Hindawi
Journal of Advanced Transportation
Volume 2023, Article ID 8298068, 14 pages
https://doi.org/10.1155/2023/8298068

Research Article

Study on the Shortest Reliable Path of Stochastic Time-Dependent Transportation Networks considering Waiting Time at Signalized Intersections

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Received 3 May 2023; Revised 16 July 2023; Accepted 20 July 2023; Published 29 July 2023

Academic Editor: Antonino Vitetta

Theoretical research is conducted on finding the shortest path with stochastic and time-dependent characteristics of link travel time in urban road networks. Considering the influence of signalized intersections on travel time, the research first presents a function of waiting time at signalized intersections and analyzes its characteristics. Then, considering the reliability of travel time, the travel time model under the min-max theorem is established, and a mathematical proof that the stochastic time-dependent traffic networks can be reduced to a deterministic time-dependent network is presented by using the first mathematical induction. Finally, based on analyzing the characteristics of the shortest path and minimum travel time, which vary with the start time, we propose solving the shortest path problem with a shortest path algorithm based on Dijkstra's algorithm that takes the waiting time at signalized intersections into consideration. The research results showed that the algorithm proposed in this study does not depend on the acquisition of the probability distribution of travel time compared with the traditional algorithm. The range of uncertain travel time can be derived from historical data and travelers' experiences, and the obtained shortest path has more robust time reliability.

1. Introduction

With the rapid development of the economy and the acceleration of urbanization, the number of motor vehicles and road traffic has dramatically increased, which has led to a series of problems such as traffic congestion. Choosing the optimal path can, to a certain extent, reduce travel time, cost waste, and resource loss. Therefore, travelers must formulate the optimal travel plan and route before departure [1].

The urban traffic network is dynamic rather than static. In the stochastic time-dependent (abbreviated as STD) traffic networks, the travel time changes with traffic flow. For example, the sudden increase of traffic flow in the morning, noon, and evening rush hours and the decrease of traffic flow at night will more or less lead to corresponding changes in travel time, thus showing the time-dependent characteristics of travel time [2]. In addition, urban traffic networks are susceptible to bad weather, emergencies, traffic control, and occasional traffic congestion, thus showing stochastic time characteristics [3]. The time dependence and stochasticity of traffic networks constitute the stochastic time-dependent characteristics of the travelers' travel time. The existence of such uncertainty leads to the uncertainty of travelers' travel time, significantly affecting travelers' travel plans and increasing travel time and cost as well as waste and consumption of resources. As a result, travelers demand not only safety, convenience, and punctuality for each road section but also economy and reliability for each intersection of the road system, that is, higher requirements for the reliability of path travel time. In the classical shortest path (SP) problem in graph theory, the intersection is usually studied as a location point, and only the sum of travel times of each segment on the path is considered, but the waiting time delay at the signal intersection is not considered [4].

However, in the urban road network, the driver's waiting time at the signalized intersection accounts for a large proportion of its travel time, and most of the optimal path planning algorithms only consider the travel time of the section, resulting in a significant difference between the desired optimal path and the actual optimal path. Therefore, the waiting time at the signalized intersection must be taken into account when calculating travel time to reduce the difference between predicted travel time and actual travel time. This article focuses on studying the shortest reliable route in urban commuting where the travel time is random and uncertain, but the travel time has a time window requirement. A holistic consideration of the influence of signalized intersections on travel time allows the presentation of the waiting time function of signalized intersections and the road network transformation method, which transforms a stochastic time-dependent traffic network into a deterministic time-dependent traffic network to solve the shortest reliable path problem.

In the following, Section 2 analyzes the research status at home and abroad. Section 3 describes the shortest path problem and the waiting time at signalized intersections under STD networks and expresses the characteristics of waiting time at signalized intersections through functions. Then, the shortest reliable path model is established and a mathematical proof that the STD network can be transformed into a deterministic time-dependent network is put forward. Section 4 proposes a shortest path algorithm that accounts for waiting time at signalized intersections. Section 5 sets up numerical experiments. Section 6 analyses the experimental results. Section 7 summarizes the full text.

2. Literature Review

The research on the SP problem in the field of transportation started relatively early. Since it was put forward, domestic and foreign traffic scholars have done extensive research on the problem, continuously extending many variants based on existing research, such as the K-short path problem and reliable path problem. With the development and deepening of relevant research, many scholars not only continuously improve and innovate on the model but also continuously optimize the model-solving algorithm, providing a theoretical basis and new methods for other research in the field of transportation.

In 1959, Dijkstra [5] first studied the SP problem and put forward the traditional Dijkstra's algorithm to solve the problem in static traffic networks. After that, many scholars conducted in-depth research and improvement on this topic. Xue and Chai [6] improved Dijkstra's algorithm and verified the effectiveness of the algorithm in solving the shortest path under uncertain weather conditions through experiments. Liu et al. [7] applied the Dijkstra algorithm as the global path planning algorithm and the dynamic window approach as its local path planning algorithm to the smart car, enabling it to successfully avoid obstacles from the planned initial position and reach the designated position. Wang et al. [8] proposed a three-dimensional Dijkstra's optimization algorithm, which can generate a global optimal ship path in a poor marine environment, and the solution obtained by the generated multiobjective function is exact. Wang et al. [9] put forward an improved Dijkstra-ant colony algorithm by combining a greedy algorithm with a heuristic algorithm. It turns out that the algorithm has strong global search ability and good convergence performance, thus improving global search efficiency. However, these traditional SP problem algorithms all take static, deterministic transportation networks as the research object. The obtained solutions are exact solutions with high computational complexity, so they are not suitable for large-scale networks.

Considering the stochasticity and uncertainty of the urban road network, many scholars [10–13] have begun to study the stochastic network. They have studied the routing optimization problem under a class of uncertain conditions to find the optimal route solution that meets the time window requirements under the condition of uncertain travel time. However, the above scholars do not consider the time-dependent characteristics of link weights, which cannot meet the travel needs of travelers.

Liu [14] established the sum of the carbon emission cost, transportation cost, penalty cost for exceeding the time window, and the damage cost of the cold chain cargo as the objective function and established a route optimization model of cold chain container multimodal transportation. Xu and Li [15] researched the time-dependent vehicle routing problem and proposed an unconventional path optimization approach, known as the fissile ripple spreading algorithm (FRSA). Liu and Zhang [16] considered the time dependence of link weights and optimized the shortest path within a time window with time-dependent driving risk. By analyzing the time-dependent characteristics of urban road networks, Zhao et al. [17] designed an electric vehicle routing problem (EVRP) model under time-dependent traffic conditions for route planning of fresh products in urban cold chains. However, the above scholars have only considered the stochastic or time-dependent characteristics of link weights but ignored the comprehensive stochastic time-dependent characteristics.

Few scholars comprehensively consider the stochastic and time-dependent characteristics of urban road networks. Many scholars [18, 19] built optimization models under stochastic time-dependent networks and used improved labeling algorithms to find optimal paths. Considering the difficulty of solving the optimal path problem in the stochastic time-dependent networks, Sun et al. [2] adopted a robust optimization method to take the path with the highest cost as the optimal path and transformed the stochastic time-dependent networks into a deterministic time-dependent network. Because it is difficult to know the probability distribution of travel time in the travel process of a section, Yang et al. [20] put forward a route selection algorithm considering risks and actual traffic flow conditions on the current road. Although the above scholars have considered the stochastic time-dependent characteristics of urban road networks, they have not yet considered the reliability of travel time, and therefore they could not realize that path planning meets the reliability requirements.

Asakura and Kashiwadani [21] first proposed a definition of travel time reliability in 1991, using probability to express travel time reliability and studying the probability of travelers arriving at destinations within the desired travel time to measure potential travel delays. The Florida Department of Transportation studied the Florentine method of travel time reliability and defined travel time reliability as the probability that the travel time of a road section is less than or equal to the sum of the desired travel time and the acceptable delay time. In 2002, the Texas Department of Transportation used the Buffer Index to describe trip time reliability, defined as the ratio of travel time to the average travel time required to ensure that travelers arrived at their destination on time. Later on, Tu et al. [22] investigated the constrained reliable shortest path problem for electric vehicles in the urban transportation network. They proposed an algorithm that can obtain the exact solutions within satisfactory computational time. Arun Prakash [23] presented a decreasing order-of-time algorithm and a label-correcting network pruning algorithm to determine the most reliable routes on stochastic and time-dependent networks. The above scholars have studied the shortest reliable path problem of STD networks, but they usually regard intersections as nodes on the path, without considering the link weight on the nodes. However, in real road conditions, because of the existence of signalized intersections, the vehicle will generate waiting time when passing the node, and the waiting time at signalized intersections accounts for a large proportion of the overall vehicle travel time, so the waiting time at signalized intersections must be considered in the travel time of the road network.

In the existing shortest path problem considering intersection delay, different researchers have proposed various algorithm models from different angles [24]. Qin et al. [25] used a reliable energy consumption path-finding algorithm for signalized traffic networks, and the result showed that accounting for the delays at signalized intersections can improve the prediction accuracy of vehicle energy consumption. Ju and Du [26] suggested the periodic change of intersection signals in urban road systems which is the leading cause of uncertain delays and proposed a hyperpath search method based on intersection signal timing. Therefore, the application of the algorithm to the traffic field has limitations. Kamal et al. [27] presented a novel adaptive traffic signal control scheme, which aims at minimizing the total crossing time of all vehicles. Liang et al. [28] extended a real-time traffic signal control algorithm based on the Internet for vehicles at isolated intersections to minimize the average vehicle delay, which can improve fairness at intersections with minimal loss of efficiency. However, this study looked at individual signalized intersections and did not take into account signal timing at intersections, where vehicles may change routes based on delays encountered.

Based on the previous research on the routing problem in STD networks, this article further considers the waiting time at signalized intersections, studies the shortest reliable path in the worst case, and transforms the STD network into a deterministic time-dependent network to solve the shortest reliable path problem.

3. Problem Description

3.1. Shortest Path Problem of Stochastic Time-Dependent Networks. The shortest path problem of travelers from the starting point to the destination can be described as the optimal path selection problem in an STD network. Given a directed graph $G(N, A, t_{ij})$, where $N = \{N_1, N_2, \ldots, N_n\}$ is the set of the nodes and $A = \{A_1, A_2, \ldots, A_m\}$ is the set of links. Assume that the number of nodes is n and the number of links is m, $(m, n = 1, 2, 3 \cdots)$, as shown in Figure 1. The path is the non-empty node set of the directed graph, which is arranged in order from the starting point through the nodes until the end, where $i, j = 1, 2, \ldots, n$ and $i \neq j$. T is the set of time periods, which can be discretized into a set of periods $t \in T$ with the same time interval. $t_{ij}(t)(i \neq j)$ is a piecewise function associated with the departure time, which indicates the travel time on the link (N_i, N_j) and its value will change with different departure moments, reflecting the time-dependent characteristics of the STD network. To reflect the stochastic properties of the stochastic time-dependent network, let $t_{ij}(t) = C_{ij}^t + R_{ij}^t$, where C_{ij}^t is a constant in period t, and R_{ij}^t is a non-negative random variable not more than the constant $e_{ij}^t (e_{ij}^t \geq 0)$, that is, $0 \leq R_{ij}^t \leq e_{ij}^t$, so $C_{ij}^t \leq t_{ij}(t) \leq C_{ij}^t + e_{ij}^t$.

3.2. Waiting Time Function at Signalized Intersections. The time dissipation at signalized intersections can be divided into two parts: one is the time delay, mainly manifested as low-speed driving and parking caused by vehicles queuing at intersections or slowing down upstream of the intersections; the other is the waiting time, mainly reflected in the time when vehicles wait for the permission to pass at the signalized intersections. The waiting time at the signalized intersection is related to waiting for the target phase and selecting the subsequent path [4]. In this article, by selecting the phase to represent the vehicle's turning condition, vehicles passing through a signalized intersection necessitate representation through phases, such as "go straight," "turn left," and "turn right." As shown in Figure 2, taking a typical intersection as an example, when a vehicle is waiting for a signal light instruction, because only vehicles in one direction are allowed to pass in each phase, the selection of the subsequent path is also related to the waiting time, which is not considered in the traditional shortest path algorithm. Therefore, this research will consider the influence of this factor while studying the waiting time at the signalized intersections.

The waiting time at the signalized intersections is related to the time of arrival at the intersection, and the parameter settings are shown in Table 1.

$$t_j^{\text{arrive}} = t_i^{\text{leave}} + t_{ij}(t), \tag{1}$$

$$t_i^{\text{leave}} = t_i^{\text{arrive}} + t_i^{\text{wait}}, \tag{2}$$

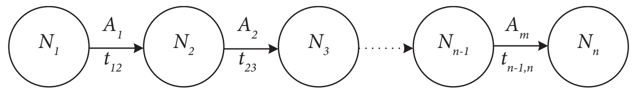

Figure 1: A graphical representation of a path.

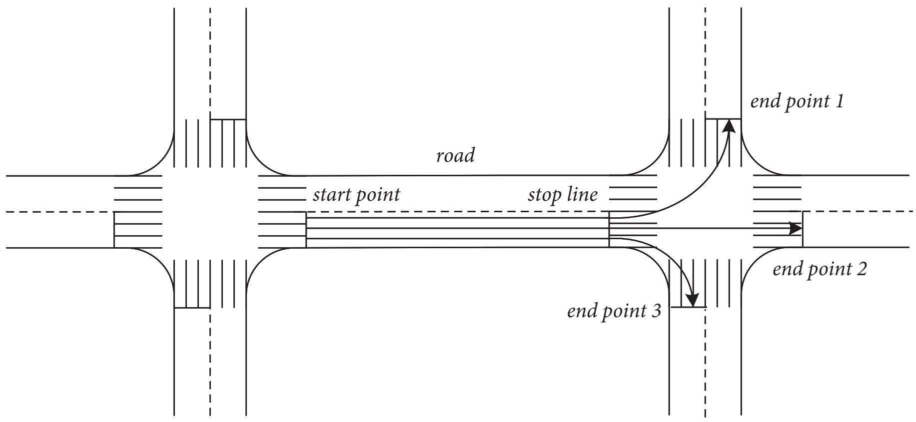

Figure 2: Schematic diagram of intersection path selection.

Table 1: Parameters and interpretations.

Parameters	Interpretations
t_i^{arrive}	The moment the vehicle reaches node N_i
t_i^{leave}	The moment the vehicle leaves node N_i
t_i^{wait}	The waiting time for the vehicle at node N_i
T_i	Signal cycle length of node N_i
T_i'	The time spent on the current phase when the vehicle reaches node N_i
$t_{L,i}$	Green time of the L phase of node N_i, $L = 1, 2, \ldots, k$
$t_{ij}(t)$	The time the vehicle passes through link (N_i, N_j)

$$\sum_{L=1}^{k} t_{L,i} = T_i. \tag{3}$$

Assuming that the required phase when the vehicle arrives at a signalized intersection is x, the current phase when the vehicle arrives at the signalized intersection is y, and meeting the condition that $x \le k$, $y \le k$. The waiting time at the signalized intersection can be discussed in two cases:

(1) When $x = 1$, the required phase of the vehicle is the first phase; when $y = 1$, the vehicle can pass without waiting; when $y \neq 1$, the vehicle needs to wait from phase y until the end of the signal cycle and the beginning of the next cycle, as shown in Figure 3(a). Also, the waiting time at the signalized intersection is

$$t_i^{\text{wait}} = \begin{cases} 0, & x = y = 1, \\ \sum_{L=y}^{k} t_{L,i} - T_i', & x = 1 < y. \end{cases} \tag{4}$$

(2) When $x \neq 1$, the waiting time at the signalized intersection can be discussed in three cases:

(i) When $x = y$, indicating that the vehicle can pass without waiting when it arrives at the signalized intersection, the waiting time at the signalized intersections is $t_i^{wait} = 0$.

(ii) When $x < y$, the vehicle needs to wait from the current phase y to the phase x of the next signal cycle after arriving at the signalized intersection, as shown in Figure 3(b). The waiting time at signalized intersections is

$$t_i^{\text{wait}} = \sum_{L=1}^{x-1} t_{L,i} + \sum_{L=y}^{k} t_{L,i} - T_i'. \tag{5}$$

(iii) When $x > y$, the vehicle arrives at the signalized intersection to wait from phase y to the phase x of the cycle, as shown in Figure 3(c). The waiting time at signalized intersections is

$$t_i^{\text{wait}} = \sum_{L=y}^{x-1} t_{L,i} - T_i'. \tag{6}$$

Assume that a signalized intersection N_i has k independent phases and each phase duration of the signalized intersection is $t_1, t_2, \ldots, t_k$. The waiting time function of each phase at the signalized intersection N_i is shown in Figure 4: the horizontal coordinate is the time t_i^{arrive} when vehicles arrive at the signalized intersection, and the vertical

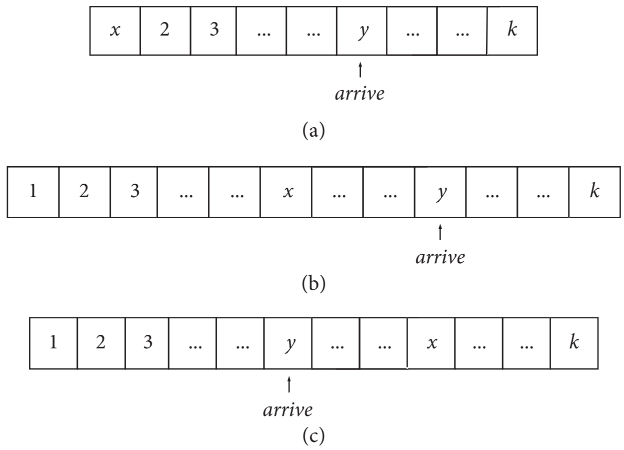

Figure 3: Phase diagram. (a) $x=1<y$, (b) $x<y$, (c) $x>y$.

coordinate is the waiting time required at different phases at the signalized intersection.

The following conclusions can be drawn from the waiting time function image of signalized intersection:

(1) At any signalized intersection N_i, within a signal cycle, the signalized intersection waiting time function is a piecewise function. When the required phase is x, the waiting time function is

$$t_i^{\text{wait}} = \begin{cases} \sum_{L=1}^{x-1} t_{L,i} - t_i^{\text{arrive}}, & 0 \le t_i^{\text{arrive}} \le \sum_{L=1}^{x-1} t_{L,i}, \\ 0, & \sum_{L=1}^{x-1} t_{L,i} \le t_i^{\text{arrive}} \le \sum_{L=1}^{x} t_{L,i}, \\ \sum_{L=1}^{x-1} t_{L,i} + T_i - t_i^{\text{arrive}}, & \sum_{L=1}^{x} t_{L,i} \le t_i^{\text{arrive}} \le T_i. \end{cases} \tag{7}$$

(2) The waiting time at the signalized intersection is related to the current phase of the vehicle and the required phase.

(3) The waiting time range of signalized intersection is

$$\left[0, \max\left\{T_i - t_1, T_i - t_2, \ldots, T_i - t_k, \sum_{L=2}^{k-1} t_{L,i}\right\}\right]. \tag{8}$$

3.3. Shortest Reliable Path Model. Average commuting time consumption significantly affects residents' intuitive perception of commuting and quality of life. That is why it has been included in the "urban experience indicator system" as a prominent indicator to measure urban traffic convenience. The traditional SP problem usually takes travel time, cost, or resource consumption as the definition criteria of the optimal path. According to the *Commuting Monitoring Report of China's Major Cities* in 2021, the average one-way commuting time in major cities in China is 36 minutes. When faced with uncertain travel conditions, the commuting time may be longer. For travelers, lateness will ruin their established plan and result in penalty costs, loss of opportunities, etc., while on the other extreme, early arrival will waste travelers' time. Therefore, this research introduces the early arrival time penalty factor $\alpha (\alpha > 0)$ and the in-transit time factor $\beta (\beta > 0)$ to find the path with the most reliable travel time from the starting point to the destination as the optimal path under the condition that lateness is not allowed.

Let $T_{\gamma,t}$ be the travel time from the starting point W to the endpoint γ of the selected path N_1 starting from period N_n and be time-dependent. Under the STD traffic network, the shortest reliable path is studied in this research to find the path with the minimum travel time in the worst case among the candidate paths as the optimal path, so as to realize that the route will not be deteriorated by the change of uncertainties. The shortest reliable path model in this study is as follows.

Objective function:

$$Z = \min_{\gamma \in P} \max\left(\alpha\left(t^* - T_{\gamma,t} - t_1\right) + \beta T_{\gamma,t}\right), \tag{9}$$

which can be rewritten as

$$\begin{aligned} Z &= \min_{\gamma \in P}\left(\alpha t^* - \alpha t_1 + (\beta - \alpha)\max T_{\gamma,t}\right) \\ &= \min_{\gamma \in P}\left(\alpha t^* - \alpha t_1 + (\beta - \alpha)\max\left(\sum_{(i,j)\in\gamma} \sum_{t\in T}\left(C_{ij}^t + e_{ij}^t\right) + \sum_{i\in\gamma} t_i^{\text{wait}}\right) v_{ij}^t\right), \end{aligned} \tag{10}$$

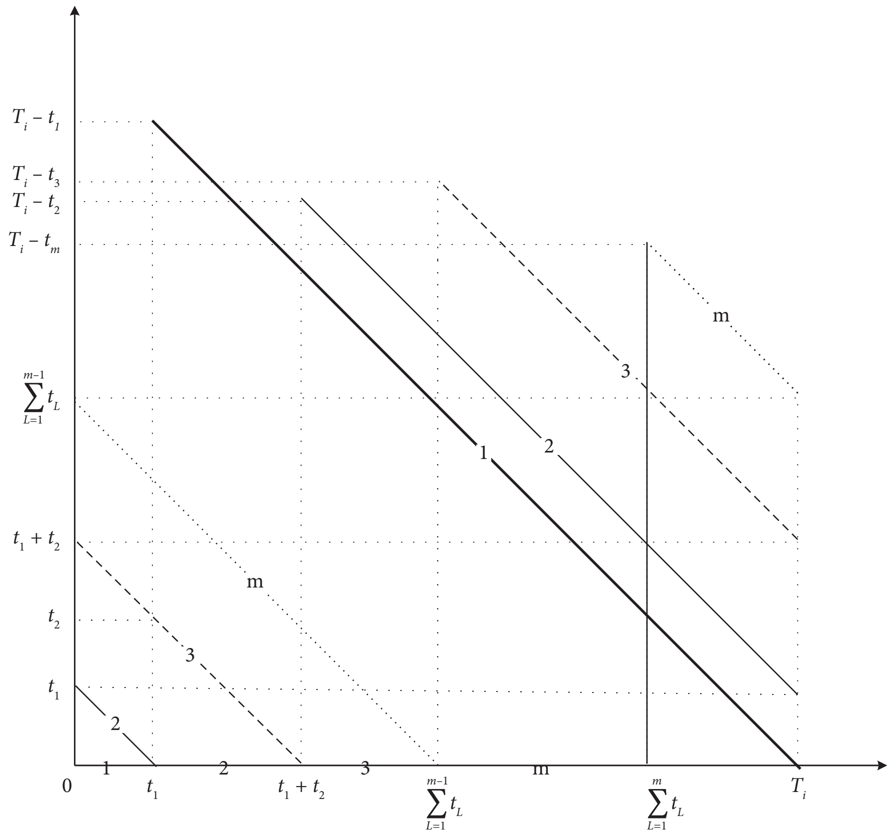

Figure 4: Waiting time function of signalized intersection.

S.t:

$$v_{ij}^{t} = \begin{cases} 1, & \text{Road segment } (i, j) \text{ is occupied at time } t, \\ 0, & \text{Road segment } (i, j) \text{ is not occupied at time } t, \end{cases}$$
$$v_{ij}^{t} - v_{ji}^{t} = \begin{cases} 1, & i < j, \\ -1, & i > j. \end{cases} \tag{11}$$

3.4. Network Conversion. Due to fluctuation in the STD network, the description of the traffic conditions of the real-time road network and problem solving have a certain complexity. Therefore, this study uses the first mathematical induction to derive the proposition that "*A stochastic time-dependent network can be simplified into a deterministic time-dependent network*," so as to simplify the real-time traffic problem.

Proof Process. Assume that the time of arrival of the vehicle at node N_i is t_i^{arrive}, the departure time from node N_i is period t_i^{leave}, the waiting time of the vehicle at node N_i is t_i^{wait}, and the travel time of road section (N_i, N_j) is $t_{ij}(t)$. The travel time is now studied using the first mathematical induction method.

Step 1. Assuming that the vehicle starts from node N_1 at time t_1^{leave}, the travel time through the link (N_1, N_2) is $t_{12}(t)$, and $t_{12}(t) \in [C_{12}^t, C_{12}^t + e_{12}^t]$, and then the uncertain t_2^{leave} can be converted into a deterministic period with an interval range: $[t_1^{\text{leave}} + t_2^{\text{wait}} + C_{12}^t, t_1^{\text{leave}} + t_2^{\text{wait}} + C_{12}^t + e_{12}^t]$.

Step 2. Assuming that the vehicle starts from node N_2 at time t_2^{leave}, the travel time through the link (N_2, N_3) is $t_{23}(t)$, and $t_{23}(t) \in [C_{23}^t, C_{23}^t + e_{23}^t]$, $t_2^{\text{leave}} = t_1^{\text{leave}} + t_2^{\text{wait}} + t_{12}(t)$.

Therefore,

$$\begin{aligned} t_3^{\text{leave}} &= t_2^{\text{leave}} + t_3^{\text{wait}} + t_{23}(t) \\ &= t_1^{\text{leave}} + t_2^{\text{wait}} + t_{12}(t) + t_3^{\text{wait}} + t_{23}(t) \\ &= t_1^{\text{leave}} + \sum_{i=2}^{3} t_i^{\text{wait}} + \sum_{\substack{i=1 \\ j=i+1}}^{2} \sum_{t \in T} t_{ij}(t). \end{aligned} \tag{12}$$

Then,

$$t_1^{\text{leave}}+\sum_{i=2}^{3}t_i^{\text{wait}}+\sum_{\substack{i=1\\j=i+1}}^{2}\sum_{t\in T}C_{ij}^t\le t_3^{\text{leave}}\le t_1^{\text{leave}}+\sum_{i=2}^{3}t_i^{\text{wait}}+\sum_{\substack{i=1\\j=i+1}}^{2}\sum_{t\in T}\left(C_{ij}^t+e_{ij}^t\right). \tag{13}$$

So, the uncertain t_3^{leave} can be converted into a deterministic period with interval range:

$$\left[t_1^{\text{leave}}+\sum_{i=2}^{3}t_i^{\text{wait}}+\sum_{\substack{i=1\\j=i+1}}^{2}\sum_{t\in T}C_{ij}^t,\ t_1^{\text{leave}}+\sum_{i=2}^{3}t_i^{\text{wait}}+\sum_{\substack{i=1\\j=i+1}}^{2}\sum_{t\in T}\left(C_{ij}^t+e_{ij}^t\right)\right]. \tag{14}$$

Step $k-2$. Assuming that the vehicle starts from node N_{k-2} at time t_{k-2}^{leave}, the travel time through the link (N_{k-2},N_{k-1}) is $t_{k-2,k-1}(t)$, and $t_{k-2,k-1}(t)\in[C_{k-2,k-1}^t, C_{k-2,k-1}^t+e_{k-2,k-1}^t]$.

Therefore,

$$\begin{aligned} t_{k-1}^{\text{leave}}&=t_{k-2}^{\text{leave}}+t_{k-1}^{\text{wait}}+t_{k-2,k-1}(t)\\ &=t_1^{\text{leave}}+\sum_{i=2}^{k-1}t_i^{\text{wait}}+\sum_{\substack{i=1\\j=i+1}}^{k-2}\sum_{t\in T}t_{ij}(t). \end{aligned} \tag{15}$$

Then,

$$t_1^{\text{leave}}+\sum_{i=2}^{k-1}t_i^{\text{wait}}+\sum_{\substack{i=1\\j=i+1}}^{k-2}\sum_{t\in T}C_{ij}^t\le t_{k-1}^{\text{leave}}\le t_1^{\text{leave}}+\sum_{i=2}^{k-1}t_i^{\text{wait}}+\sum_{\substack{i=1\\j=i+1}}^{k-2}\sum_{t\in T}\left(C_{ij}^t+e_{ij}^t\right). \tag{16}$$

So, the uncertain t_{k-1}^{leave} can be converted into a deterministic period with interval range: $\left[t_1^{\text{leave}}+\sum_{i=2}^{k-1}t_i^{\text{wait}}+\sum_{\substack{i=1\\j=i+1}}^{k-2}\sum_{t\in T}C_{ij}^t, t_1^{\text{leave}}+\sum_{i=2}^{k-1}t_i^{\text{wait}}+\sum_{\substack{i=1\\j=i+1}}^{k-2}\sum_{t\in T}(C_{ij}^t+e_{ij}^t)\right]$.

Step $k-1$. Assuming that the vehicle starts from node N_{k-1} at time t_{k-1}^{leave}, the travel time through the link (N_{k-1},N_k) is $t_{k-1,k}(t)$, and $t_{k-1,k}(t)\in[C_{k-1,k}^t, C_{k-1,k}^t+e_{k-1,k}^t]$.

Therefore,

$$\begin{aligned} t_k^{\text{leave}}&=t_{k-1}^{\text{leave}}+t_k^{\text{wait}}+t_{k-1,k}(t)\\ &=t_1^{\text{leave}}+\sum_{i=2}^{k-1}t_i^{\text{wait}}+\sum_{\substack{i=1\\j=i+1}}^{k-2}\sum_{t\in T}t_{ij}(t)+t_k^{\text{wait}}+t_{k-1,k}(t)\\ &=t_1^{\text{leave}}+\sum_{i=2}^{k}t_i^{\text{wait}}+\sum_{\substack{i=1\\j=i+1}}^{k-1}\sum_{t\in T}t_{ij}(t). \end{aligned} \tag{17}$$

Then,

$$t_1^{\text{leave}}+\sum_{i=2}^{k}t_i^{\text{wait}}+\sum_{\substack{i=1\\j=i+1}}^{k-1}\sum_{t\in T}C_{ij}^t\le t_k^{\text{leave}}\le t_1^{\text{leave}}+\sum_{i=2}^{k}t_i^{\text{wait}}+\sum_{\substack{i=1\\j=i+1}}^{k-1}\sum_{t\in T}\left(C_{ij}^t+e_{ij}^t\right). \tag{18}$$

So, the uncertain t_k^{wait} can be converted into a deterministic period with interval range: $\left[t_1^{\text{leave}}+\sum_{i=2}^{k} t_i^{\text{wait}}+\sum_{\substack{i=1\\ j=i+1}}^{k-1}\sum_{t\in T} C_{ij}^t, t_1^{\text{leave}}+\sum_{i=2}^{k} t_i^{\text{wait}}+\sum_{\substack{i=1\\ j=i+1}}^{k-1}\sum_{t\in T}\left(C_{ij}^t+e_{ij}^t\right)\right]$.

Therefore, a propositional STD network can be simplified into a deterministic time-dependent network.

$$\text{Total time}=t_k^{\text{leave}}-t_1^{\text{arrive}}=t_k^{\text{leave}}+t_1^{\text{wait}}-t_1^{\text{leave}},$$
$$\max\ \text{Total time}=\sum_{i=1}^{k} t_i^{\text{wait}}+\sum_{\substack{i=1\\ j=i+1}}^{k-1}\sum_{t\in T}\left(C_{ij}^t+e_{ij}^t\right). \tag{19}$$

Therefore, the objective function (9) can be rewritten as

$$Z=\min_{\gamma\in P}\left(\alpha t^*-\alpha t_1+(\beta-\alpha)\max\left(\sum_{i=1}^{k} t_i^{\text{wait}}+\sum_{\substack{i=1\\ j=i+1}}^{k-1}\sum_{t\in T}\left(C_{ij}^t+e_{ij}^t\right)\right)v_{ij}^t\right), \tag{20}$$

S.t:

$$v_{ij}^t=\begin{cases}1, & \text{Road segment } (i,j) \text{ is occupied at time } t,\\ 0, & \text{Road segment } (i,j) \text{ is not occupied at time } t,\end{cases}$$
$$v_{ij}^t-v_{ji}^t=\begin{cases}1, & i<j,\\ -1, & i>j,\end{cases}$$
$$\sum_{(i,j)\in\gamma}\sum_{t\in T}\left(C_{ij}^t+e_{ij}^t\right)v_{ij}^t+\sum_{i\in\gamma} t_i^{\text{wait}}+t_1\le t^*. \tag{21}$$

The travel time of the path in the STD network can be transformed into a deterministic time-dependent network by the first mathematical induction. $C_{ij}^t+e_{ij}^t$ is a deterministic variable, which is the maximum value of the link travel time $t_{ij}(t)$. Therefore, the objective function can be converted to solve the deterministic time-dependent network under the constraint of the time window t^*, seeking the route with the minimum $\sum_{i=1}^{k} t_i^{\text{wait}}+\sum_{\substack{i=1\\ j=i+1}}^{k-1}\sum_{t\in T}\left(C_{ij}^t+e_{ij}^t\right)$.

4. The Shortest Path Algorithm

The shortest path algorithm that accounts for waiting time at signalized intersections based on Dijkstra's algorithm is used to solve the model. For a given origin node in the graph, the algorithm can find the shortest reliable path between it and every other node.

Step 1: initialization.

Before implementing the algorithm, it is necessary to initialize each signalized crossing node and assign every node a tentative distance value. Set the initial node to zero, which is the starting coordinate of the traveler, and all other nodes to infinity. Create sets W, P_i, and S. W is the set of unvisited nodes, consisting of all the unvisited nodes except the source node. Let S be the set of visited nodes and P_i be the set of all predecessors of node i.

$$\text{Let } \forall i\ne j, l_i=0, l_j=+\infty, P_i=\{0\}, l_i^{\max}=l_i^{\min}=0. \tag{22}$$

Step 2: update labels.

For the current node i, consider all nodes except the nodes in P_i. Then, the departure time, tentative maximum travel time $l_i^{\max}$, and minimum travel time $l_i^{\min}$ of subsequent nodes are calculated according to equations (1)–(6). If $l_i^{\min}$ is less than the previously recorded time, then overwrite the tag. Even if the node has been checked or marked as visited, it should remain in the unvisited set at this point.

Step 3: move to the next node.

Set the node with the best travel time marked $l_i^{\min}$ as the next "current node i" and add this node to the set S. Consider all nodes in the set W except those nodes in P_i to find the node with the minimum travel time from the initial node. Calculate the tentative minimum travel time $l_j^{\min}$, and if $l_i^{\min}$ is less than the previously recorded distance, update the label and update the set W.

Step 4: stop and find the best path.

If W is empty and all node labels cannot be updated in Step 2, then stop. Otherwise, go back to Step 3 and continue updating the label to find a better path. After completing the label update of the last node in set W, the optimal path from the starting point to the end point is found in the opposite direction starting from the last node.

5. Computational Test

To solve the shortest reliable path problem in the STD networks, this research conducts a computational test with the small traffic network described in Figure 5. The network contains 12 one-way road sections and 9 signalized intersections. The set of signalized intersection nodes is $W=\{A,B,C,D,E,F,G,H,I\}$, including a cross-shaped intersection and 8 T-shaped intersections.

In order to ensure fairness between each phase, the green light duration of each phase is evenly allocated, so it is assumed that the green light duration of each phase at the same signalized intersection is the same. The signal starts at the moment of arrival at the starting point of 0, the phase sequence appears randomly, the properties of each signalized intersection are shown in Table 2, and the link weights are shown in Table 3. Now search for the shortest reliable path from the starting point A to the end point I.

Traditional Dijkstra's algorithm and the improved shortest path algorithm based on Dijkstra's algorithm considering the waiting time of signalized intersection are, respectively, used to calculate the shortest reliable path problem of this small traffic network. According to the results of the proof in Section 3 that the STD networks can be transformed into a deterministic time-dependent network, the original problem can be transformed into the shortest path problem in a deterministic time-dependent network. The transformed traffic network is shown in Figure 6, and the link weight is the upper limit of the original range.

5.1. Solving the Shortest Reliable Path. Visual Studio C/C++ 6.0 is used to calculate the shortest path and travel time between the starting and ending nodes for different departure times with the shortest path algorithm that accounts for the waiting time of signalized intersections based on Dijkstra's algorithm, as shown in Table 4.

From Table 4, the shortest paths at different departure times can be obtained. As can be seen from Figure 7, the travel time varies periodically at different departure times, and the period duration is 48. The minimum travel time is 35, and the maximum travel time is 50. For the same origin destination (OD) matrix, 141 sets of experimental results are selected from the table to compare with the traditional Dijkstra's algorithm without accounting for waiting time at signalized intersections, and the results are shown in Figure 7. The travel time obtained by the traditional shortest path algorithm is a fixed value of 35, which does not vary with the departure time, while the shortest travel time obtained by improved Dijkstra's algorithm without accounting for waiting time at signalized intersections varies with the departure time.

5.2. Solving Travel Time Range. The shortest path and travel time range at different departure times can be obtained by using the upper and lower limits of road weight, respectively. The lower limit weights of the section are used to find the arrival moments and shortest path at different departure moments, as shown in Table 5.

From Table 5, the shortest path and travel time obtained by the lower limit of link weights can be obtained. The travel time range is the time interval obtained by the upper and lower limits of the road weight.

As can be seen from Figure 8, the travel time varies periodically at different departure times, and the period duration is 48. The minimum travel time is 29, and the maximum travel time is 39. For the same OD matrix, 142 sets of test results in the table are compared with traditional Dijkstra's algorithm without accounting for the waiting time at signalized intersections. The travel time obtained by the traditional shortest path algorithm is a fixed value of 29, which does not change with the departure time, while the shortest travel time obtained by improved Dijkstra's algorithm with intersection waiting time varies with the departure time. Figure 9 shows the travel time range that accounts for the waiting time at signalized intersections and the traditional travel time range that does not account for the waiting time at signalized intersections, respectively.

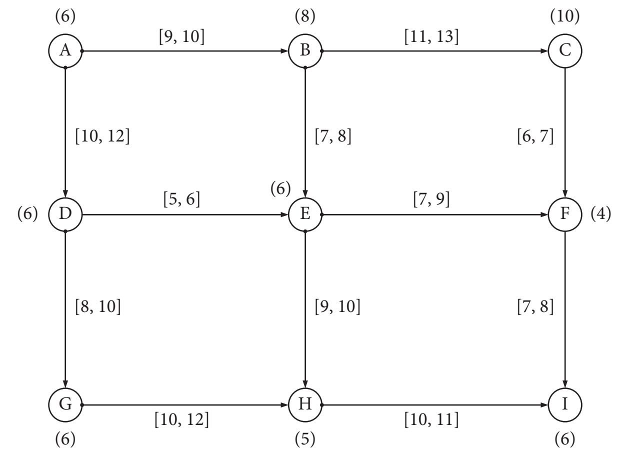

Figure 5: STD network test example (the figure in the chart is the weight of link).

Table 2: Property table of signalized intersections.

Node	Phase quantity	Green time of each phase	Signal cycle
A	2	6	12
B	2	8	16
C	1	10	10
D	2	6	12
E	4	6	24
F	2	4	8
G	1	6	6
H	2	5	10
I	Null	Null	Null

Table 3: Link weight table.

Section number	Nodes	Weight (s)
1	0-1	[9, 10]
2	1-2	[11, 13]
3	0–3	[10, 12]
4	1–4	[7, 8]
5	2–5	[6, 7]
6	3-4	[5, 6]
7	4-5	[7, 9]
8	3–6	[8, 10]
9	4–7	[9, 10]
10	5–8	[7, 8]
11	6-7	[10, 12]
12	7-8	[10, 11]

5.3. Contrast Trials with Varying Weights. A road section is randomly selected in the road network to simulate road congestion, and the experiment is repeated several times by changing the weight of the road section. The weight of the E-F section was set as [7-15], and the obtained results were

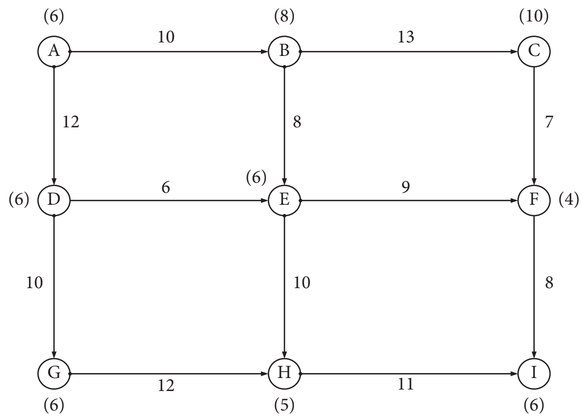

FIGURE 6: Deterministic time-dependent network test example.

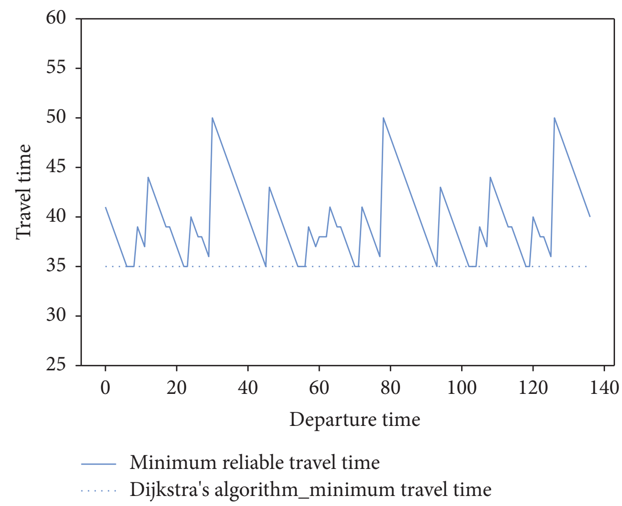

FIGURE 7: Minimum reliable travel time, obtained by the upper limits of link weight.

TABLE 4: The shortest reliable path and corresponding travel time at different departure time.

Departure time	Arrival time	Route
0	41	A-D-E-F-I
1–3	41	A-D-E-F-I
4–6	41	A-B-E-F-I
7	42	A-B-E-F-I
8	43	A-B-E-F-I
9–11	48	A-B-E-F-I
12-13	56	A-B-E-F-I
14–17	56	A-D-E-F-I
18–22	57	A-B-E-F-I
23	58	A-B-E-F-I
24–26	64	A-B-C-F-I
27–29	65	A-B-E-F-I
30–45	80	A-B-E-F-I
46–51	89	A-D-E-F-I
52–54	89	A-B-E-F-I
55	90	A-B-E-F-I
56	91	A-B-E-F-I
57–59	96	A-B-E-F-I
60	98	A-B-C-F-I
61	99	A-B-C-F-I
62	100	A-D-E-F-I
63–65	104	A-D-E-F-I
66–70	105	A-B-E-F-I
71	106	A-B-E-F-I
72–77	113	A-B-E-F-I
78–93	128	A-B-E-F-I
94–99	137	A-D-E-F-I
100–102	137	A-B-E-F-I
103	138	A-B-E-F-I
104	139	A-B-E-F-I
105–107	144	A-B-E-F-I
108–110	152	A-B-E-F-I
111–113	152	A-D-E-F-I
114–118	153	A-B-E-F-I
119	154	A-B-E-F-I
120–122	160	A-B-C-F-I
123–125	161	A-B-E-F-I
126–141	176	A-B-E-F-I

compared with previous experimental results. The shortest path and travel time between the starting and ending points under different departure times were obtained, as shown in Table 6.

From Table 6, the shortest reliable path and travel time can be obtained by randomly changing the E-F weight value of the road section. As can be seen from Figure 10, the minimum travel time varies periodically at different departure times, and the cycle time is about 48. The experimental results of 152 groups in the selected table are compared with the traditional Dijkstra algorithm results that do not consider the waiting time at the intersection, and the results are shown in Figure 10. The travel time obtained by the traditional shortest path algorithm is a fixed value of 38, regardless of departure time. However, the minimum travel time obtained by the improved Dijkstra algorithm, which includes waiting time at intersections, varies with the departure time. The comparison test shows that the travel time can meet the requirements of rigid arrival time after randomly changing the weight of the road section, and the effectiveness and robustness of the algorithm are verified.

6. Result Analysis

The accuracy and effectiveness of the model and algorithm were demonstrated through computational testing. The analysis of the results of the computational testing led to the following conclusions:

(1) A stochastic time-dependent network can be transformed into a deterministic time-dependent network through the maximum-minimum criterion, and the travel time range from the starting point to the end point can be obtained through the link weights.

Table 5: The shortest path and corresponding travel time at different departure time.

Departure time	Arrival time	Route
0–2	31	A-D-E-F-I
3–5	39	A-D-E-F-I
6–9	39	A-B-E-F-I
10	40	A-B-E-F-I
11	41	A-B-E-F-I
12	42	A-B-E-F-I
13	47	A-B-E-F-I
14	47	A-D-E-F-I
15–19	50	A-D-E-F-I
20–25	55	A-B-E-F-I
26	55	A-D-E-F-I
27-28	63	A-B-C-F-I
29-30	63	A-B-E-F-I
31	63	A-D-E-F-I
32–38	71	A-D-E-F-I
39-40	73	A-B-C-F-I
41–44	74	A-B-E-F-I
45-46	79	A-B-E-F-I
47–50	79	A-D-E-F-I
51–53	87	A-D-E-F-I
54–57	87	A-B-E-F-I
58	88	A-B-E-F-I
59	89	A-B-E-F-I
60	90	A-B-E-F-I
61	95	A-B-E-F-I
62	95	A-D-E-F-I
63–67	98	A-D-E-F-I
68–73	103	A-B-E-F-I
74	103	A-D-E-F-I
75–78	111	A-B-E-F-I
79	111	A-D-E-F-I
80–86	119	A-D-E-F-I
87	120	A-B-C-F-I
88	121	A-B-C-F-I
89–92	122	A-B-E-F-I
93-94	127	A-B-E-F-I
95–98	127	A-D-E-F-I
99–101	135	A-D-E-F-I
79	111	A-D-E-F-I
80–86	119	A-D-E-F-I
87	120	A-B-C-F-I
88	121	A-B-C-F-I
89–92	122	A-B-E-F-I
93-94	127	A-B-E-F-I
95–98	127	A-D-E-F-I
99–101	135	A-D-E-F-I
102–105	135	A-B-E-F-I
106	136	A-B-E-F-I
107	137	A-B-E-F-I
108	138	A-B-E-F-I
109	143	A-B-E-F-I
110	143	A-D-E-F-I
111–115	146	A-D-E-F-I
108	138	A-B-E-F-I
109	143	A-B-E-F-I
110	143	A-D-E-F-I
111–115	146	A-D-E-F-I
116–121	151	A-B-E-F-I
122	151	A-D-E-F-I

Table 5: Continued.

Departure time	Arrival time	Route
123-124	159	A-B-C-F-I
125-126	159	A-B-E-F-I
127	159	A-B-E-F-I
128–134	167	A-D-E-F-I
135	167	A-B-E-F-I
136–140	170	A-B-E-F-I
141-142	175	A-B-E-F-I

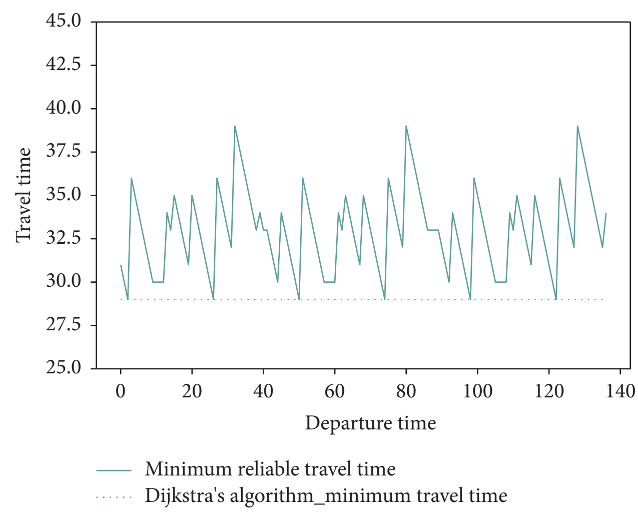

Figure 8: Minimum reliable travel time, obtained by the lower limits of link weight.

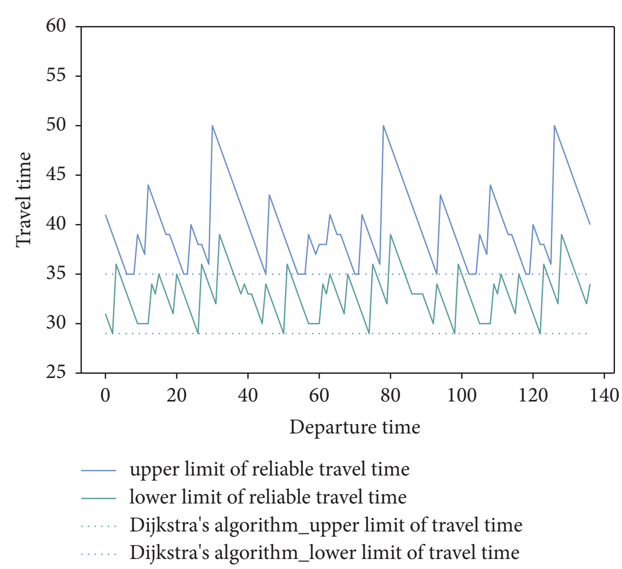

Figure 9: Travel time range.

(2) The change of departure time will cause the shortest path of the vehicle from the starting point A to the endpoint I to change. For example, when the departure time is 0, the shortest path is A-D-E-F-I, and when the departure time is 20, the shortest path is A-

Table 6: The shortest path and the shortest travel time after changing the weight of the section.

Departure time	Arrival time	Route
0–3	45	A-D-E-H-I
4–6	45	A-B-E-H-I
7	48	A-B-E-H-I
8	49	A-B-E-H-I
9	50	A-B-E-H-I
10-11	51	A-B-E-H-I
12-13	56	A-B-C-F-I
14–17	59	A-D-E-F-I
18–22	61	A-B-E-H-I
23	62	A-B-E-H-I
24–26	64	A-B-C-F-I
27–29	71	A-B-E-H-I
30–42	80	A-B-C-F-I
43	81	A-B-C-F-I
44	82	A-B-C-F-I
45	83	A-B-C-F-I
46–51	93	A-D-E-H-I
52–54	93	A-B-E-H-I
55	94	A-B-E-H-I
56	95	A-B-E-H-I
57-58	96	A-B-C-F-I
59	97	A-B-C-F-I
60	98	A-B-C-F-I
61	99	A-B-C-F-I
62–65	105	A-D-E-H-I
66–71	111	A-B-E-H-I
72–77	115	A-B-C-F-I
78–86	128	A-B-C-F-I
87–92	131	A-B-E-H-I
93	132	A-B-E-H-I
94–99	141	A-D-E-H-I
100–102	141	A-B-E-H-I
103	142	A-B-E-H-I
104	143	A-B-E-H-I
105	144	A-B-E-H-I
106	144	A-B-C-F-I
107	151	A-B-E-H-I
108-109	153	A-B-E-H-I
110–113	153	A-D-E-H-I
114–119	160	A-B-E-F-I
120–122	160	A-B-C-F-I
123	161	A-B-C-F-I
124	162	A-B-C-F-I
125	163	A-B-C-F-I
126–138	176	A-B-C-F-I
139	177	A-B-C-F-I
140	178	A-B-C-F-I
141	179	A-B-C-F-I
142–147	191	A-D-E-H-I
148–152	191	A-B-E-H-I

B-E-F-I. This is mainly because different departure times cause the vehicle to reach the signalized intersections at different times, which makes a significant change in the waiting time at the signalized intersections according to the phase change and the subsequent path selection.

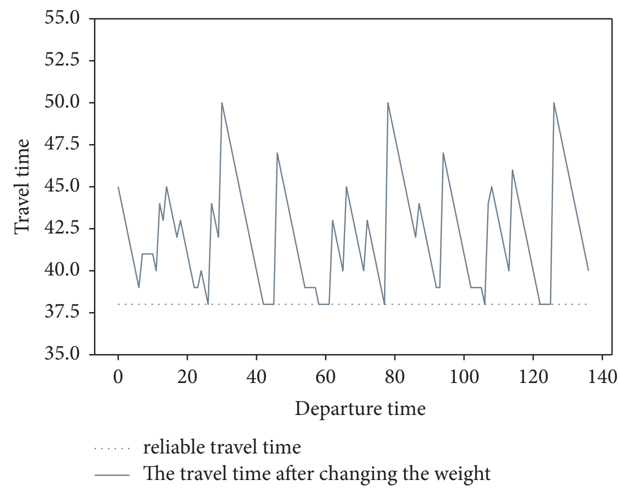

Figure 10: Minimum travel time after changing the E-F weight value.

(3) The change of departure time will cause the shortest travel time of the vehicle from the starting point A to the endpoint I to change periodically. For example, when the departure time is 0, the travel time range is [31, 41], and when the departure time is 20, the travel time range is [55, 57]. As shown in Figure 7, the shortest travel time shows a periodic change within a certain range, and the change period is 48. The reason for this change is that the waiting time for any phase at the signal intersection is a periodic function. The waiting time for any desired phase is characterized by a periodic cycle with the periodic duration of the signal intersection.

(4) With different departure time, the minimum travel time shows a monotonic variation within a certain time range. The reason is that different departure times lead to different time for vehicles to arrive at the signalized intersection, which makes the waiting time at the signalized intersection change significantly according to the phase change and the subsequent path selection.

(5) The minimum travel time of the optimal path is 29, and the worst case can be calculated as 50, indicating that the reliability of the travel time on the optimal path is poor, which may violate the constraints of the time window and cause commuters to be late. For example, the traveler needs to arrive at the destination before $t = 35$ under the constraint of a time window; otherwise, it will be regarded as late. If the traveler only considers the minimum travel time and does not consider the worst case and reliability of travel time, it may not meet the requirements of rigid arrival time. Thus, although the optimal path enables the traveler to reach the destination with the minimum expected travel time, for conservative travelers, the rigid arrival time requirements are not surely met due to the lack of consideration for the worst-case travel time and reliability.

7. Conclusion

Considering the impact of stochastic time-dependent characteristics of urban road networks on daily travel, the shortest reliable path problem in stochastic time-dependent networks was studied. First, in the STD network, this article proposes a waiting time function at the signalized intersection and analyzes the waiting time characteristics at the signalized intersection and draws the conclusion that the waiting time at the signalized intersection is a continuous non-differentiable periodic function. Then, a method of network transformation is proposed, and the first mathematical induction method is used to prove that the stochastic time-dependent network can be reduced to a deterministic time-dependent network, and the feasibility of this method is verified. Finally, a shortest path algorithm that accounts for waiting time at signalization intersections is proposed. Compared with the traditional probabilistic methods, this article does not need the accurate probability distribution of uncertainty data, since the range of uncertainty can be derived from the historical data and the experience of decision makers. By setting up a simple numerical experiment that takes into account the reliability requirements of the time window and the travel time, the reliable travel time and path corresponding to different departure times are obtained, and the rigid arrival time requirements of travelers are satisfied. Therefore, the algorithm can more accurately describe the real traffic conditions.

The deficiency of the paper is that the treatment of phases at signal intersections does not take into account the yellow light time and all-red time, which is still a restrictive assumption in this case. Moreover, the test lacks the analysis of the effect of dynamic real-time traffic volume on travel time. How to apply the algorithm proposed in this paper to a larger traffic network to verify the effectiveness of the proposed model and algorithm may be the content of subsequent research.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This study was supported by the Natural Science Foundation of Shandong Province (no. ZR2021MG021).

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