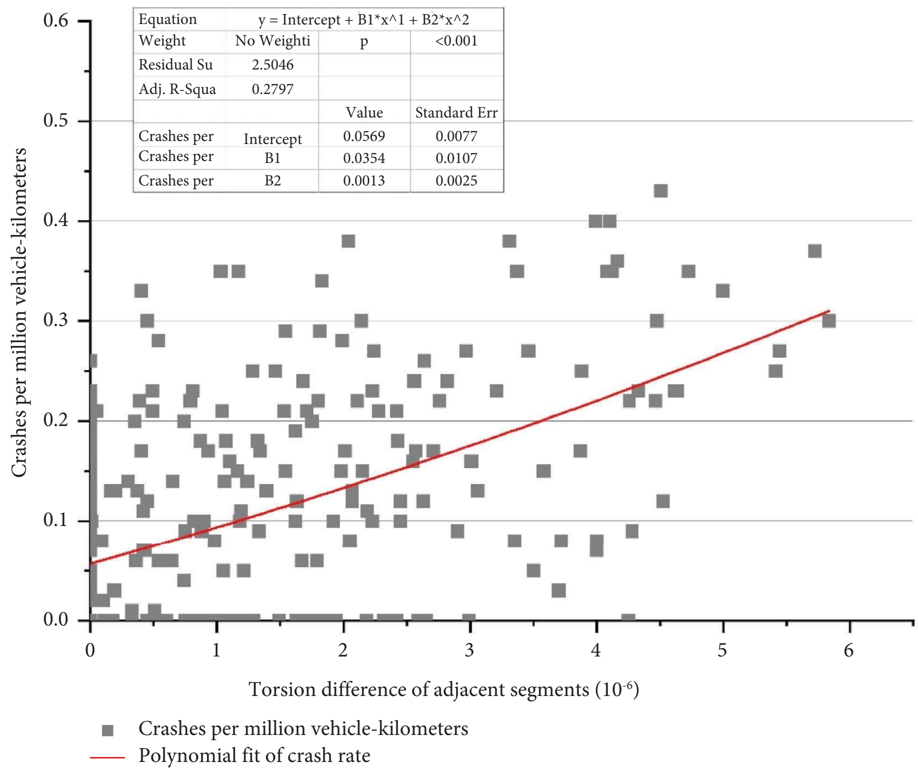

FIGURE 6: Effect analysis of crash rate and torsion difference.

TABLE 4: Results of the correlation analysis of crash rate, curvature difference, and torsion difference.

	Crash rate	Curvature difference	Torsion difference
Crashes per million vehicle-kilometers	1	0.607**	0.533**
Curvature difference of adjacent segments	0.607**	1	0.601**
Torsion difference of adjacent segments	0.533**	0.601**	1

* $p < 0.05$, ** $p < 0.01$.

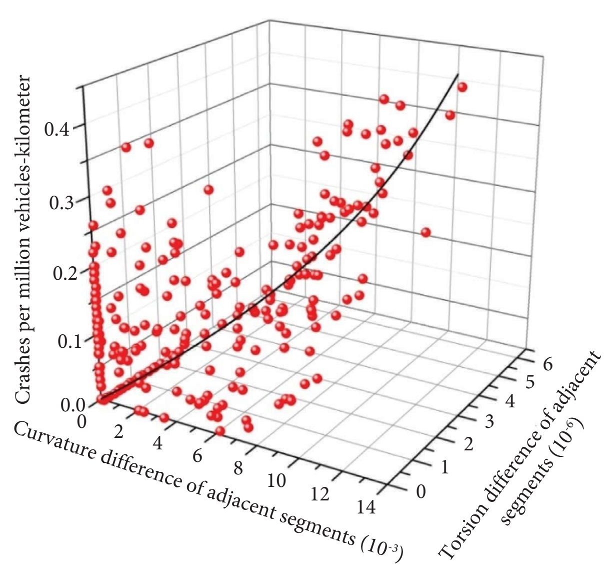

FIGURE 7: Coupling effect of curvature and torsion difference on crash rates.

Table 5: Crash rate and spatial curve properties of different roads.

No.	CR	Curvature			Torsion		
		AV	SD	CV	AV	SD	CV
1	0.095	0.001789	0.005869	3.281	0.00000825	0.00012100	14.667
2	0.159	0.001538	0.004338	2.821	0.00000609	0.00007141	11.717
3	0.113	0.001325	0.003220	2.430	0.00000452	0.00003959	8.751
4	0.021	0.000279	0.000143	0.512	0.00000023	0.00000132	5.870
5	0.030	0.000391	0.000280	0.717	0.00000039	0.00000156	3.965
6	0.235	0.002537	0.011804	4.653	0.00001990	0.00031290	15.723
7	0.036	0.000626	0.002054	1.148	0.00000289	0.00004235	5.133
8	0.067	0.001252	0.004108	2.297	0.00000578	0.00008470	10.267
9	0.111	0.001077	0.003037	1.975	0.00000426	0.00004999	8.202
10	0.079	0.000928	0.002254	1.701	0.00000316	0.00002771	6.126
11	0.015	0.000195	0.000100	0.358	0.00000016	0.00000092	4.109
12	0.021	0.000274	0.000196	0.502	0.00000027	0.00000109	2.776
13	0.165	0.001776	0.008263	3.257	0.00001393	0.00021903	11.006
14	0.138	0.000746	0.001456	1.367	0.00000235	0.00020209	6.014
15	0.214	0.001989	0.010369	3.648	0.00001457	0.00025531	12.268
16	0.099	0.000645	0.001089	1.182	0.00000153	0.00000722	3.306
17	0.235	0.001519	0.006713	2.496	0.00003528	0.00025823	10.732
18	0.197	0.001065	0.002080	1.953	0.00000336	0.00028870	8.591
19	0.305	0.002842	0.014813	5.212	0.00002081	0.00036473	17.526
20	0.141	0.000921	0.001556	1.689	0.00000219	0.00001032	4.723
21	0.336	0.001899	0.009590	3.566	0.00005040	0.00036890	15.332

Note. CR = crashes per million vehicle-kilometers; AV = average; SD = standard deviation; CV = coefficient of variation.

was orderly divided into many spatial segments in terms of 6 types of combination rules (see Figure 4). The relationship between crash rate and spatial curvature and torsion was analyzed through statistics and regression analysis. The applications of local and global evaluation in the form of case studies are shown as follows, respectively.

The total length of the selected roads is 1072.07 kilometers. All roads were divided into 1345 space curve segments based on the abovementioned 6 types of combination rules. According to the analysis of the crash data, some accidents are not relevant to the road alignment changing or the correlation is weak and are excluded from the statistics, such as drunk driving or misoperation. In addition, some nonstatistical accidents are also removed to ensure the correlation between accidents and space alignments. The detailed analysis results are described as follows. The effect analysis of crash rate and curvature difference is shown in Figure 5. And the effect analysis of crash rate and torsion difference is shown in Figure 6.

The statistical results showed there is a positive correlation between curvature/torsion difference and crashes per million vehicles-kilometers (Table 4), especially for curvature difference (crash rate is more sensitive to curvature change of adjacent segments). With the increase in the curvature difference, the trend of crashes is ascending notably. And with the increase in curvature and torsion differences, not only the crash rate increases but also the influence of curvature and torsion on the crash rate increases. The results indicate that controlling curvature/torsion differences can reduce crash rates.

From the perspective of 3D space, the crash rate shows a rising trend as curvature and torsion difference increases simultaneously. It is also verified that the changes in physical properties of spatial alignments, curvature, and torsion are significantly positively correlated with the crash rate. However, a crash is a random event in general, especially when displayed in the beginning space within the curvature difference range from 0 to 7×10^{-3} and torsion difference range from 0 to 3×10^{-6}. As shown in Figure 7, those special points can be observed that fail to meet the integral tendency. The analysis shows that although these accidents are affected by the factors of road alignment space changes, other accident factors may play a leading role, such as the driver 's fatigue driving in a small radius curve section or speeding through the curve.

After highway path construction, the spatial alignment of any road and its alignment combination have been determined. In spatial curve building, each road has a fixed combination of curvature and torsion. The average, standard deviation, and coefficient of variation of curvature/torsion, as well as crash rates can be calculated according to the collected data. The difference in curvature/torsion of each road will correspond to different crashes. Table 5 and Figure 8 reveal the relationship of the crash rate and spatial curve under different road conditions.

Obviously, computed results show crashes have a significant fluctuation following the changes in curvature and torsion. On the whole, there is a positive correlation between crash rate and mathematical statistical information on curvature/torsion (Table 6). The crash rate has a stronger correlation with the AV of curvature than the AV of torsion. The SD of torsion is more important to the accident rate than curvature. The correlation between CV of torsion and crash rate is weaker than that of CV of curvature. Among the

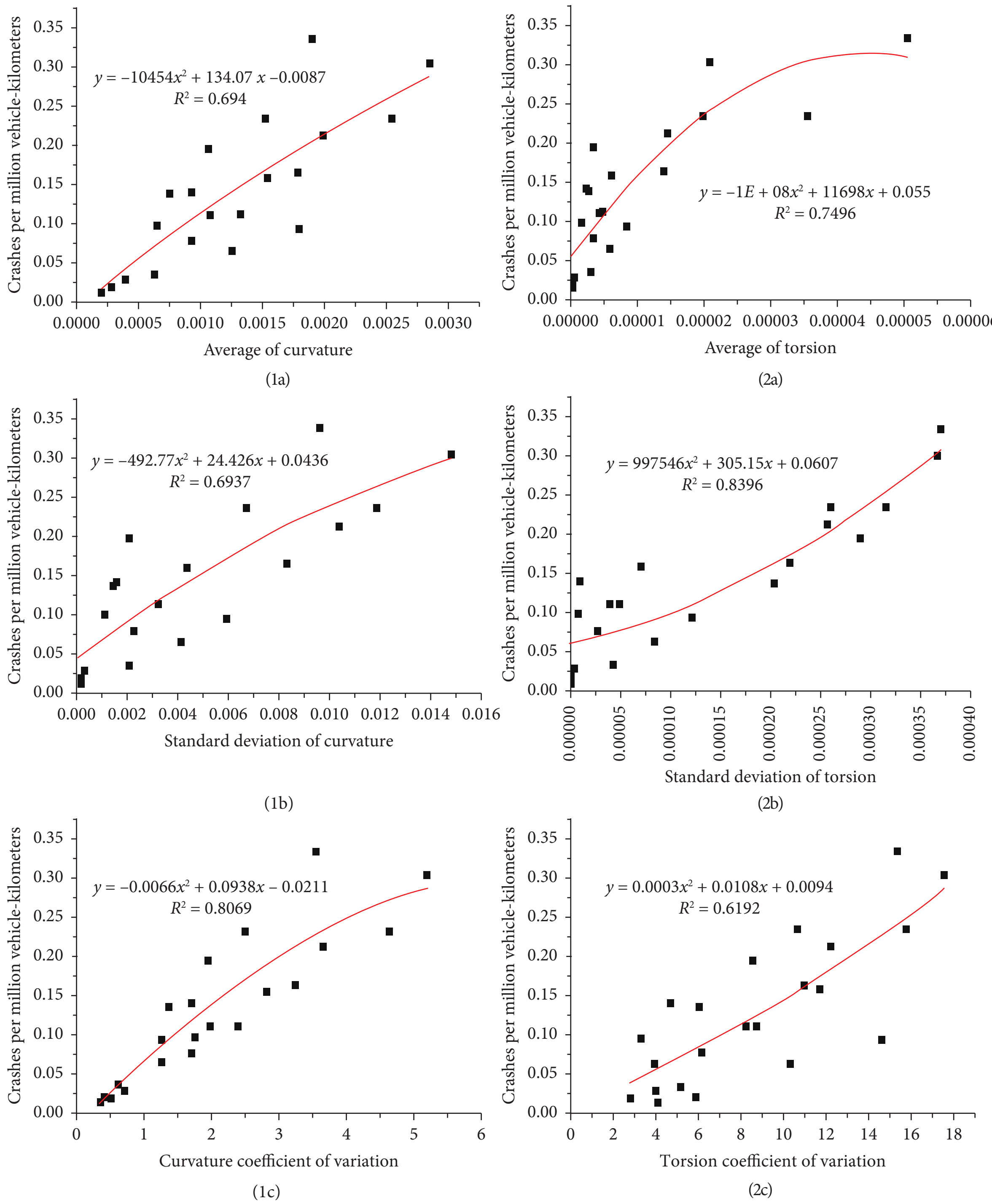

Figure 8: Interrelations of crash rate and spatial curve under different road conditions.

parameters related to curvature, the crash rate has the strongest correlation with the AV of curvature, and the CV of curvature has the weakest correlation. Among the relevant parameters of torsion, the crash rate has the strongest correlation with the SD of torsion, and the correlation with CV is the weakest. And the nonlinear relationship between road parameters and crash rate is further estimated by the regression model (Figure 8 and Table 7).

Interrelations between the crash rate and curvature are shown in Figures 8 1(a)–1(c). For curvature, the crash rate increases with the increase in AV, SD, and CV. However, with the increase in curvature, the influence of curvature on the crash rate is somewhat reduced. What can also be noticed is that curvature change has a significant influence on crashes within a certain range, such as average in the intervals [0 to 0.002] and standard deviation in [0 to 0.006],

Table 6: Results of correlation analysis of crash rate and spatial curve under different road conditions.

	CR	AV of curvature	SD of curvature	CV of curvature	AV of torsion	SD of torsion	CV of torsion
CR	1	0.830**	0.828**	0.826**	0.823**	0.908**	0.785**
AV of curvature	0.830 ∗ ∗	1	0.963**	0.999**	0.649**	0.797**	0.949**
SD of curvature	0.828**	0.963**	1	0.961**	0.719**	0.831**	0.915**
CV of curvature	0.826**	0.999**	0.961**	1	0.638**	0.792**	0.950**
AV of torsion	0.823**	0.649**	0.719**	0.638**	1	0.769**	0.697**
SD of torsion	0.908**	0.797**	0.831**	0.792**	0.769**	1	0.798**
CV of torsion	0.785**	0.949**	0.915**	0.950**	0.697**	0.798**	1

* $p < 0.05$, ** $p < 0.01$.

Table 7: Results of regression models of Figure 8.

Model	R^2	F	P	Intercept	B_1	B_2
1a	0.694	20.408	≤0.01	−0.009	134.070	−10453.677
1b	0.694	20.383	≤0.01	0.044	24.426	−492.772
1c	0.686	19.702	≤0.01	−0.008	0.073	−0.003
2a	0.750	26.948	≤0.01	0.055	11698.158	−131228923.350
2b	0.840	47.101	≤0.01	0.061	305.151	997546.050
2c	0.619	14.637	≤0.01	0.009	0.011	0.0003

seen from Figures 8 1(b) and 1(c). However, the number of accidents may reduce as the dangerous degrees of the road continue to increase (generally, a greater discrete degree of curvature change indicates more dangerous and difficult road conditions), the reason for which may be that poor road conditions usually require drivers with a high level of caution and a lower operating speed.

Figures 8 2(a)–2(c) are compared with crash rate with the torsion change, which mainly influences the longitudinal gradient in the vertical plane. A larger torsion angle will bring a higher risk of collision, as shown in Figure 8 2(a). Frequent and sharp changes of slope are detrimental to driving safety, with greater torsion bringing a surge in collision risk as shown in Figures 8 2(b) and 2(c). In other words, the higher the discrete degree of torsion change is, the more crashes there will be. However, for the AV of torsion, the greater the AV of torsion, the lower the impact of the AV of torsion on the crash rate. For the SD and CV of torsion, with the increase of SD and CV, not only the accident rate will increase but also the effect of SD and CV on the accident rate will become stronger.

5. Conclusions

The spatial configuration should not be determined separately, as the location of a highway is comprised of horizontal alignment and profile. Curvature and torsion, two invariants of a curve in space, can define the spatial characteristics of highway alignments. This study develops an evaluation method for determining highway geometric design and safety operations based on spatial curve properties. The detailed methods and findings of the research are summarized as follows.

(1) The spatial curve path of a road is segmented into six types based on spatial combination regularities. The three-dimensional location of each point on the curve is written by a parametric representation. Thus, three-dimensional numerical models of highway alignments are built.

(2) Based on a parametric representation of a spatial curve, two geometric invariants of curvature κ and torsion τ are calculated to determine the spatial shape and trends. The dispersion of curvature/torsion change between adjacent segments on a low-grade highway is greater than on a motorway, according to the results of a calculation.

(3) Using global and local approximations, the interrelationships between collisions and curvature/torsion distribution are determined. The results of an analysis show that the regularity of curvature/torsion change has significant impacts on crash frequency. By comparison, curvature distribution has more obvious correlation to crashes than torsion spatial variation. The segments with a higher dispersion of curvature/torsion difference correspond to a higher crash rate.

(4) The proposed method can be used to identify the spatial structure of curvature/torsion for in-plan or in-service highways, providing a basis for the determination of crash frequency and optimizing highway geometric design to improve safety operation levels. The evaluation model building will allow for improved accessibility for future research to explore the 3D space design of route.

The safety of road alignment design is a primary consideration in this study. The physical properties of spatial alignment, curvature, and torsion are used to characterize the quality of road alignment design, beginning with the actual situation of road alignment space change. This study establishes a safety evaluation method for analysing the interrelation between crash rate and spatial curve properties.

It also offers a new perspective for evaluating road alignment design from a three-dimensional state. Then, a genuine three-dimensional evaluation of the road's safety is realized. However, the design of roads does not account for the equilibrium between all sustainability factors. The social, economic, and environmental aspects of road design should also be examined in depth, despite the fact that safety is a significant factor that directly affects people and indirectly affects other social factors, such as economic loss and social concern. Thus, although safety is a top priority, economic, environmental, and social factors need to be considered more thoroughly in future research to promote urban sustainability.

Notations

2D: Two-dimensional
3D: Three-dimensional
&[†]: Superposition of two line elements
(x, y, z): Any coordinate point of spatial curve
(x_0, y_0, z_0): A starting point for calculating
l: Length, m
i: Slope of spatial straight line, %
α: Angle, degree
R: Corresponding radius, m
β: Deflection angle of this segment in radians, degree
A: Spiral parameter
(m, n): Center of a circle
θ: Parameter of circle equation that denotes the azimuth angle
a, b: Coefficient of a parabolic equation
κ: Curvature, m^{-1}
ρ: Radius of curvature, m
τ: Torsion, m^{-1}
$\overline{T}$: The curve tangent vector
$\overline{N}$: The curve normal vector
γ: Changing rate of grade
S: Length of vertical curve, m
i_1, i_2: Grades of the former and latter tangents, %
Δx_j: The curvature/torsion difference of adjacent combination segments, m^{-1}
$\overline{x_i}, \overline{x_{i-l}}$: The average values of curvature/torsion in section i or $i-1$, m^{-1}
CR_i: The crash rate of the section, i
$\doteqdot$: A certain correlation
$\overline{x}$: Mean value of spatial curvature/torsion, m^{-1}
σ: Standard deviation of spatial curvature/torsion, m^{-1}
CV: Coefficient of variation
AV: Average
SD: Standard deviation
CR: Crashes per million vehicle-kilometers
TT: A spatial combination alignment consisting of straight line in the horizontal projection plane and straight line in the vertical projection plane
TP: A spatial combination alignment consisting of straight line in the horizontal projection plane and parabola in the vertical projection plane
ST: A spatial combination alignment consisting of Euler spiral in the horizontal projection plane and straight line in the vertical projection plane
SP: A spatial combination alignment consisting of Euler spiral in the horizontal projection plane and parabola in the vertical projection plane
CT: A spatial combination alignment consisting of circular curve in the horizontal projection plane and straight line in the vertical projection plane
CP: A spatial combination alignment consisting of circular curve in the horizontal projection plane and parabola in the vertical projection plane.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] A. Dhanuka, A. Srivastava, L. Khadke, and N. L. Kushwaha, *Smart Geometric Design Of Highways Using Html Programming For Sustainable And Climate Resilient Cities Bt-Urban Commons, Future Smart Cities And Sustainability*, U. Chatterjee, N. Bandyopadhyay, M. D. Setiawati, and S. Sarkar, Eds., Springer International Publishing, Berlin, Germany, 2023.
[2] T. S. Khuzan and M. A. Al-Jumaili, "A review of studying the relationship of rural road accidents with geometric design," *AIP Conference Proceedings*, vol. 2787, 2023.
[3] C. Rindone and P. Panuccio, "Planning for risk reduction in the transport system at urban level," *International journal of transport development and integration*, vol. 7, no. 1, pp. 27–34, 2023.
[4] S. N. Nurjannah, A. I. Rifai, and A. Fajarika Akhir, "Geometric design for relocation of national road sei duri-mempawah section," *West Kalimantan using AutoCAD ® 2D*, vol. 2, no. 5, p. 2022, 2022.
[5] L. Wang, J.-C. Cheng, and Y.-L. Zhang, "Reliability-based specification on critical length of highway sections with near-maximum grade," *KSCE Journal of Civil Engineering*, vol. 22, no. 4, pp. 1406–1417, 2018a.
[6] K. You, Q. Yu, W. Huang, and Y. Hu, "Safety-based optimization model for highway horizontal alignment design,"

Mathematical Problems in Engineering, vol. 2022, Article ID 6214910, 15 pages, 2022.

[7] J. Wang, S. He, X. Zhai, Z. Wang, and X. Fu, "Estimating mountainous freeway crash rate: application of a spatial model with three-dimensional (3D) alignment parameters," *Accident Analysis & Prevention*, vol. 170, Article ID 106634, 2022.

[8] J. Wang, L. Wang, H. Zhou, W. Jing, and J. Cheng, *A Three-Dimensional Method for Safety Evaluation of Highway Geometric Design*, Transportation Research Board 97th Annual Meeting, Washington, DC, USA, 2018, https://trid.trb.org/view/1495085.

[9] A. Iannantuono, W. Hare, and Y. Lucet, "Optimization with regularization to create sensible vertical alignments in road design," *Decision Analytics Journal*, vol. 6, Article ID 100183, 2023.

[10] M. B. Sushma, S. Roy, and A. Maji, "Exploring and exploiting ant colony optimization algorithm for vertical highway alignment development," *Computer-Aided Civil and Infrastructure Engineering*, vol. 37, no. 12, pp. 1582–1601, 2022.

[11] M. B. Sushma and A. Maji, "A modified motion planning algorithm for horizontal highway alignment development," *Computer-Aided Civil and Infrastructure Engineering*, vol. 35, no. 8, pp. 818–831, 2020.

[12] Y. Zhou, R. Huang, T. Jiang, Z. Dong, and B. Yang, "Highway alignments extraction and 3D modeling from airborne laser scanning point clouds," *International Journal of Applied Earth Observation and Geoinformation*, vol. 102, Article ID 102429, 2021.

[13] S. Talmaki, V. R. Kamat, and H. Cai, "Geometric modeling of geospatial data for visualization-assisted excavation," *Advanced Engineering Informatics*, vol. 27, no. 2, pp. 283–298, 2013.

[14] S. Zheng, Y. Meng, H. Cai, J. Luo, and S. Sun, "Research on highway accident prediction based on interactive highway safety design model," in *Proceedings of the International Conference on Smart Transportation and City Engineering 2021*, Bellingham, WA, USA, June 2021.

[15] D. Llopis-Castelló, D. J. Findley, and A. García, "Comparison of the highway safety manual predictive method with safety performance functions based on geometric design consistency," *Journal of Transportation Safety & Security*, vol. 13, no. 12, pp. 1365–1386, 2021.

[16] I. Hamilton, S. Himes, R. J. Porter, and E. Donnell, "Safety evaluation of horizontal alignment design consistency on rural two-lane highways," *Transportation Research Record*, vol. 2673, no. 2, pp. 628–636, 2019.

[17] K. K. Tottadi and A. Mehar, "Operating speed: review and recommendations for future research," *Innovative Infrastructure Solutions*, vol. 7, no. 1, p. 67, 2021.

[18] T. Choudhari, G. Sil, and A. Maji, *Speed-Based Safety Evaluation Of Horizontal Curves In Rural Highways Bt- Transportation Research In India: Practices And Future Directions*, A. K. Maurya, L. D. Vanajakshi, S. S. Arkatkar, and P. K. Sahu, Eds., Springer, Singapore, 2022.

[19] A. Ahmed, A. F. M. Sadullah, and A. S. Yahya, "Analysis of the effect of directional traffic volume and mix on road traffic crashes at three-legged unsignalized intersections," *Transport Engineer*, vol. 3, Article ID 100052, 2021.

[20] A. K. Høye and I. S. Hesjevoll, "Traffic volume and crashes and how crash and road characteristics affect their relationship – a meta-analysis," *Accident Analysis and Prevention*, vol. 145, Article ID 105668, 2020.

[21] H. Zhang, B. Heng, and W. Zhao, "Path tracking control for active rear steering vehicles considering driver steering characteristics," *IEEE Access*, vol. 8, pp. 98009–98017, 2020.

[22] P. M. Chaudhari, J. Goyani, S. Arkatkar, G. Joshi, and S. M. Easa, "Design consistency evaluation of two-lane rural highways in hilly terrains," *Transportation Research Procedia*, vol. 62, pp. 75–82, 2022.

[23] A. Afolayan, O. Abiola Samson, S. Easa, F. Modupe Alayaki, and O. Folorunso, "Reliability-based analysis of highway geometric Elements: a systematic review," in *Cogent Engineering*, M. Giunta, Ed., vol. 9, no. 1, Article ID 2004672, 2022.

[24] G. M. Gibreel, S. M. Easa, Y. Hassan, and I. A. El-Dimeery, "State of the art of highway geometric design consistency," *Journal of Transportation Engineering*, vol. 125, no. 4, pp. 305–313, 1999.

[25] Y. Hassan, S. M. Easa, and A. O. Abd El Halim, "State-of-the-art of three-dimensional highway geometric design," *Canadian Journal of Civil Engineering*, vol. 25, no. 3, pp. 500–511, 1998.

[26] M. M. Garnaik, J. P. Giri, and A. Panda, "Impact of highway design on traffic safety: how geometric elements affect accident risk," *Ecocycles*, vol. 9, no. 1, pp. 83–92, 2023.

[27] A. Shalkamy, K. El-Basyouny, and Y. Li, "Calibrating safety-based design charts for horizontal curves using system reliability analysis and multivariate models," *Journal of Transportation Safety and Security*, vol. 14, no. 12, pp. 1997–2028, 2022.

[28] M. Moradi, A. Abdi Kordani, and M. Zarei, "New geometric design approach to reduce vehicle's speed in accident-prone downgrade highways using dynamic vehicle modeling," *Journal of Transportation Engineering, Part A: Systems*, vol. 147, p. 1, 2021.

[29] N. Zulfa, A. I. Rifai, and M. Taufik, "Road geometric design used AutoCAD® Civil 3D: a case study jalan campaka-wanaraja garut, Indonesia," *Citizen: Jurnal Ilmiah Multidisiplin Indonesia*, vol. 2, no. 5, pp. 843–850, 2022.

[30] X. Wang, S. Li, T. Shen, Y. Wang, W. Qi, and J. Yao, "The impact of the combination equilibrium of horizontal and sag-vertical curves on safety," *International Journal of Transportation Science and Technology*, vol. 12, 2022.

[31] M. Jin, C. Ynchausti, and L. L. Howell, "Zero-curvature deformation properties and 3R pseudo-rigid-body model of large-deflection Euler spiral beams," *Mechanism and Machine Theory*, vol. 183, Article ID 105261, 2023.

[32] P. S. March, *Geometric-based Spatial Path Planning*, Dissertations and Theses- Gradworks, The University of Texas at Austin, Austin, TX, USA, 2008.

[33] M. Frego, "Closed form parametrisation of 3D clothoids by arclength with both linear varying curvature and torsion," *Applied Mathematics and Computation*, vol. 421, pp. 126907–126920, 2022.